NNGPD: Neural Network Generalized Parton Distributions

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On behalf of the Exclusives for AI and ML (Excl**AI**m) Collaboration

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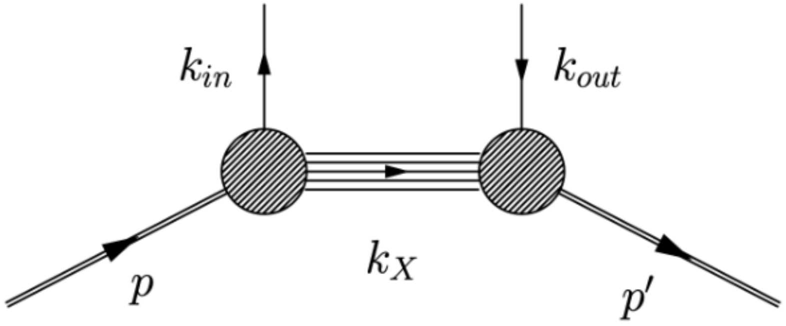
Generalized parton distributions (GPDs)

GPDs parameterize off-forward matrix elements of QCD correlation functions

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{i(k^+_{in} z^-_{in} - \mathbf{k}_{in,T}\cdot \mathbf{z}_{in,T})} e^{-i(k^+_{out} z^-_{out} - \mathbf{k}_{out,T}\cdot \mathbf{z}_{out,T})}$$
$$\times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda \rangle$$

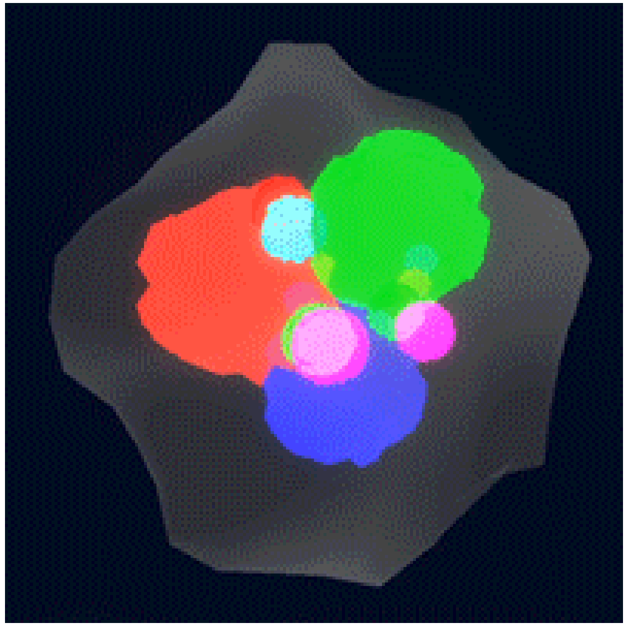


proton + quark → proton + quark correlation function

$$\begin{aligned} W^{\gamma^+}_{\Lambda,\Lambda'} &= \int \frac{dz^-}{2\pi} e^{iXp^+z^-} \langle P', \Lambda' | \bar{\psi}(0,0,0) \gamma^+ \psi(0, z^-, 0) | P, \Lambda \rangle \\ &= \frac{1}{2P^+} \left[H^q(X,\zeta,t) \bar{u}(p',\Lambda') \gamma^+ u(p,\Lambda) + E^q(X,\zeta,t) \bar{u}(p',\Lambda') \frac{\sigma^{i+}\Delta_i}{2M} u(p,\Lambda) \right] \end{aligned}$$

Similar relations for the gluon GPDs

Tools for seeing inside the proton



Access to the proton's intrinsic angular momentum structure

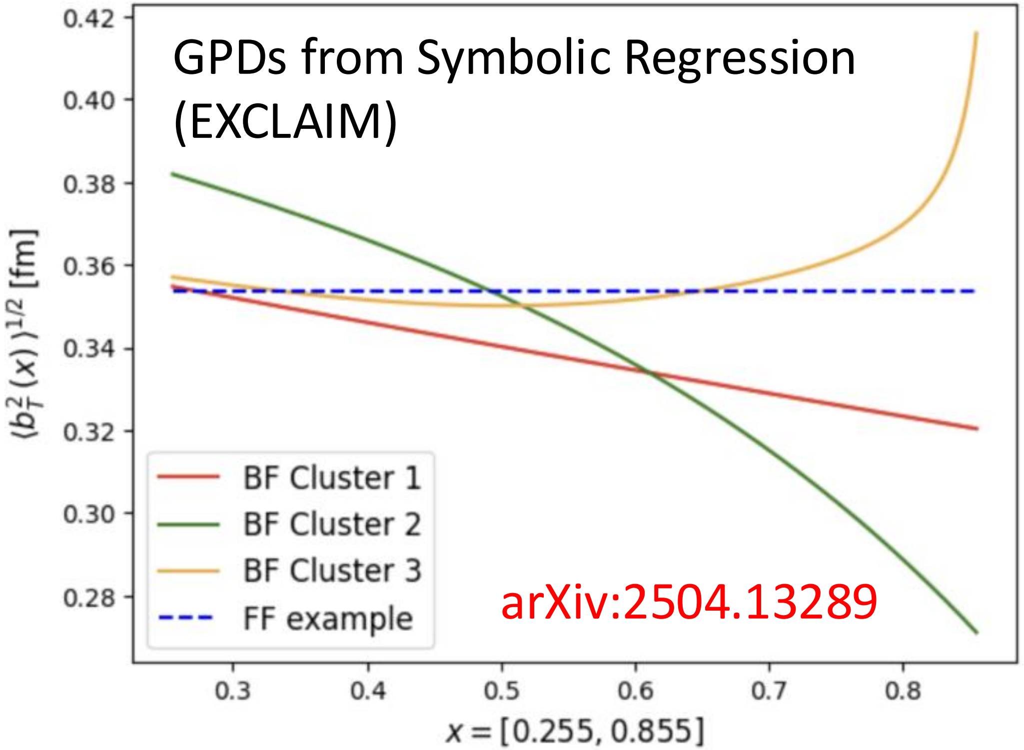
$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\int dx x\Big(H_q(x,0,0) + E_q(x,0,0) + H_g(x,0,0) + E_g(x,0,0)\Big)$$

