Two “New” Precision Observables in Kaon Physics

Joachim Brod

UNIVERSITY OF Cincinnati

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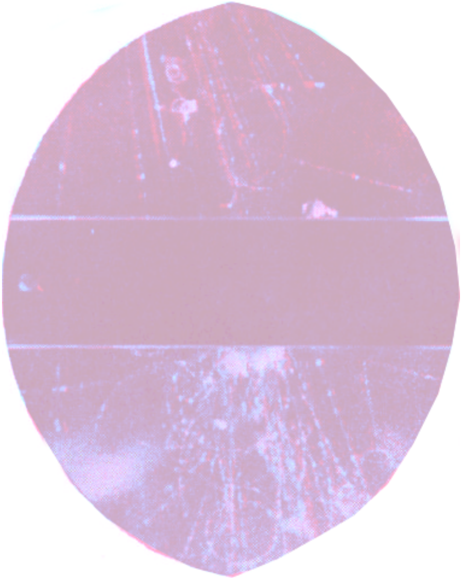
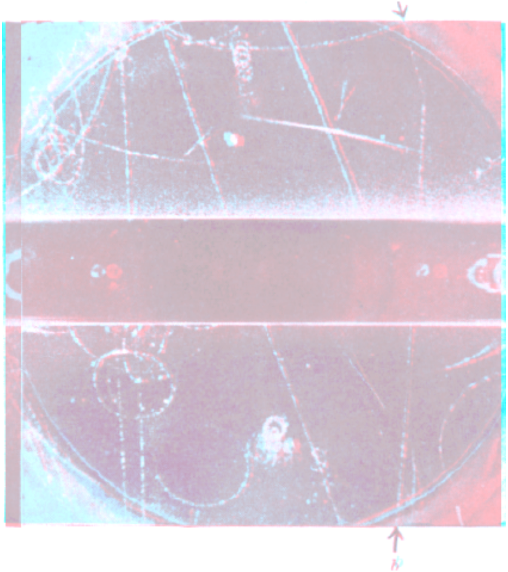
Kaons at the Frontier of Science and Technology



$K_S \to \pi^+\pi^-$ [Rochester & Butler, 1947]



$K^{\pm} \to \pi^0 e^{\pm} \nu_e$ [Rochester & Butler, 1947]

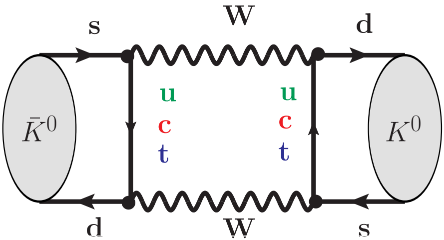

AND NOW FOR
SOMETHING
COMPLETELY
DIFFERENT

Test short-distance dynamics with kaons

- Most kaon observables are long-distance dominated
- Few exceptions (mostly CP violation and rare decays)
- CKM structure and QCD effects make kaons sensitive probes of high-energy scales
  - In the SM
  - Beyond the SM
- Need both precise measurements and solid theory predictions

Neutral Kaons

Reminder: neutral kaons



$\bar{K}^0 \sim [s\bar{d}]$ $K^0 \sim [\bar{s}d]$

Neutral Kaon Mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle \\ |\bar{K}^0(t)\rangle\end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix}|K^0(t)\rangle \\ |\bar{K}^0(t)\rangle\end{pmatrix}$$

Neutral Kaon Mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}=\left(M-i\frac{\Gamma}{2}\right)\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}.$$

- The Hamiltonian is diagonalized by

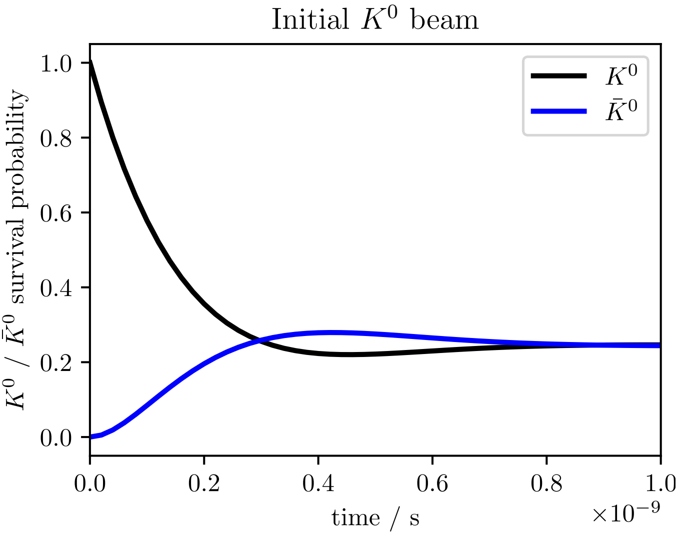
$$|K_S(t)\rangle=e^{-(iM_S+\Gamma_S)t}|K_S\rangle \qquad |K_L(t)\rangle=e^{-(iM_L+\Gamma_L)t}|K_L\rangle$$

