Poincaré-friendly tracking with complex tune modulation

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October 8, 2025

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Outline

Goal: Construct a tune modulation "probe" element that can be added to a lattice to better evaluate transverse vulnerability to complex tune modulation.

Physical problem: Bad things can happen when a tune modulation spectrum *drive line* aligns with a resonance island *response line:* $Q_M \approx Q_I$

Complexity:

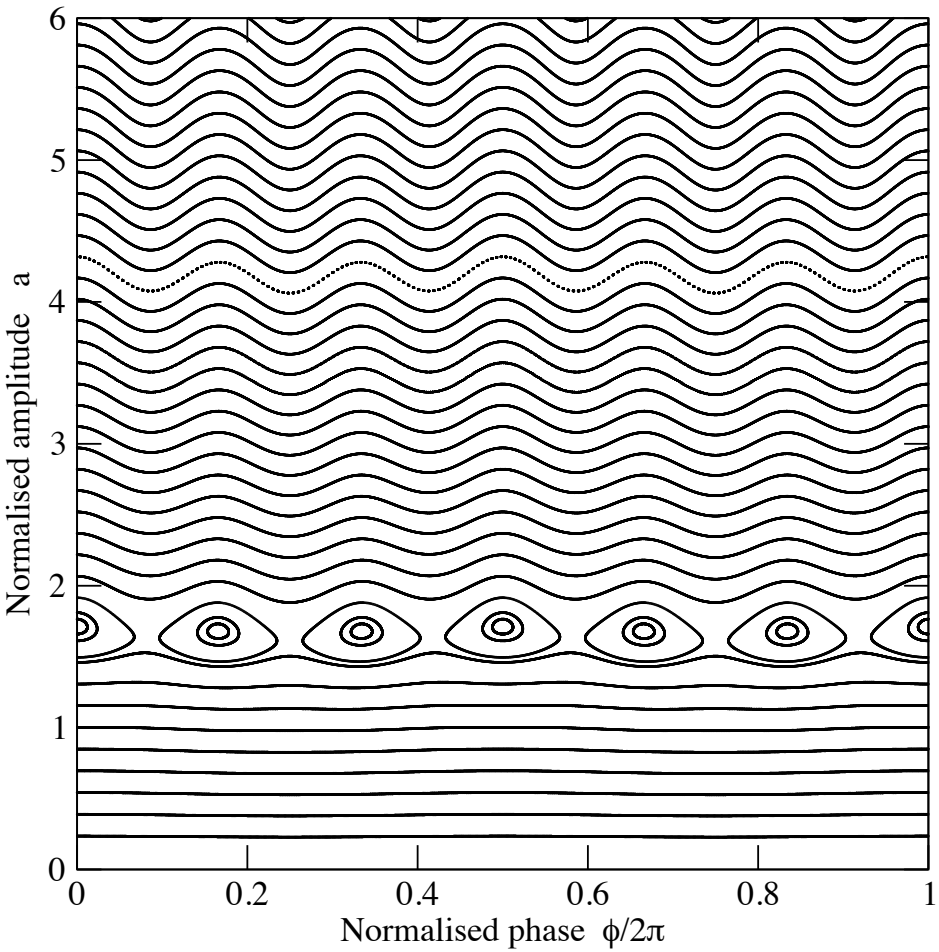
- Many resonances co-exist, weak & strong
- Polynomial chromaticities
- Multi-RF bucket structure (eg RHIC)
- Transverse chaos in (eg) H drives diffusion in V

Key subsidiary goal: re-enable Poincaré surface-of-section graphic insight.

Past

3 key tune domain parameters

1) Base tune Q_0 for small amplitude a particles
2) Resonance tune, eg $Q_R = 1/6$ when $Q(a_R) = 1/6$
3) Island tune Q_I



Near island center on turn t

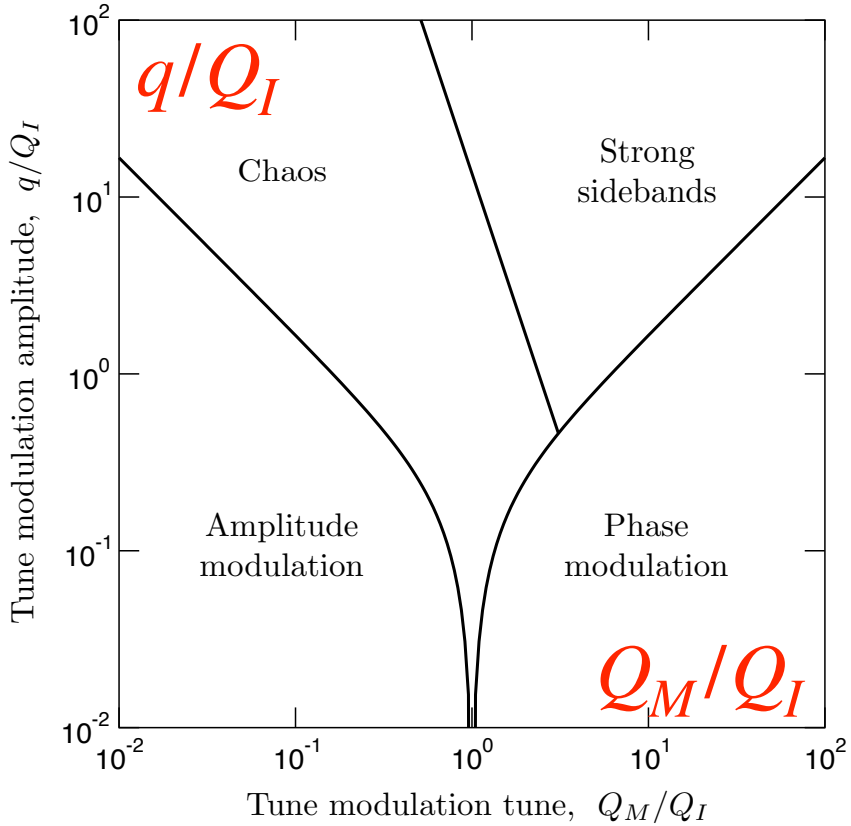
$$a(t) \approx a_R + \hat{a} \sin(2\pi Q_I t)$$

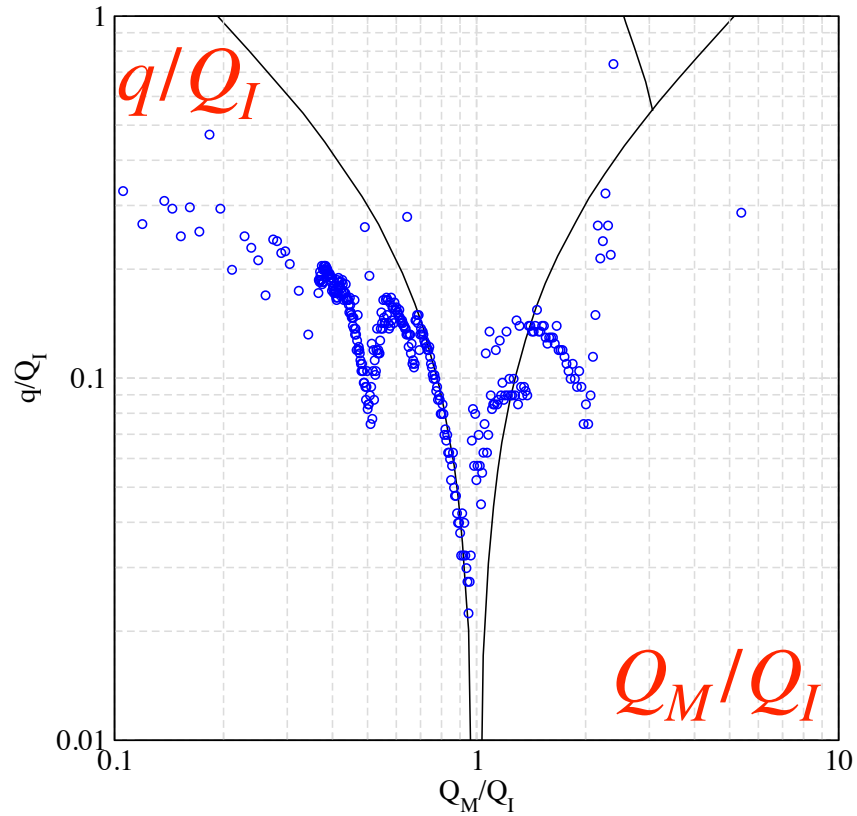
$$\phi(t) \approx \phi_R + \hat{\phi} \cos(2\pi Q_I t)$$