Access to the proton's spatial structure

$$H_q(X,0,t) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\boldsymbol{\Delta}}\, \tilde{\phi}^*(X,\mathbf{b})\, \tilde{\phi}(X,\mathbf{b})$$

$$\rho_q(X,\mathbf{b}) = \tilde{\phi}^*(X,\mathbf{b})\, \tilde{\phi}(X,\mathbf{b})\,.$$

$$\langle b_T^2(X)\rangle^{1/2} = \frac{\int d^2 b_T b_T^2 \rho^{q,g}(X,b_T)}{\int d^2 b_T \rho^{q,g}(X,b_T)}$$



Sources of GPD information

- Experiment (Compton form factors)

$$F_{UU,T} = 4[(1-\xi^2)[(\Re e\mathcal{H})^2 + (\Im m\mathcal{H})^2 + (\Re e\tilde{\mathcal{H}})^2 + (\Im m\tilde{\mathcal{H}})^2] + \frac{t_o - t}{2M^2}[(\Re e\mathcal{E})^2 + (\Im m\mathcal{E})^2 + \xi^2(\Re e\tilde{\mathcal{E}})^2 + \xi^2(\Im m\tilde{\mathcal{E}})^2]$$
$$-\frac{2\xi^2}{1-\xi^2}(\Re e\mathcal{H}\Re e\mathcal{E} + \Im m\mathcal{H}\Im m\mathcal{E} + \Re e\tilde{\mathcal{H}}\Re e\tilde{\mathcal{E}} + \Im m\tilde{\mathcal{H}}\Im m\tilde{\mathcal{E}})]$$

1. Inverse problem from the cross section to CFFs. See, e.g, Adams et al. arXiv:2410.23469 (2025) (EXCLAIM)

$$\mathcal{F} = \Re\mathrm{e}\mathcal{F} + i\Im\mathrm{m}\mathcal{F} = PV\int_{-1}^{1} dx\left(\frac{1}{x-\xi} - \frac{1}{x+\xi}\right)F(x,\xi,t) + i\pi[F(\xi,\xi,t) - F(-\xi,\xi,t)]$$

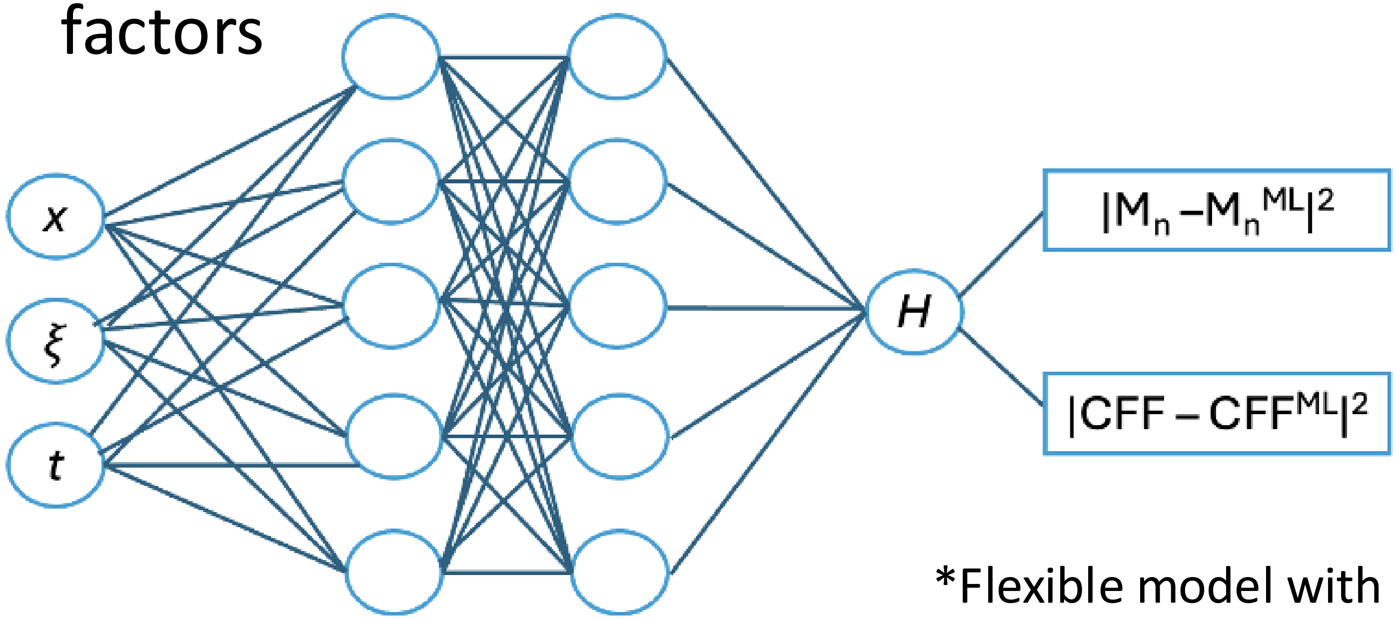
2. Inverse problem from the CFFs to GPDs → Address with NNGPD!