- where

$$|K_S\rangle=p|K^0\rangle+q|\bar{K}^0\rangle \qquad |K_L\rangle=p|K^0\rangle-q|\bar{K}^0\rangle$$

- Mass difference $\Delta M_K \equiv M_L - M_S = 3.484\times 10^{-15}\,\text{GeV}$

Kaon mixing



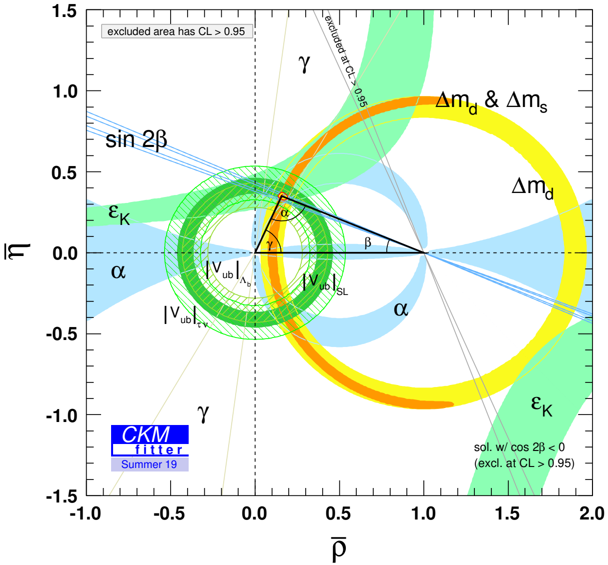
Initial K^0 beam
1.0
0.8
0.6
0.4
0.2
0.0
K^0 / $\bar{K}^0$ survival probability
0.0
0.2
0.4
0.6
0.8
1.0
time / s
$\times 10^{-9}$
K^0
$\bar{K}^0$

CP properties

- CP transformation $(|K^0\rangle \sim |\bar{s}d\rangle, \quad |\overline{K^0}\rangle \sim |\bar{d}s\rangle)$
  - $CP|K^0\rangle = -|\overline{K^0}\rangle$
  - $CP|\overline{K^0}\rangle = -|K^0\rangle$
- CP eigenstates
  - $|K_1\rangle \equiv \left(|K^0\rangle - |\overline{K^0}\rangle\right)/\sqrt{2}$ (CP even)
  - $|K_2\rangle \equiv \left(|K^0\rangle + |\overline{K^0}\rangle\right)/\sqrt{2}$ (CP odd)
- $K_2^0 \to \pi\pi$ is forbidden by CP
  - $1/\Gamma_S = \tau_S = 8.954 \times 10^{-11}\,\mathrm{s} = 2.68\,\mathrm{cm}$
  - $1/\Gamma_L = \tau_L = 5.116 \times 10^{-8}\,\mathrm{s} = 15.3\,\mathrm{m}$
  - $\Gamma_{K_2} \ll \Gamma_{K_1}$