Single tune modulation tune

Base tune modulation amplitude & tune are q and Q_M, so

$$Q_0(t) = Q_{00} + q\cos(2\pi Q_M t)$$

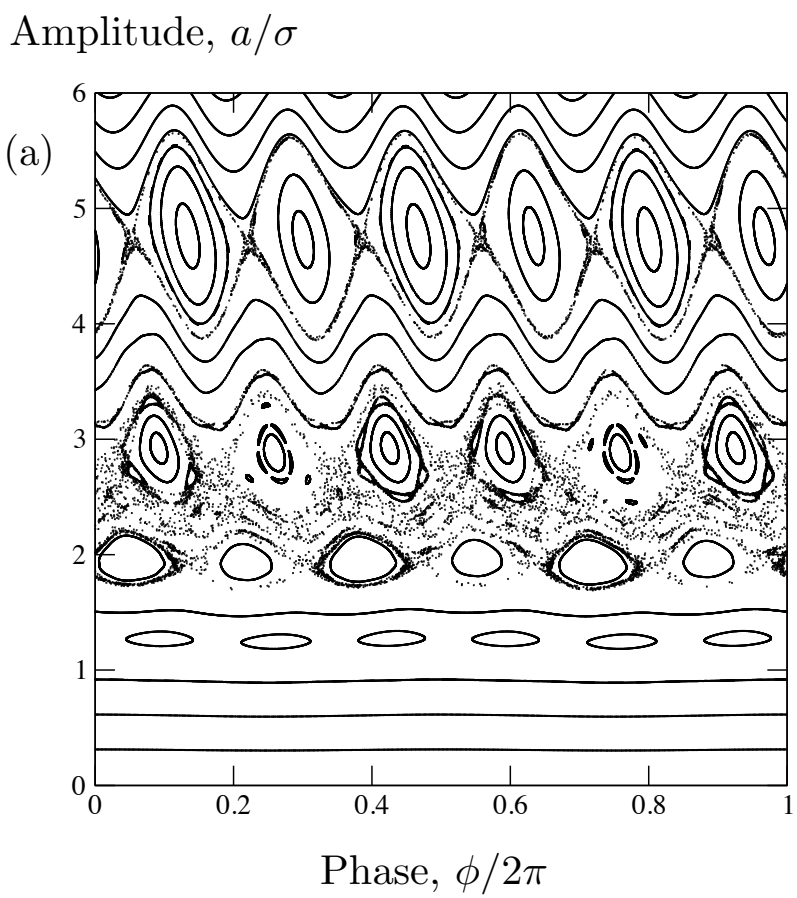


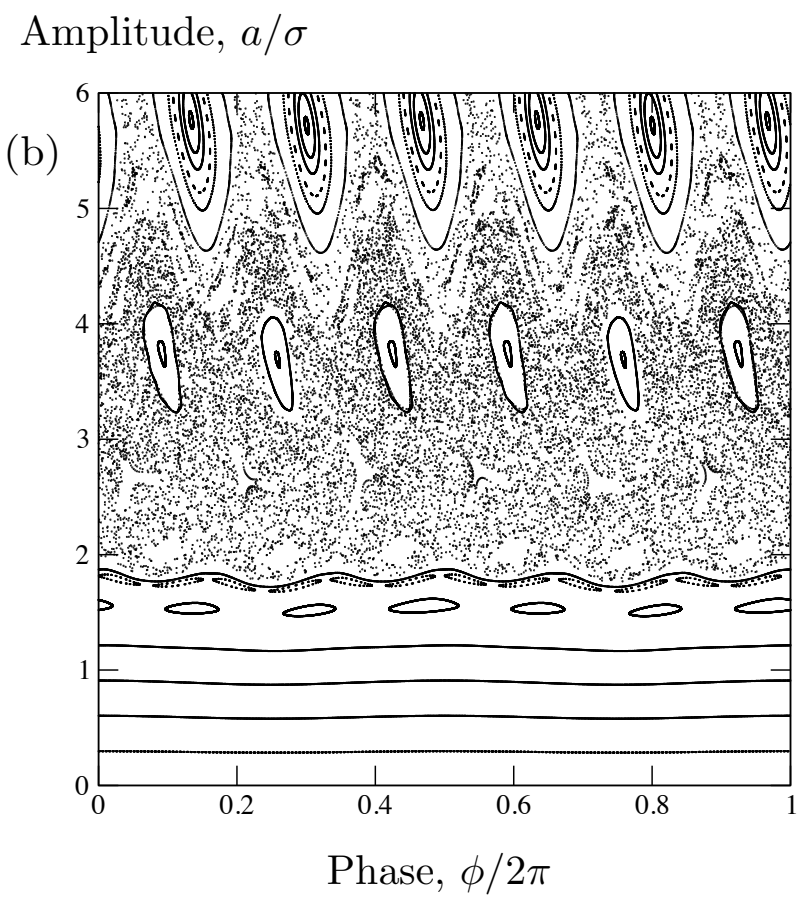


Generic theory & particular simulation agree … plus structure

Tune domain dynamical zones

$(q, Q_M) = (0.001,\ 0.0052)$ below, while beam-beam tune shift parameter $\xi = 0.0042$ (Left) & 0.006 (Right)





