

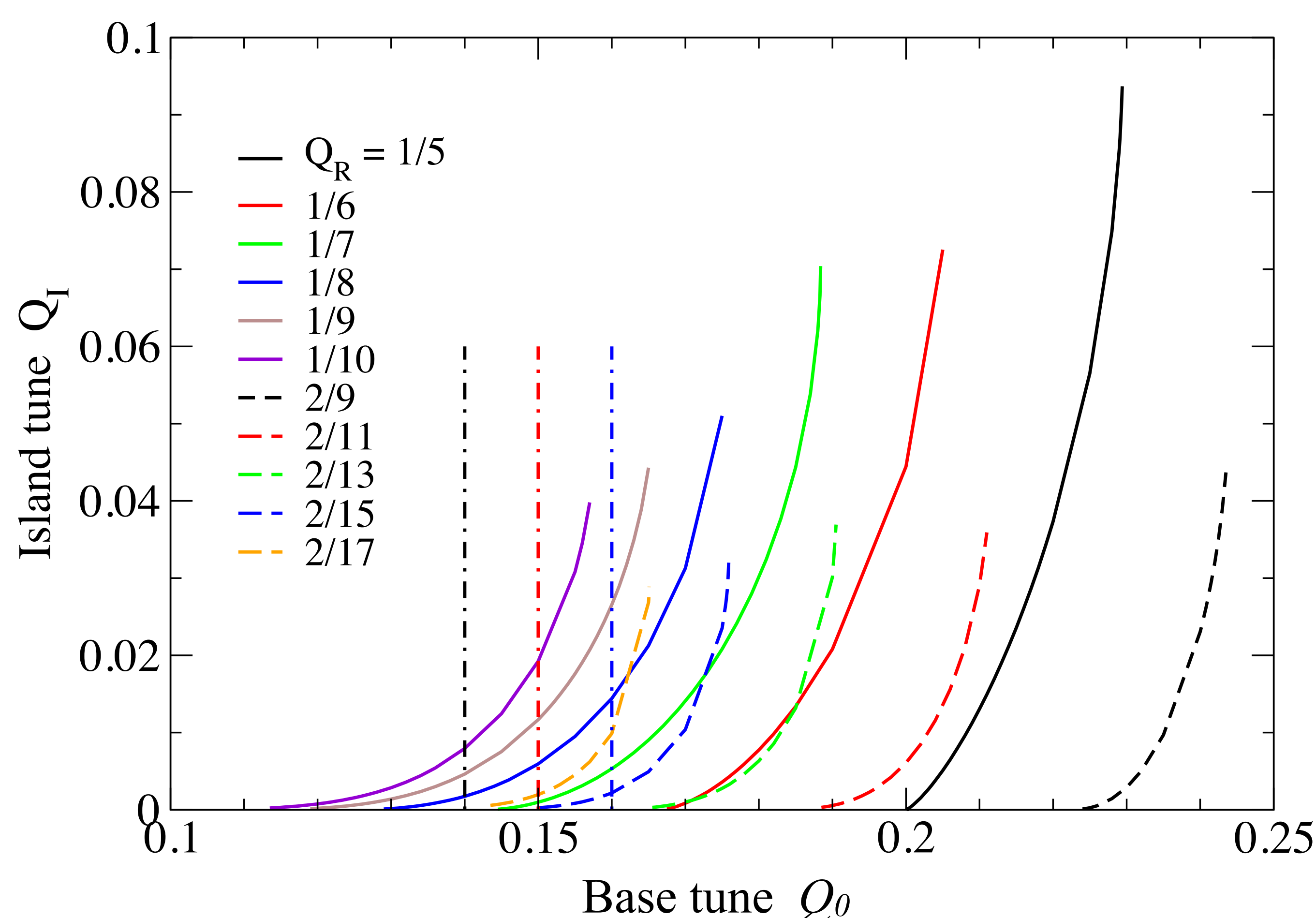
Single-magnet tracking with complex tune modulation

