Deep Neural Networks for extracting the 3D Structure of Nucleon at EIC

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AI4EIC 2025

October 27- 29, 2025

Boston, MA

This work is supported by DOE contract DE-FG02-96ER40950

Outline

- Sivers function extraction with DNNs ★ First-ever application of Neural Networks in any TMD in the literature

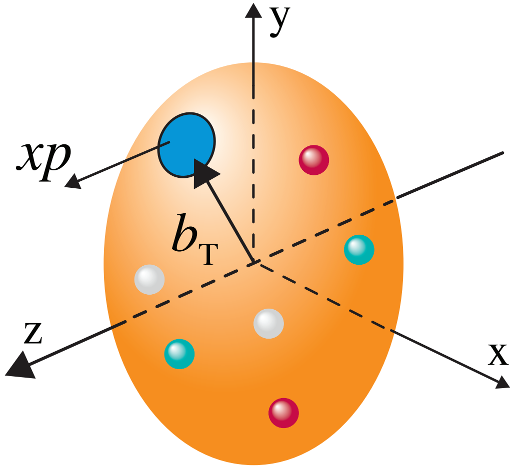
  DOI: https://doi.org/10.1103/PhysRevD.108.054007

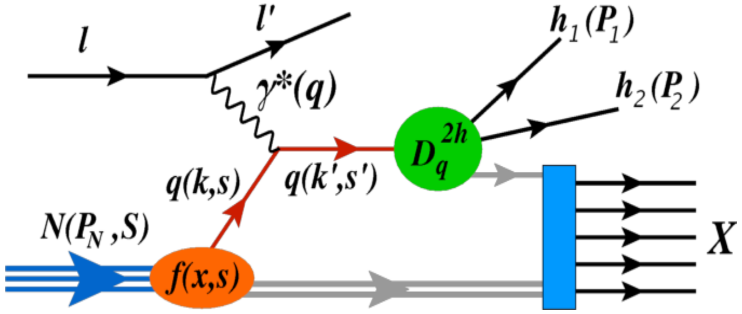
- Unpolarized TMDs extraction with DNN

  DOI: https://doi.org/10.48550/arXiv.2510.17243

- Summary & Outlook

TMDPDFs





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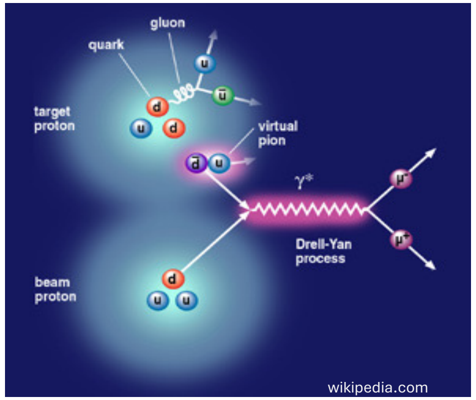
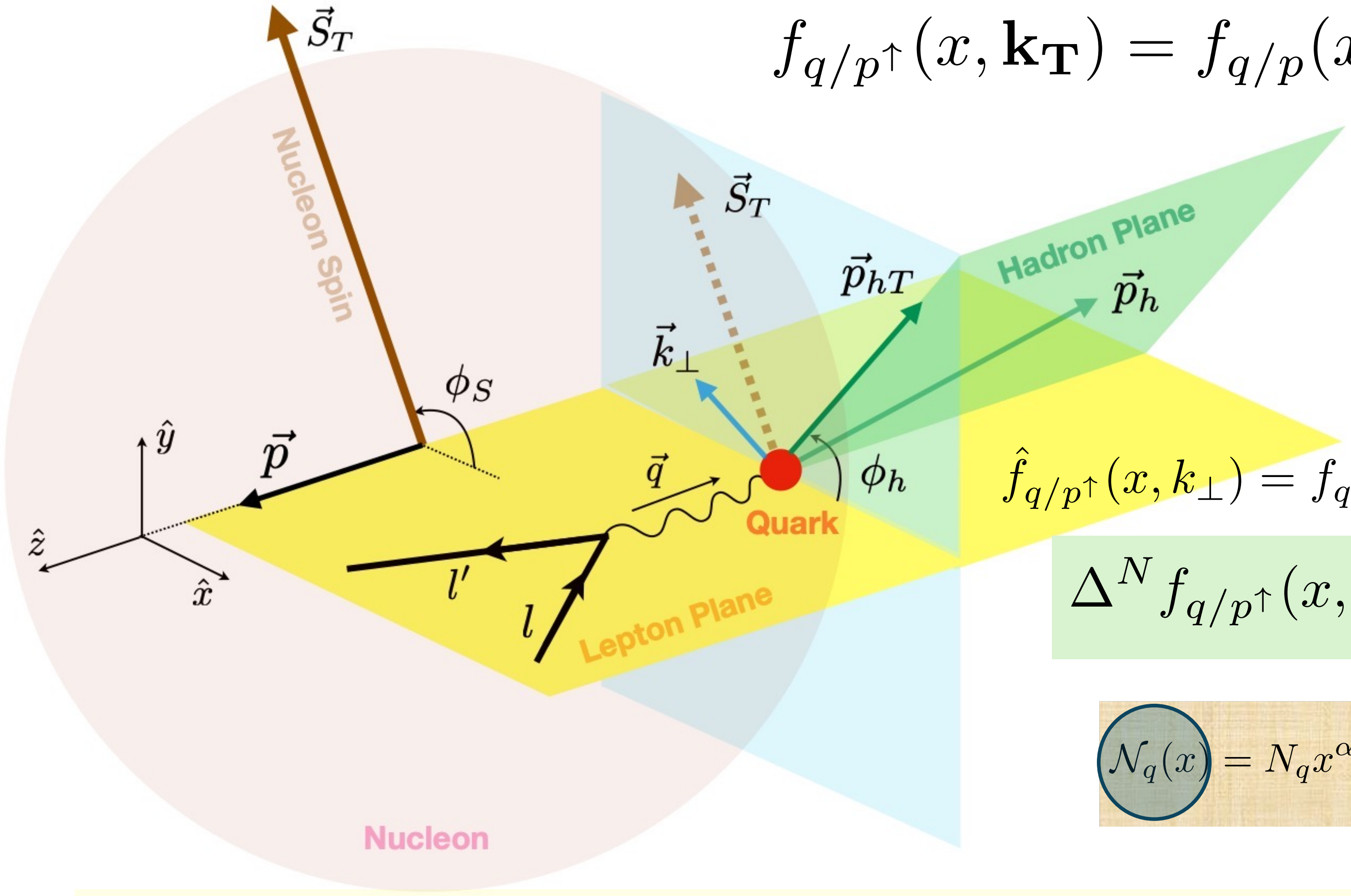


Leading Twist		Quark Polarization		
		Unpolarized [U]	Longitudinal [L]	Transverse [T]
Target Polarization	U	f_1 Unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^\perp$ Worm-gear 1
	T	$f_{1T}^\perp$ Sivers	g_{1T} Worm-gear 2	h_1 Transversity $h_{1T}^\perp$ Pretzelosity
	TENSOR	$f_{1LL}(x, \boldsymbol{k_T^2})$ $f_{1TT}(x, \boldsymbol{k_T^2})$ $f_{1LT}(x, \boldsymbol{k_T^2})$	$g_{1TT}(x, \boldsymbol{k_T^2})$ $g_{1LT}(x, \boldsymbol{k_T^2})$	$h_{1LL}^\perp(x, \boldsymbol{k_T^2})$ $h_{1TT}(x, \boldsymbol{k_T^2})$ $h_{1TT}^\perp(x, \boldsymbol{k_T^2})$ $h_{1LT}(x, \boldsymbol{k_T^2})$ $h_{1LT}^\perp(x, \boldsymbol{k_T^2})$

Sivers Asymmetry from SIDIS

$$f_{q/p^\uparrow}(x, \mathbf{k_T}) = f_{q/p}(x, \mathbf{k_T}) + f_{1T}^{\perp}(x, \mathbf{k_T})\mathbf{S}.(\hat{\mathbf{P}} \times \hat{\mathbf{k_T}})$$



- Unpolarized electron beam
- Polarized Proton (target)
- The scattered electron is measured
- An outgoing fragmented hadron (pion/kaon) is measured

$$\hat{f}_{q/p^\uparrow}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x, k_\perp)\vec{S}_T \cdot (\hat{p} \times \hat{k}_\perp)$$

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp)$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

Anselmino et al. (2017)

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \frac{d\sigma^{l\uparrow p \to hlX} - d\sigma^{l\downarrow p \to lhX}}{d\sigma^{l\uparrow p \to hlX} + d\sigma^{l\downarrow p \to hlX}} \equiv \frac{d\sigma\uparrow - d\sigma\downarrow}{d\sigma\uparrow + d\sigma\downarrow}$$

Deep Neural Networks Approach

First-ever application of Neural Networks in any TMD in the literature

PHYSICAL REVIEW D **108,** 054007 (2023)

Extraction of the Sivers function with deep neural networks

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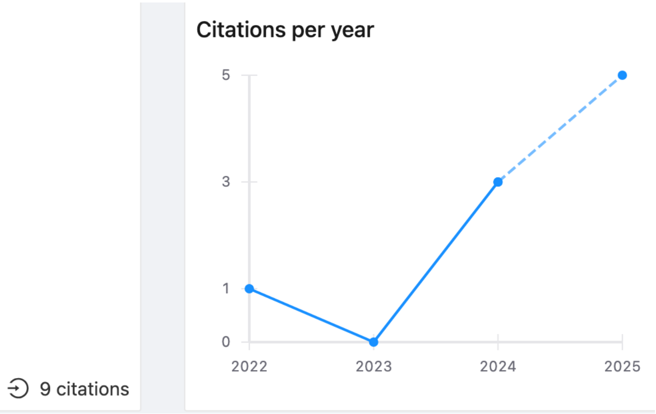
Department of Physics, University of Virginia, Charlottesville, Virginia 22904, USA

(Received 10 March 2023; accepted 9 August 2023; published 8 September 2023)

Deep neural networks (DNNs) are a powerful and flexible tool for information extraction and modeling. In this study, we use DNNs to extract the Sivers functions by globally fitting semi-inclusive deep inelastic scattering (SIDIS) data. To make predictions of this transverse momentum-dependent distribution, we construct a minimally biased model using data from COMPASS and HERMES. The resulting Sivers function model, constructed using SIDIS data, is also used to make predictions for Drell-Yan kinematics specific to the valence and sea quarks, with careful consideration given to experimental errors, data sparsity, and complexity of phase space.

DOI: 10.1103/PhysRevD.108.054007



https://inspirehep.net/literature/2654773

DOI: https://doi.org/10.1103/PhysRevD.108.054007

Deep Neural Networks Approach

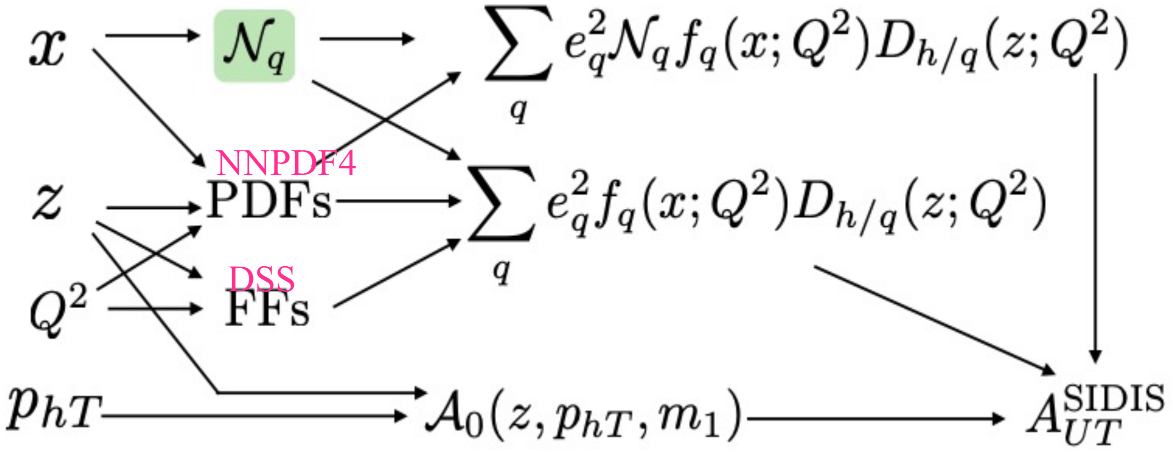
First-ever application of DNNs in any TMD in the literature

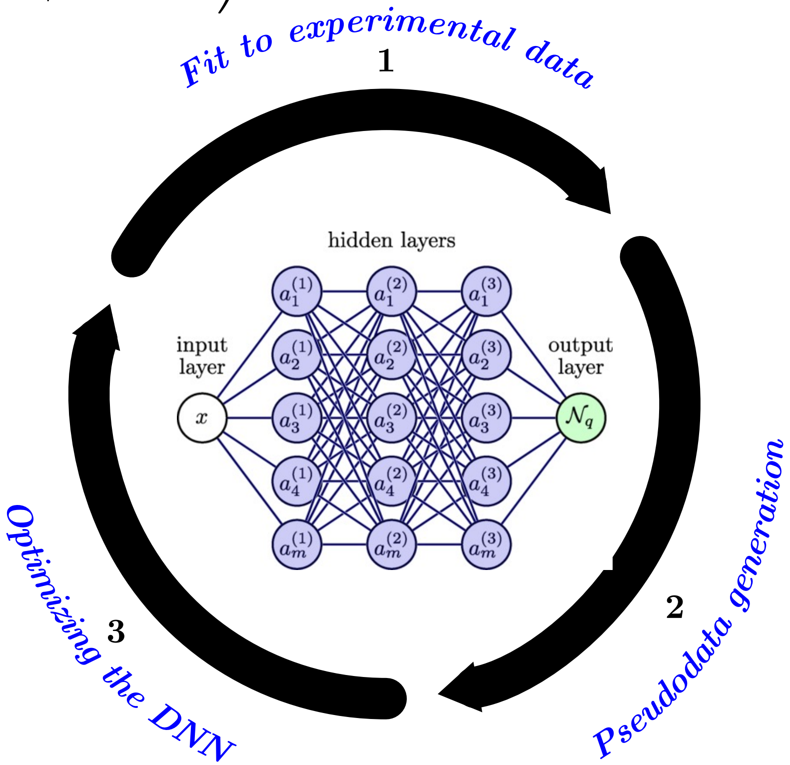
I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp)$$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, m_1) \left(\frac{\sum_q \mathcal{N}_q(x) e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$

- ✓ Data Driven
- ✓ Minimal biases, assumptions, limitations
- ✓ Capacity to handle complex patterns