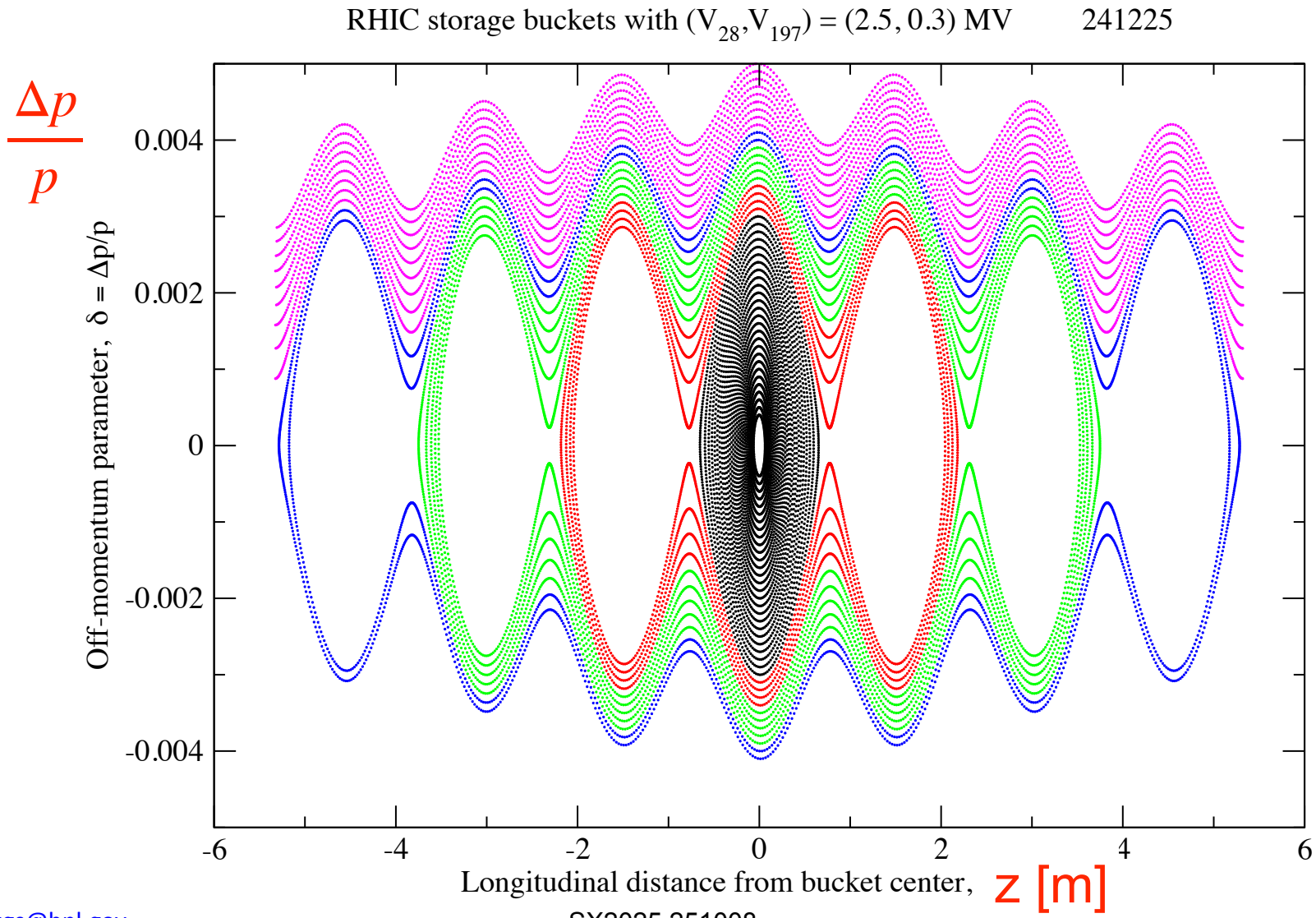
Left: strong sidebands

Right: chaos

Present

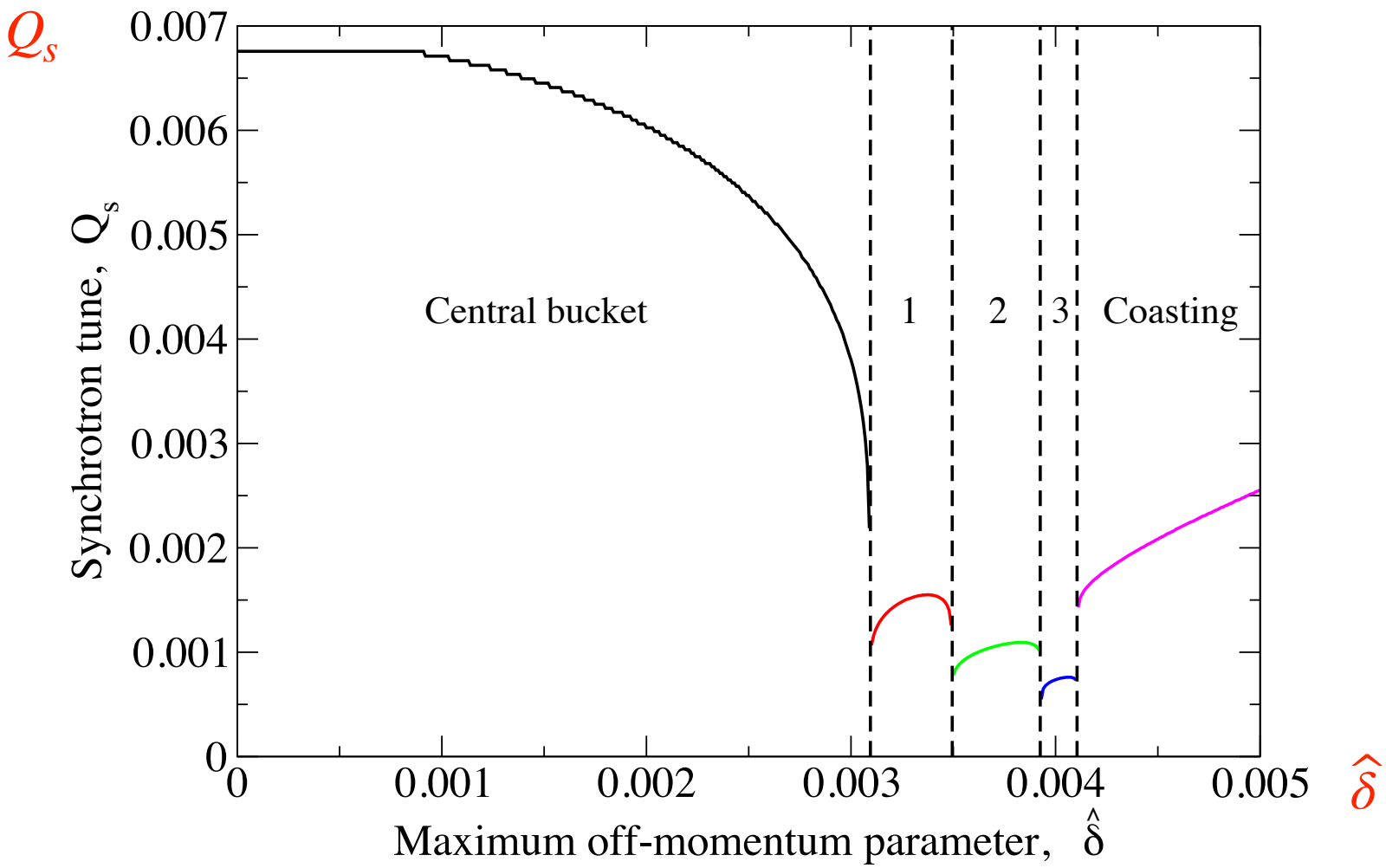
Complex storage buckets

RHIC uses 2 RF systems in collision, helping IBS but …



Synchrotron tune versus $\hat{\delta}$

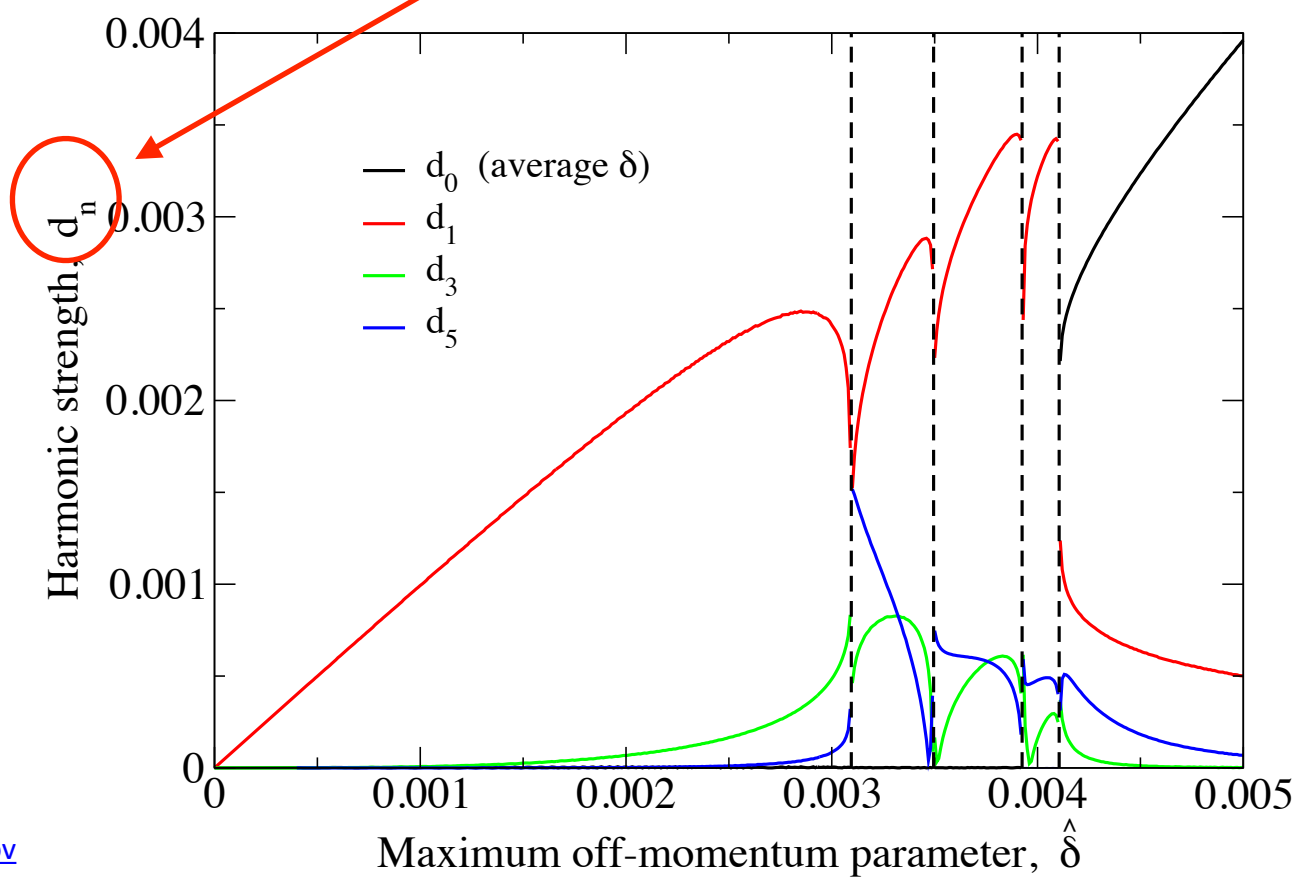
… causing Q_s to vary significantly with amplitude $\hat{\delta}$