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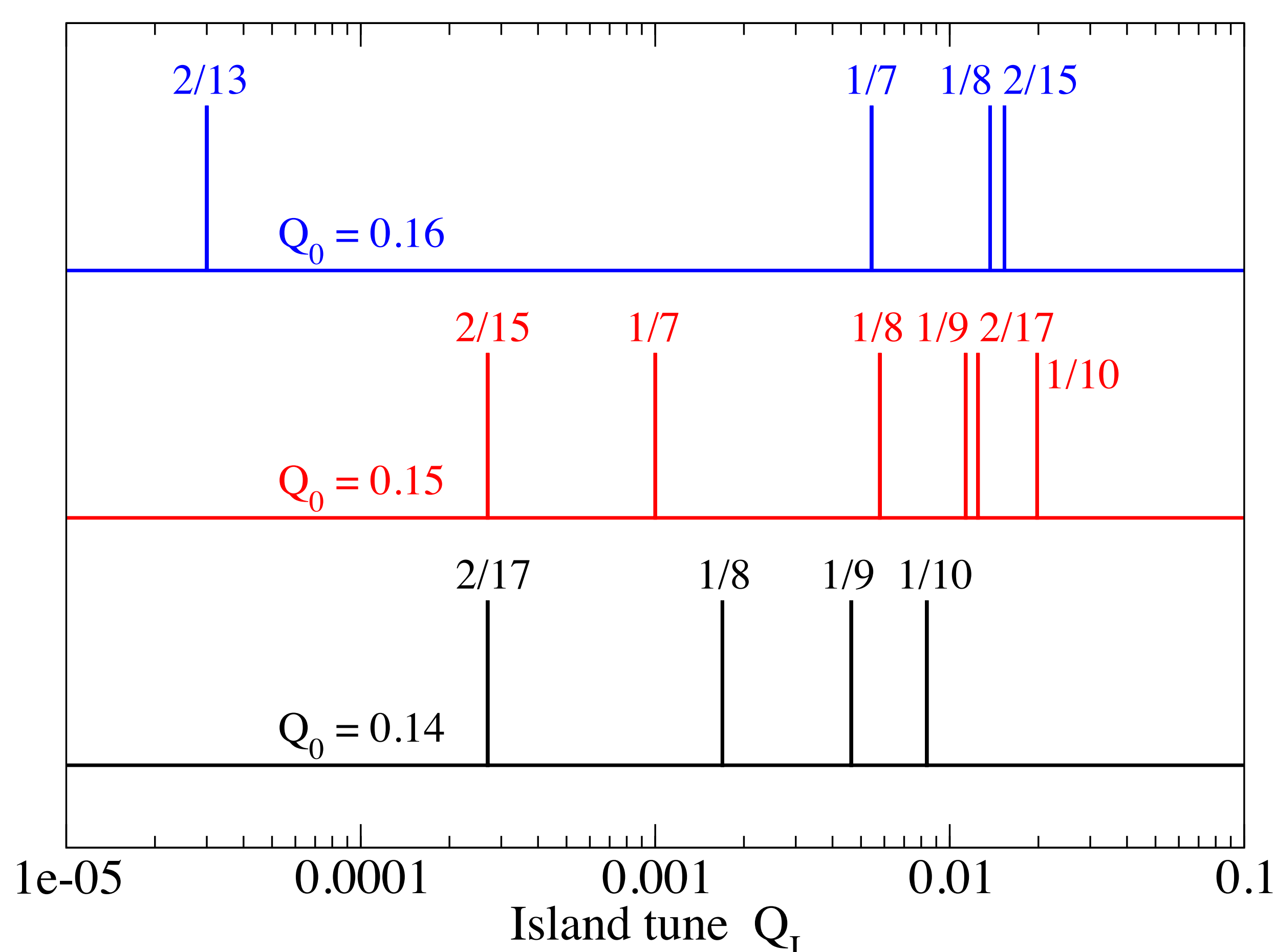
Single-sextupole and single-octupole lattices “*exhibit .. all the typical properties of more complicated mappings and dynamical issues*” (Hénon), e.g. resonances of all orders N with island tunes Q_I

Five key **tune domain** parameters:

- **Base tune** Q_0 eg 0.24
- **Resonance tune** Q_R eg 1/4
- **Island tune** Q_I eg 0.05
- **Tune modulation amplitude** q eg 0.005
- **Tune modulation tune** Q_M eg 0.01