- Ab initio theory (Mellin moments from lattice QCD)

$$M_n^{q,H}(\xi,t,Q^2) = \sum_{i,even}^{n} (2\xi)^i A_{n+1,i}(t,Q^2) + \mod(n,2)(2\xi)^{n+1}C_{n+1,i}(t,Q^2)$$
$$M_n^{q,E}(\xi,t,Q^2) = \sum_{i,even}^{n} (2\xi)^i B_{n+1,i}(t,Q^2) - \mod(n,2)(2\xi)^{n+1}C_{n+1,i}(t,Q^2)$$

ML model for capturing experimental and lattice constraints

- Feed-forward neural network*: predict the x, ξ, and t dependence of GPDs using their Mellin moments M_n of order n and Compton form factors



Compare to the scheme of NNPDF:

$$\hat{f}_i(x, Q_0) = x^{-\alpha_i}(1-x)^{\beta_i}\,\mathrm{NN}_i(x)$$

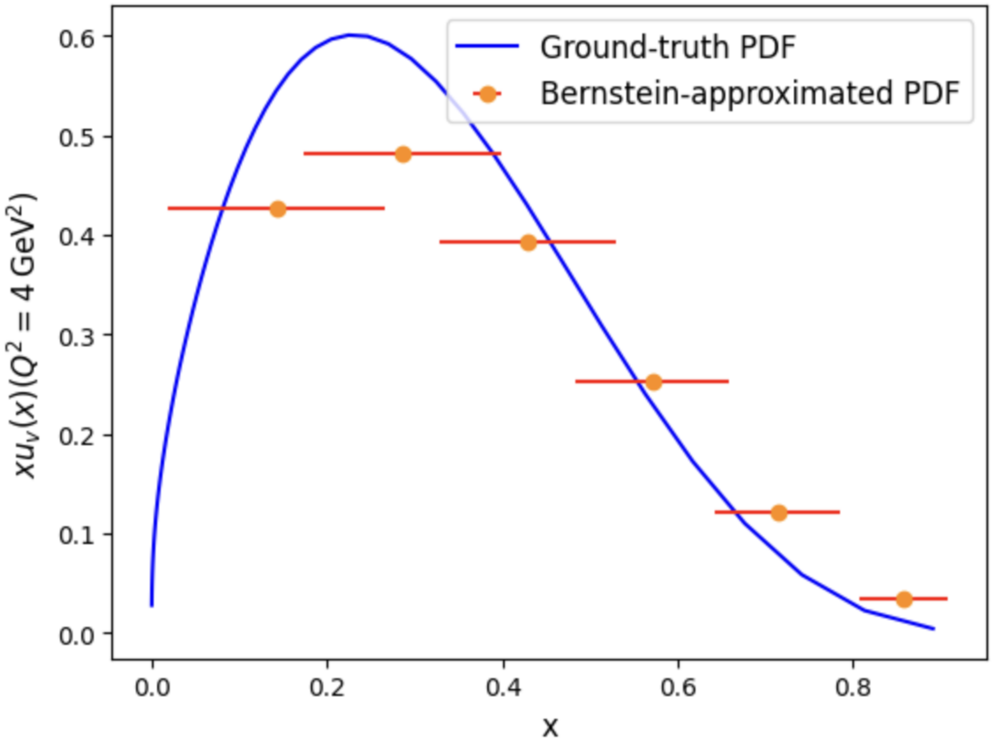
The NNPDF Collaboration, Ball et al. JHEP 2015, 40 (2015)

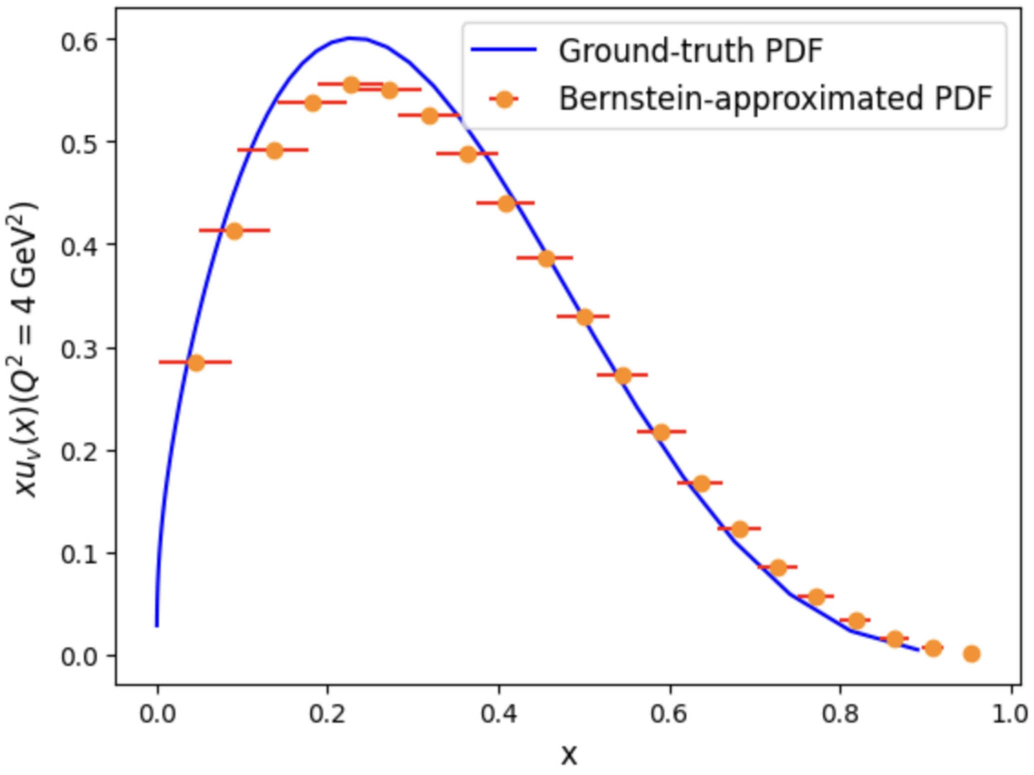
*Flexible model with minimal physics bias

$$\mathcal{L} = \sum_{\xi,t}\sum_{n}\left|M_n(\xi,t) - M_n^{\mathrm{ML}}(\xi,t)\right|^2 + \left|\mathrm{CFF}(\xi,t) - \mathrm{CFF}^{\mathrm{ML}}(\xi,t)\right|^2 + \text{symmetry constraints}$$