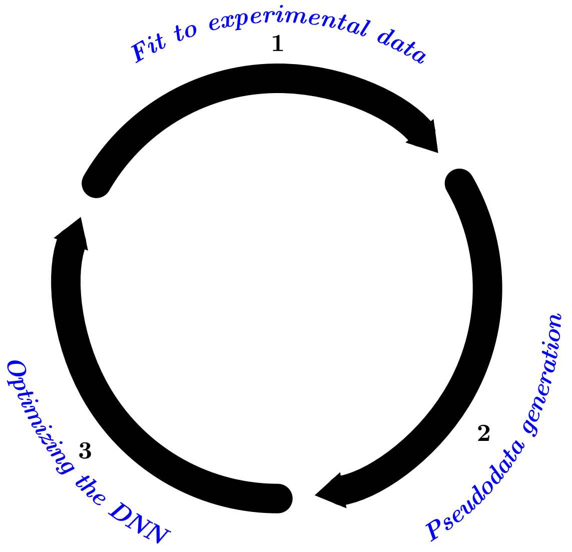


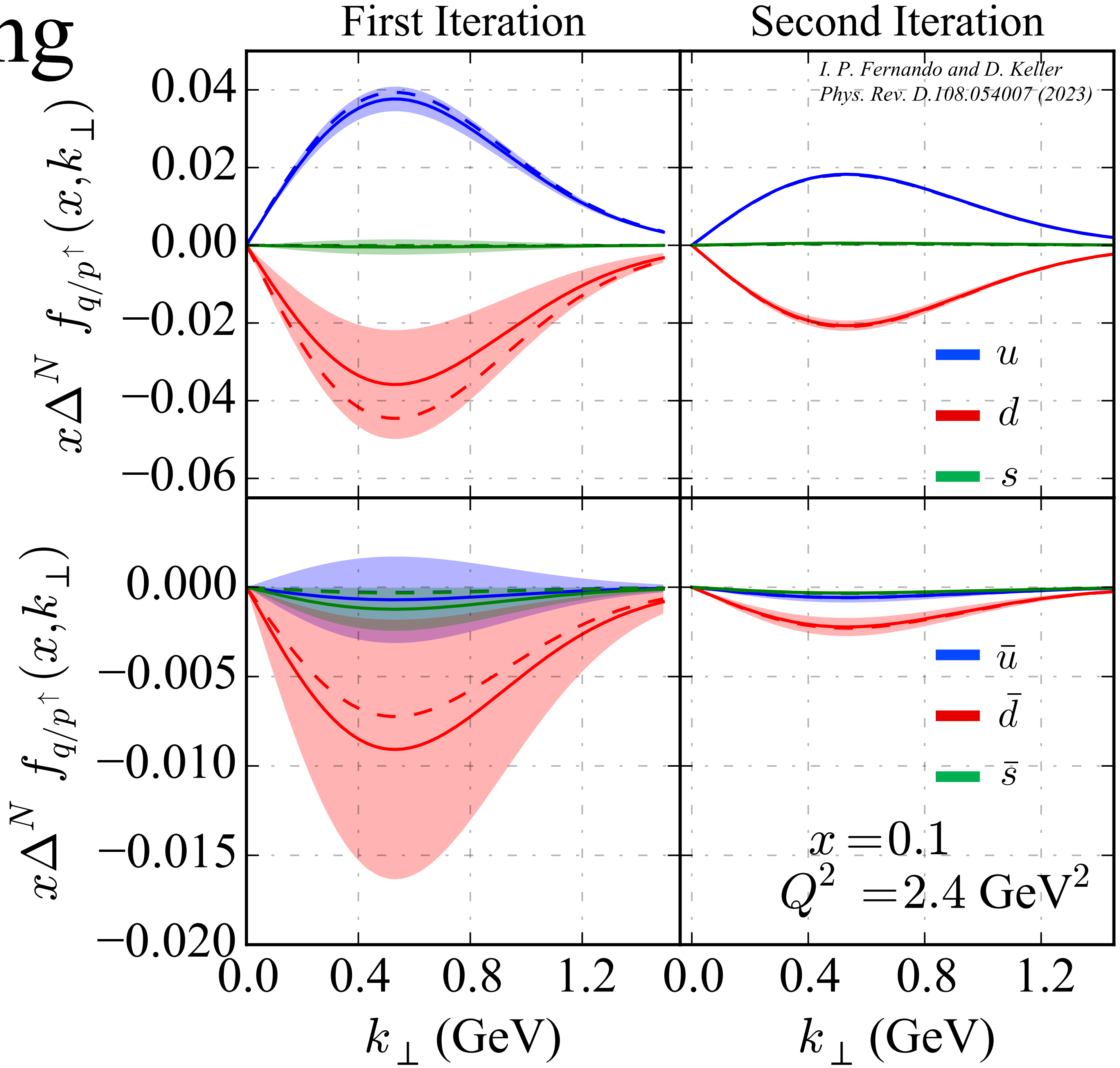
- ✓ Recursive Improvements to the DNN
- ✓ Systematic Studies of the method

$$\epsilon_q(x, k_\perp) = \left(1 - \frac{|\Delta^N f^{(\text{true})}_{q/p^\uparrow} - \Delta^N f^{(\text{mean})}_{q/p^\uparrow}|}{\Delta^N f^{(\text{true})}_{q/p^\uparrow}}\right) \times 100\%$$

$$\sigma_q(x, k_\perp) = \sqrt{\frac{\sum_i \left(\Delta^N f^{(i)}_{q/p^\uparrow} - \Delta^N f^{(\text{mean})}_{q/p^\uparrow}\right)^2}{N}}$$

DNN Method testing





- Dashed lines represent the ***generating function*** in each iteration.
- A comparison:
  Improving the ***generating functio***n
  Fine-tuning the hyperparameters
- Solid-lines and the band represent the mean and 68% CL with 1000 replicas of the DNN model.

Proton DNN Fit Results

I. P. Fernando and D. Keller
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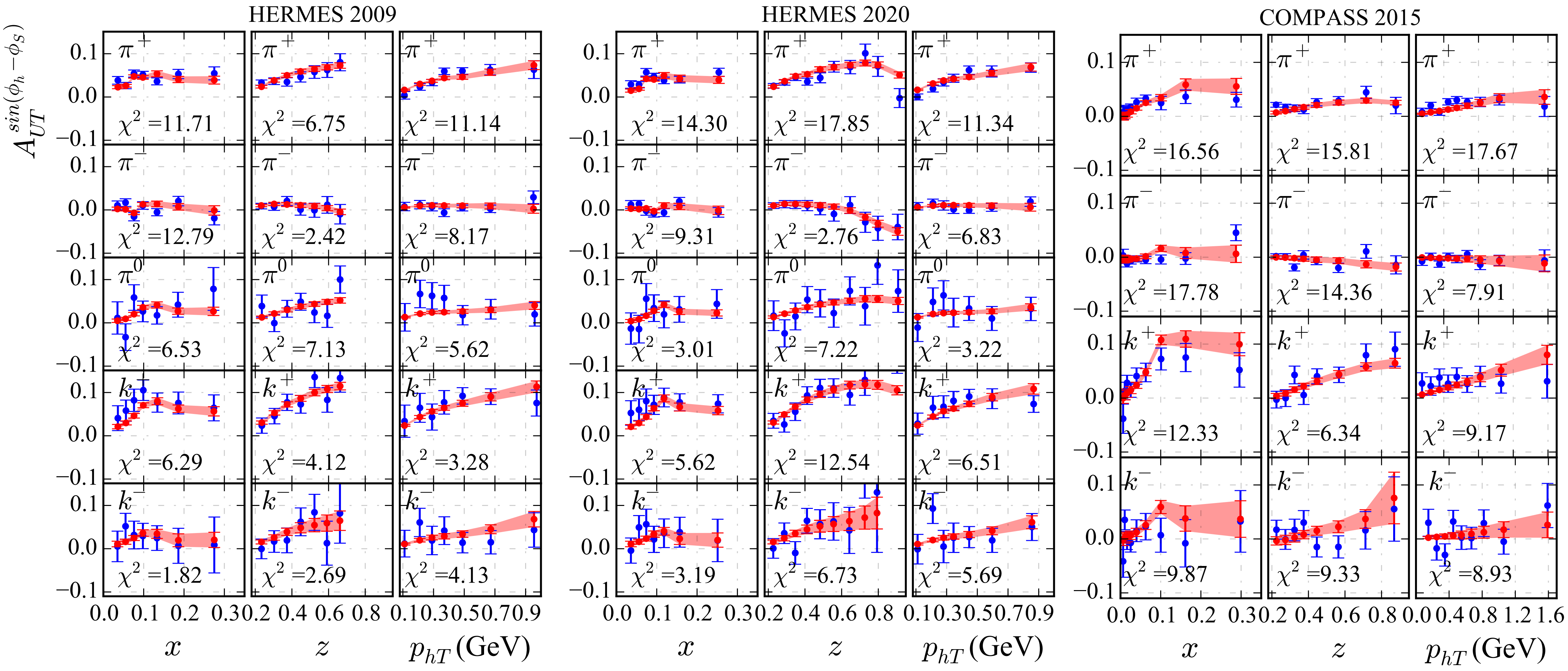
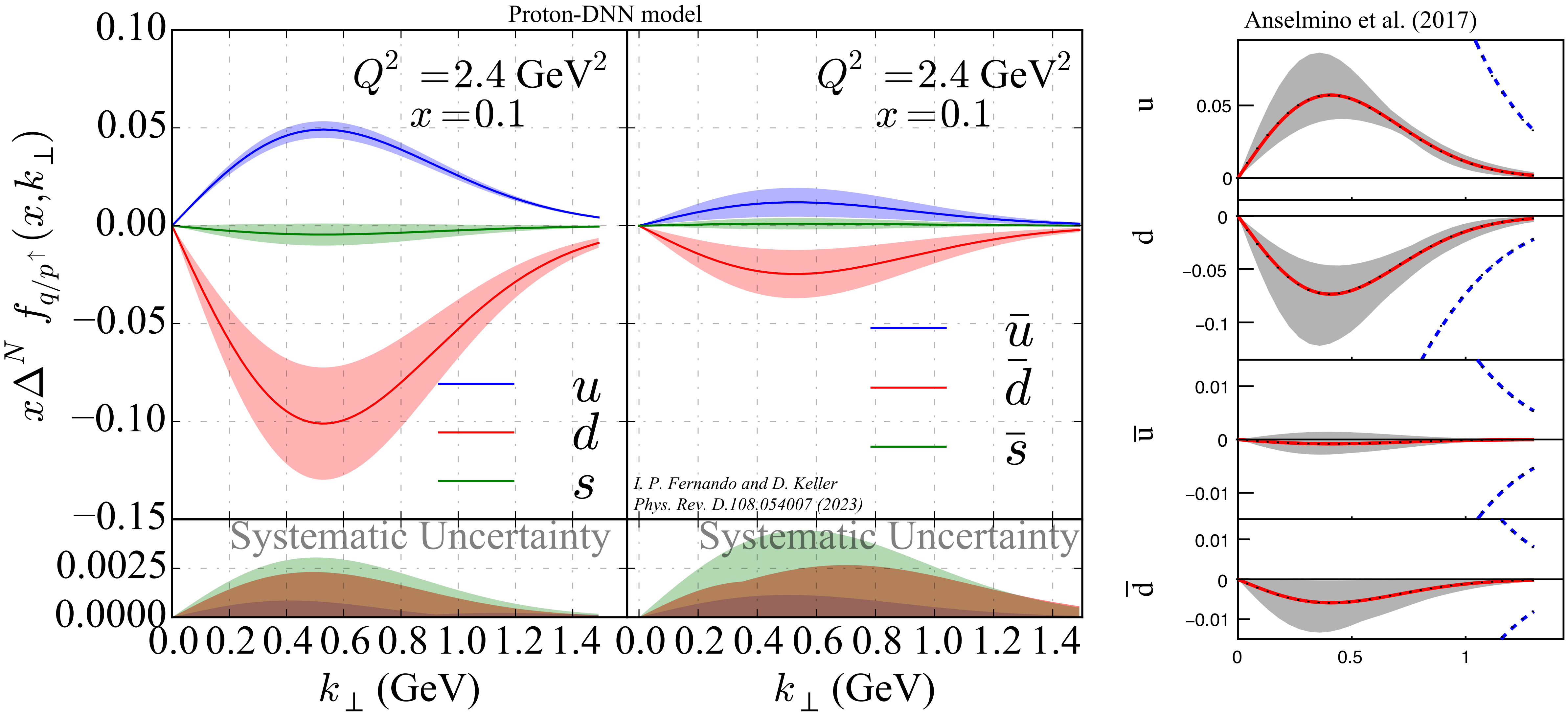


- All data points are well-described by the proton-DNN model.
- No kinematic cuts were implemented.

Calculated $\chi_{total}^2/N_{pt} = 1.04$

Sivers functions from the "Proton" DNN Model



3D Tomography from the "Proton" DNN Model

$$\rho^{a}_{p\uparrow}(x, k_x, k_y; Q^2) = f^{a}_{1}(x, k^2_{\perp}; Q^2) - \frac{k_x}{m_p} f^{\perp a}_{1T}(x, k^2_{\perp}; Q^2)$$

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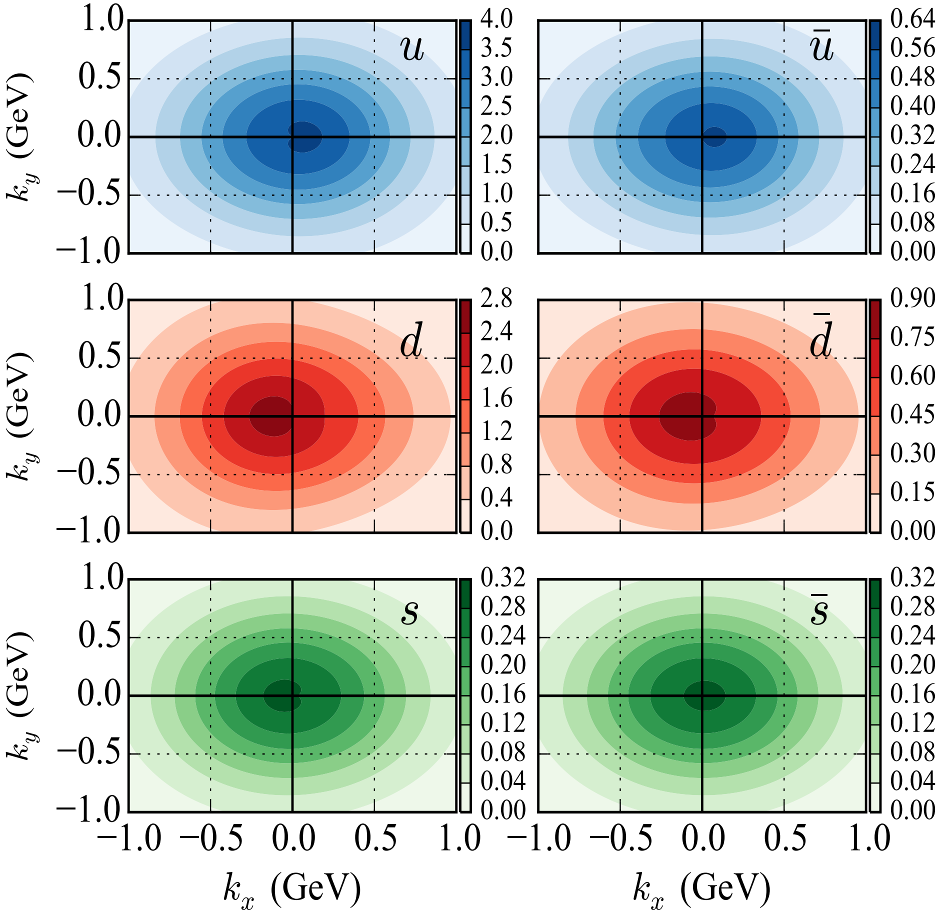
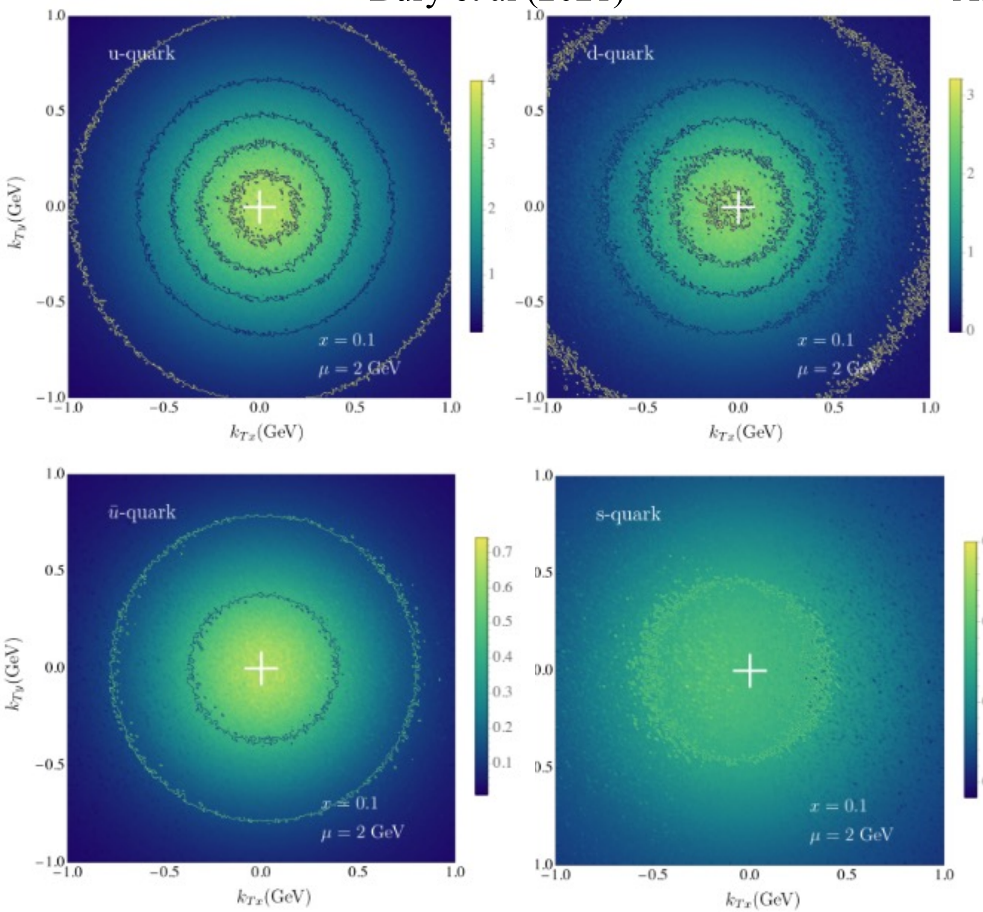
Proton-DNN model