Resonance trajectory fingerprint with **one** sextupole



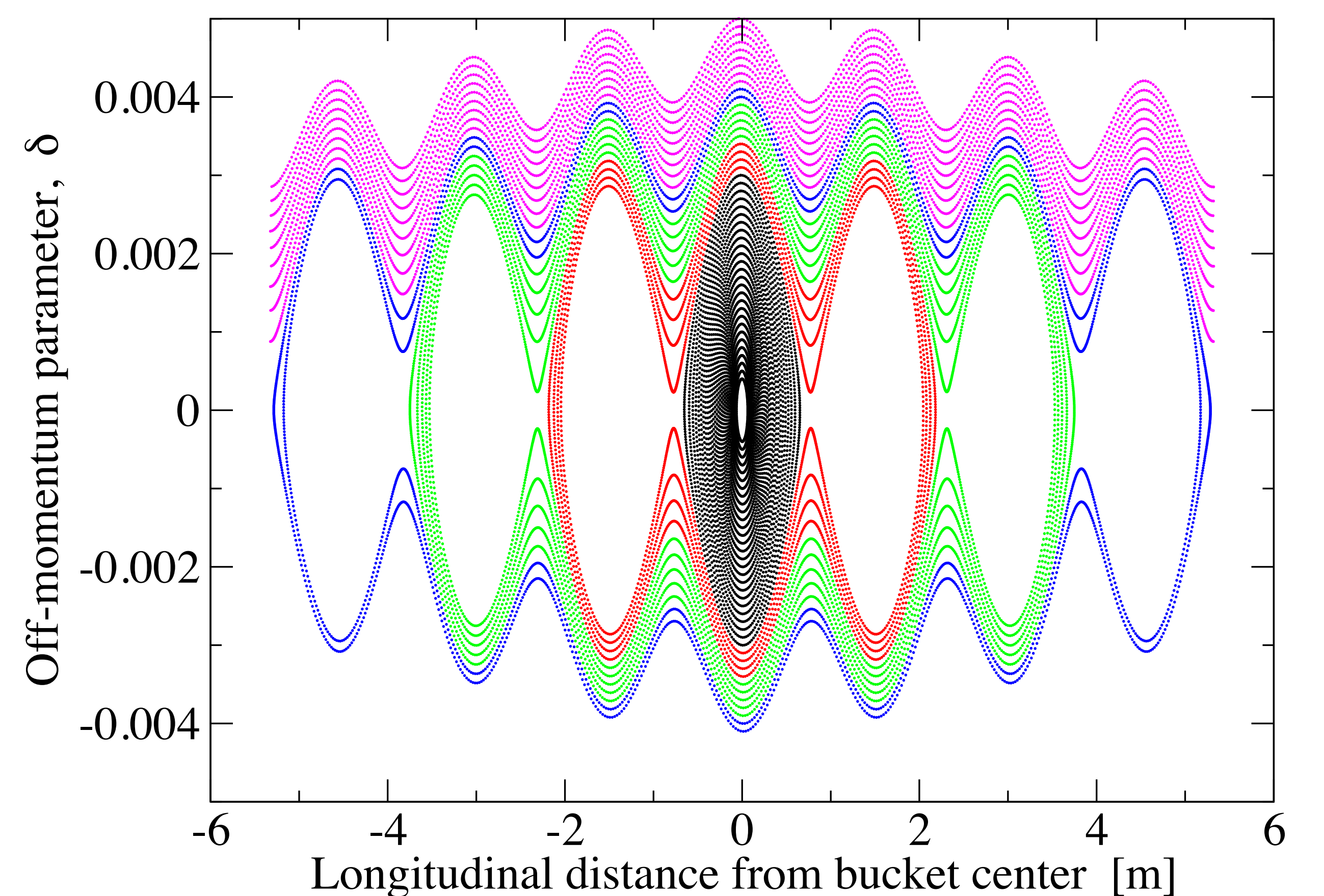
Island tune **response spectra** at 3 base tunes with **one** sextupole

Two strong nonlinear sources in 2024 RHIC:

