The Theory Space* of the Flavour Problem

Based on
'Mapping and probing Froggatt-Nielsen solutions to the quark flavor puzzle' (***Cornella***, *DC, Neil, Thompson, 2306.08026*)
'Testing the Froggatt-Nielsen Mechanism with Lepton Violation' (***Cornella***, *DC, Krnjaic,* ***Mellors****, 2501.00629*)

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David Curtin
University of Toronto

UNIVERSITY OF TORONTO

Introduction

The Flavour Problem

In the SM, the fermion masses span 12 orders of magnitude from neutrinos to the top quark. Even just in the quark sector, the quark masses span vary by 10^5.

This hierarchy is *technically natural*, but still really *weird* and suggestive, given that in all other respects, different fermions seem identical copies of each other.

There are also intriguing differences between the quark and lepton sectors: quark mixings are hierarchical, neutrino mixings seem more anarchic.

Many possible explanations of this "flavour problem" have been proposed:

- Froggatt Nielsen — Froggatt, Nielsen, 1979
- Partial Compositeness / 5D localization — Modern review: Altmannshofer, Zupan 2203.07726
- Discrete symmetries
- …

The Theory Space* of the Flavour Problem

* let's focus on one very popular solution: the **Froggatt-Nielsen mechanism**.
(This exercise will teach us lessons that could ***eventually*** *realize the lofty title…)*

In the (simplest version of the) Froggatt-Nielsen mechanism, we postulate a U(1)$_X$ flavour symmetry broken by a 'flavon' scalar ϕ with $X_\phi = 1$ and vev $\langle \phi \rangle$.

SM fermions have different charges under U(1)$_X$ (put aside Higgs for now), making the Yukawa couplings generation-dependent non-renormalizable operators:

$$L_Y \supset - c^u_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_F} \right)^{|X_{Q_i} - X_{u_j}|} \bar{Q}_i \, \tilde{H} \, u_j \quad - \quad c^d_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_F} \right)^{|X_{Q_i} - X_{d_j}|} \bar{Q}_i \, H \, d_j$$

Typically take $X_{Q_3} = X_{u_3} = 0$ so $y_t \sim 1$, but higher charges will suppress Yukawas.

Froggatt Nielsen Review

This is very elegant, and you an easily generate hierarchical masses and mixings with some **reasonable guesses** at possible fermion charges.

E.g. a standard choice of charge assignment is: Leurer, Nir, Seiberg, hep-ph/9310320, hep-ph/9212278

$X_Q = (3,2,0),\ X_u = (-3,-1,0),\ X_d = (-3,-2,-2)$

Defining $\epsilon \equiv \langle \phi \rangle / \Lambda_F \sim 0.1$, the Yukawas become

$$L_Y \supset -c^u_{ij}\epsilon^{n^u_{ij}}\ \bar{Q}_i\ \tilde{H}\ u_j \quad - \quad c^d_{ij}\epsilon^{n^d_{ij}}\ \bar{Q}_i\ H\ d_j$$

$$n^u_{ij} = \begin{pmatrix} 6 & 4 & 3 \\ 5 & 3 & 2 \\ 3 & 1 & 0 \end{pmatrix} \qquad n^d_{ij} = \begin{pmatrix} 6 & 5 & 5 \\ 5 & 4 & 4 \\ 3 & 2 & 2 \end{pmatrix}$$

Froggatt Nielsen Review

Neglecting O(1) coefficients with $\epsilon = 0.12$, diagonal entries give masses

$$m_u \sim (0.5\ \mathrm{MeV},\ 0.3\ \mathrm{GeV},\ 174\ \mathrm{GeV})$$
$$m_d \sim (0.5\ \mathrm{MeV},\ 30\ \mathrm{MeV},\ 2.5\ \mathrm{GeV})$$

Froggatt Nielsen Review

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SM parameter	Value
m_u	0.00117(35)
m_c	0.543(72)
m_t	148.1(1.3)
m_d	0.0024(42)
m_s	0.049(15)
m_b	2.41(14)
$\lvert V_{12}\rvert$	0.22450(67)
$\lvert V_{13}\rvert$	0.00382(11)
$\lvert V_{23}\rvert$	0.04100(85)
$\lvert J\rvert$	$3.08(14)\ \times 10^{-5}$

Running quark masses (GeV) at $\mu = 1$ TeV

Xing, Zhang, Zhou 1112.3112

Froggatt Nielsen Review

You can convince yourself that the Yukawas are diagonalized by left and right rotations with parametric size

$$(U_u)_{ij} \sim \begin{cases} 1 & i = j \\ \epsilon^{n^u_{ij} - n^u_{jj}} & i < j \\ \epsilon^{n^u_{ji} - n^u_{ii}} & i > j \end{cases} \qquad (W_u)_{ij} \sim \begin{cases} 1 & i = j \\ \epsilon^{n^u_{ji} - n^u_{jj}} & i < j \\ \epsilon^{n^u_{ij} - n^u_{ii}} & i > j \end{cases}$$

Again ignoring all O(1) coefficients, this charge assignment predicts

$$V_{CKM} = U_u^\dagger U_d \sim \begin{pmatrix} 1.0144 & 0.240025 & 0.005184 \\ 0.240025 & 1.01461 & 0.0290074 \\ 0.005184 & 0.0290074 & 1.00021 \end{pmatrix}$$

Froggatt Nielsen Review

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$$(U_u)_{ij} \sim \begin{cases} 1 & i=j \\ \epsilon^{n^u_{ij}-n^u_{jj}} & i<j \\ \epsilon^{n^u_{ji}-n^u_{ii}} & i>j \end{cases} \qquad (W_u)_{ij} \sim \begin{cases} 1 & i=j \\ \epsilon^{n^u_{ji}-n^u_{jj}} & i<j \\ \epsilon^{n^u_{ij}-n^u_{ii}} & i>j \end{cases}$$

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$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