d_n-harmonics

Complex buckets require multiple harmonics $d_n(\hat{\delta})$ of $Q_s(\hat{\delta})$ to describe the turn-by-turn evolution of δ

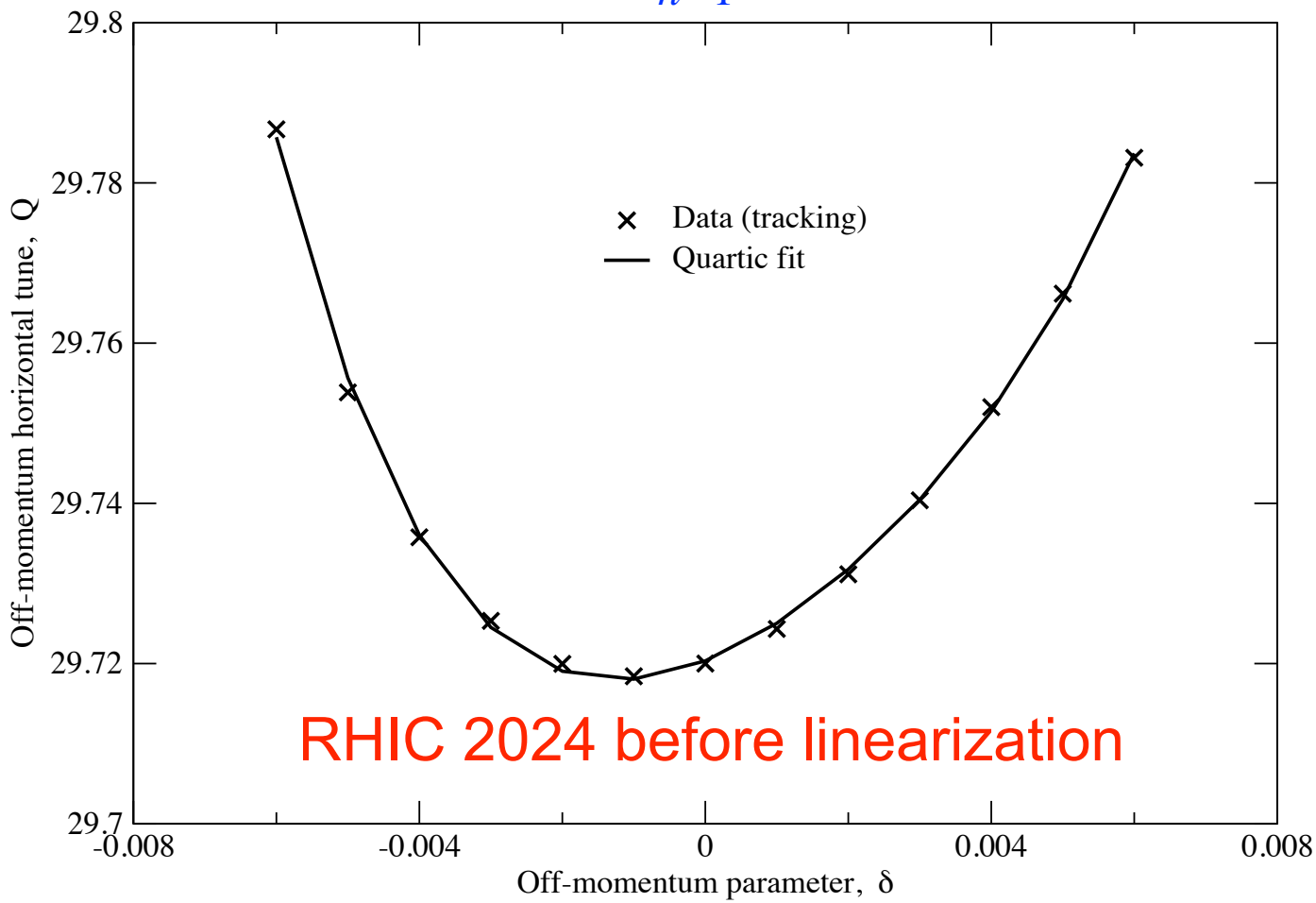
$$\delta(\hat{\delta}, t) = \sum_{n=0}^{\infty} d_n(\hat{\delta}) \cos(2\pi n \,.\, Q_s(\hat{\delta})\, t)$$