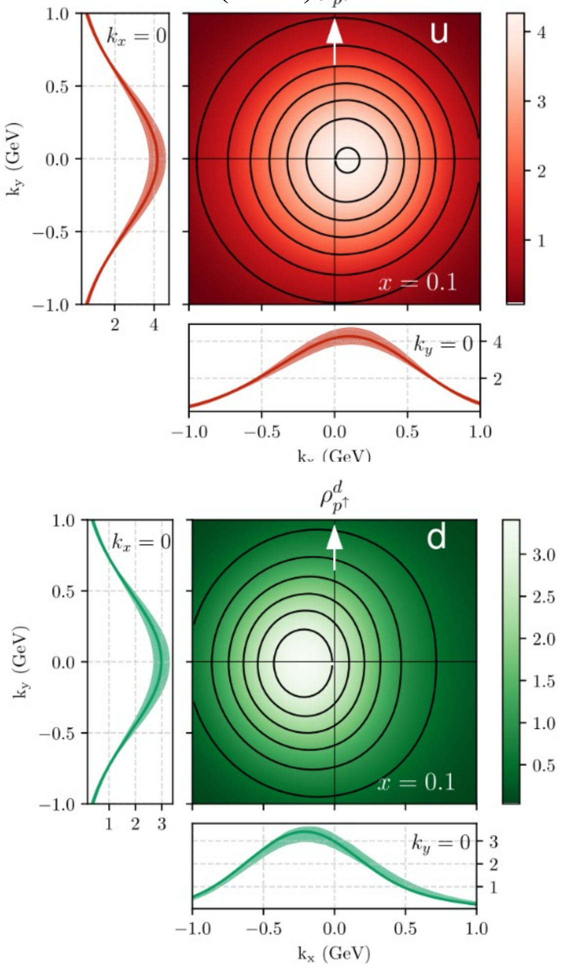
$Q^2 = 2.4$ GeV^2

$x = 0.1$

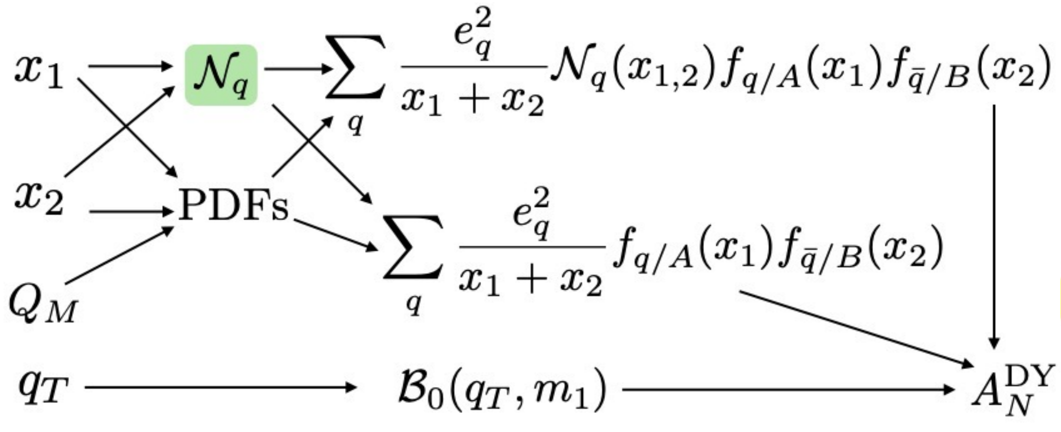
Bury et al (2021)

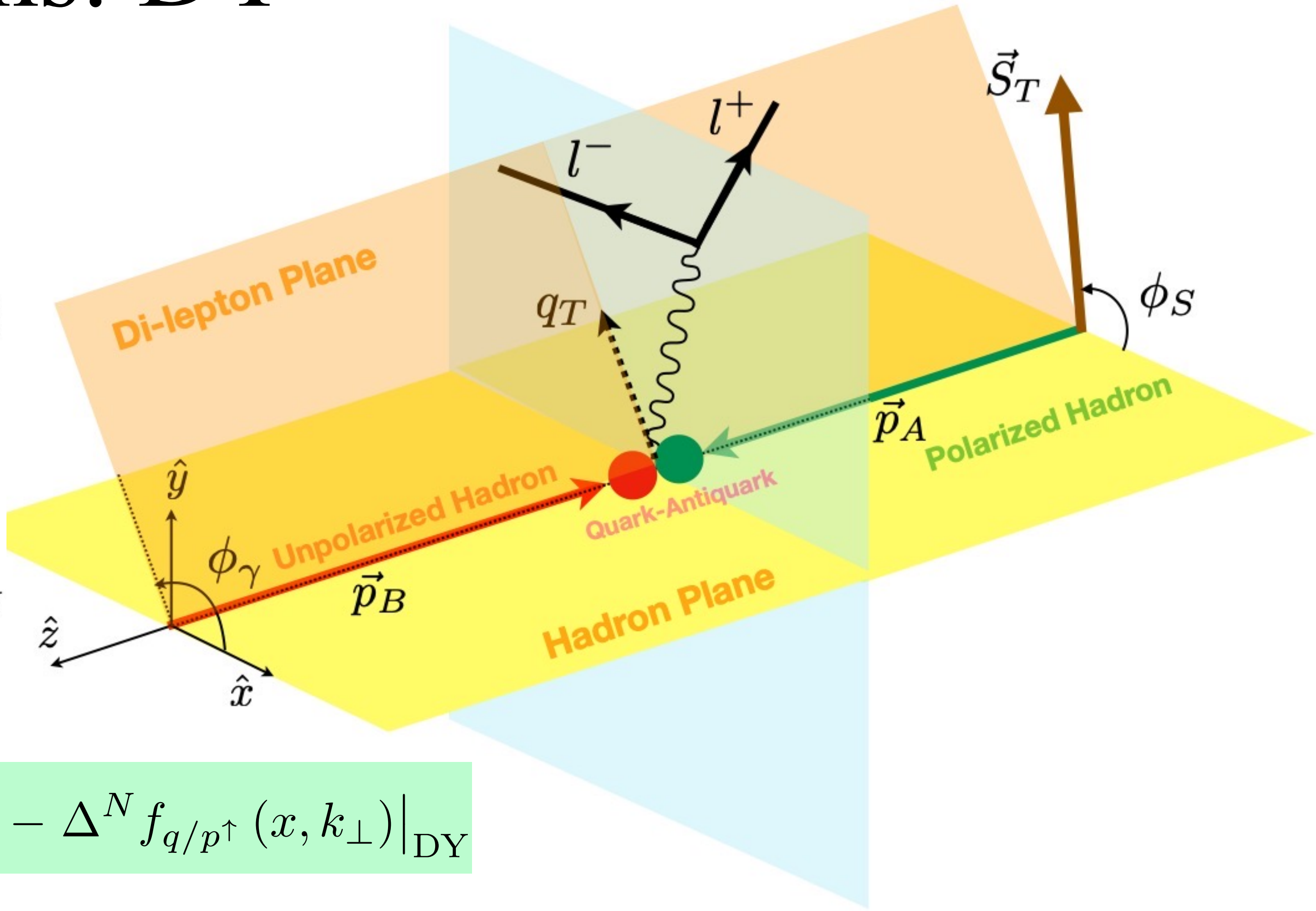


A. Bacchetta et al (2021) $\rho^{u}_{p\uparrow}$



DNN Model Projections: DY





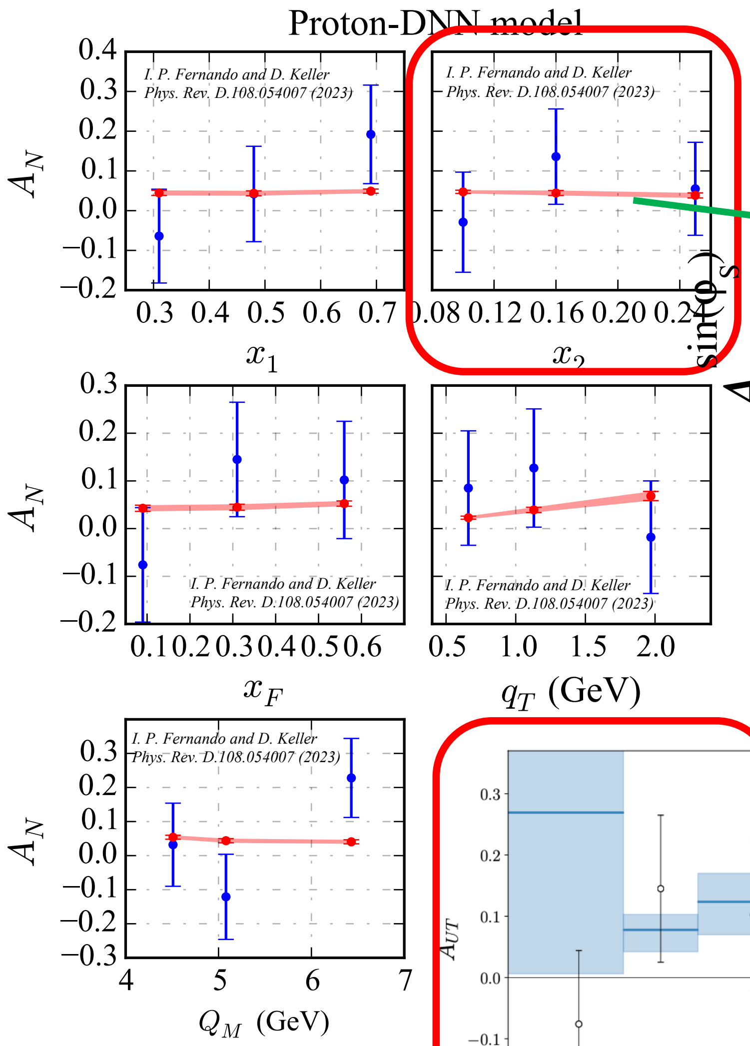
$$\Delta^N f_{q/p^\uparrow}(x, k_\perp)\big|_{\mathrm{SIDIS}} = -\,\Delta^N f_{q/p^\uparrow}(x, k_\perp)\big|_{\mathrm{DY}}$$

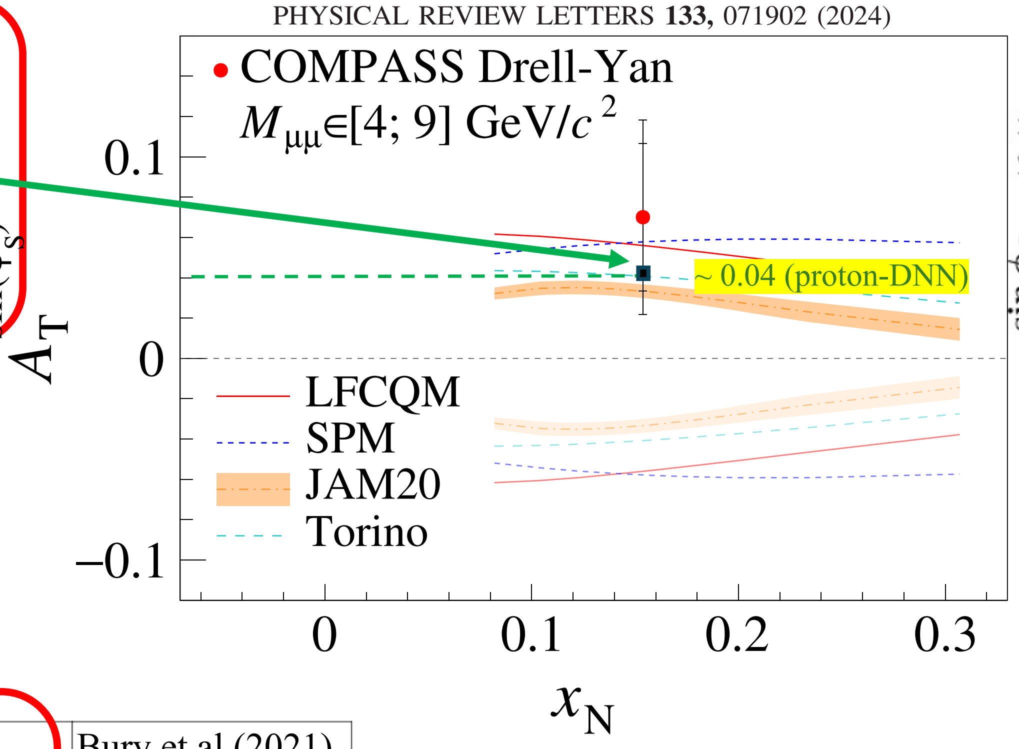
Based on Anselmino et al. (2017)

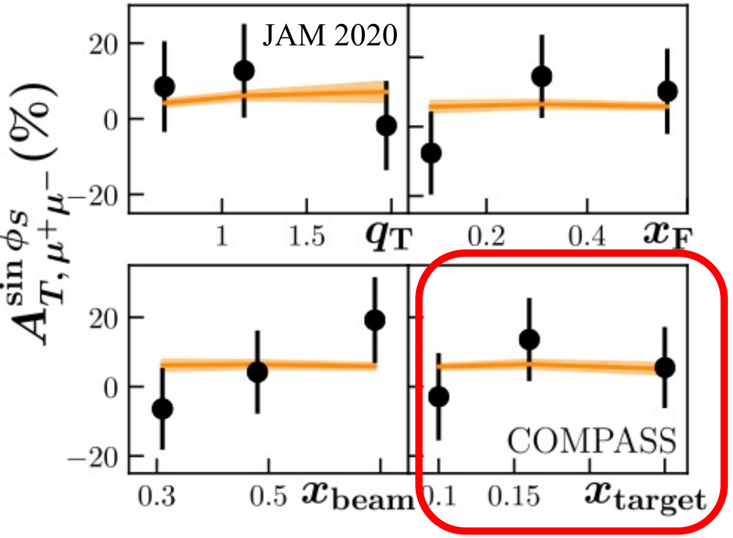
$$A_N^{\sin(\phi_\gamma - \phi_S)}(x_F, M, q_T) \quad = \mathcal{B}_0(q_T, m_1) \frac{\sum_q \frac{e_q^2}{x_1+x_2} \mathcal{N}_q(x_1) f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}{\sum_q \frac{e_q^2}{x_1+x_2} f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}$$