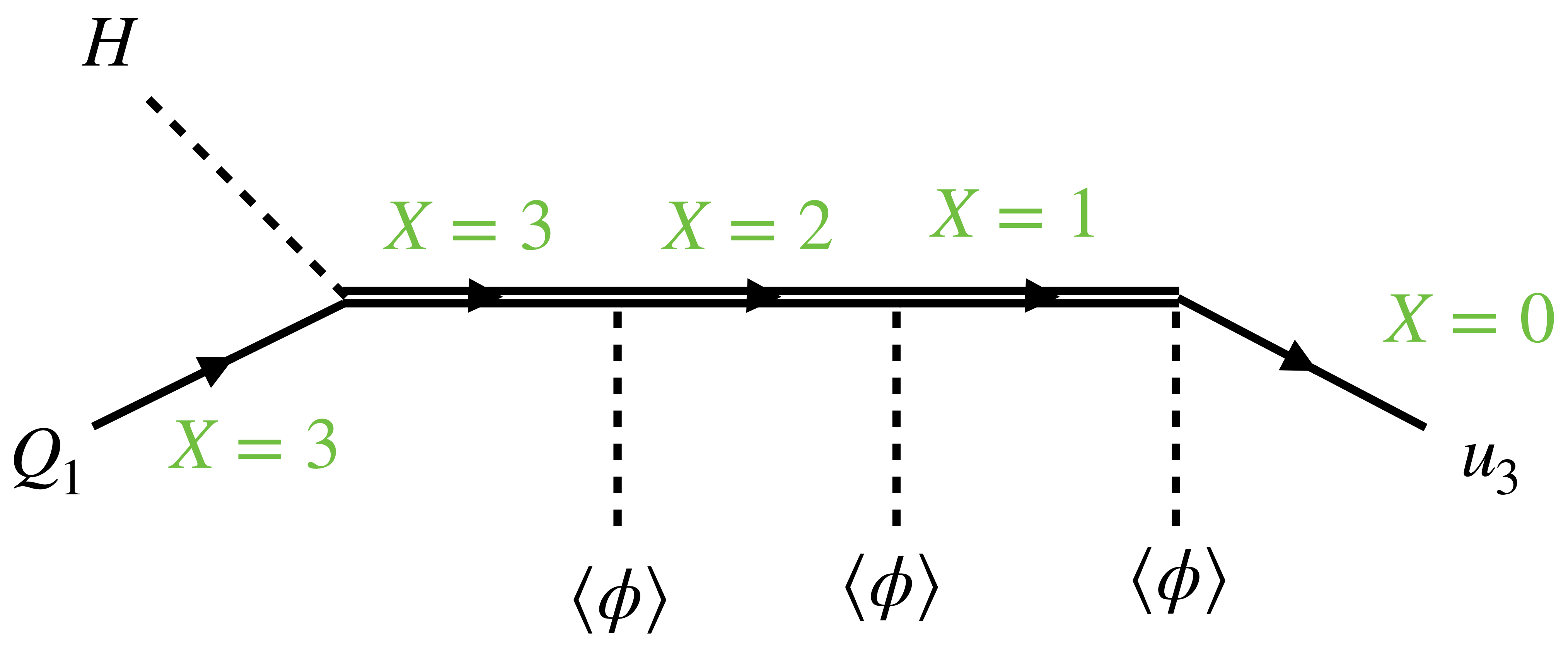
PDG

UV Completion

What actually happens at the "flavour scale" Λ_F?

Many possible scenarios, but simple one is existence of many **heavy vector-like fermions** with SM gauge charges and many different U(1)$_X$ flavour charges to generate the Yukawas like this:

$$c^u_{13}\left(\frac{\langle\phi\rangle}{\Lambda_F}\right)^3 \bar{Q}_1\, \tilde{H}\, u_3 \quad \sim$$



The many parameters of the FN model are generated in the deep UV by even more particles and parameters.

Flavour-Violating Signatures

The same selection rules apply to (e.g.) flavour-violating dimension-6 operators:

$$\mathcal{O}^A_{ijkl} = \frac{1}{(\Lambda^A_{\text{eff},ijkl})^2}(\bar{q}_i q_j)(\bar{q}_k q_l) \qquad \text{with} \qquad \frac{1}{(\Lambda^A_{\text{eff},ijkl})^2} = c^A_{ijkl}\frac{\epsilon^{|-X_{q_i}+X_{q_j}-X_{q_k}+X_{q_l}|}}{\Lambda_F^2}$$

(Various possible Lorentz structures suppressed)

Hence there are generic predictions for new flavour-violating processes that bear the fingerprint of the particular FN theory.

What makes a good solution to the flavour problem?

It's easy to see that FN can "work" for some obvious choices of flavour-charges.

What does 'work' mean?

Two **different** necessary features of a real solution to the flavour problem:

1. The theory should generically produce SM-like hierarchies no matter what O(1) coefficients ($c_{ij}^{u,d}$) you plug in.
    - For FN, this is a function of the charge assignments.
    - What does O(1) mean? Exact definition shouldn't matter for *hierarchies*!

2. In theories that generically produce 'SM-like hierarchies', it should then be possible to obtain (many possible) exact SM fits with *untuned O(1)* choices of the coefficients.

A birds-eye view of the FN mechanism

It is clearly interesting to understand the most **generic** predictions of FN (and other flavour mechanisms), **assuming only that the structure of the Yukawa matrices is dictated purely by the horizontal symmetry**.

→ **However, it is not so obvious that these predictions have been exhausted by the handful of charge assignments typically analyzed in the literature, or that these analyses imposed these two necessary features of a real flavour-problem-solution.**

Simply scanning over the many parameters of FN models to find e.g. SM fits seems to naive, we want to carefully impose these necessary features of a good flavour mechanism.

A birds-eye view of the FN mechanism

We start with these necessary features to assemble a general and global picture of the FN mechanism and its generic phenomenological predictions.

This can serve as a template for studying other flavour mechanisms in generality.