Complex chromaticity

Polynomial chromaticity has coefficients C_n where

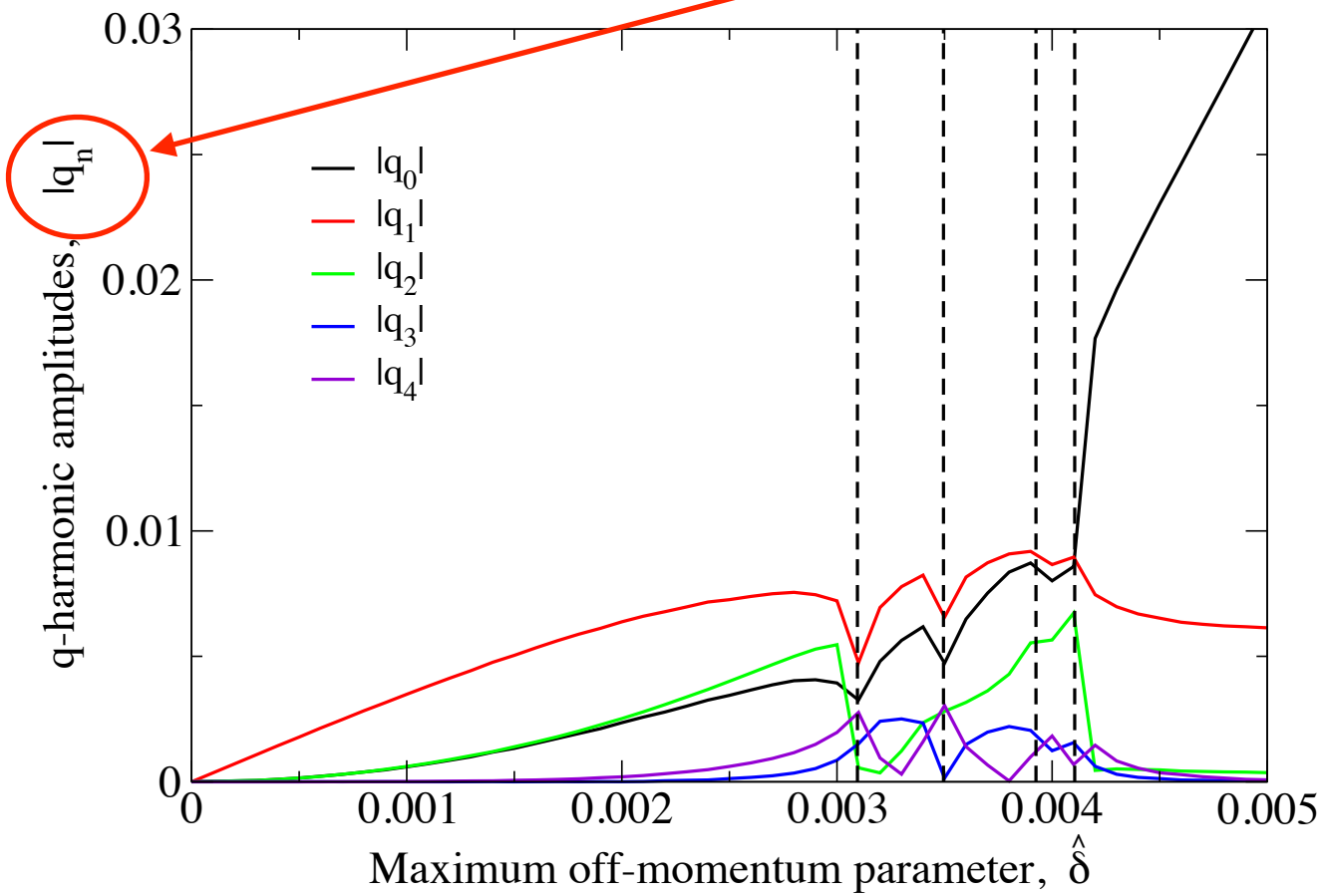
$$\Delta Q(t) = \sum_{n=1}^{\infty} C_n \delta(t)^n$$



q_n-harmonics

Putting this together delivers the q_n-harmonics

$$\Delta Q(\hat{\delta},t) = \sum_{m=1}^{\infty} C_m \left(\sum_{n=0}^{\infty} d_n(\hat{\delta}) \cos(2\pi n \,.\, Q_s(\hat{\delta}) \,.\, t) \right)^m = \sum_{n=0}^{\infty} q_n(\hat{\delta}) \cos(2\pi n \,.\, Q_s(\hat{\delta}) \,.\, t)$$



Single magnets

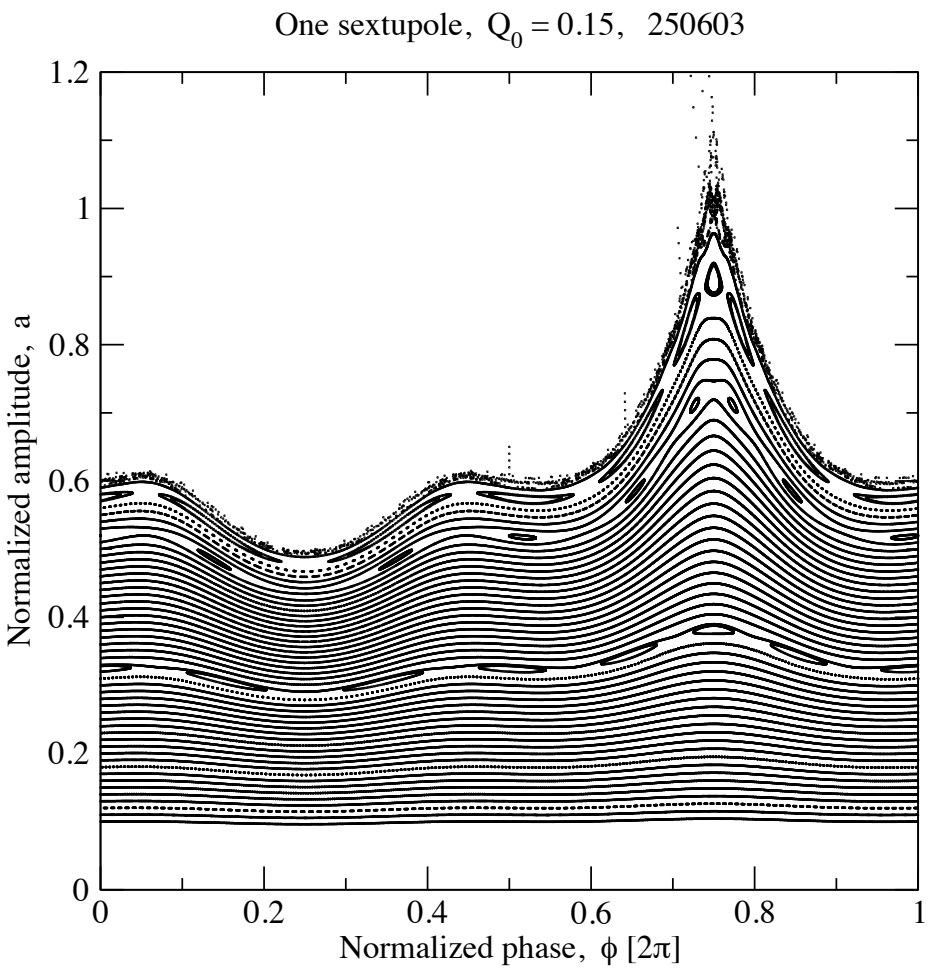
Hénon said that "[a single sextupole lattice] exhibits … all the typical properties of more complicated mappings and dynamical issues".