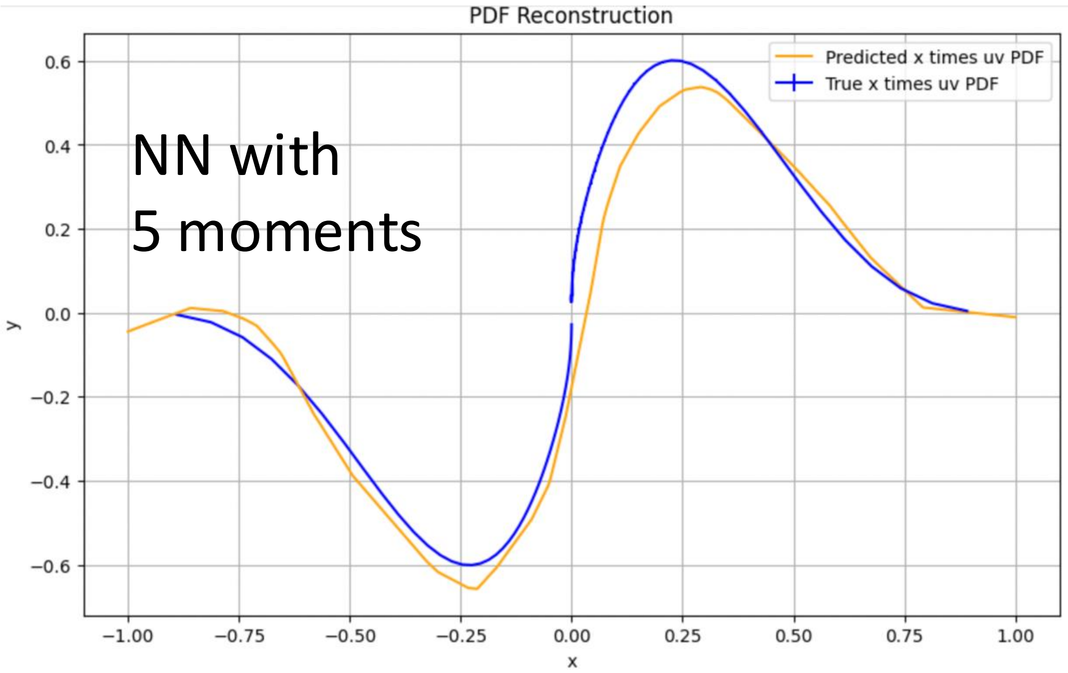
Why jump to ML?

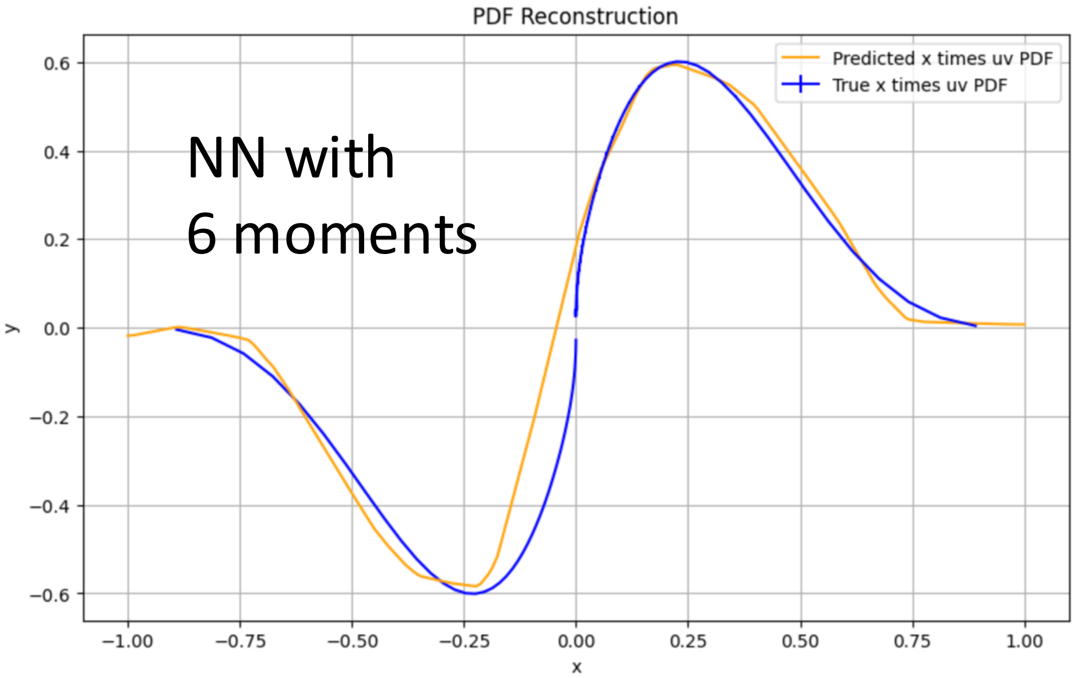
See, e.g., Ahmad et al., *Eur. Phys. J. C* (2009) 63: 407–421 for non-zero skewness GPDs, and Yndurain, PLB (1978) for DIS structure functions