Quark sector:

'Mapping and probing Froggatt-Nielsen solutions to the quark flavor puzzle' (**Cornella**, DC, Neil, Thompson, 2306.08026)

Lepton sector:

'Testing the Froggatt-Nielsen Mechanism with Lepton Violation' (**Cornella**, DC, Krnjaic, **Mellors**, 2501.00629)

Especially want to highlight my excellent junior collaborators:

Claudia Cornell (CERN)

Micah Mellors (Toronto)

Other approaches

Philosophically very similar approach to ours, but focusing on lepton CPV in MSSM:
Aloni, Asadi, Nakai, Reece, Suzuki 2104.02679

An alternative approach just looks for natural SM-fits, but we argue that this misses the global naturalness criterion on a given charge assignment:
Fedele, Mastroddi, Valli 2009.05587

Our approach is very Bayesian-inspired in exploring many charge assignments **and** specific models within a given charge assignment. Previous analyses applied more strictly Bayesian analyses to *a few specific* charge assignments:
Altarelli, Feruglio, Masina hep-ph/0210342
Altarelli, Feruglio, Masina, Merlo, 1207.0587
Bergstrom, Meloni, Merlo, 1403.4528

Coincident with our lepton paper, Ibe, Shirai, Watanabe (2412.19484) published a Bayesian analysis scanning over quark and lepton charge assignments, highly complementary to our approach (they can consider nucleon decay, our analysis is more general and comprehensive in the lepton sector)

Methodology

Basic Idea

You know you can always fit any FN model to the SM, but that's not the point.

The model should 'want' to look like the SM with 'O(1)' parameters.

Let's implement a numerical method to implement this sentiment and make it quantitative.

This method should generalize to other solutions of the flavour problem.

Methodology

For a given *type* of FN model (quarks, leptons, 2HDM, ...):

1) Scan over all possible flavour charges for the fermions, typically to some maximum $|X_f|$ (and grouping together physically equivalent assignments, charge orderings, etc)

2) For each charge assignment, generate 1000s of individual models by randomly sampling the coefficients in Yukawa couplings from an "O(1)" distribution.

$$c^u_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_F} \right)^{|X_{Q_i} - X_{u_j}|} \bar{Q}_i \, \tilde{H} \, u_j$$

What is the prior for the coefficients? Default is log-normal (spread $10^{\pm 0.3}$), but we check that results are robust to changing that "O(1)" prior to another one.

Methodology

3) To assess how 'SM-like' a given random model (choice of coeffs) is, define

$$\delta_{\max} \equiv \max_{\mathcal{O}} \left[\frac{\mathcal{O}_{\mathrm{FN}}}{\mathcal{O}_{\exp}}, \frac{\mathcal{O}_{\exp}}{\mathcal{O}_{\mathrm{FN}}} \right]$$

where $\mathcal{O}_{\exp}$ = measured mass/mixing parameter

e.g. $\delta_{\max} = 3.1$ means that all observables are at worst deviating from exp values by factor of 3.1

4) For each *charge assignment*, you can then define parameter

$F_a \equiv$ fraction of randomly generated models $\delta_{\max} < a$, for $a = 5$ or 2 or 1.1…

E.g. $F_2 = 0.5$ means that half the generated models give masses/mixings within factor of 2 of the observed SM values.

F_a gives some estimate of the 'volume fraction of parameter space' for a given charge assignment that is "SM-like"

Methodology

5) Different charge assignments can then be ranked by F_2 (or smaller deviations, if statistics permits).

Intermediate objective: you now have, for a given type of FN model, a way of ranking charge assignments by how SM-like they want to be.

You can then interrogate the top SM-like charge assignments in more detail for their phenomenological predictions.

Aside 1: F_a is a Global Flavour Naturalness Criterion

We call a FN charge assignment "SM-like" if it has a "high" value of F_a.

Obviously $F_a \to 0$ *as* $a \to 1$*, but top charge assignments tend to give* $F_5 \sim 50\,\%$, $F_2 \sim \text{few}\,\%$.

This makes intuitive sense, but we have to make sure it is robust. We verified:

a) F_a (and associated rankings of charge assignments) does **<u>not</u>** depend strongly on the prior chosen for generating random coefficients.

b) High F_5 predicts high F_2, which in turn (almost always) predicts high $F_{1.1}$
→ useful in practice, you can very quickly get a feeling for how good a charge assignment is by just trying ~10 models and seeing if a few of them are within a factor of a few of the SM.

c) You can perturb the coefficients of a random FN model with $\delta_{\max} \sim 1.1$ by O(10%) to obtain an exact SM fit, **and that SM fit is 'natural'.** $\leftarrow ??$

Aside 2: How to tell if a SM fit is "natural"?

1. The fit should only involve O(1) coefficients.

More quantitatively, given prior for selecting a single coefficient, you can exactly define how 'unlikely' a given fit's (max coefficient)/(min coefficient) ratio is.

2. The fit should not be tuned, using a custom tuning measure $\Delta_{\rm tot} \sim \mathcal{O}(1)$.

A standard approach is to use the Barbieri-Giudice measure over all observables K

$$\Delta^K_{\rm BG} \equiv \max_k |\delta_{K,k}| \;\;, \;\; \delta_{K,k} \equiv \frac{\delta \log \mathcal{O}_K}{\delta \log c_k} \quad \Rightarrow \quad \Delta_{\rm BG} \equiv \max_K \Delta^K_{\rm BG} \quad \Rightarrow \quad \Delta_{\rm BG} \equiv \max_K \Delta^K_{\rm BG}$$

But this is not suitable for flavour models that derive from a rich UV completion, where **jiggling one UV parameter will jiggle all the FN coefficients**. Instead:

$$\Delta^K_{\rm tot} \equiv \left[\sum_s (\lambda^K_s)^2\right]^{1/2} \text{ where the } \lambda^K_s \text{ are the eigenvalues of } \Delta^K_{kl} \equiv \frac{\delta^2 \log \mathcal{O}_K}{\delta \log c_k \delta \log c_l} \Rightarrow \Delta_{\rm tot} = \left[\sum_K (\Delta^K_{\rm tot})^2\right]^{1/2}$$

Indeed: numerically jiggling all coeffs by $\mathcal{O}(\delta)$ will perturb observables by $\mathcal{O}(\Delta_{\rm tot}\delta)$