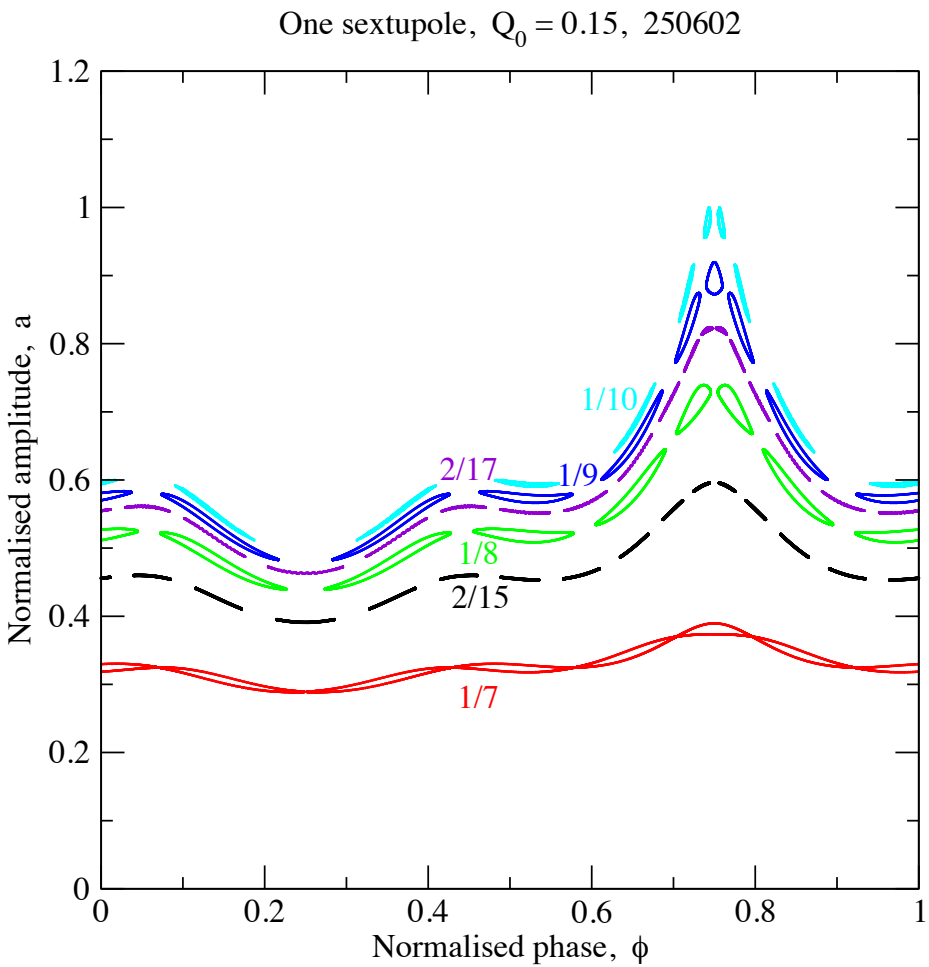
Agreed!

We are using single sextupole (or octupole) lattices to begin debugging software that will soon have:

1) Poincaré-friendly tracking
2) Multi-sextupoles & octupoles (RHIC & HSR)
3) Transverse diffusion probes