1. Nonlinear chromaticity

$$\Delta Q(t) = C(\delta(t)) = \sum_{n=1}^{\infty} C_n \delta(t)^n$$

2. Complex RF buckets (below)

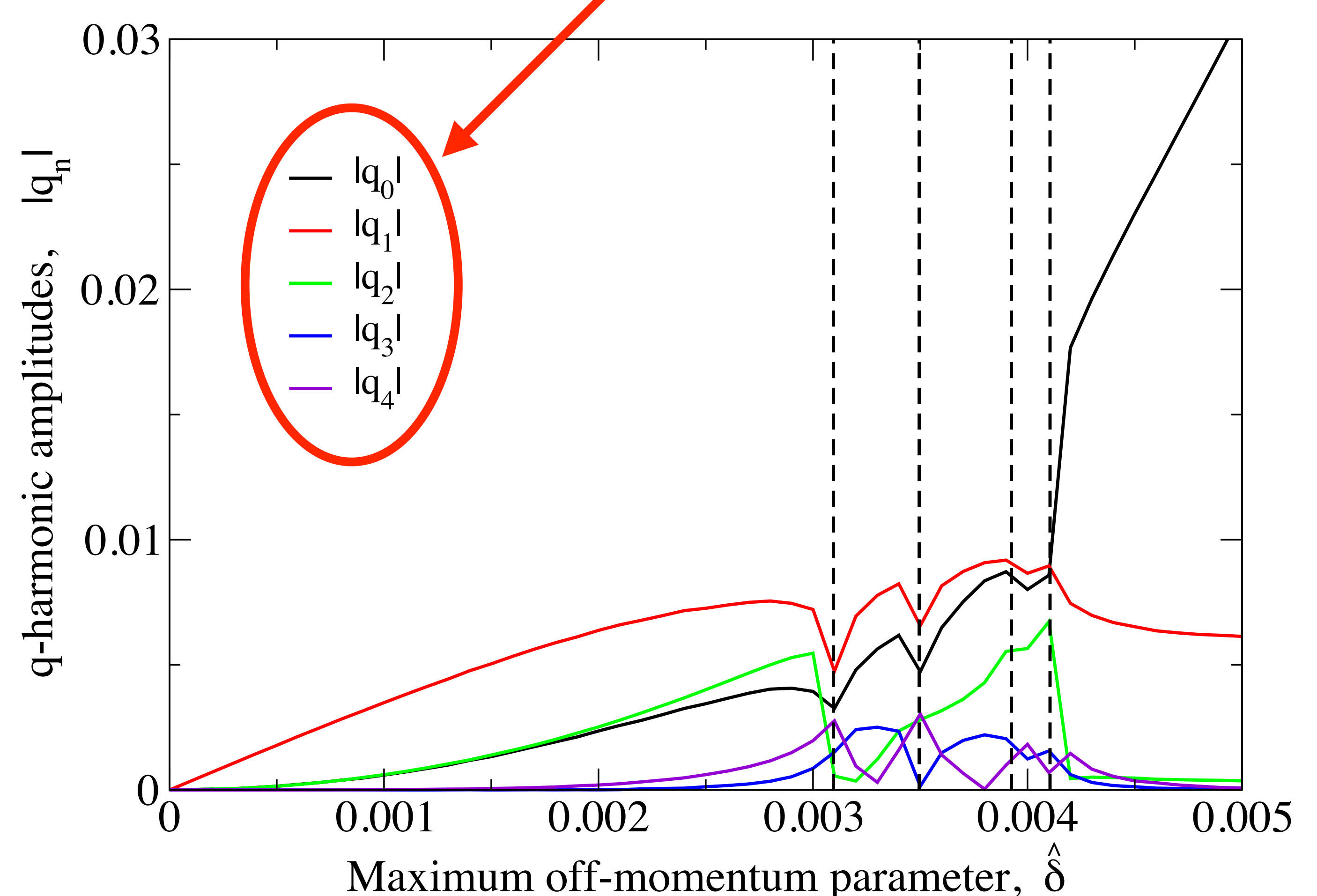


Combining both sources gives a double sum

$$\Delta Q(\hat{\delta}, t) = \sum_{m=1}^{\infty} C_m \left(\sum_{n=0}^{\infty} d_n(\hat{\delta}) \cos(2\pi n \cdot Q_s(\hat{\delta}) \cdot t) \right)^n$$

that delivers **many** tune modulation **drive strengths** q_n

$$\Delta Q(\hat{\delta}, t) = \sum_{n=0}^{\infty} q_n(\hat{\delta}) \cos(2\pi n \cdot Q_s(\hat{\delta}) \cdot t)$$



Future work: Develop a Tunemod lattice element for use in 4D tracking as an extrinsic signal generator, which is Poincaré-friendly and independent of other effects that confuse 6D tracking, e.g. dispersion in RF cavities.