Methodology

6) For the ‘most SM-like’ charge assignments, you can now obtain predictions for other flavour-violating observables:

a) For a given charge assignment, find an ensemble of SM fits starting from the coefficient choices with small $\delta_{\max}$

b) For each SM fit, generate random O(1) coefficients for (e.g.) dim-6 4-fermi ops in the FLAVOUR BASIS (then transform to that model’s mass basis & match to SMEFT for exp predictions)

$$\mathcal{O}^A_{ijkl} = \frac{1}{(\Lambda^A_{\mathrm{eff},ijkl})^2}(\bar{q}_i q_j)(\bar{q}_k q_l) \qquad \frac{1}{(\Lambda^A_{\mathrm{eff},ijkl})^2} = \boxed{c^A_{ijkl}}\,\frac{\epsilon^{|-X_{q_i}+X_{q_j}-X_{q_k}+X_{q_l}|}}{\Lambda_F^2}$$

`flavio, smelli`

c) For each SM-like charge assignment, can then get **distributions** of predictions

Methodology — Summary

For a given *type* of FN model (quarks, leptons, 2HDM, …):

1) Scan over possible flavour charge assignments
2) For each charge assignment, generate many random models with O(1) coeffs
3) Assess how SM-like each model is by computing $\delta_{\rm max}$ (fractional diff from SM)
4) Assess how SM-like each charge assignment "wants" to be by computing F_5, F_2, etc (i.e. fractions of models that give SM within x5, x2, etc)
5) Rank charge assignments by F_2 (or another F_a).
6) For the 'most SM-like" charge assignments, add higher-dimensional FV operators with random O(1) coeffs to each of the generated models, and compute predictions.

This will give a global and comprehensive view of the predictions of the FN mechanism.

Aside 3: Analytical checks

Our brute-force numerical scans "discover" many new natural SM-like textures that were not considered in the literature previously, but for this, we did not need the full numerical machinery.

Parametrically estimating fermion mass/mixing hierarchies for each charge assignment would yield the same natural charge assignments, provided you scan over all possible FN charges.

However, computing the distribution of FV predictions requires the full numerical approach.

Froggatt-Nielsen in the Quark Sector

'Mapping and probing Froggatt-Nielsen solutions to the quark flavor puzzle' (***Cornella***, *DC, Neil, Thompson, 2306.08026)*

FN in the Quark Sector

Basically the example we've shown previously.

$$L_Y \supset - c^u_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_F} \right)^{|X_{Q_i} - X_{u_j}|} \bar{Q}_i \; \tilde{H} \; u_j \quad - \quad c^d_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_F} \right)^{|X_{Q_i} - X_{d_j}|} \bar{Q}_i \; H \; d_j$$

Assume single Higgs, single flavon ϕ. Assume Higgs has no flavour charge.

If $X_H \neq 0$, both the SM quark mass matrices and dim6 FV operators can be mapped to a model with $X_H = 0$, so our results are general.

Scan all integer quark flavour charges up to $|X_q| \leq 4$.
Compare to quark masses, mixings, Jarlskog.

Most SM-like FN charge assignments

All tables public as csv.

"Rediscover" many classic textures (#3 is the Leurer, Nir, Seiberg favourite), but also find many new ones.

The good textures really are good. Throw a dart, it looks like the SM.

Num.	X_{Q_1}	X_{Q_2}	X_{u_1}	X_{u_2}	X_{d_1}	X_{d_2}	X_{d_3}	$\mathcal{F}_2$ (%)	$\mathcal{F}_5$ (%)	ϵ
1	3	2	-4	-2	-3	-3	-3	2.7	67	0.17
2	3	2	-4	-2	-4	-3	-3	2.5	66	0.18
3	3	2	-3	-1	-3	-2	-2	1.9	56	0.12
4	3	2	-4	-1	-3	-3	-3	1.5	65	0.16
5	4	3	-4	-2	-4	-3	-3	1.2	52	0.23
6	3	2	-4	-1	-3	-3	-2	1.1	63	0.15
7	4	2	-4	-2	-4	-3	-3	1.1	47	0.21
8	3	2	-3	-1	-2	-2	-2	0.9	41	0.11
9	3	2	-3	-1	-3	-3	-2	0.9	55	0.14
10	3	2	-4	-2	-3	-3	-2	0.9	59	0.16
11	2	1	-3	-1	-2	-2	-2	0.8	52	0.06
12	4	3	-4	-1	-4	-3	-3	0.8	52	0.22
13	4	3	-4	-2	-4	-4	-3	0.8	50	0.24
14	3	2	-4	-2	-4	-3	-2	0.7	56	0.17
15	4	3	-4	-2	-3	-3	-3	0.7	43	0.22
16	4	3	-4	-1	-3	-3	-3	0.6	45	0.21
17	4	2	-4	-2	-4	-4	-3	0.6	48	0.22
18	4	2	-4	-2	-3	-3	-3	0.6	38	0.2
19	3	2	-4	-1	-3	-2	-2	0.6	56	0.14
20	3	2	-3	-2	-3	-3	-2	0.6	45	0.15

Most SM-like FN charge assignments

All tables public as csv.

“Rediscover” many classic textures (#3 is the Leurer, Nir, Seiberg favourite), but also find many new ones.

The good textures really are good. Throw a dart, it looks like the SM.

Notice: SM does need **near-degenerate flavour charges in the down sector!**

Num.	X_{Q_1}	X_{Q_2}	X_{u_1}	X_{u_2}	X_{d_1}	X_{d_2}	X_{d_3}	$\mathcal{F}_2$ (%)	$\mathcal{F}_5$ (%)	ϵ
1	3	2	-4	-2	-3	-3	-3	2.7	67	0.17
2	3	2	-4	-2	-4	-3	-3	2.5	66	0.18
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4	3	2	-4	-1	-3	-3	-3	1.5	65	0.16
5	4	3	-4	-2	-4	-3	-3	1.2	52	0.23
6	3	2	-4	-1	-3	-3	-2	1.1	63	0.15
7	4	2	-4	-2	-4	-3	-3	1.1	47	0.21
8	3	2	-3	-1	-2	-2	-2	0.9	41	0.11
9	3	2	-3	-1	-3	-3	-2	0.9	55	0.14
10	3	2	-4	-2	-3	-3	-2	0.9	59	0.16
11	2	1	-3	-1	-2	-2	-2	0.8	52	0.06
12	4	3	-4	-1	-4	-3	-3	0.8	52	0.22
13	4	3	-4	-2	-4	-4	-3	0.8	50	0.24
14	3	2	-4	-2	-4	-3	-2	0.7	56	0.17
15	4	3	-4	-2	-3	-3	-3	0.7	43	0.22
16	4	3	-4	-1	-3	-3	-3	0.6	45	0.21
17	4	2	-4	-2	-4	-4	-3	0.6	48	0.22
18	4	2	-4	-2	-3	-3	-3	0.6	38	0.2
19	3	2	-4	-1	-3	-2	-2	0.6	56	0.14
20	3	2	-3	-2	-3	-3	-2	0.6	45	0.15