Progress in ϵ_K

$$|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$$


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
0.5
Δm_d
ε_K
α
β
γ
α
$|V_{ub}|_{\Lambda_b}$
$|V_{ub}|_{SL}$
$|V_{ub}|_{\tau\nu}$
α
γ
ε_K
CKM fitter
Summer 19
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
$\bar{\eta}$
$\bar{\rho}$
1.5
1.0
0.0
-0.5
-1.0
-1.5
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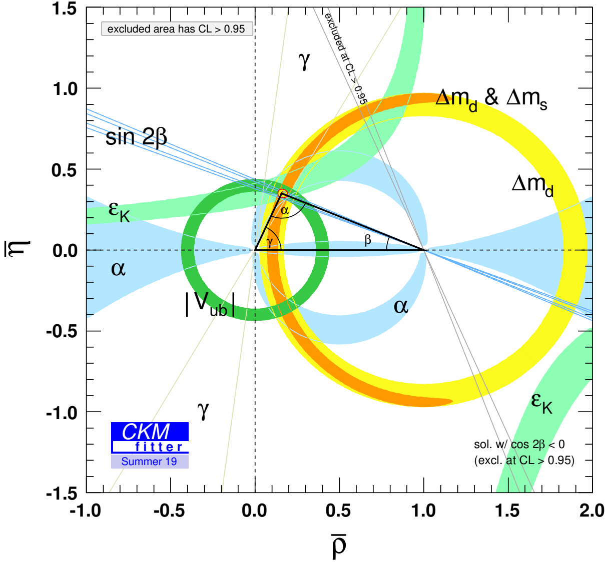
-0.5
0.0
0.5
1.0
1.5
2.0


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
γ
α
|V_ub|
α
γ
ε_K
CKM fitter
Summer 19
sol. w/ cos 2β < 0
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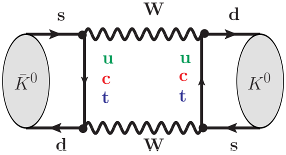
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
η̄
ρ̄

Definition of ϵ_K

$$\epsilon_K \equiv \frac{\eta_{00} + 2\eta_{+-}}{3} \approx \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle} \approx e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im} M_{12}}{2\mathrm{Re} M_{12}} - \frac{\mathrm{Im}\Gamma_{12}}{2\mathrm{Re}\Gamma_{12}}\right)$$

- $\phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta\Gamma_K/2} = (43.5 \pm 0.5)^\circ$
- M_{12} – dispersive part of $|\Delta S| = 2$ amplitude
- Γ_{12} – absorptive part of $|\Delta S| = 2$ amplitude
- Approximations good up to

  $|A_2|^2/|A_0|^2 \sim 0.002, \qquad \Gamma_L/\Gamma_S \sim 0.002.$

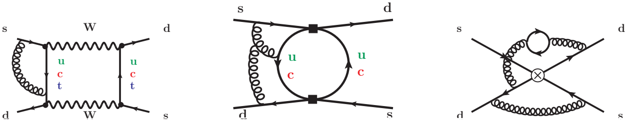


Calculating ϵ_K – outline

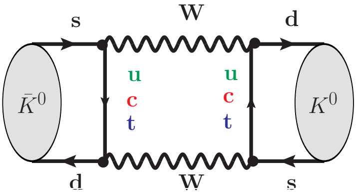
- ϵ_K receives short-distance and long-distance contributions
  - Split using weak effective Hamiltonian
  - Two options: Above and below charm mass
- Integrating out charm works well for ϵ_K
  - Perturbation theory seems to converge very well
  - Single hadronic matrix element B_K very well known
  - Additional long-distance contributions of order few percent
- Keeping dynamical charm has many benefits:
  - Cross check of "local" approach
  - Naturally include more long-distance effects
  - Essential for calculation of ΔM_K

Local SD contribution to ϵ_K

- Top quark dominates the SD contribution to ϵ_K
- Initial condition and RG evolution known to NLO
  [Buras et al., Nucl.Phys. B347 (1990) 491-536]
  - $\eta_{tt} = 0.55 \pm 0.02$
- Historically, charm contribution was artifically split into two
- Both known at NNLO
  - Charm-top (ϵ_K) [Brod, Gorbahn 1007.0684]
  - Charm (ϵ_K, ΔM_K) [Herrlich, Nierste, hep-ph/9310311; Brod, Gorbahn 1108.2036]



SD contribution to ϵ_K: CKM Unitarity



$$\lambda_u = V_{us} V_{ud}^*\,, \quad \lambda_c = V_{cs} V_{cd}^*\,, \quad \lambda_t = V_{ts} V_{td}^*\,, \quad \lambda_u + \lambda_c + \lambda_t = 0$$

c-*t* unitarity

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_c \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$

u-*t* unitarity

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_u \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_u^2	0	$\sim \lambda^2$