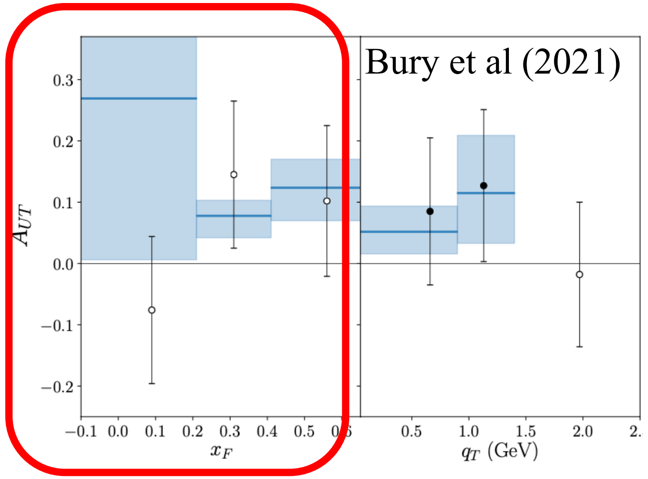
DNN Model Projections: DY

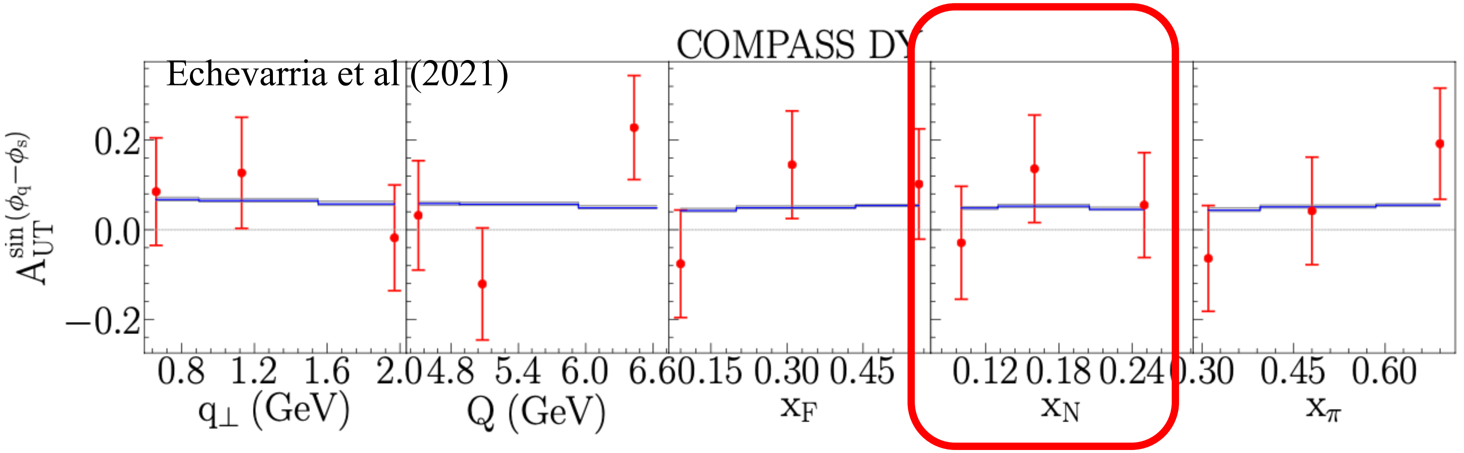
In Comparison with COMPASS 2024 Final



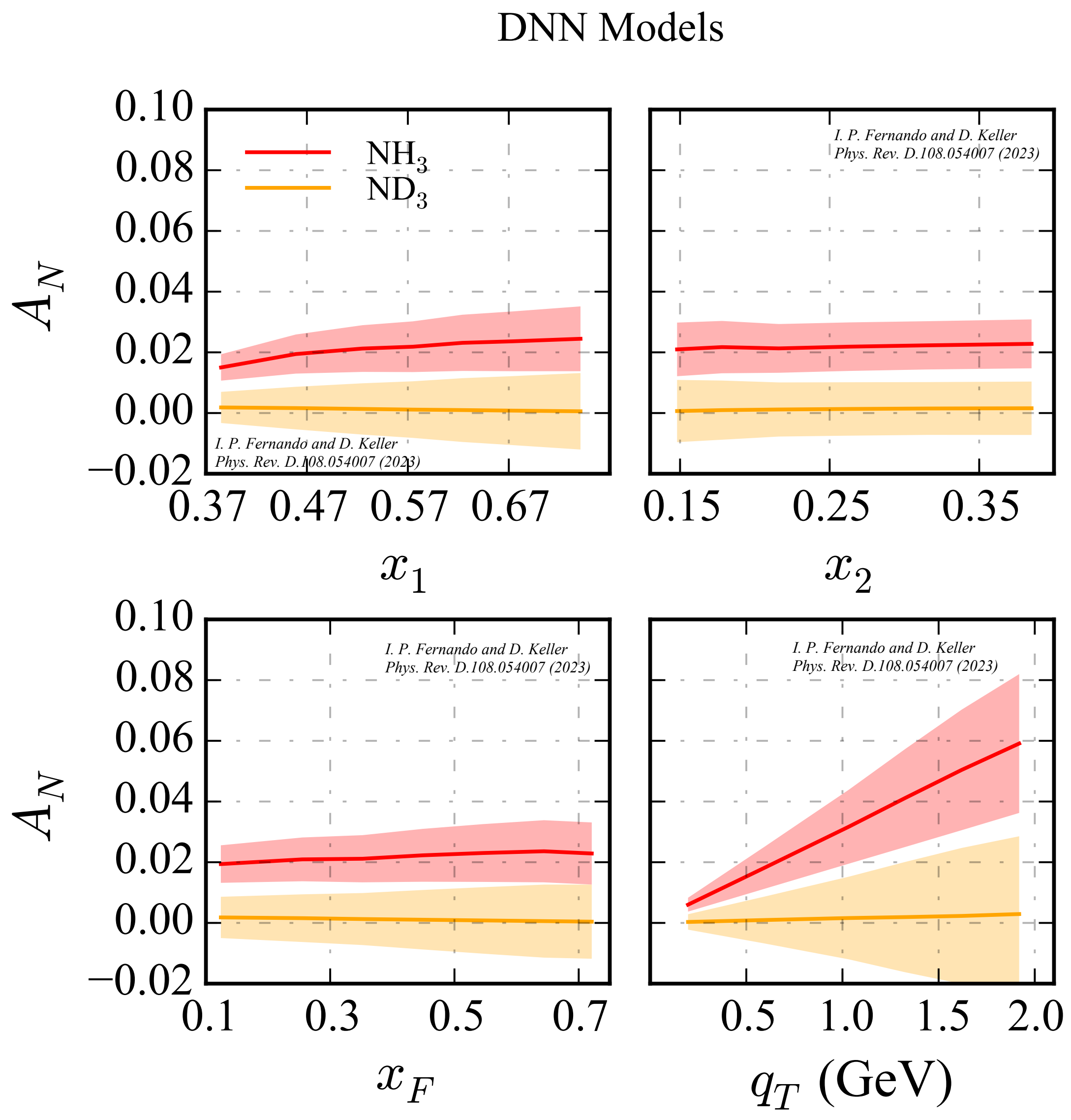








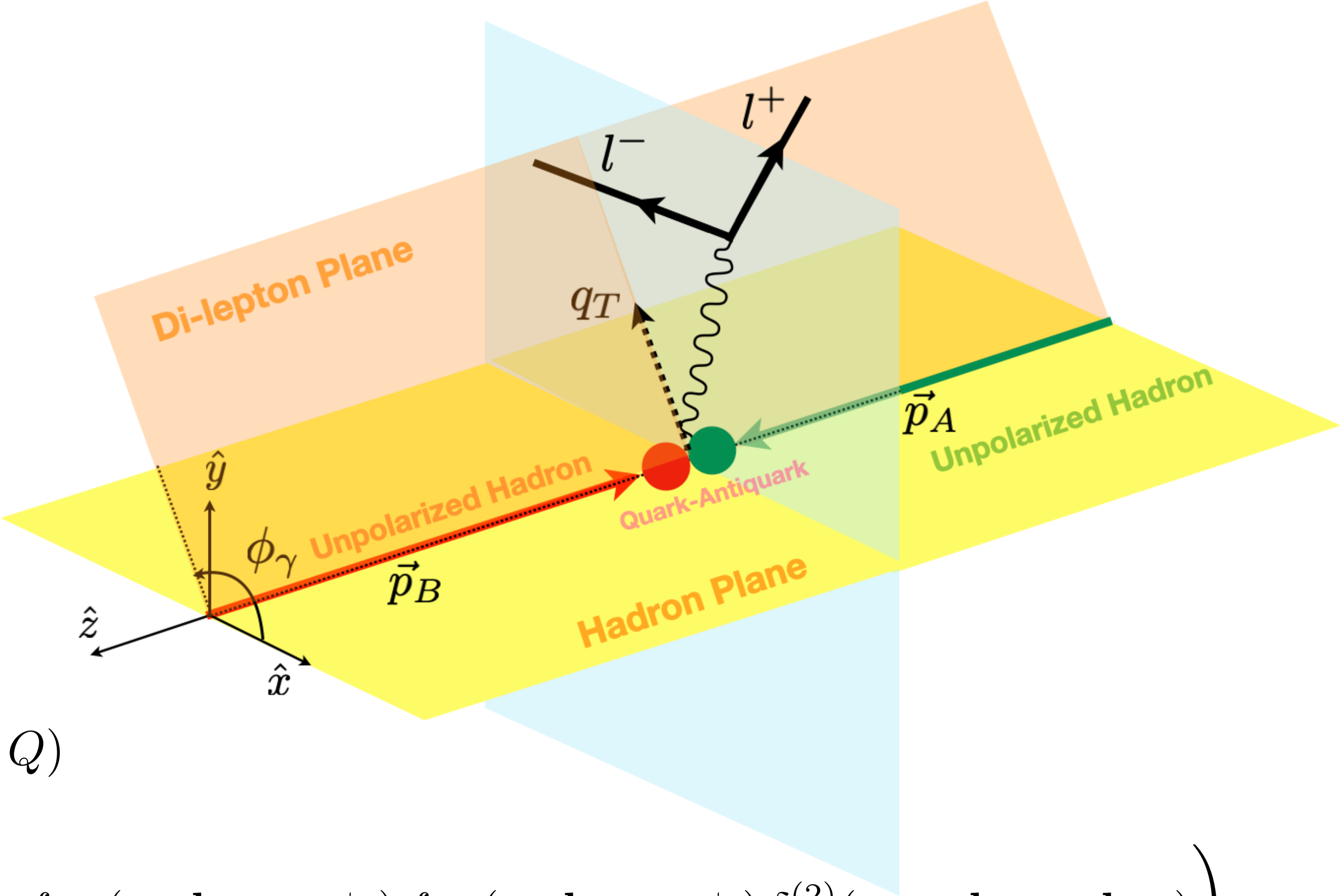
DNN Model Projections: DY @ SpinQuest



- SpinQuest (E1039) experiment at Fermilab is aiming to extract the Sivers function for the light-sea quarks.
- Unpolarized 120 GeV proton beam with polarized proton and deuteron targets (separately).
- Proton-DNN model predictions (Red)
  Deuteron-DNN model predictions (Orange)

Unpolarized TMD

https://arxiv.org/html/2510.17243v1



Let's consider the DY Process

$$\frac{d\sigma}{dq_T\, dQ_M\, dy} = \frac{16\pi^2\alpha^2}{9\, Q_M^3}\, q_T\, F_{UU}^1(x_a, x_b, |q_T|, Q)$$

$$F_{UU}^1 = x_a x_b \sum_q e_q^2\, \mathcal{H}^{\mathrm{DY}}(Q,\mu) \left(\int_0^\Lambda d^2\mathbf{k}_{aT}\, d^2\mathbf{k}_{bT}\, f_{q/a}(x_a, \mathbf{k}_{aT}; \mu, \zeta_a)\, f_{\bar{q}/b}(x_b, \mathbf{k}_{bT}; \mu, \zeta_b)\, \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_{aT} - \mathbf{k}_{bT}) \right)$$

$$+(x_a \leftrightarrow x_b)$$

$$x_a = \frac{Q_M}{\sqrt{s}}\, e^{+y},\;\; x_b = \frac{Q_M}{\sqrt{s}}\, e^{-y},\;\; x_a x_b = \frac{Q_M^2}{s}$$

The progress so far…

$$f(x,k_\perp;Q^2) = f(x;Q^2)\,\frac{1}{\pi\langle k_\perp^2\rangle}\exp\left(-\frac{k_\perp^2}{\langle k_\perp^2\rangle}\right)$$

Anselmino et al

Apologies for picking only few examples here…

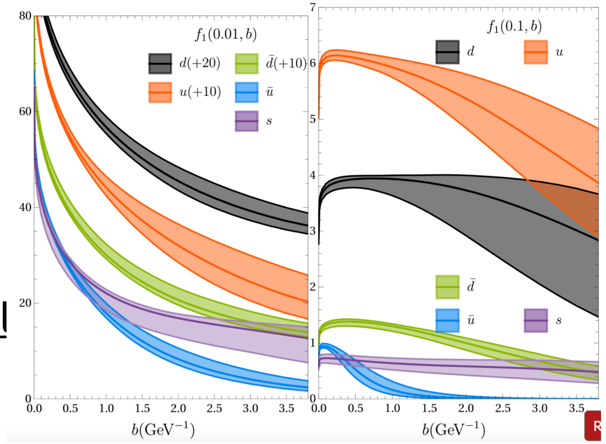
<u>bT-space formalism</u>

$$\hat{f}_1^a(x,|\boldsymbol{b}_T|;\mu,\zeta) = \int d^2\boldsymbol{k}_\perp\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

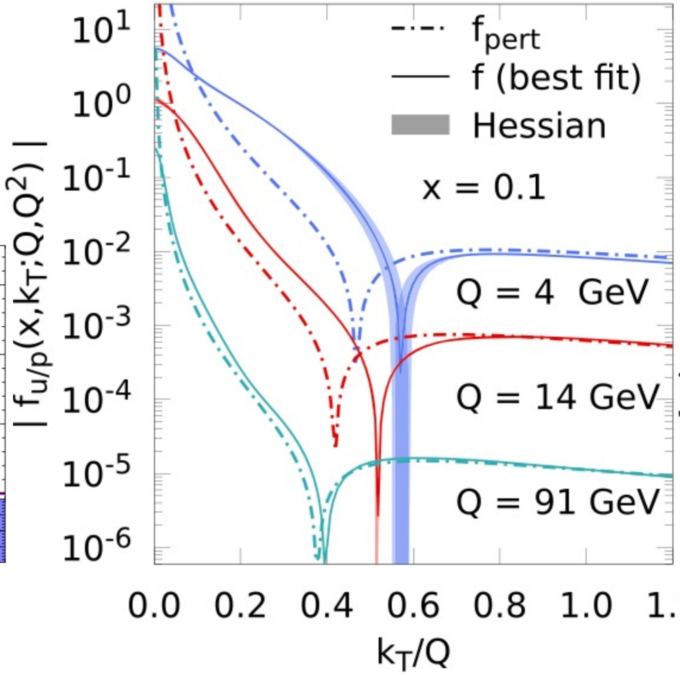
$$= 2\pi\int_0^\infty d|\boldsymbol{k}_\perp|\,|\boldsymbol{k}_\perp| J_0(|\boldsymbol{b}_T||\boldsymbol{k}_\perp|)\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu,\zeta)$$

<u>with TMD-evolution via CSS kernel</u>

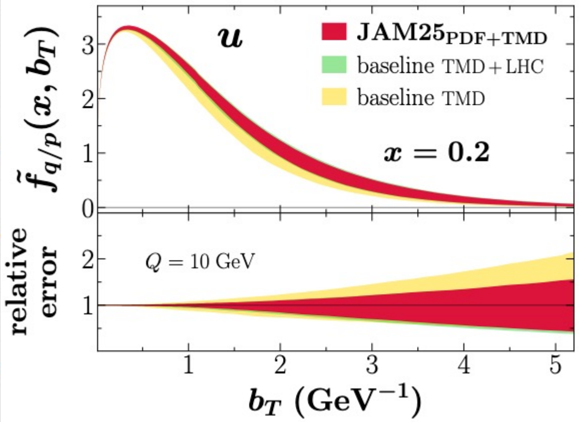
Moose et al (2025)



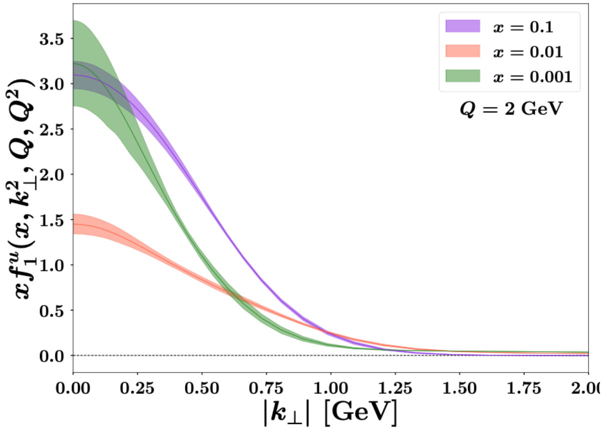
Aslan et al (2024)



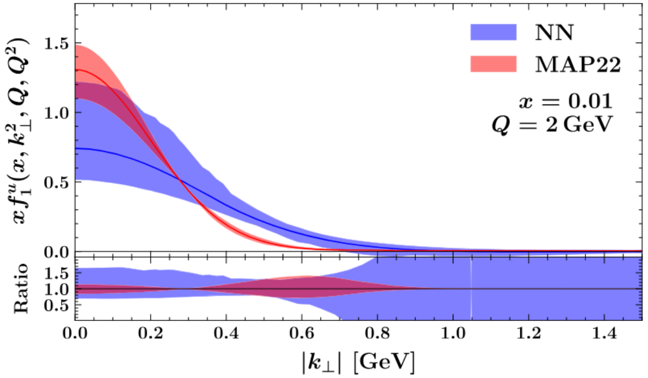
JAM (2025)



MAP (2022)



MAP (2025)



Phenomenological approach with DNNs

$$f_{q/N}(x, k_\perp; Q, Q^2) = f_q(x; Q^2)\, s(x, k_\perp; Q)$$

Collinear Effects
with DGLAP

Transverse Effects
with transverse-evolution

Stage 1: Fitting with Cross-Section data

DNN

$$\frac{d\sigma}{dq_T dQ_M dy} = \frac{16\pi^2\alpha^2}{9Q_M^3} q_T \boxed{\mathcal{S}(q_T, x_a, x_b; Q_M)} \sum_q e_q^2 x_a f_q(x_a, Q_M) x_b f_{\bar{q}}(x_b, Q_M) + (x_a \leftrightarrow x_b)$$

$$f_q(x; Q_M) = \int d^2\mathbf{k}_\perp f_q(x, k_\perp; Q_M)$$

Stage 2: Fits for inverse-integration (momentum-space auto-convolution with DNNs)

DNN

$$S(q_T; x_a, x_b, Q) = \int_0^{2\pi} d\phi \int_0^{k_{max}} dk\, k\, \boxed{s(x_a, k, Q)}\, s\left(x_b, \sqrt{q_T^2 + k^2 - 2q_T k\cos\phi},\, Q\right)$$

https://arxiv.org/html/2510.17243v1

FiLM (Feature-wise Linear Modulation): modulations sensitive to x and Q

Closure-test with cross-sections (pseudo-data)