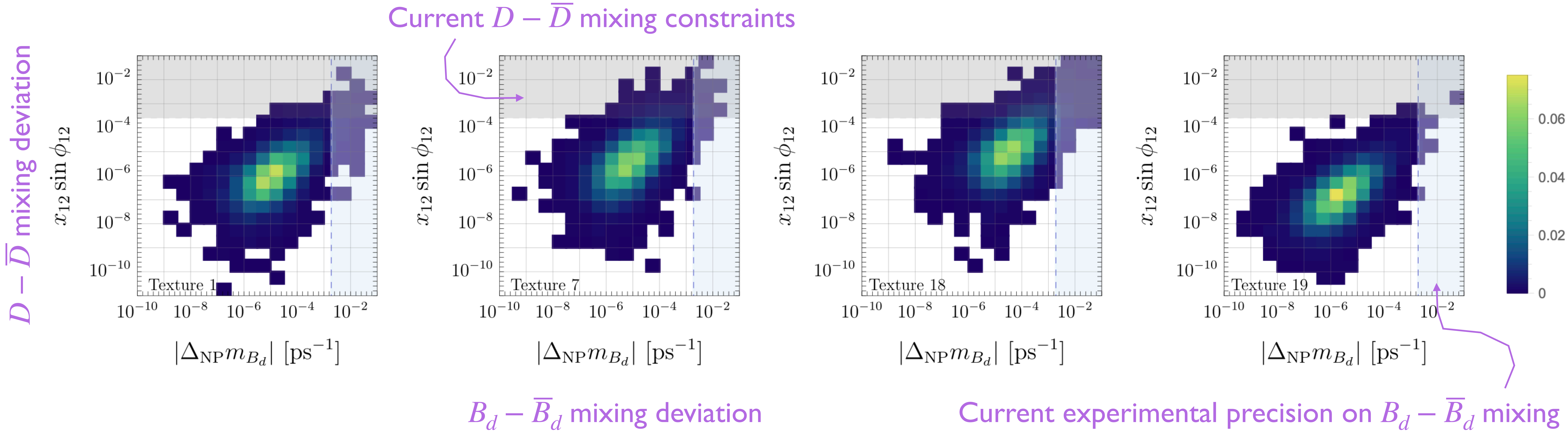
Phenomenology: bounds on flavour scale Λ_F

In basically all SM-like textures, most stringent constraint on flavour scale comes from $K-\overline{K}$ mixing: $\Lambda_F \gtrsim 10^5$ TeV. **Charge-independent FN prediction.**

Very close to flavour-anarchic prediction, hence texture-universal. Why?

→ degenerate d_R flavour charges make RH rotations to mass basis anarchic, undermines suppression of dim6 operators in flavour basis.

Phenomenology: correlated deviations in D, B mixing



Set Λ_F to current constraint → predict correlated D, B$_d$ mixing deviations from SM

Single measurement could favour some textures over others, but this would require significant improvement of SM prediction for $B_d-\overline{B}_d$ mixing.

Other FV processes (rare B, K decays) are many orders of magnitude too small.

Upshot for the quark sector

Some things are very universal amongst top textures: we show FN is extremely *predictive. (Degenerate X_d, near-anarchic limit from $K-\overline{K}$ mixing)*

Even so, correlated measurement of B, D mixing may favour some textures and reveal details of the FN model.

Global SMEFT fits incorporating specific FN models may be more constraining: our list of motivated FN models is a natural target for such an analysis.

Froggatt-Nielsen in the Lepton Sector

*'Testing the Froggatt-Nielsen Mechanism with Lepton Violation' (**Cornella**, DC, Krnjaic, **Mellors**, 2501.00629)*

FN in the Lepton Sector

The lepton sector offers interesting challenges and opportunities compared to the quark sector, and the leptonic FN mechanism has hence been much less studied:

- The precise values and the mechanism of generating the neutrino masses are currently unknown.
- The neutrino mixing matrix U_{PMNS} does not appear to have large hierarchies, unlike V_{CKM}.
- Certain charged lepton flavour violating (CLFV) signatures are *exceedingly* clean, providing hope of future detection
- Cosmological measurements and direct searches for neutrino-less double-beta decay ($0\nu\beta\beta$) will soon provide more detailed information on the neutrino sector.

Setup

Consider FN *only* in the lepton sector for now, independent of quarks.

For the charged lepton sector, implement FN analogously to quarks:

$$\mathcal{L}_Y \supset -c^{\ell}_{ij} L_i H^\dagger \bar{e}_j \epsilon^{|X_{L_i}+X_{\bar{e}_j}|} \qquad \epsilon \equiv \frac{\phi}{\Lambda} \ll 1$$

We then separately study three ways of generating the neutrino mass:
1) Dirac Neutrinos
2) Majorana Neutrinos
3) Type-1 Seesaw

Setup

1) **Dirac Neutrinos:** most straightforward, and small m_ν comes *entirely* from the FN mechanism and large flavour charges of the RH neutrinos.

$$\mathcal{L}_D \supset c^\nu_{ij}\epsilon^{n^\nu_{ij}} H L_i N_j \quad , \quad n^\nu_{ij} \equiv |X_{L_i} + X_{N_j}|$$

2) **Majorana Neutrinos:** add Weinberg operator at scale Λ_W that may or may not be related to the flavour scale Λ_F.

$$\mathcal{L}_W \supset -\frac{c^W_{ij}\,\epsilon^{n^W_{ij}}}{\Lambda_W}(L_i H)(L_j H), \qquad n^W_{ij} \equiv |X_{Li} + X_{Lj}| \qquad \Rightarrow \qquad \hat{m}_\nu = U^T_\nu \left[c^W \epsilon^{n^W}\left(\frac{v^2}{\Lambda_W}\right)\right] U_\nu$$