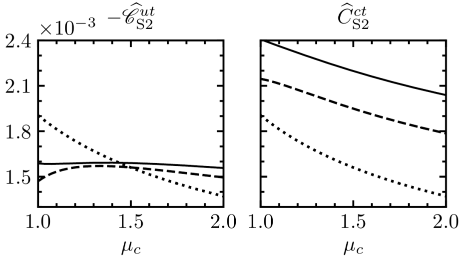
$$\mathrm{Im}(M_{12}) \to \epsilon_K \qquad \mathrm{Re}(M_{12}) \to \Delta M_K$$

SD contribution to ϵ_K – charm

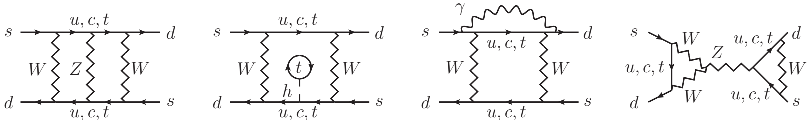
- The use of "u-t unitarity" was proposed in 2012
  [Christ et al., 1212.5931; see also Barbieri 2007]

- Application to ϵ_K [Brod et al. 1911.06822]
  - Can "recycle" NNLO anomalous dimensions and matching conditions

- "Simple" rearrangement of effective Hamiltonian has reduced perturbative uncertainty from order 30% to order 1%



SD contribution to ϵ_K – EW corrections



- Electroweak effects are large because of
  - Large couplings (e.g. top Yukawa) [Brod, Kvedaraite, Polonsky, Youssef 2207.07669]
  - Large logarithms [Brod, Kvedaraite, Polonsky, Youssef 2207.07669]
- Fix scheme of electroweak input parameters
  - Removes $\sim 5\%$ ambiguity
- Caveat: Scheme dependence due to missing QED contribution to B_K
  - Effect is expected to be tiny, $\mathcal{O}(\alpha/4\pi)$

SD contribution to ϵ_K – What's Next?

- NNLO QCD corrections to the top contribution [Gorbahn, Stamou, W.I.P.]
  - Residual error expected below percent level
- N^3LO QCD corrections to the charm contribution [Brod, Stamou, Steudtner, W.I.P.]
  - 4-loop operator mixing (current-current sector sufficient)
  - 3-loop initial conditions
  - Residual error expected below percent level

LD contribution to ϵ_K – hadronic matrix element

- Traditionally parameterized as deviations from V.I.A.

$$B_K(\mu) \equiv \frac{\langle \bar{K}^0 | Q^{|\Delta S|=2} | K^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}$$

- $\hat{B}_K = 0.7533(91)$ ($N_f = 2+1$, NLO conversion)[FLAG Review 2024]

- RI-(S)MOM – $\overline{\text{MS}}$ conversion has recently been calculated at NNLO [Gorbahn, Jäger, Kvedaraite 2411.19861]
  - They quote $\hat{B}_K = 0.7627(60)$
  - N^3LO perturbative QCD correction would require three-loop conversion

LD contribution to ϵ_K – power corrections

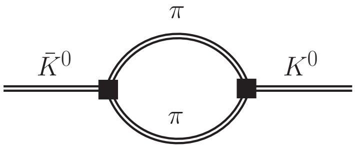
- What is the contribution of dimension-eight operators?
  - Naive estimate gives $m_K^2/m_c^2 \sim 15\%$ correction to charm contribution
- Leading term ($SU(3)$ chiral limit, large N_c): [Cata, Peris 0303162]
  - $O_4^{(8)} = g_s(\bar{s}_L\gamma^\mu \widetilde{G}_{\mu\nu} d_L)(\bar{s}_L\gamma^\nu d_L)$
  - gives $\sim 1\%$ correction to ϵ_K
- Contribution of all eight operators gives $+(1 \pm 0.3)\%$ shift in ϵ_K [Ciuchini et al. 2111.05153]
  - Matrix elements calculated in V.I.A. approximation
  - Can be improved by lattice QCD

Bilocal LD contribution to ϵ_K

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im} M_{12}}{2\mathrm{Re} M_{12}} - \frac{\mathrm{Im}\Gamma_{12}}{2\mathrm{Re}\Gamma_{12}} \right)$$

Dispersive (real) and

absorptive (imaginary) part of



$$\int d^4x \langle \bar{K}^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | K^0 \rangle$$