A simplified version for testing purposes

$$F_{UU}^{1} = \sum_{q} e_q^2 \, \mathcal{H}^{\mathrm{DY}}(Q,\mu) \times \left[x_a f_{q/a}(x_a;Q)\, x_b f_{\bar{q}/b}(x_b;Q) \right] S(q_T)\, \mathcal{B}^2(Q) + (x_a \leftrightarrow x_b)$$

$$\mathcal{B}^2(Q_M) = mQ_M$$

$$\mathcal{S}(q_T) = \frac{1}{2\pi m^2} e^{-\frac{q_T^2}{2m^2}}$$

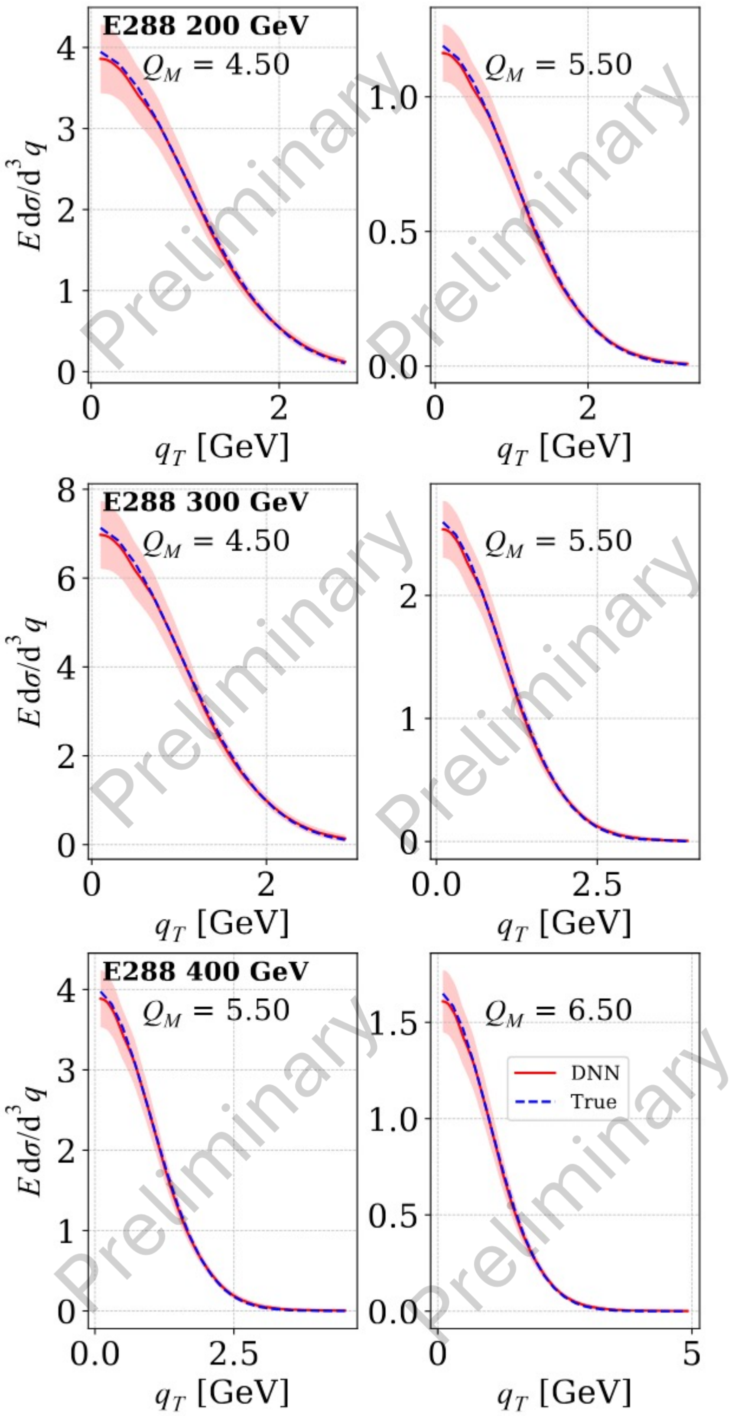
$$\mathcal{B}^2(Q_M) = aQ_M$$

$$s(k_\perp) = \frac{e^{k_\perp^2/m^2}}{\pi m^2}$$

$$S(q_T) = \int_0^{k_{\max}} \int_0^{2\pi} s(k) \cdot s(k') \cdot k \, dk \, d\phi$$

$$k' = \sqrt{q_T^2 + k^2 - 2q_T k \cos(\phi)}$$

https://arxiv.org/html/2510.17243v1

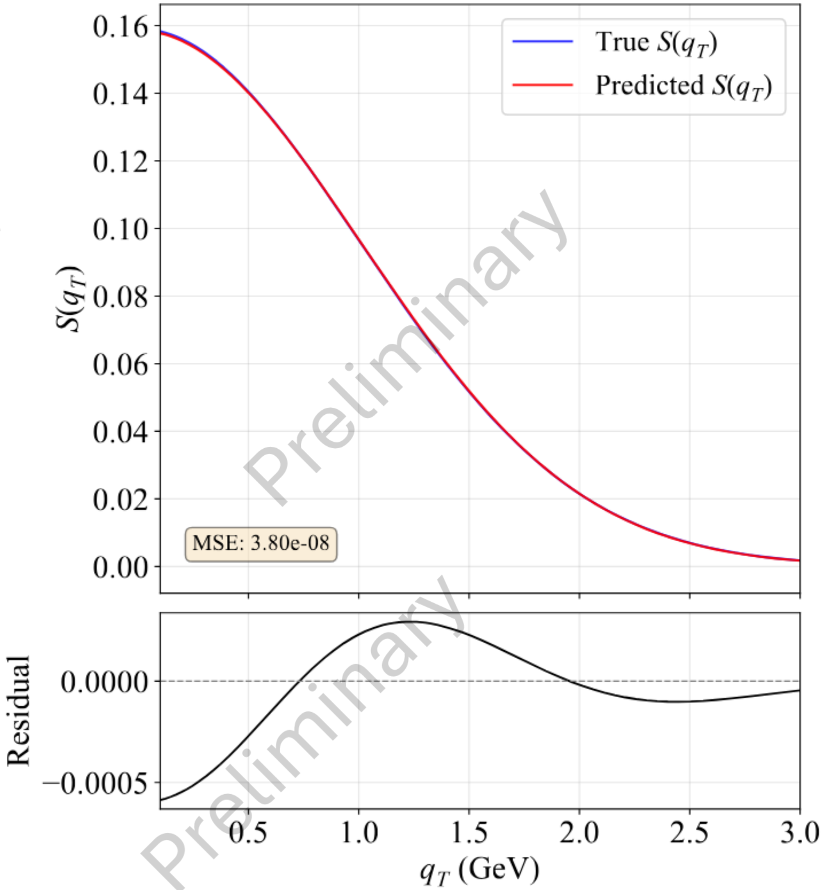


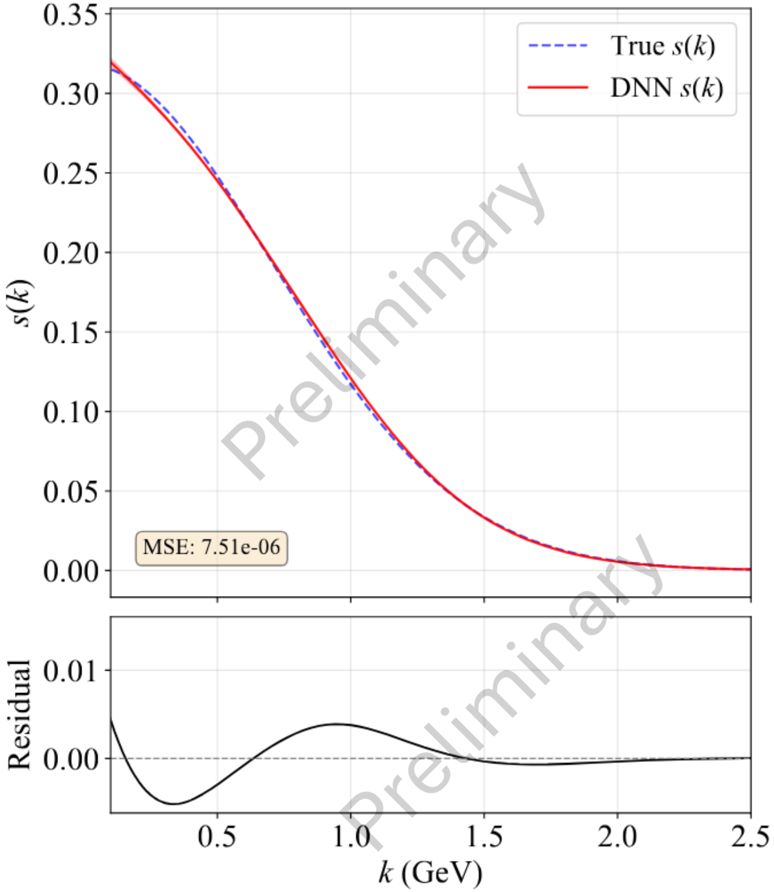
Closure Test for inverse integration

$$S(q_T) = \int_0^{k_{\max}} \int_0^{2\pi} s(k) \cdot s(k') \cdot k \, dk \, d\phi \qquad k' = \sqrt{q_T^2 + k^2 - 2 q_T k \cos(\phi)}$$

$$S(q_T) = \frac{1}{2\pi m^2} e^{-\frac{q_T^2}{2m^2}}$$

True Integral



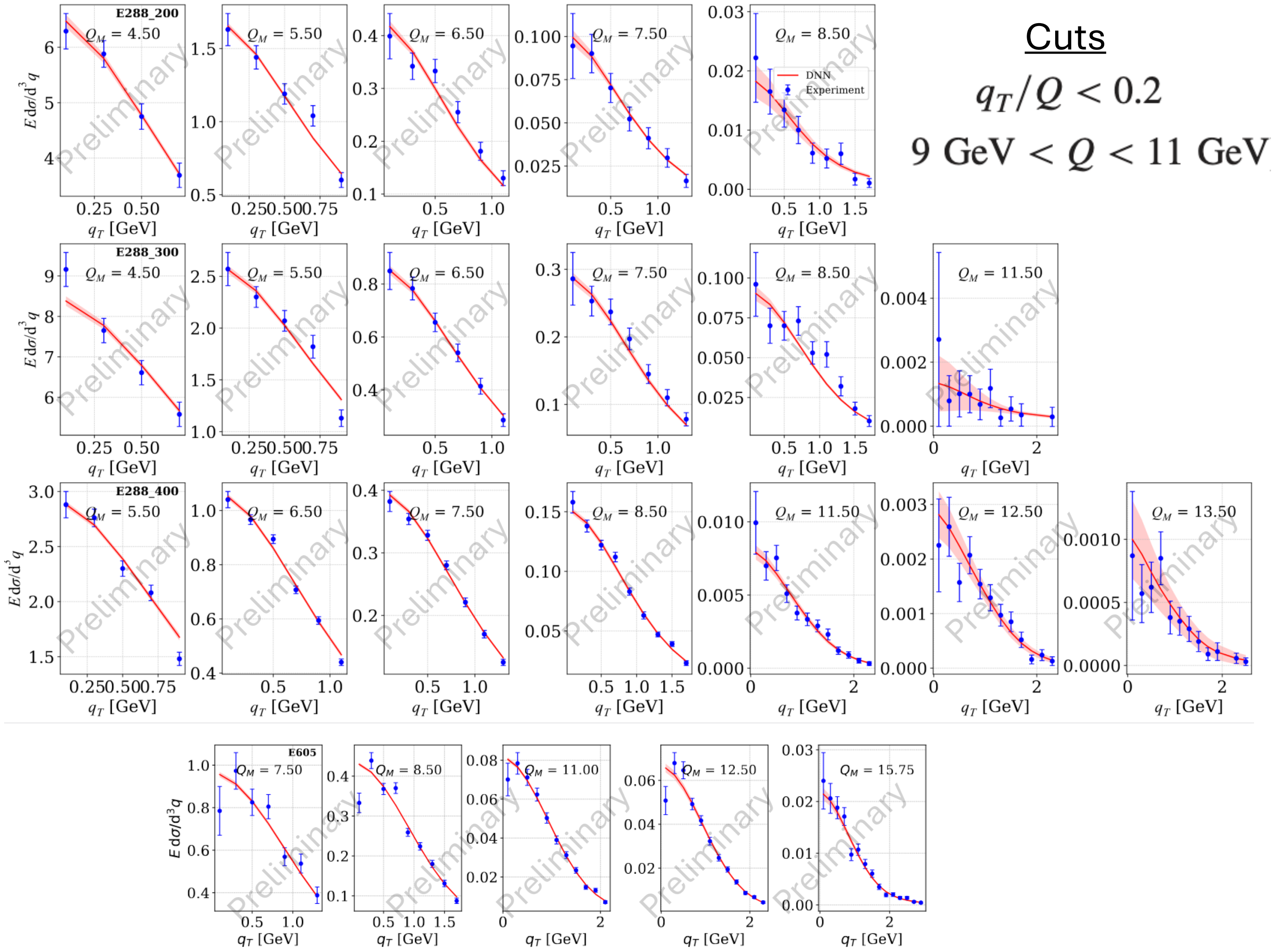


$$s(k_\perp) = \frac{e^{k_\perp^2/m^2}}{\pi m^2}$$

True Integrand

https://arxiv.org/html/2510.17243v1

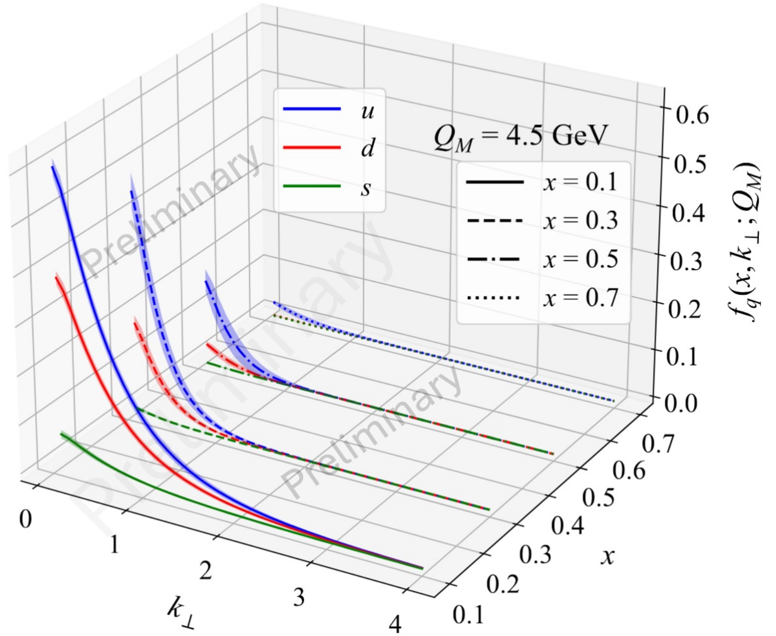
Preliminary Results

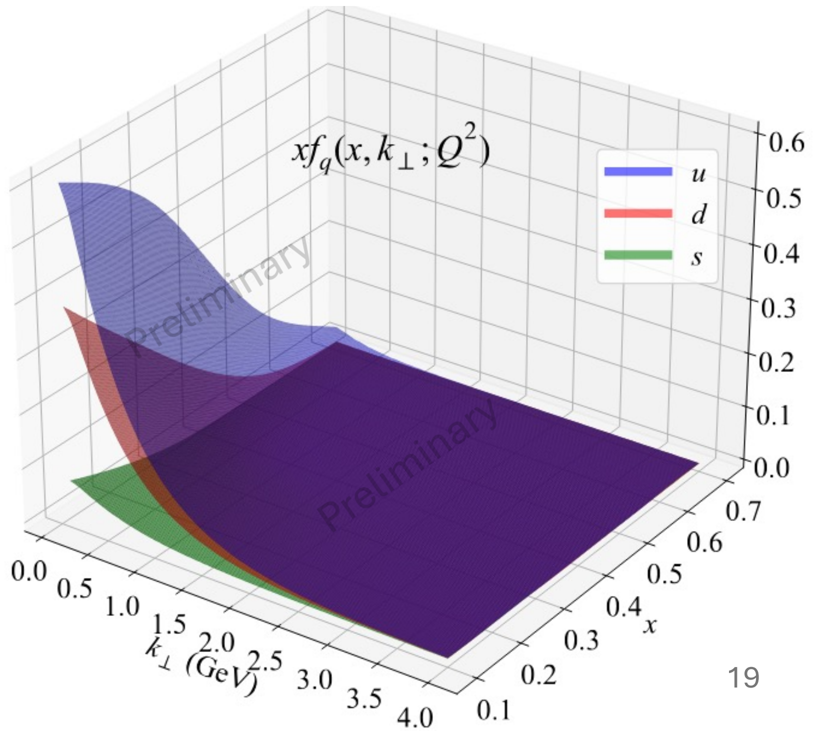


Cuts

$$q_T/Q < 0.2$$

$$9\ \text{GeV} < Q < 11\ \text{GeV}$$



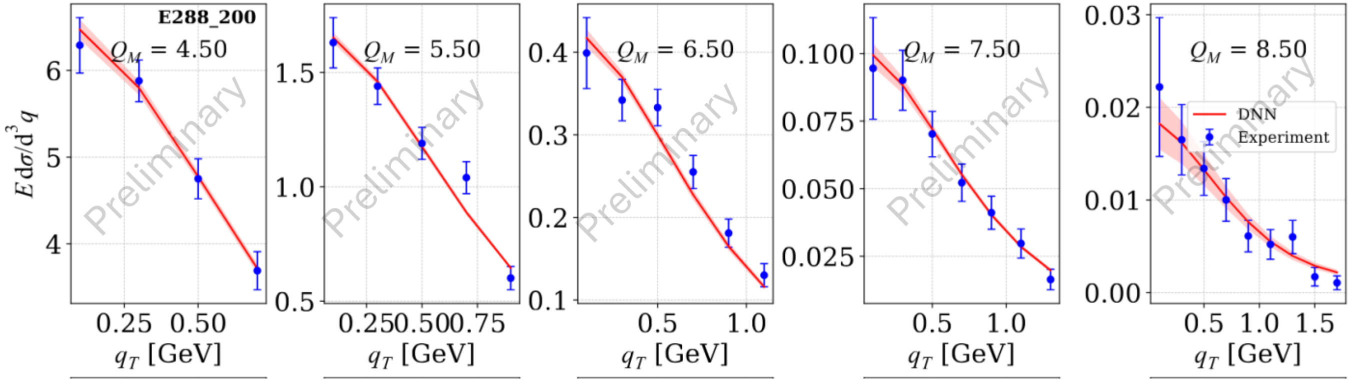


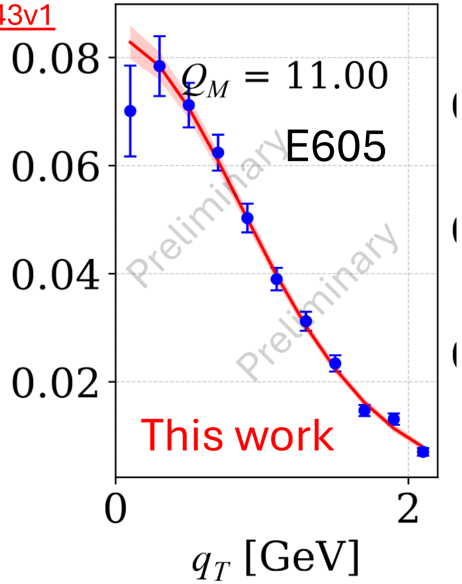
https://arxiv.org/html/2510.17243v1

Preliminary Results (a comparison…)