3) **Type-1 Seesaw**, where we identify the RH neutrino mass scale with Λ_F

$$\mathcal{L}_{\rm SS} \supset -c^\nu_{ij}\epsilon^{n^\nu_{ij}} H L_i N_j - c^M_{ij}\,\epsilon^{n^M_{ij}}\frac{M}{2} N_i N_j \qquad \Rightarrow \qquad \hat{m}_\nu \approx \frac{v^2}{2M} U^T_\nu \left(c^\nu \epsilon^{n^\nu}\right)\left(c^M \epsilon^{n^M}\right)^{-1}\left(c^\nu \epsilon^{n^\nu}\right)^T U_\nu$$

$$n^\nu_{ij} \equiv |X_{L_i} + X_{N_j}| \qquad n^M_{ij} \equiv |X_{N_i} + X_{N_j}|$$

Setup

1) **Dirac Neutrinos:** most straightforward, and small m_ν comes *entirely* from the FN mechanism and large flavour charges of the RH neutrinos.

$$\mathcal{L}_D \supset c^{\nu}_{ij}\epsilon^{n^{\nu}_{ij}} H L_i N_j \quad , \quad n^{\nu}_{ij} \equiv |X_{L_i} + X_{N_j}|$$

2) **Majorana Neutrinos:** add Weinberg operator at scale Λ_W that may or may not be related to the flavour scale Λ

$$\mathcal{L}_W \supset -\frac{c^W_{ij}\,\epsilon^{n^W_{ij}}}{\Lambda_W}(L_i H)(L_j H),$$

$$U_\nu^T \left[c^W \epsilon^{n^W} \left(\frac{v^2}{\Lambda_W} \right) \right] U_\nu$$

Generates effective Weinberg operator, but with flavour-dependent Λ_W^{ij}

3) **Type-I Seesaw**, where we identify the RH neutrino mass scale with Λ_F

$$\mathcal{L}_{\rm SS} \supset -c^{\nu}_{ij}\epsilon^{n^{\nu}_{ij}} H L_i N_j - c^M_{ij}\,\epsilon^{n^M_{ij}}\frac{M}{2} N_i N_j$$

$$\Rightarrow \quad \hat{m}_\nu \approx \frac{v^2}{2M} U_\nu^T \left(c^\nu \epsilon^{n^\nu}\right)\left(c^M \epsilon^{n^M}\right)^{-1}\left(c^\nu \epsilon^{n^\nu}\right)^T U_\nu$$

$$n^{\nu}_{ij} \equiv |X_{L_i} + X_{N_j}| \qquad n^{M}_{ij} \equiv |X_{N_i} + X_{N_j}|$$

Numerical Scan

Again: Assume single Higgs ($X_H = 0$ again w.l.o.g.), single flavon ϕ with $X_\phi = 1$.

Scan all integer lepton flavour charges up to $|X_\ell| \leq 7$ (≤ 9 for Dirac)

In computing $\delta_{\max}$, compare to experimental values for

- charged lepton masses
- measured IO/NO neutrino mass differences
- *all* off-diagonal UPMNS matrix elements
- sum of neutrino masses, either from cosmology or from lab measurements: no effect on results (!!)

Parameter	Exp. Value	Ref.
$m_e(1\text{ TeV})$	$0.489535765^{+0.000000013}_{-0.000000012}$ MeV	[55]
$m_\mu(1\text{ TeV})$	$103.3441945 \pm 0.0000059$ MeV	[55]
$m_\tau(1\text{ TeV})$	1756.81 ± 0.16 MeV	[55]
Δm^2_{21}	$(7.53 \pm 0.18) \times 10^{-5}\text{ eV}^2$	[46]
Δm^2_{32} (IO)	$(-2.536 \pm 0.034) \times 10^{-3}\text{ eV}^2$	[46]
Δm^2_{32} (NO)	$(2.453 \pm 0.033) \times 10^{-3}\text{ eV}^2$	[46]
$\sum m_\nu$ (cosmo)	≤ 0.12 eV (95% CL)	[46]
$\sum m_\nu$ (KATRIN)	≤ 1.35 eV (90% CL)	[47]
$\lvert V_{12}\rvert$	$[0.513, 0.579]$	[56]
$\lvert V_{13}\rvert$	$[0.143, 0.155]$	[56]
$\lvert V_{23}\rvert$	$[0.637, 0.776]$	[56]
$\lvert V_{21}\rvert$	$[0.234, 0.500]$	[56]
$\lvert V_{31}\rvert$	$[0.271, 0.525]$	[56]
$\lvert V_{32}\rvert$	$[0.477, 0.694]$	[56]

Small detail: the definition of $\delta_{\max}$ is tweaked to take the large experimental uncertainties into account. $\delta_{\max} = 1$ means all observables within uncertainties, while $\delta_{\max} = 2$ means that at least one observable is x2 above (below) experimental upper (lower) bound.

Most SM-like charge assignments

Dirac

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	N_1	N_2	N_3	ϵ	NO
6	5	5	-3	-2	0	9	8	8	0.10	96
3	3	3	2	-1	-6	9	9	8	0.07	99
3	3	3	2	-5	-6	9	9	8	0.07	99
7	7	6	-4	-2	0	9	9	9	0.14	99
7	7	6	-4	-3	-1	9	7	7	0.11	99
3	3	3	2	0	-5	9	9	8	0.07	99
3	3	3	2	0	-1	9	9	8	0.07	99
6	5	5	-3	-2	0	9	7	7	0.08	97
7	3	3	2	0	-5	9	9	9	0.08	93
6	6	6	-4	-3	-1	9	6	5	0.07	99

Majorana

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	ϵ	$\log\Lambda$	NO
2	0	-1	7	6	4	0.24	15	91
5	5	-2	7	-2	-3	0.08	12	3
4	4	3	5	2	0	0.23	11	96
7	6	5	7	3	0	0.39	11	97
6	6	5	5	1	-1	0.30	10	96
7	7	6	2	-1	-3	0.23	7.6	96
5	5	4	6	2	0	0.30	11	96
7	7	6	4	0	-2	0.30	9	96
5	5	-2	7	-2	-7	0.08	12	3
1	1	-1	-7	-5	-4	0.18	15	2