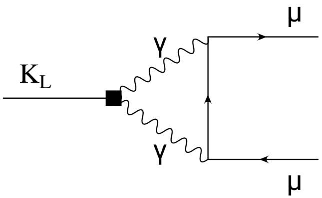
- Estimate $\mathrm{Im}\Gamma_{12}/2\mathrm{Re}\Gamma_{12} = -\mathrm{Im}A_0/\mathrm{Re}A_0$ from ϵ'/ϵ: $-(6 \pm 2)\%$ [Nierste, 0201071; Buras et al., 0805.3887]
- Estimate dispersive part in ChPT: $+(2.4 \pm 1.2)\%$ [Buras et al., 1002.3612]
- A lattice calculation with dynamical charms seems to be the way forward [Laiho et al., 0910.2928; Blum et al., 1502.00263; Bai et al. 1505.07863, 2309.01193]


AND NOW FOR
SOMETHING
COMPLETELY
DIFFERENT

$$K \to \mu^+\mu^-$$

Brief overview

- $\mathrm{Br}(K_L \to \mu^+\mu^-) = 6.84(11) \times 10^{-9}$
- $\mathrm{Br}(K_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}$



- $K_L \to \mu^+\mu^-$:
  - completely dominated by absorptive part of two-photon amplitude
  - (Small) dispersive SD contribution; very difficult to disentangle from LD
  - Lattice calculation aims at 10% error [Chao, Christ 2406.07447]
- $K_S \to \mu^+\mu^-$:
  - SD component CP violating [Isidori, Unterdorfer hep-ph/0311084]
  - Difficult to disentangle from LD

A new golden mode?

	$(\mu^+\mu^-)_{\ell=0}$	$(\mu^+\mu^-)_{\ell=1}$
K_L	CP conserving	(CP violating)
K_S	CP violating	CP conserving

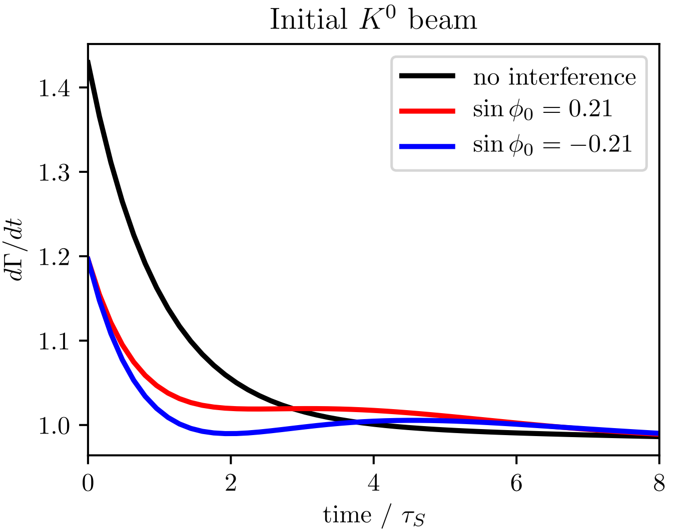
- Experiment cannot distinguish between $\ell = 0$ and $\ell = 1$
- Notation: write
  - $A_\ell{}^L = A[K_L \to (\mu^+\mu^-)_\ell]$
  - $A_\ell{}^S = A[K_S \to (\mu^+\mu^-)_\ell]$
- Can extract CPV contribution from data!
- $K_S \to (\mu^+\mu^-)_0$ is purely CPV only for $\epsilon_K = 0$

Time-dependent decay rate of neutral kaons

$$\frac{d\Gamma(K(t) \to \mu^+\mu^-)}{dt} \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\big[C_{\sin}\sin(\Delta M t) + C_{\cos}\cos(\Delta M t)\big] e^{-\Gamma t}$$

- C_i related to decay amplitudes:
  - $C_L = |A^L_{\ell=0}|^2 + |A^L_{\ell=1}|^2$
  - $C_S = |A^S_{\ell=0}|^2 + |A^S_{\ell=1}|^2$
  - $C_{\sin} = \mathrm{Im}\big\{\big(A^S_{\ell=0}\big)^* A^L_{\ell=0}\big\} + \mathrm{Im}\big\{\big(A^S_{\ell=1}\big)^* A^L_{\ell=1}\big\}$
  - $C_{\cos} = \mathrm{Re}\big\{\big(A^S_{\ell=0}\big)^* A^L_{\ell=0}\big\} + \mathrm{Re}\big\{\big(A^S_{\ell=1}\big)^* A^L_{\ell=1}\big\}$
- Interference terms $C_{\text{int}}^2 = C_{\cos}^2 + C_{\sin}^2$ sensitive to short-distance component!
  [D'Ambrosio, Kitahara 1707.06999]

Time-dependent decay rate of neutral kaons



A useful relation

- Single assumption: $A^L_{\ell=1} = 0$

- It follows immediately ("in two lines!")