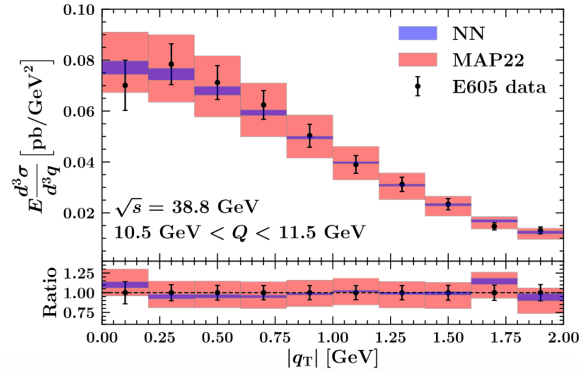
This work

https://arxiv.org/html/2510.17243v1

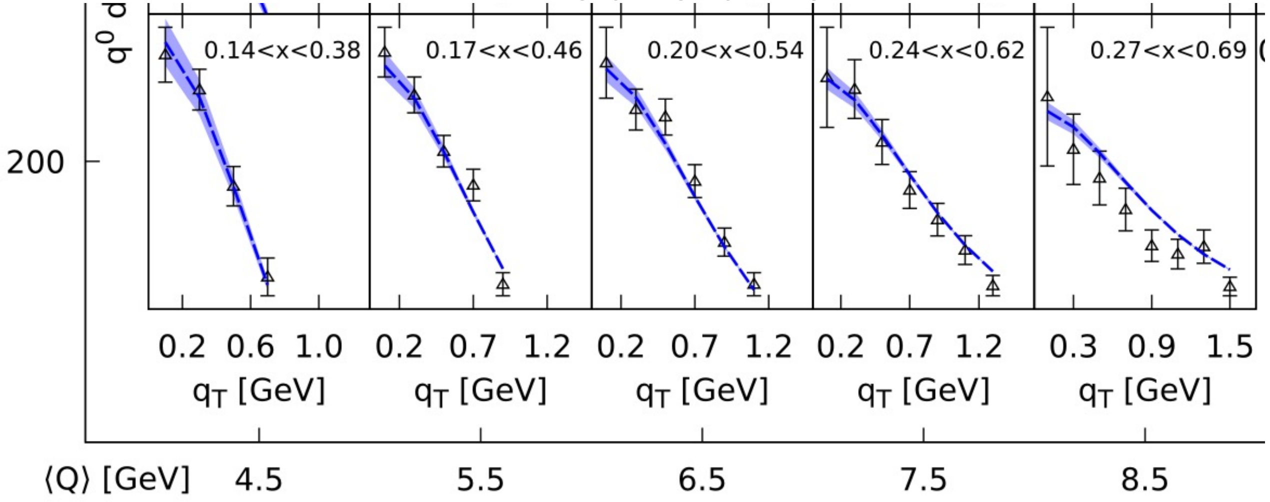


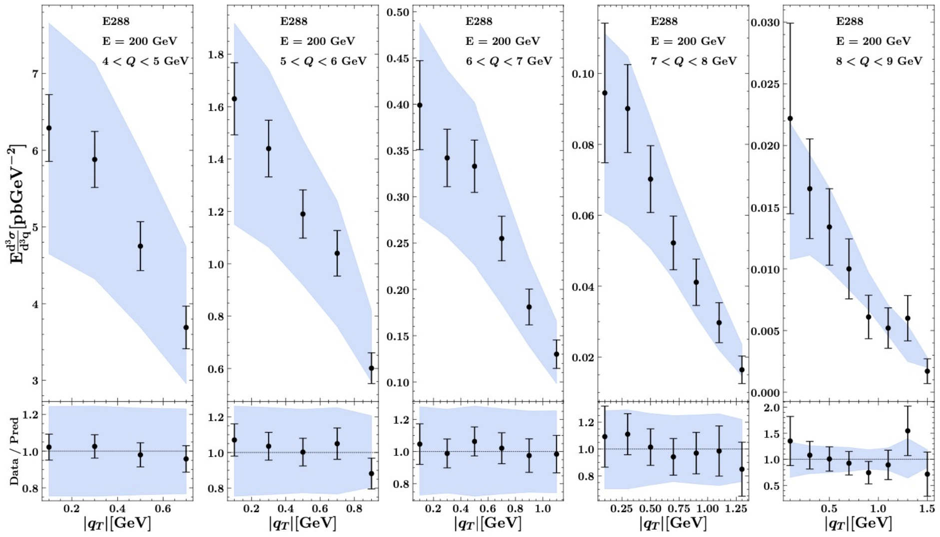


MAP 2025



Aslan et al 2024





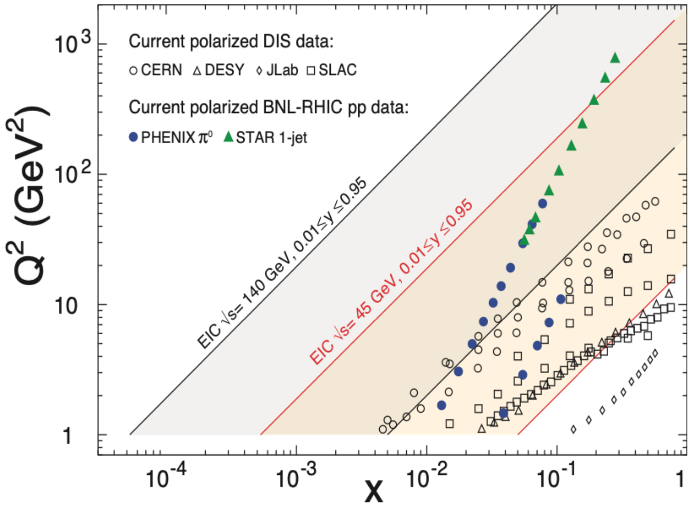
MAP 2022

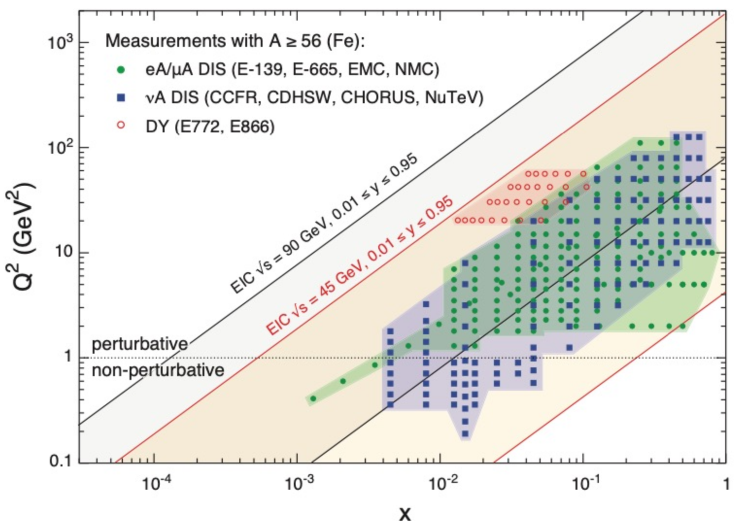
Summary & Outlook

- We proposed DNN method for performing global fits to extract TMDs for the first-time: used Sivers function as an example: extracted the Sivers functions for all light quark flavors in SU(3).
- We have successfully tested our method with pseudo-data, also demonstrated reproducibility with a systematic study.
- We projected SIDIS and DY Sivers asymmetries: for existing (as a validation check) and upcoming experiments.
- Performed fits to DY data with DNN techniques to extract unpolarized TMDs

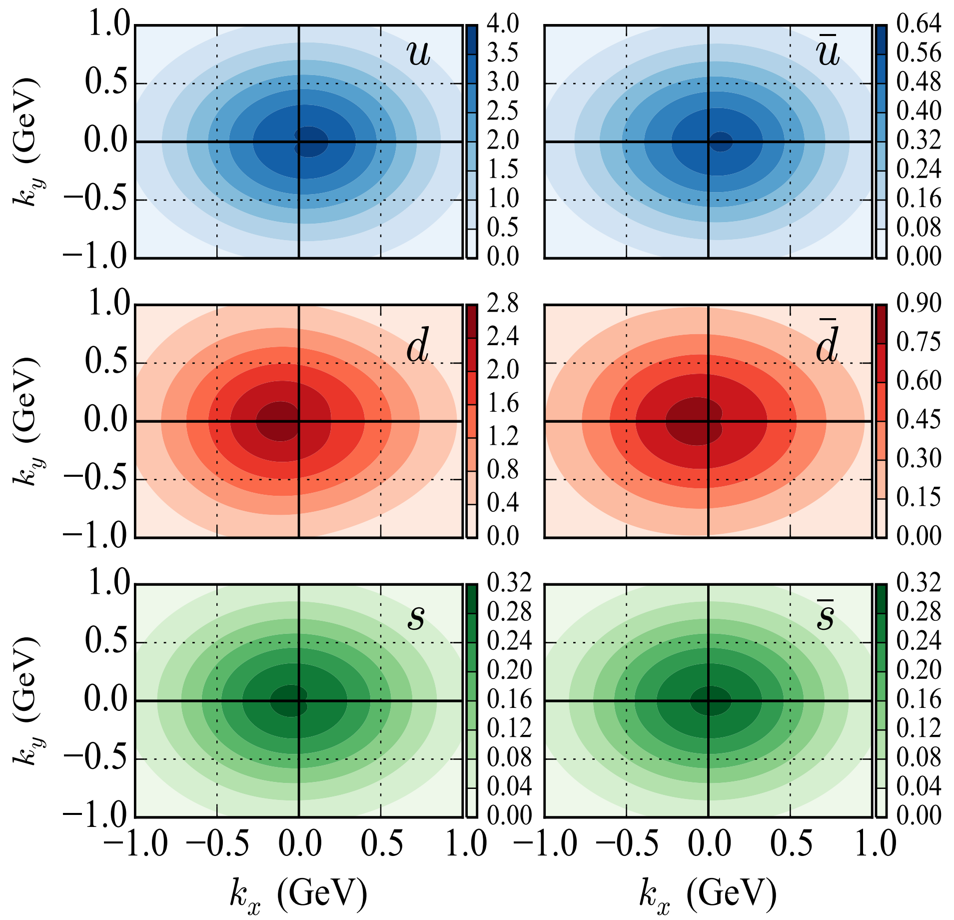
Next / On-going:

- Nuclear TMDs with DNNs…
- Applying the “DNN method” to extract other TMDs such as Transversity, Boer-Mulders function, Spin-1 TMDs…
- Making predictions for EIC kinematics…





I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)



Thank you

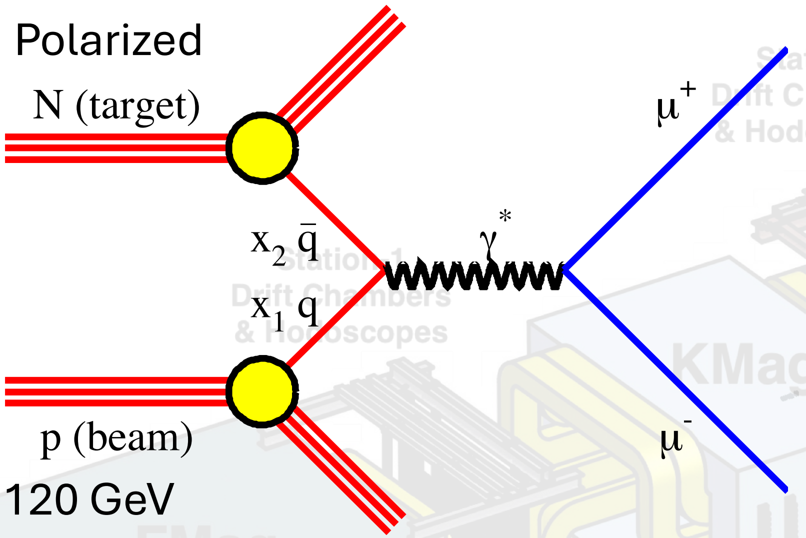
This work is supported by DOE contract DE-FG02-96ER40950

SpinQuest (E1039) Experiment at Fermilab

- Measurement of ‘sea’ quark Sivers function

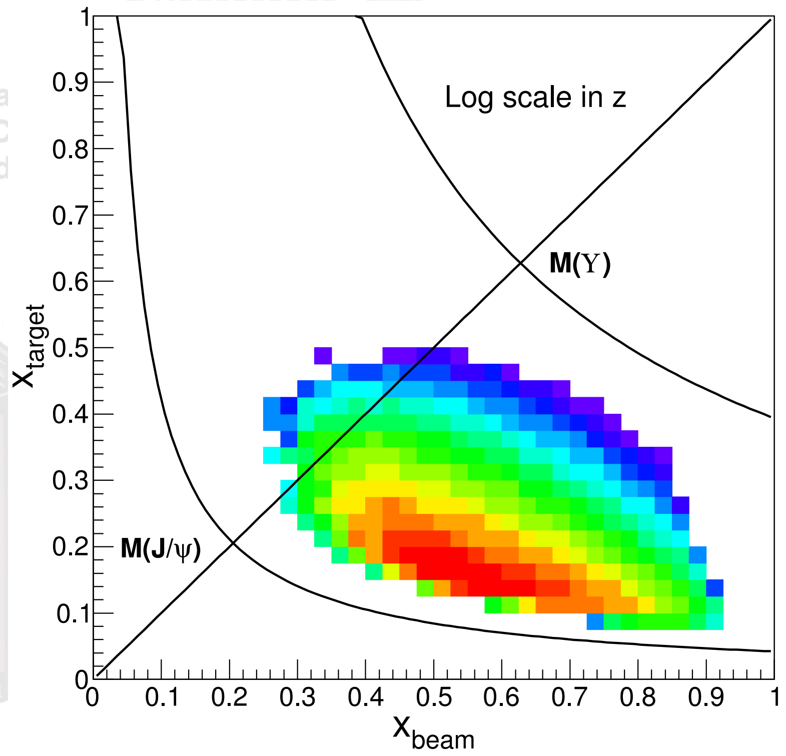
$$pp^{\uparrow}(d^{\uparrow}) \rightarrow \mu^+\mu^- X,\ 4 < M_{\mu\mu} < 9\ GeV$$

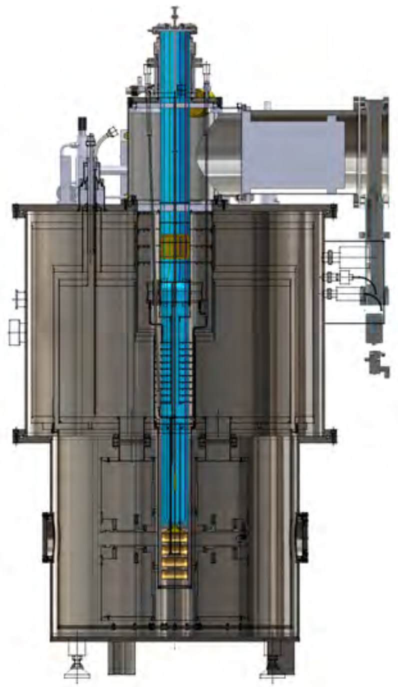
Polarized



120 GeV proton beam

$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_i e_i^2 (q_i^B(x_1, Q^2)\bar{q}_i^T(x_2, Q^2) + \bar{q}_i^B(x_1, Q^2) q_i^T(x_2, Q^2))$$





LANL-UVA Polarized Target

https://spinquest.fnal.gov/
http://twist.phys.virginia.edu/E1039/

Please Join The Effort

Dustin Keller (dustin@virginia.edu)[Spokesperson]
Kun Liu (liuk@lanl.gov)[Spokesperson])

Highest beam intensity on a polarized target ever!