Type-I Seesaw

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	N_1	N_2	N_3	ϵ	$\log\Lambda$	NO
6	1	-1	7	7	6	3	0	-4	0.36	14	93
6	1	-1	6	6	6	3	0	-4	0.34	14	93
6	1	-2	7	7	7	5	0	-4	0.37	14	93
7	2	-1	7	7	7	4	0	-5	0.40	14	95
6	2	-6	2	1	1	3	2	-4	0.16	12	90
4	1	-1	6	5	5	6	0	-3	0.27	14	93
4	1	-1	7	5	5	4	0	-3	0.29	14	93
7	2	-1	7	7	6	4	0	-5	0.39	14	95
6	1	-1	7	6	6	3	0	-4	0.35	14	93
5	1	-1	5	5	5	2	0	-3	0.27	14	79

TABLE I: Some of the most natural and realistic FN textures for Dirac, Majorana, and type-I seesaw neutrinos, reproducing masses and mixings with a relative experimental deviation factor $\delta_{\max} < 5,\ 2,\ 1.35$ for approximately 50%, 2-5 % and 0.03% of random $\mathcal{O}(1)$ coefficient choices, respectively. Each texture is specified by the FN charges of the LH lepton doublets (X_{L_i}), RH charged leptons ($X_{\bar{e}_i}$), and RH neutrinos (X_{N_i}). For ϵ and $\log_{10}(\Lambda/\mathrm{GeV})$, we show texture-averages for coefficient choices with $\delta_{\max} < 2$. NO denotes the percentage of coefficient choices that predict normal ordered (NO) neutrino masses.

- Very high FN charges required for Dirac neutrinos.
- FN prefers normal ordering (NO), but Majorana neutrinos can generate IO.
- In Majorana and Seesaw models, $\Lambda = \Lambda_W$ or M is set by $\sum_\nu m_\nu$

Phenomenology

As for quarks, generate random dim6 FV operators for each model in each of the most SM-like charge assignments, rotate to mass basis.

Use `wilson` to run operators to relevant energy scale, and `flavio` to compute low-energy FV observables.