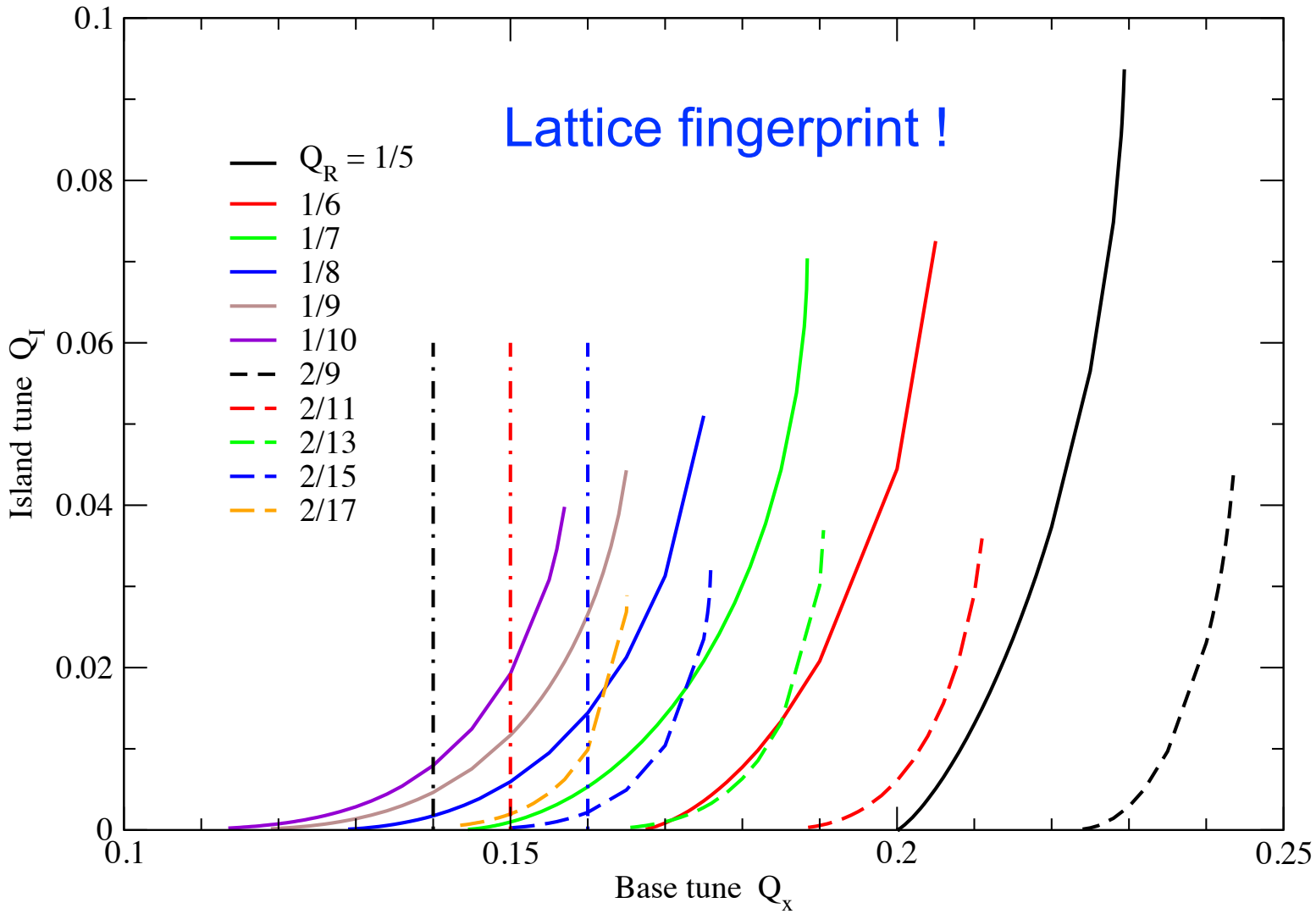
One sextupole with $Q_0 = 0.15$





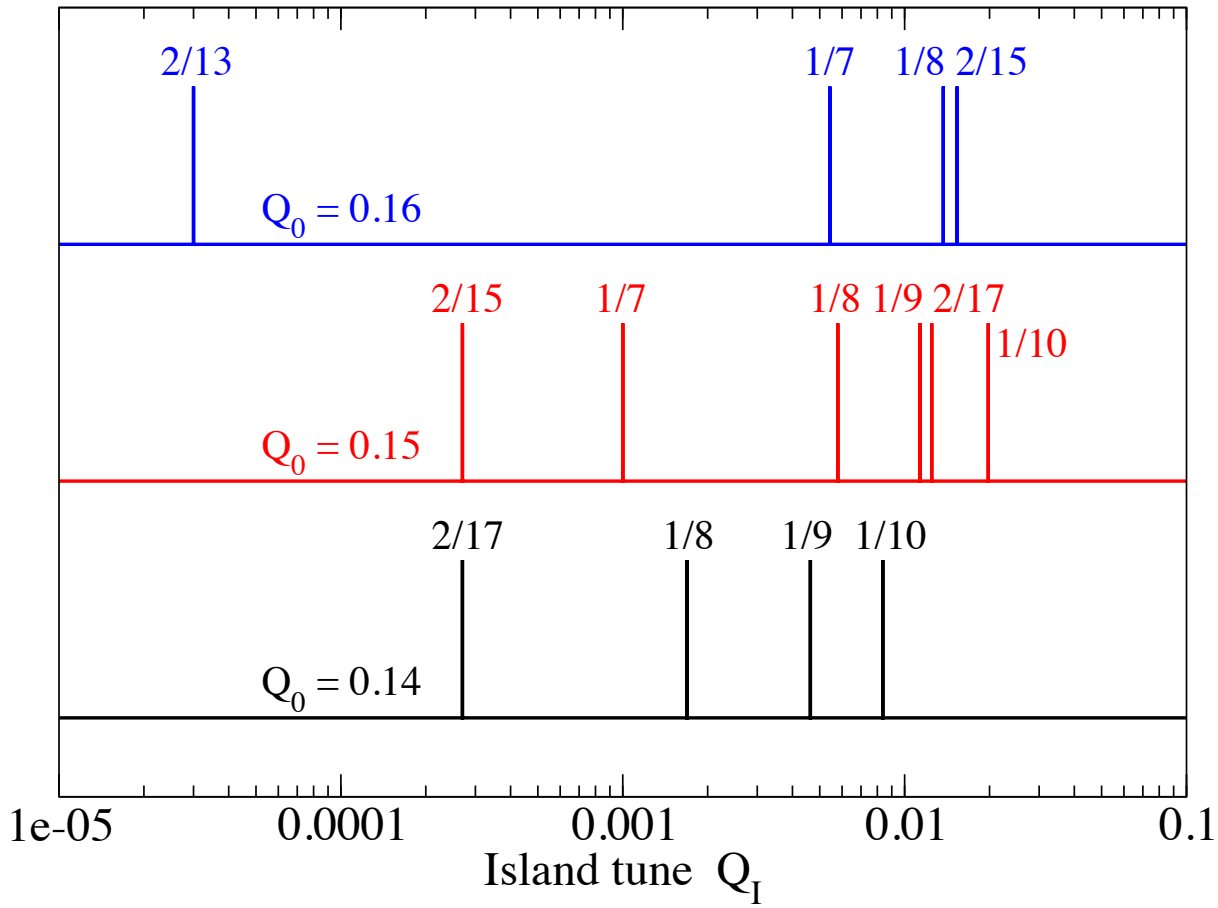
Many resonances are present, each with its own Q_I

Island tunes Q_I vs base tune Q_0



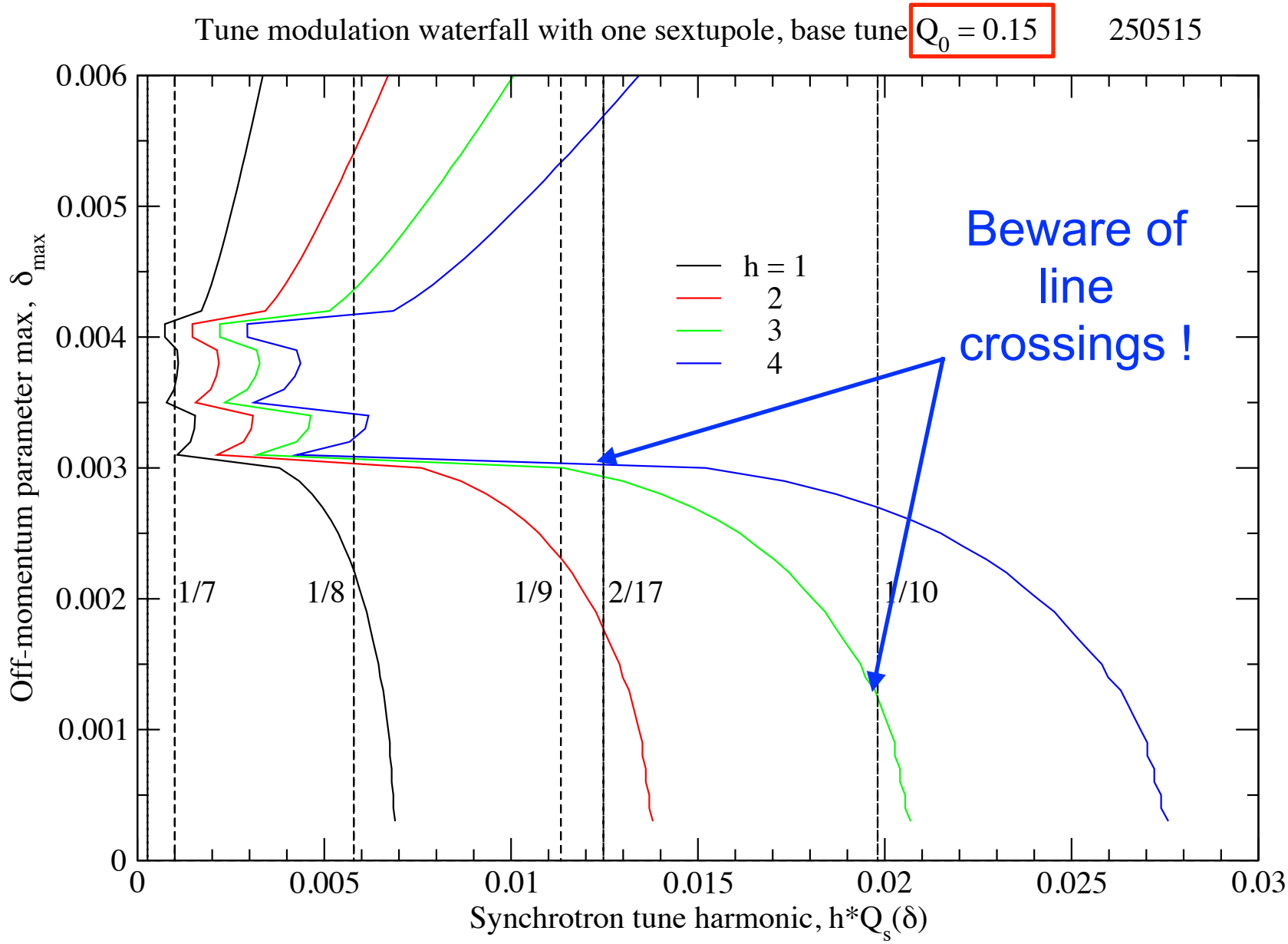
Vertical dashed lines at base tunes $Q_0 = (0.14,\ 0.15,\ 0.16)$