Backup Slides

Mitigating the over-fitting

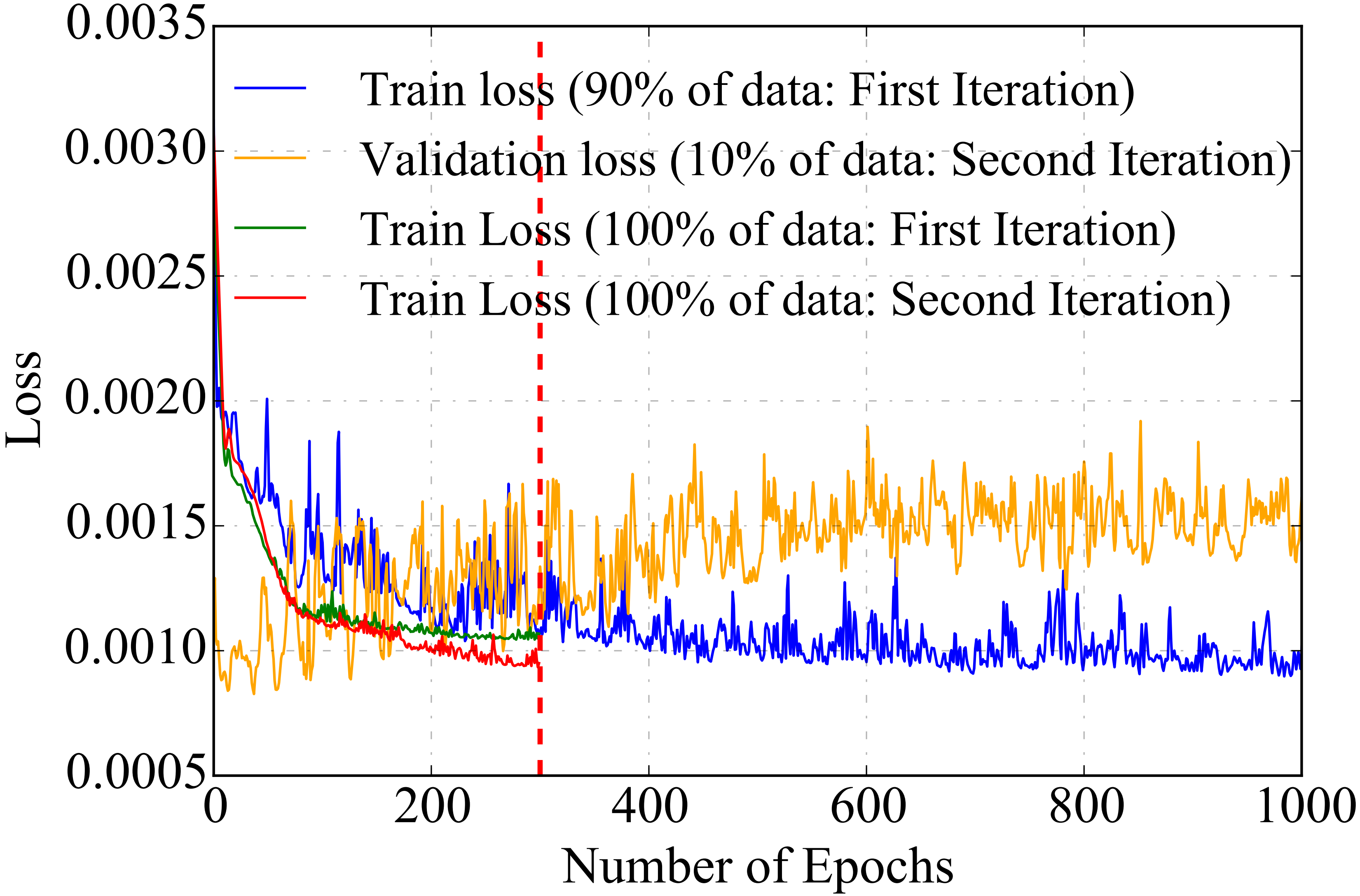


TABLE III. The summary of the optimized sets of hyperparameters: The indications in the table are $\mathcal{C}_0^i$ and $\mathcal{C}_0^f$ for results from the pseudodata from the generating function, $\mathcal{C}_p^i$, and $\mathcal{C}_p^f$ for results from SIDIS data from experiments associated with the polarized-proton target, and $\mathcal{C}_d^i$ and $\mathcal{C}_d^f$ for results from SIDIS data from experiments associated with the polarized-deuterium target, where i and f indicate the *First Iteration* and *Second Iteration* respectively. The initial learning rate is also listed ($\times 10^{-4}$) as is the final training loss ($\times 10^{-3}$). The accuracy and precision in each case are the maxima over the phase space.

Hyperparameter	$\mathcal{C}_0^i$	$\mathcal{C}_0^f$	$\mathcal{C}_p^i$	$\mathcal{C}_p^f$	$\mathcal{C}_d^i$	$\mathcal{C}_d^f$
Hidden layers	5	7	5	7	5	8
Nodes/layer	256	256	550	550	256	256
Learning rate	1	0.125	5	1	10	1
Batch size	200	256	300	300	100	20
Number of epochs	1000	1000	300	300	200	200
Training loss	0.6	0.05	1.5	1	2	1
$\varepsilon_u^{\max}$	95.67	99.27	55.21	94.04	56.80	93.02
$\varepsilon_{\bar{u}}^{\max}$	42.62	98.09	52.57	96.70	34.83	91.40
$\varepsilon_d^{\max}$	80.46	98.89	55.69	93.13	52.44	89.27
$\varepsilon_{\bar{d}}^{\max}$	74.59	97.08	55.37	95.04	46.60	92.58
$\varepsilon_s^{\max}$	45.53	79.27	49.54	90.64	36.34	93.41
$\varepsilon_{\bar{s}}^{\max}$	59.27	91.13	33.89	82.51	65.57	91.45
$\sigma_u^{\max}$	3	0.1	5	2	2	0.4
$\sigma_{\bar{u}}^{\max}$	2	0.2	6	2	8	2
$\sigma_d^{\max}$	10	1	20	6	2	1
$\sigma_{\bar{d}}^{\max}$	7	4	20	8	7	1
$\sigma_s^{\max}$	2	0.2	4	1	6	2
$\sigma_{\bar{s}}^{\max}$	1	0.1	4	2	6	3

Systematic Studies: data cuts

Backup

So far, the applicability of TMD factorization was ensured by applying cuts to SIDIS data based on various criteria in the literature.

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu} \times \int d^2k_\perp d^2p_\perp \delta^{(2)}(z_h k_\perp + p_\perp - p_{hT}) \times F_{f/N^\uparrow}(x, z_h k_\perp, S; \mu, \zeta_F) D_{h/f}(z_h, p_\perp; \mu, \zeta_D) + Y(p_{hT}, Q^2),$$

Examples:

1. Bury et al JHEP 05 (2021) 151
   $Q > 2$ GeV
   $\delta = p_{hT}/zQ \leq 0.3$
2. Echevarria et al JHEP 01 (2021) 126
   $q_T/Q < 0.75$
3. JAM2020
   $Q^2 > 1.63\ \text{GeV}^2,\ \ 0.2 < z < 0.6,\ \ 0.2 < p_{hT} < 0.9$ GeV

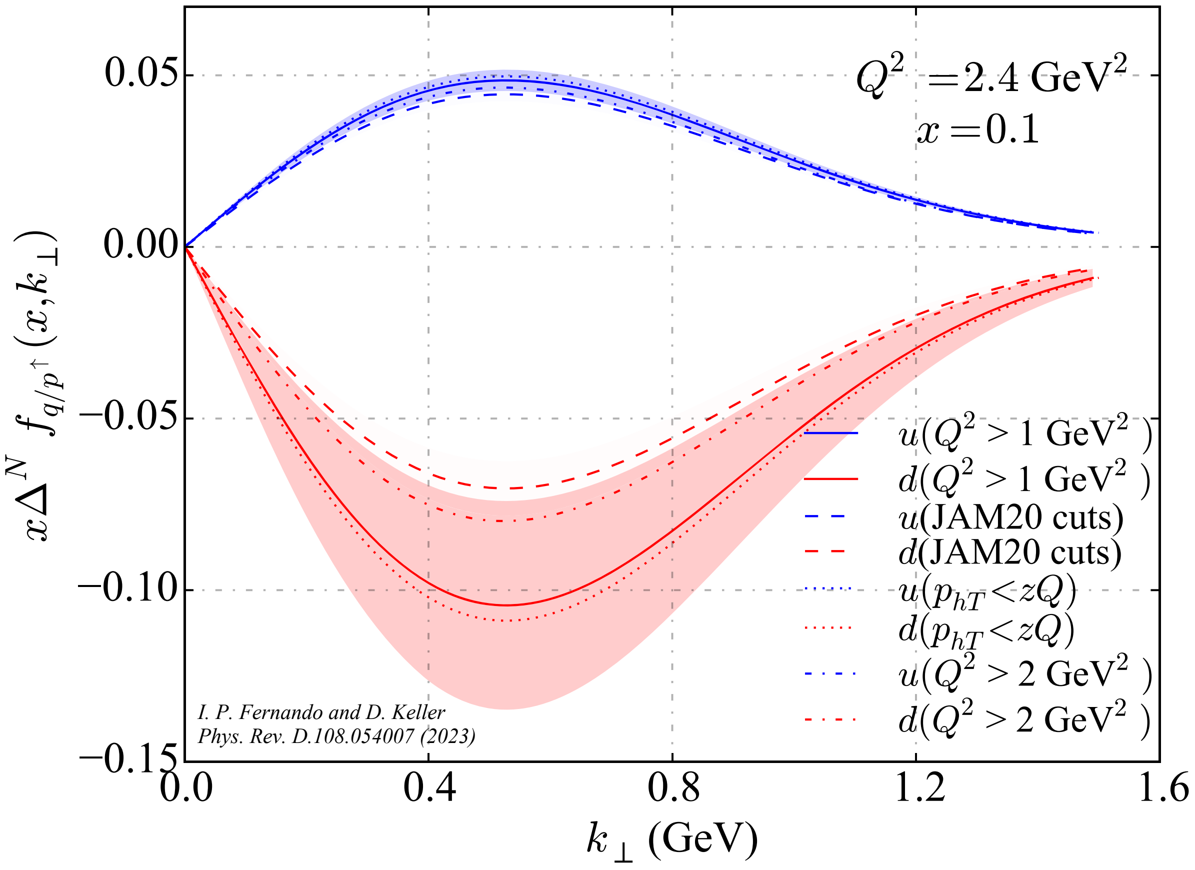


FIG. 17. Solid lines with light band represent the u (in blue), d (in red) Sivers functions using the cut $Q^2 > 1$ GeV^2. These resulting DNN models made from the cuts from all tests are also shown.

Systematic Studies: data cuts

Backup

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu} \times \int d^2k_\perp d^2p_\perp \delta^{(2)}(z_h k_\perp + p_\perp - p_{hT}) \times F_{f/N^\uparrow}(x, z_h k_\perp, S; \mu, \zeta_F) D_{h/f}(z_h, p_\perp; \mu, \zeta_D) + Y(p_{hT}, Q^2),$$

In addition to the basic data cut $Q^2 > 1$ GeV^2 we performed $Q^2 > 2$ GeV^2 and $p_{hT} < zQ$ cuts separately with the proton-DNN model to understand the impact on the extracted Sivers functions.

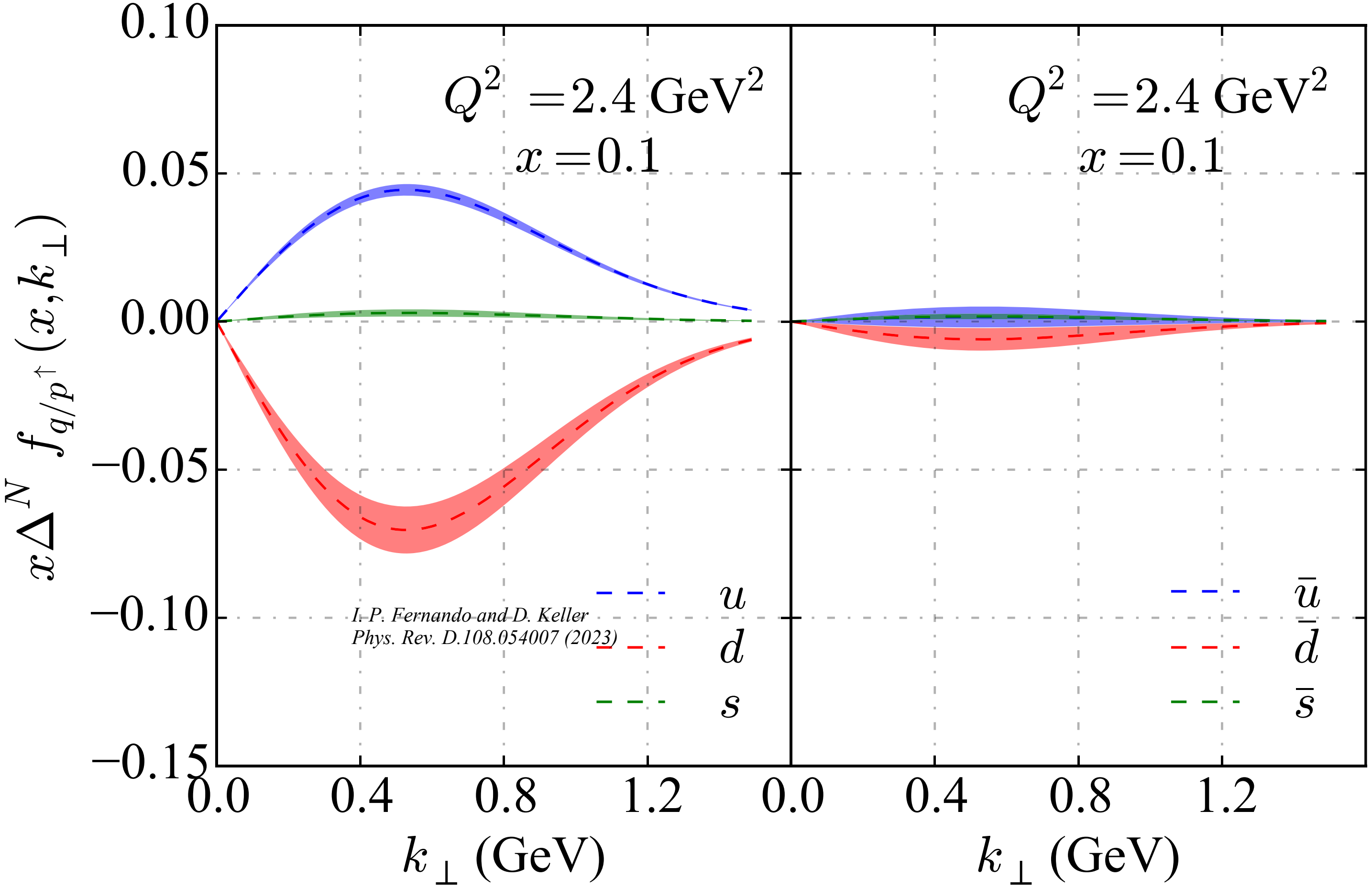


FIG. 18. Sivers functions from a retrained DNN model using the cuts [65] to the data demonstrating that being selective with the data can reduce the error bands of the fit but may also add an unintentional bias.