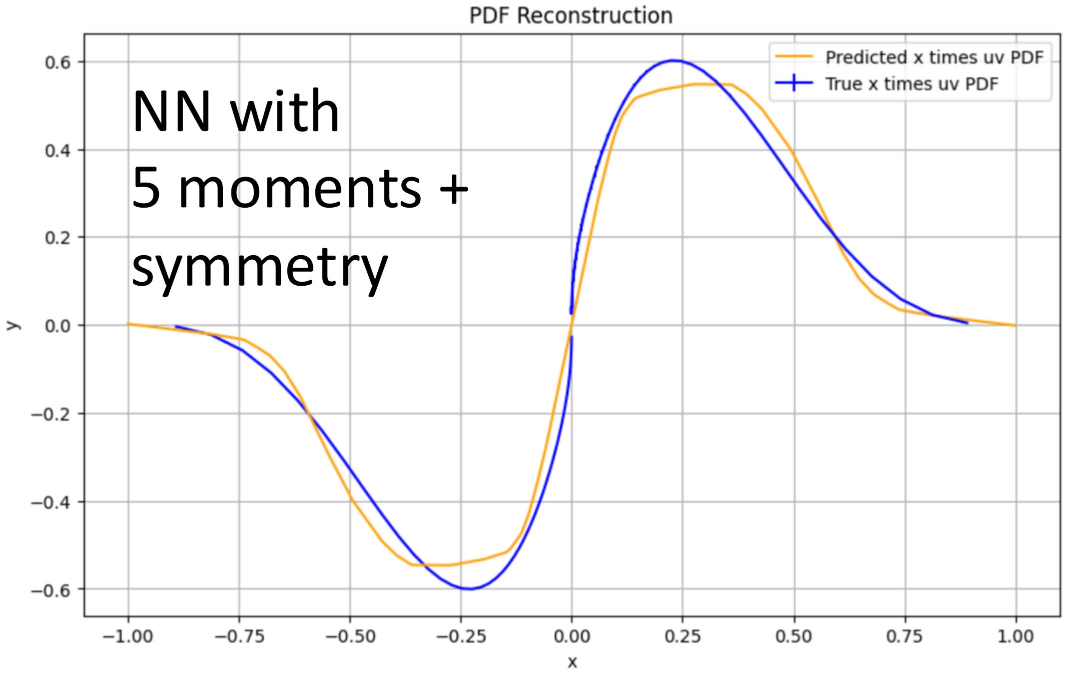


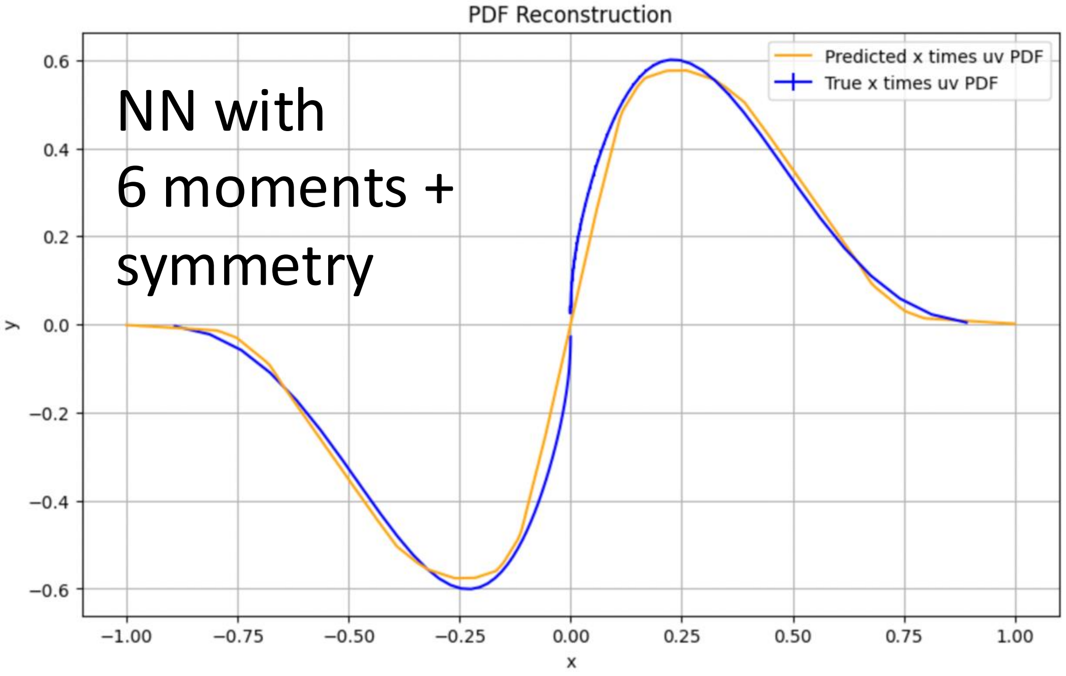


- A well-known problem in the approximation of a function from its moments: To get a proper resolution of the x dependence, a large number of moments is required. But the state-of-the-art lattice calculations only go up to around 5 or 6 moments!
- Has potential for use as a prior in our network