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

[Dery, Ghosh, Grossman, Schacht 2104.06427; Brod, Stamou 2209.07445]

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)}$$

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S}$$

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\mathrm{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2 |A_0^L|^2\,,$$

$$\frac{\mathrm{Br}(K_S \to (\mu\mu)_{\ell=0})}{\mathrm{Br}(K_L \to \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2|A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\mathrm{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2|A_0^L|^2 \,, \qquad C_L^2 = |A_0^L|^4$$

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$$C_{\mathrm{int}}^2 = \left|\left(A_0^S\right)^* A_0^L\right|^2 = |A_0^S|^2 |A_0^L|^2 \,, \qquad C_L^2 = |A_0^L|^4$$

$$\Rightarrow \mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

A useful relation

- Single assumption: $A^L_{\ell=1} = 0$
- It follows immediately ("in two lines!")

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

[Dery et al. 2104.06427; Brod, Stamou 2209.07445]

- Recall $K_S \to (\mu^+\mu^-)_{\ell=0}$ is CPV $\Rightarrow$ sensitive to UV physics!

Compare to SM

- Experiment (~~HIKE~~; LHCb?) could possibly provide:

$$\mathrm{Br}^{\mathrm{exp}}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

- We want to compare to the SM prediction!
- Three parts:
  - Hadronic matrix element is just kaon decay constant – f_K
  - SD contribution – can calculate
  - Effect of indirect CP violation – estimate from data!

SM short-distance contribution

- CPV – imaginary part of weak Hamiltonian
  - $\Rightarrow$ Only top-quark contribution relevant
- Including three-loop QCD and two-loop electroweak:
  [Bobeth et al., 1311.0903, Brod, Stamou 2209.07445]]

$$\mathrm{Br}(K_S \to \mu^+\mu^-)^{\text{pert.}}_{\ell=0} = 1.70\,(02)_{\text{QCD/EW}}(01)_{f_K}(19)_{\text{param.}} \times 10^{-13}$$

Impact of indirect CP violation

- $K_S = K_1 + \epsilon_K K_2$ (mainly CP even)
- $K_L = K_2 + \epsilon_K K_1$ (mainly CP odd)

- Can then show: $A_0^S = A_0^S|_{\epsilon_K=0} + \epsilon_K A_0^L$
- Squaring to get decay rate [Brod, Stamou 2209.07445]

$$\begin{aligned}\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = &\, \mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0}^{\text{pert.}} \\ &\times \left(1 + \sqrt{2}|\epsilon_K|\frac{|A_0^L|}{|A_0^S|}(\cos\phi_0 - \sin\phi_0)\right),\end{aligned}$$

- From $\mathrm{Br}(K_L \to \mu^+\mu^-)$, we know $|A_0^L|/|A_0^S| \sim 8.4$
- Obtain $\phi_0 = \arg\{(A_0^S)^* A_0^L\}$ from $K_L \to \mu^+\mu^-$ and $K_L \to \gamma\gamma$ [A. Dery et al., 2211.03804]

- Additional $\pm 2\%$ or $\pm 3\%$ correction to $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0}$

Is the basic relation really valid?

- How good is the assumption $A_1^L = 0$?
- Can show: $A_1^L = A_1^L|_{\epsilon_K=0} + \epsilon_K A_1^S$
- Don't really know $A_1^L|_{\epsilon_K=0}$, but can estimate $\epsilon_K A_1^S$ from data / ChPT
- The relation

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathrm{Br}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

receives a correction

$$r \sim 1 - 2\frac{|A_1^L|}{|A_0^L|}\frac{|A_1^S|}{|A_0^S|}\cos(\phi_0 - \phi_1) + \ldots$$

- For $A_1^L|_{\epsilon_K=0} = 0$, this translates to a $\lesssim$ percent-level uncertainty

Summary

- Kaons <u>can</u> provide very sensitive probes of high-scale dynamics
- Two "new" precision observables in kaon physics:
  - Indirect CPV (ϵ_K) – can we reach 0.5% error?
  - $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\ell=0}$ – can we measure it?