Systematic Studies: Choice of h(k)

Backup

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)h(k_\perp)f_{q/p}(x,k_\perp)$$

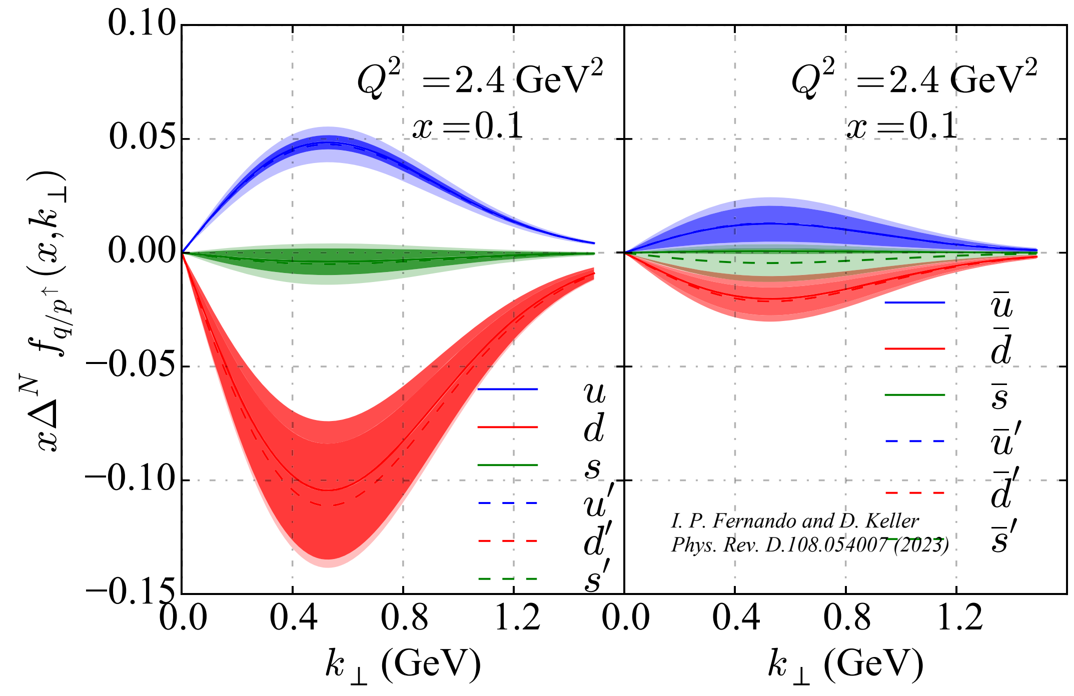


FIG. 19. Using two different $h(k_\perp)$. Solid line with dark band represents the Sivers functions with $h(k_\perp) = \sqrt{2e}\frac{k_\perp}{m_1}e^{-k_\perp^2/m_1^2}$, whereas the dashed line with light band represents the Sivers functions with $h(k_\perp) = \frac{2k_\perp m_1}{m_1^2+k_\perp^2}$.

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{m_1}\,e^{-k_\perp^2/m_1^2}$$

$$h(k_\perp) = \frac{2k_\perp m_1}{m_1^2 + k_\perp^2}$$

- It is clear that the DNN is capable of incorporating both types of h(k) without affecting the Sivers functions in the final model as well as the asymmetries (with deviation less than 1%).
- This is because DNN demonstrates that it maps to the h(k) such that the Sivers function is nearly unchanged.

Systematic Studies : TMD Evolution

Backup

$$\mathcal{N}_q(x) \longrightarrow \mathcal{N}_q(x, Q^2)$$

The solution of the TMD evolution equations

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta)$$

$$\zeta \frac{F(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta),$$

$$F(x,b;\mu,\zeta) = \left(\frac{\zeta}{\zeta_\mu(b)}\right)^{-\mathcal{D}(b,\mu)} F(x,b)$$

$$\mu \sim Q, \qquad \zeta_F \zeta_D \sim Q^4, \qquad \mu^2 = \zeta^2 = Q^2$$

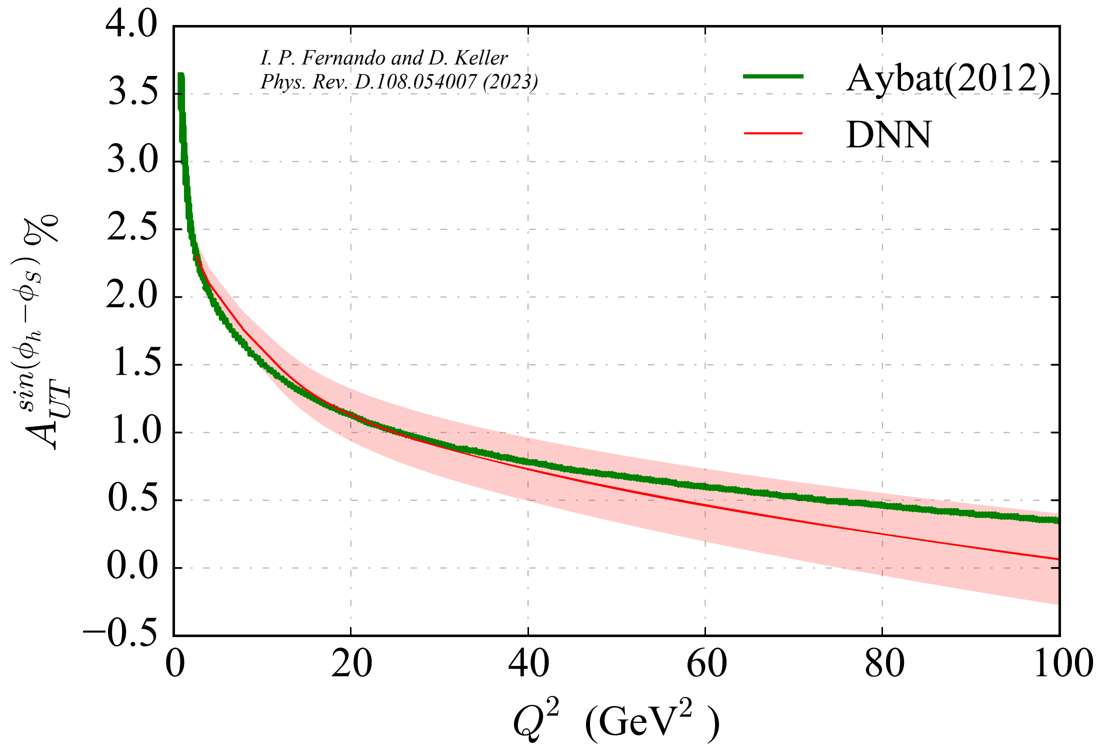


FIG. 21. The Sivers asymmetry evolution in Q^2 compared to the result from [6]. The red-colored solid line and the band represent the mean and standard deviation of the $A_{UT}^{\sin(\phi_h-\phi_S)}$ from 1000 replica models of the proton DNN at $x = 0.12$, $z = 0.32$, $p_{hT} = 0.14$ GeV.

TMD PDFs

Backup

Leading Twist		Quark Polarization		
		Unpolarized [U]	Circular [L]	Linear [T]
Target Polarization	U	f_1 Unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^\perp$ Worm-gear 1
	T	$f_{1T}^\perp$ Sivers	g_{1T} Worm-gear 2	h_1 Transversity $h_{1T}^\perp$ Pretzelosity
	TENSOR	$\theta_{LL}(x, \mathbf{k_T^2})$ $\theta_{TT}(x, \mathbf{k_T^2})$ $\theta_{LT}(x, \mathbf{k_T^2})$	$g_{1TT}(x, \mathbf{k_T^2})$ $g_{1LT}(x, \mathbf{k_T^2})$	$h_{1LL}^\perp(x, \mathbf{k_T^2})$ $h_{1TT}(x, \mathbf{k_T^2})$, $h_{1TT}^\perp(x, \mathbf{k_T^2})$ $h_{1LT}(x, \mathbf{k_T^2})$, $h_{1LT}^\perp(x, \mathbf{k_T^2})$

$$\Phi(x, k_T; S) = \int \frac{d\xi^- d\xi_T}{(2\pi)^3} e^{ik.\xi} \langle P, S|\bar{\psi}(0)\, \mathcal{U}_{[0,\xi]}\, \psi(\xi)|P, S\rangle|_{\xi^+=0}$$

At order, the Quark correlator can be decomposed into 8 components (6 T -even and 2 T -odd terms)

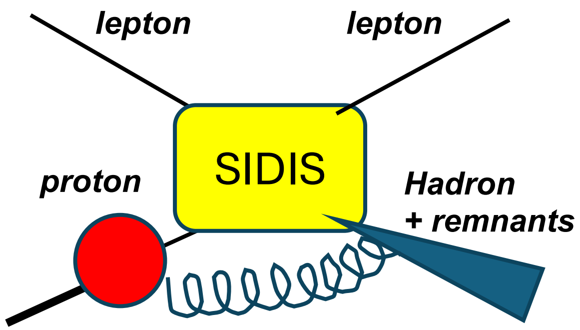
$$\Phi(x, k_T, P, S) = f_1(x, k_T^2)\frac{\not{P}}{2} + \frac{h_{1T}(x, k_T^2)}{4}\gamma_5[\not{S}_T, \not{P}] + \frac{S_L}{2} g_{1L}(x, k_T^2)\gamma_5 \not{P} + \frac{k_T.S_T}{2M} g_{1T}(x, k_T^2)\gamma_5 \not{P}$$

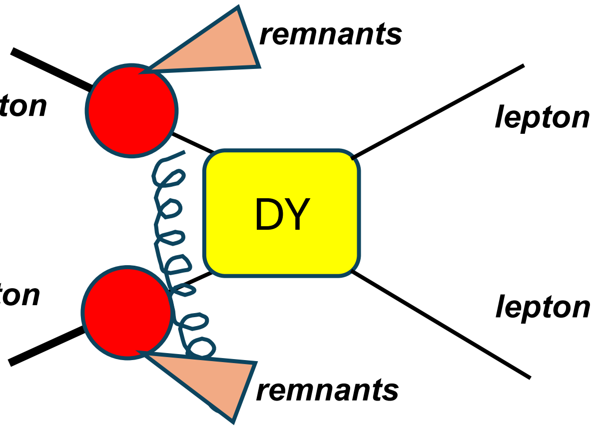
$$+ S_L h_{1L}^\perp(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M} + \frac{k_T.S_T}{2M} h_{1T}^\perp(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M}$$

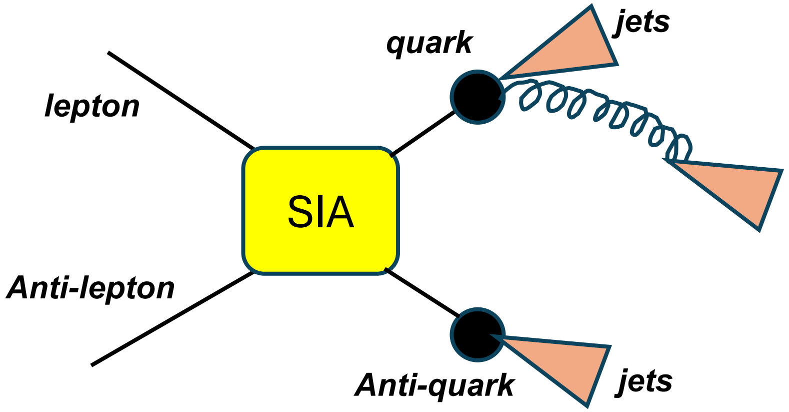
T-even

$$+ i h_1^\perp(x, k_T^2)\frac{[\not{k}_T, \not{P}]}{4M} - \frac{\epsilon_T k_T S_T}{4M} f_{1T}^\perp(x, k_T^2)\not{P}$$

T-odd



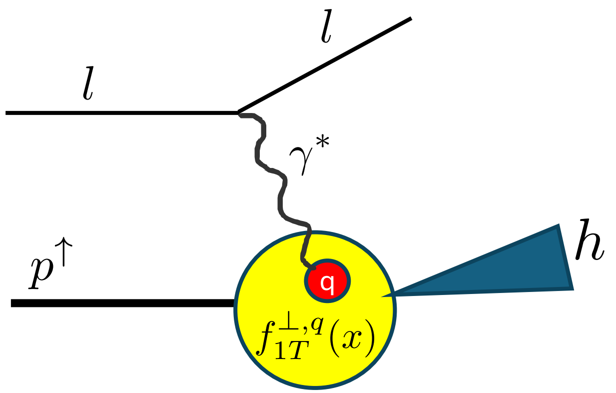




TMD PDFs

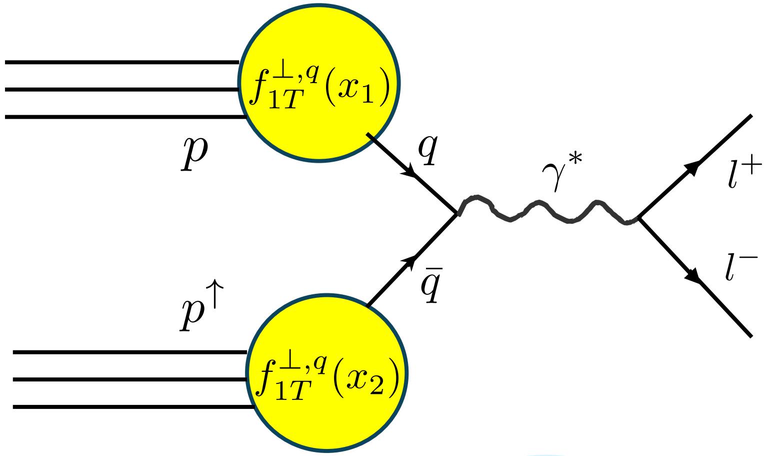
Leading Twist		Quark Polarization: Unpolarized [U]	Circular [L]	Linear [T]
Target Polarization	U	f_1 Unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^\perp$ Worm-gear 1
	T	$f_{1T}^\perp$ Sivers	g_{1T} Worm-gear 2	h_1 Transversity; $h_{1T}^\perp$ Pretzelosity
	TENSOR	$\theta_{LL}(x,\mathbf{k_T^2})$, $\theta_{TT}(x,\mathbf{k_T^2})$, $\theta_{LT}(x,\mathbf{k_T^2})$	$g_{1TT}(x,\mathbf{k_T^2})$, $g_{1LT}(x,\mathbf{k_T^2})$	$h_{1LL}^\perp(x,\mathbf{k_T^2})$, $h_{1TT}(x,\mathbf{k_T^2})$, $h_{1TT}^\perp(x,\mathbf{k_T^2})$, $h_{1LT}(x,\mathbf{k_T^2})$, $h_{1LT}^\perp(x,\mathbf{k_T^2})$

Polarized Semi Inclusive DIS



* For these two processes TMD factorization is proven

Polarized DY



$$\frac{d\sigma^{LO}_{SIDIS}}{dx\,dy\,dz\,dp_T^2\,d\phi_h\,d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{y^2}{2x}\right)\right] \times (F_{UU,T}+\epsilon F_{UU,L})\Big\{1+\cos 2\phi_h\left(\epsilon A_{UU}^{\cos 2\phi_h}\right) + S_T\Big[\sin(\phi_h-\phi_s)\left(A_{UT}^{\sin(\phi_h-\phi_s)}\right)+\sin(\phi_h+\phi_s)\left(\epsilon A_{UT}^{\sin(\phi_h+\phi_s)}\right) + \sin(3\phi_h-\phi_s)\left(\epsilon A_{UT}^{\sin(3\phi_h-\phi_s)}\right)\Big]$$

$A_{UU}^{\cos 2\phi_h} \propto h_1^{\perp q} \circledast H_{1q}^{\perp h}$ BM ⊛ CF

$A_{UT}^{\sin(\phi_h-\phi_s)} \propto f_{1T}^{\perp q} \circledast D_{1q}^{h}$ Sivers ⊛ FF

$A_{UT}^{\sin(\phi_h+\phi_s)} \propto h_1^{q} \circledast H_{1q}^{\perp h}$ Transv ⊛ CF

$A_{UT}^{\sin(3\phi_h-\phi_s)} \propto h_{1T}^{\perp q} \circledast H_{1q}^{\perp h}$ Pretz ⊛ CF

$$\frac{d\sigma^{LO}}{d\Omega} = \frac{\alpha_{em}^2}{Fq}F_v^1\Big\{1+\cos^2\theta + sin^2\theta cos2\phi_{CS}A_U^{cos2\phi_{CS}} + S_T\Big[(1+\cos^2\theta)\sin\phi_s A_T^{\sin\phi_s} + \sin^2\theta\Big(\sin(2\phi_{CS}+\phi_s)A_T^{\sin(2\phi_{CS}+\phi_s)} + \sin(2\phi_{CS}-\phi_s)A_T^{\sin(2\phi_{CS}-\phi_s)}\Big)\Big]$$

$A_T^{\cos 2\phi_{CS}} \propto h_1^{\perp q} \circledast h_1^{\perp q}$ BM ⊛ BM

$A_T^{\sin\phi_s} \propto f_1^{q} \circledast f_{1T}^{\perp q}$ PDF ⊛ Sivers

$A_T^{\sin(2\phi_{CS}-\phi_S)} \propto h_1^{\perp q} \circledast h_1^{q}$ BM ⊛ Transv

$A_T^{\sin(2\phi_{CS}+\phi_S)} \propto h_1^{\perp q} \circledast h_{1T}^{\perp q}$ BM ⊛ Pretz

$h_1^{\perp q}\big|_{SIDIS} = -h_1^{\perp q}\big|_{DY}$

$f_{1T}^{\perp q}\big|_{SIDIS} = -f_{1T}^{\perp q}\big|_{DY}$

$h_1^{q}\big|_{SIDIS} = h_1^{q}\big|_{DY}$

$h_{1T}^{\perp q}\big|_{SIDIS} = h_{1T}^{\perp q}\big|_{DY}$