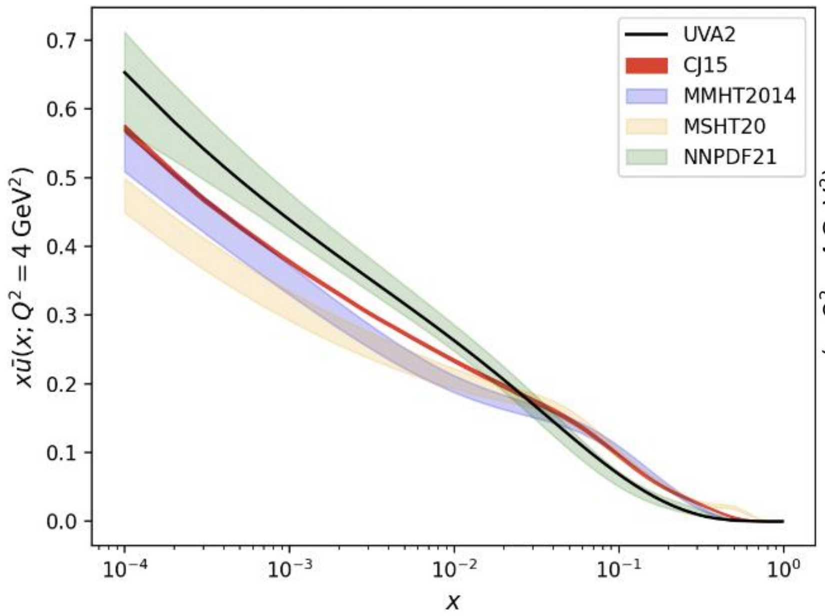


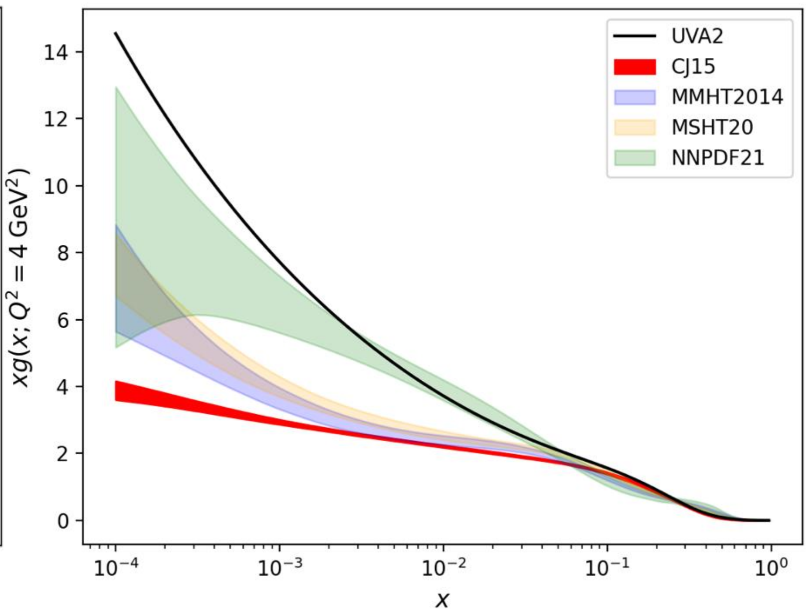


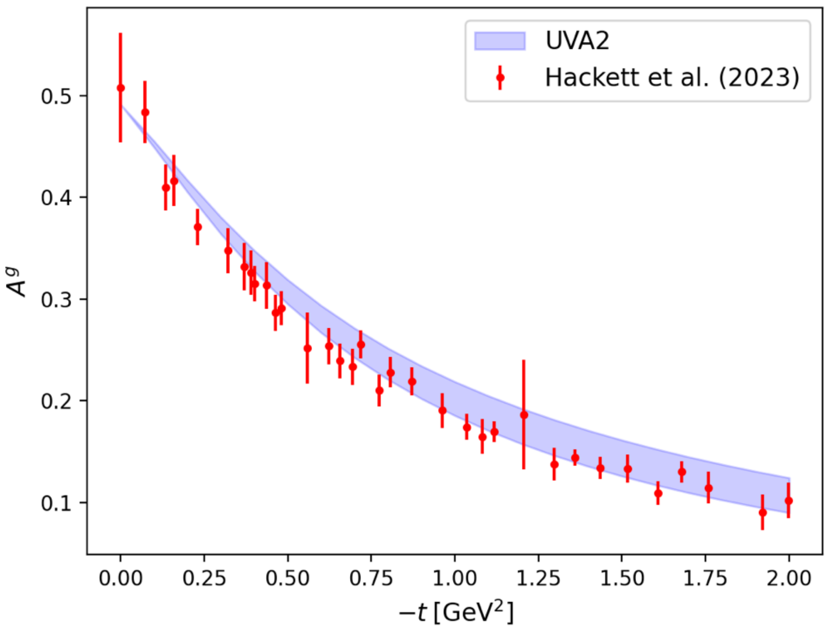
Compare to VAIM-based study: B. Kriesten & T. Hobbs PRD 111, 014028 (2025)

GPD Physics model to benchmark the ML with

- Captures lattice QCD moments, electromagnetic form factor data, inclusive scattering data, and perturbative QCD evolution equations







Updated flexible global parametrization of generalized parton distributions from elastic and deep inelastic inclusive scattering data

UVA2 soon to be posted on arXiv

Zaki Panjsheeri,[1,*] Douglas Q. Adams,[1,†] Adil Khawaja,[1,‡] Saraswati Pandey,[1,§] Kemal Tezgin,[2,¶] and Simonetta Liuti[1,**]

Compare with previous iterations, e.g. UVA1: B. Kriesten et al. PRD 105, 056022 (2022)

GPD Physics model to benchmark the ML with

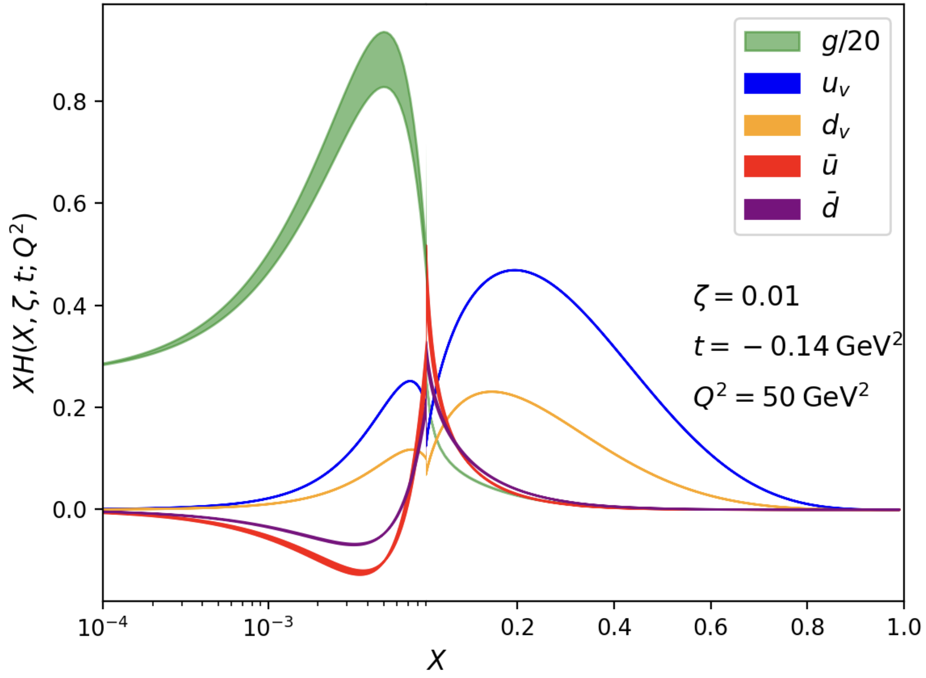
- Captures GPD symmetries in the DGLAP and ERBL regions, and predicts Compton form factors