Island tune response spectra

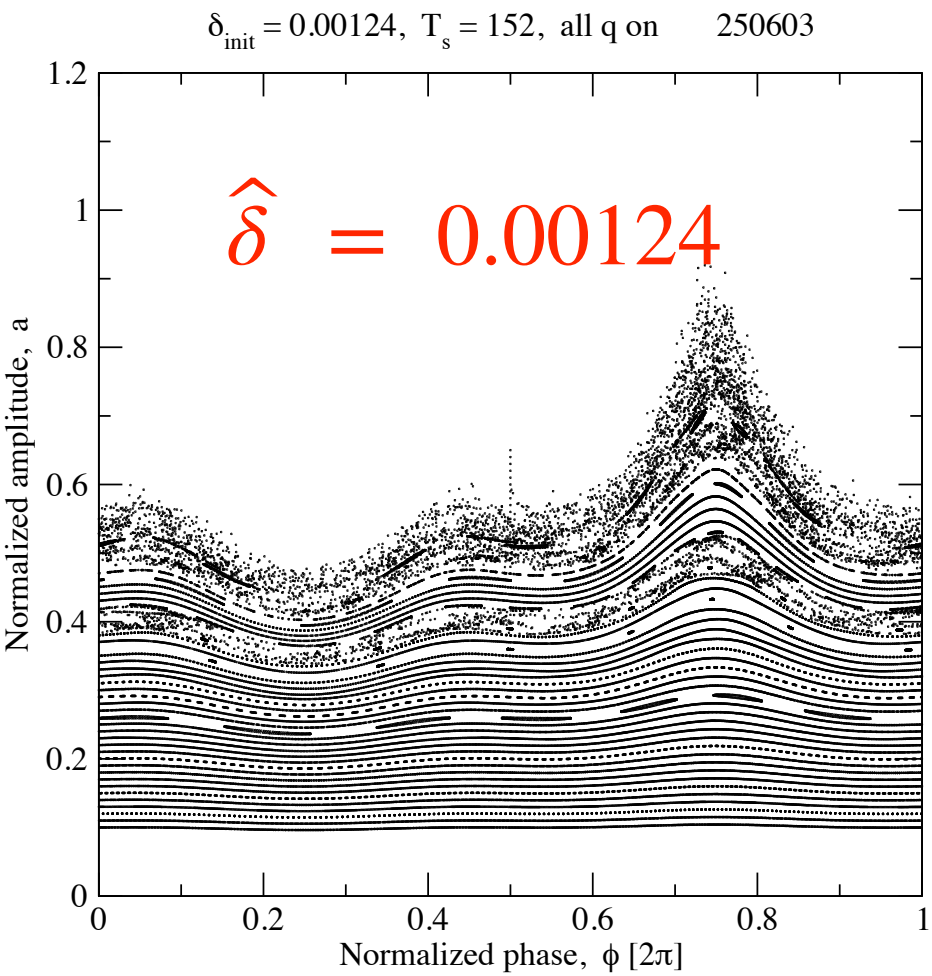


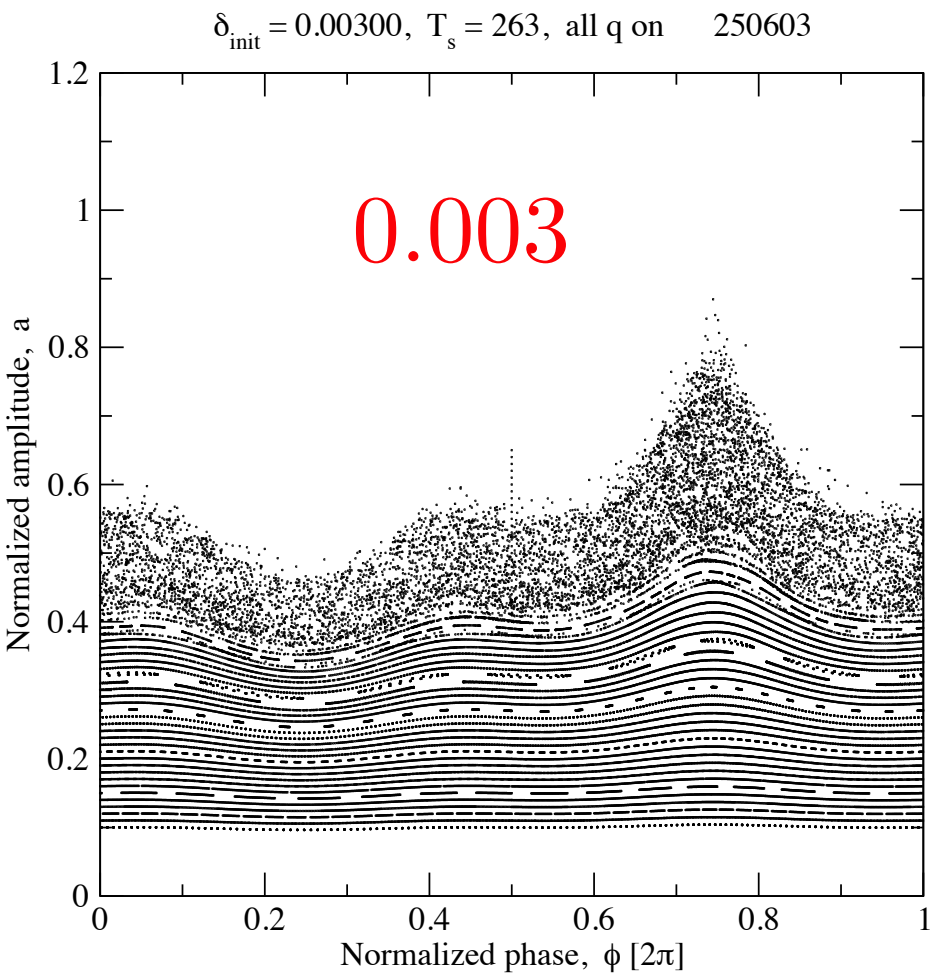
Significant trouble is possible when one of these *response* spectral lines is close to an $n \,.\, Q_s(\hat{\delta})$ *drive* line — for some values of $\hat{\delta}$ at a given Q_0

Synchrotron harmonics cross island tunes



Behavior parameterized by $\hat{\delta}$





Surfaces-of-section are plotted once per synchrotron period $T_s(\hat{\delta})$, which is forced to be an integer.

Movie

Crisp Poincaré surface-of-section plots enable visual and quantitative tune modulation probes.

The synchrotron period T_S is forced to be an integer.

Longitudinal motion is represented by externally imposed tune modulation.

The 30-second movie 2_bucket_NLC.mp4 shows a sequence of many such plots, for

$$0 < \hat{\delta} < 0.006$$

Future

Many magnets, 2 transverse dimensions

Repeat the same on-momentum probe of RHIC & HSR, with many chromaticity sextupoles (& octupoles).

Add vertical motion and complex tune modulation.

Connect to the RHIC/HSR on-line model ?

Embedded H & V `tunemod` elements are being tested with realistic lattices in the new tracking code Sparks

Later, embed in BMAD, Xsuite, … ?

Goal: develop generic tools to probe tune modulation resilience or vulnerability, independent of other longitudinal effects.

Finis