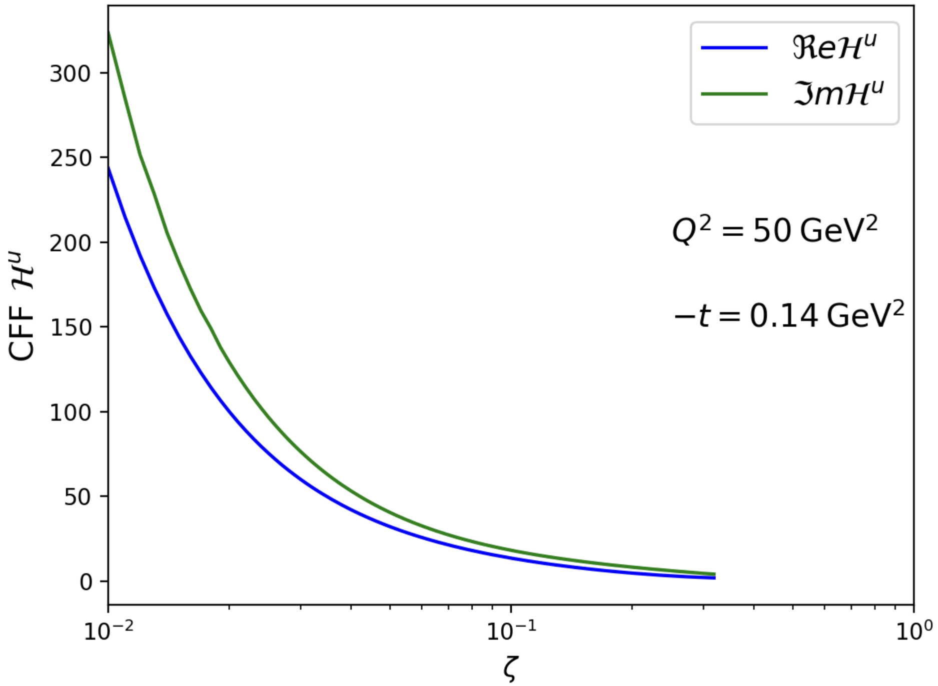
$$F^{q,g}(X,\zeta,t;Q^2) = F^{DGLAP}_{q,g}\left[\theta(X-\zeta)+\theta(-X)\right] + F^{ERBL}_{q,g}\left[\theta(-X+\zeta)+\theta(X)\right]$$

$$H_q^{DGLAP} = \mathcal{A}_H^q \, X^{-(\alpha_q+\alpha_q'(1-X)^{p_q}t)} \left[a_0 I_0 + a_1^- I_1 + I_2\right]_{m_q, M_X^q, M_\Lambda^q}$$

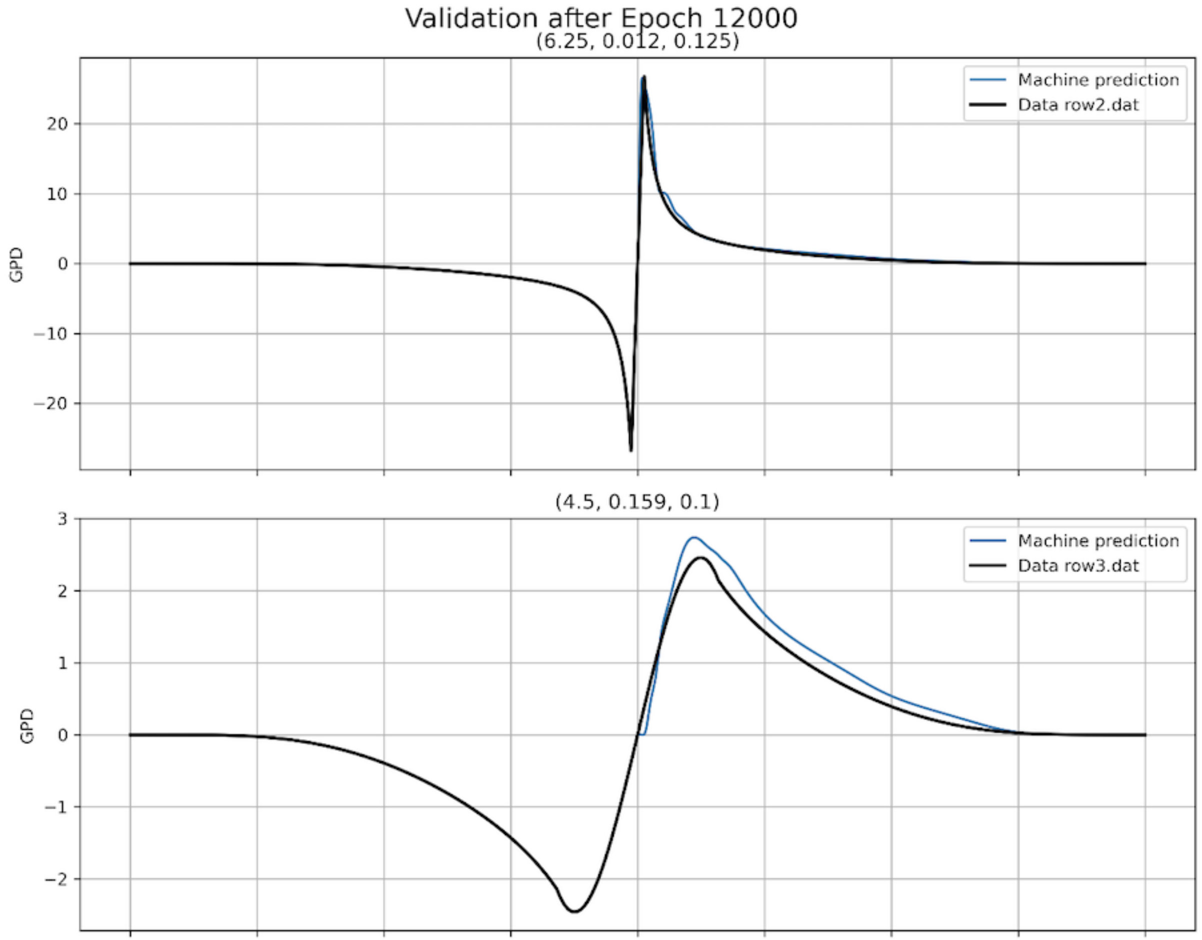
$$H_q^{ERBL} = a^- X^2 + b^- X + c^-$$

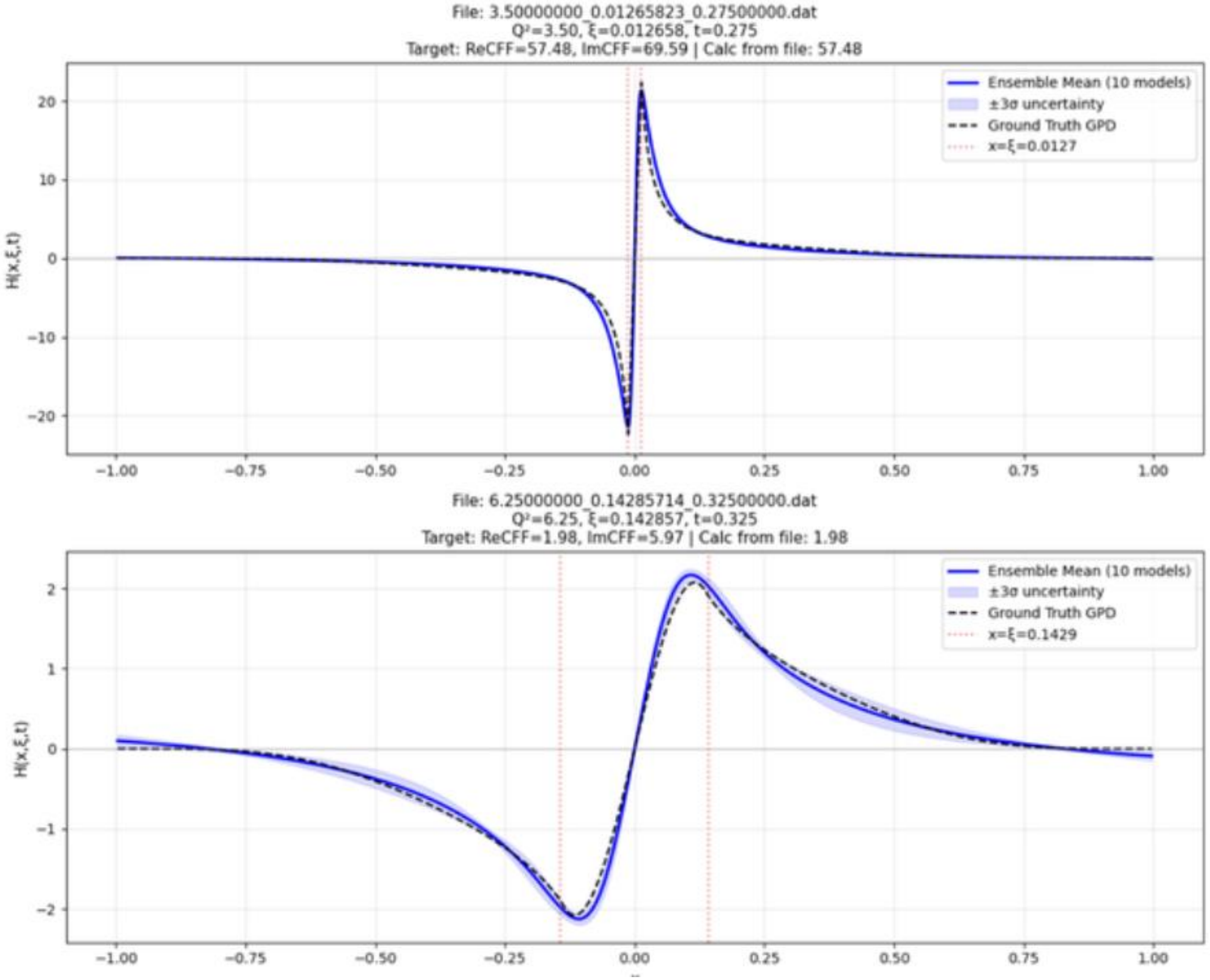
Valid for low x_{Bj} and high Q^2 → EIC kinematics





Preliminary ML results





Results from Ho (Jason) Jang, Jitao Xu, and Yaohang Li

Conclusions

- GPDs are interesting and complicated physical distributions that enter QCD in a manner which makes their extraction from experiment difficult, though not intractable
- Machine learning offers powerful methods for the extraction of GPDs from experimental data as well as ab initio lattice QCD results.
- Many theoretical constraints can be leveraged to aid in this extraction.
- Future work will entail applying more interpretable methods, such as symbolic regression, to solving this inverse problem.