→ 2- and 3-body μ, τ decays

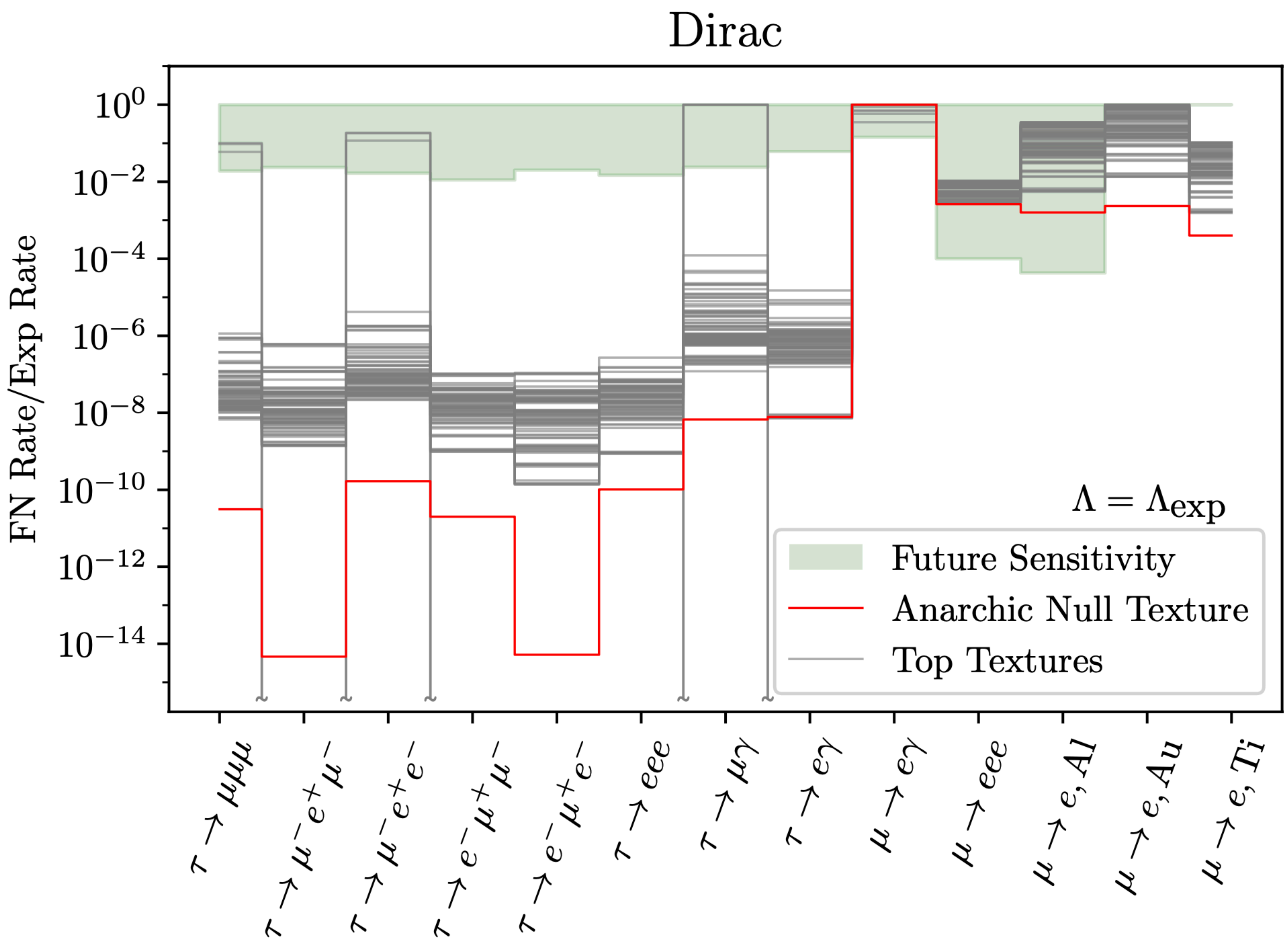
→ μ conversion in nuclei

→ $0\nu\beta\beta$

→ FV dilepton production at future muon colliders

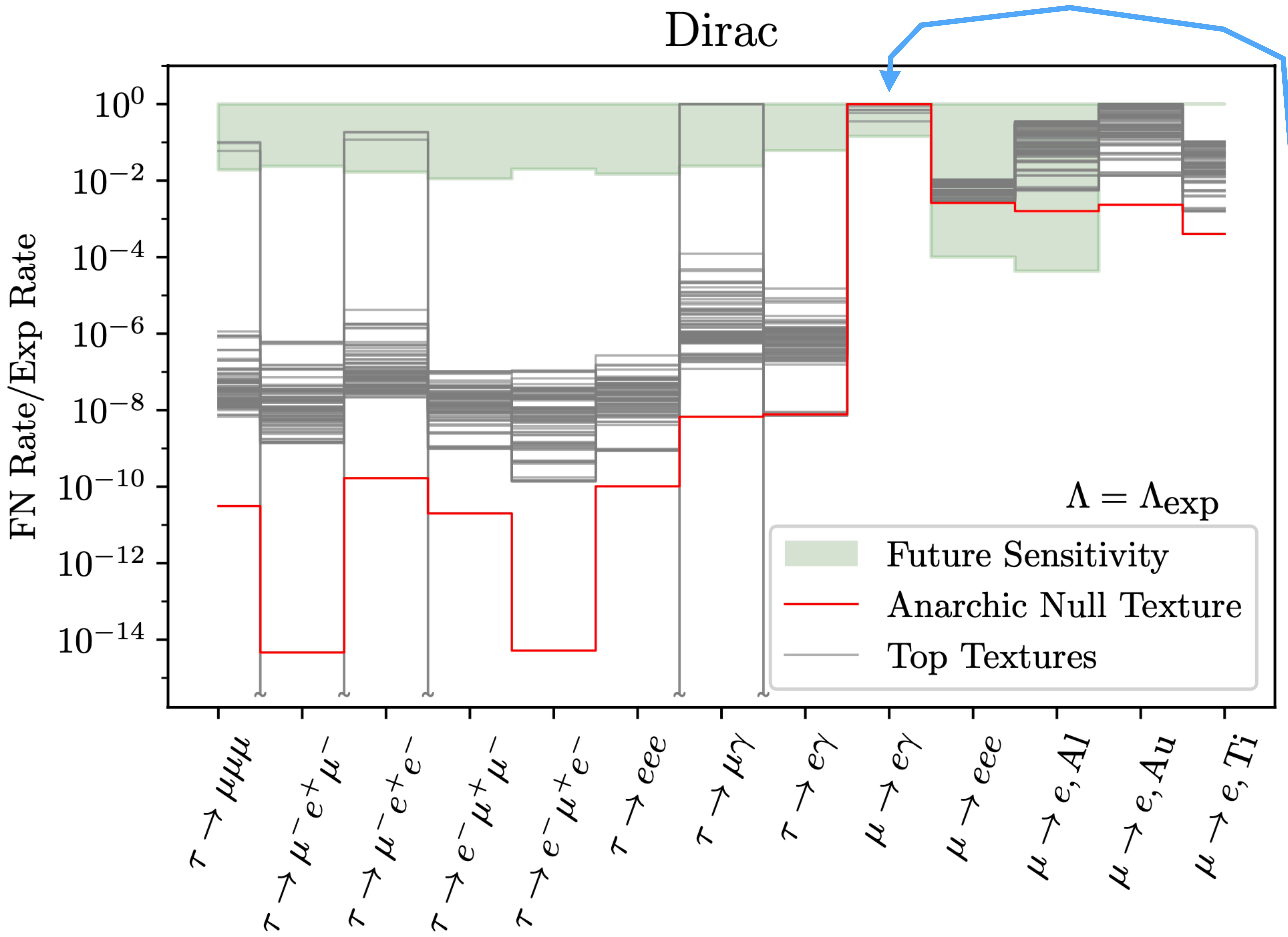
Observable	Current	Ref.	Future	Ref.
$\mathrm{BR}(\mu^+ \to e^+\gamma)$	4.2×10^{-13}	[42]	6×10^{-14}	[57]
$\mathrm{BR}(\mu^+ \to e^+e^-e^+)$	1.0×10^{-12}	[58]	10^{-16}	[59]
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	[60]	2×10^{-9}	[61]
$\mathrm{BR}(\tau \to \mu\gamma)$	4.2×10^{-8}	[62]	10^{-9}	[61]
$\mathrm{BR}(\tau \to eee)$	2.7×10^{-8}	[63]	4×10^{-10}	[61]
$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.1×10^{-8}	[63]	4×10^{-10}	[61]
$\mathrm{BR}(\tau^- \to \mu^+e^-\mu^-)$	2.7×10^{-8}	[63]	4×10^{-10}	[61]
$\mathrm{BR}(\tau^- \to e^+\mu^-\mu^-)$	1.7×10^{-8}	[63]	3×10^{-10}	[61]
$\mathrm{BR}(\tau^- \to e^+\mu^-e^-)$	1.8×10^{-8}	[63]	3×10^{-10}	[61]
$\mathrm{BR}(\tau^- \to \mu^+e^-e^-)$	1.5×10^{-8}	[63]	3×10^{-10}	[61]
$\mathrm{CR}(\mu^-\mathrm{Ti} \to e^-\mathrm{Ti})$	6.1×10^{-13}	[64]	-	-
$\mathrm{CR}(\mu^-\mathrm{Pb} \to e^-\mathrm{Pb})$	4.6×10^{-11}	[65]	-	-
$\mathrm{CR}(\mu^-\mathrm{Au} \to e^-\mathrm{Au})$	7.0×10^{-13}	[43]	-	-
$\mathrm{CR}(\mu^-\mathrm{Al} \to e^-\mathrm{Al})$	-	-	3×10^{-17}	[44]
m_{ee}	36 meV	[46]	3 meV	[48, 49]
$\mu\mu \to e\mu$	-	-	-	-
$\mu\mu \to \mu\tau$	-	-	-	-
$\mu\mu \to e\tau$	-	-	-	-

CLFV with Dirac Neutrinos



Consider 100 most SM-like textures, compare to flavour-anarchic null texture.

CLFV with Dirac Neutrinos

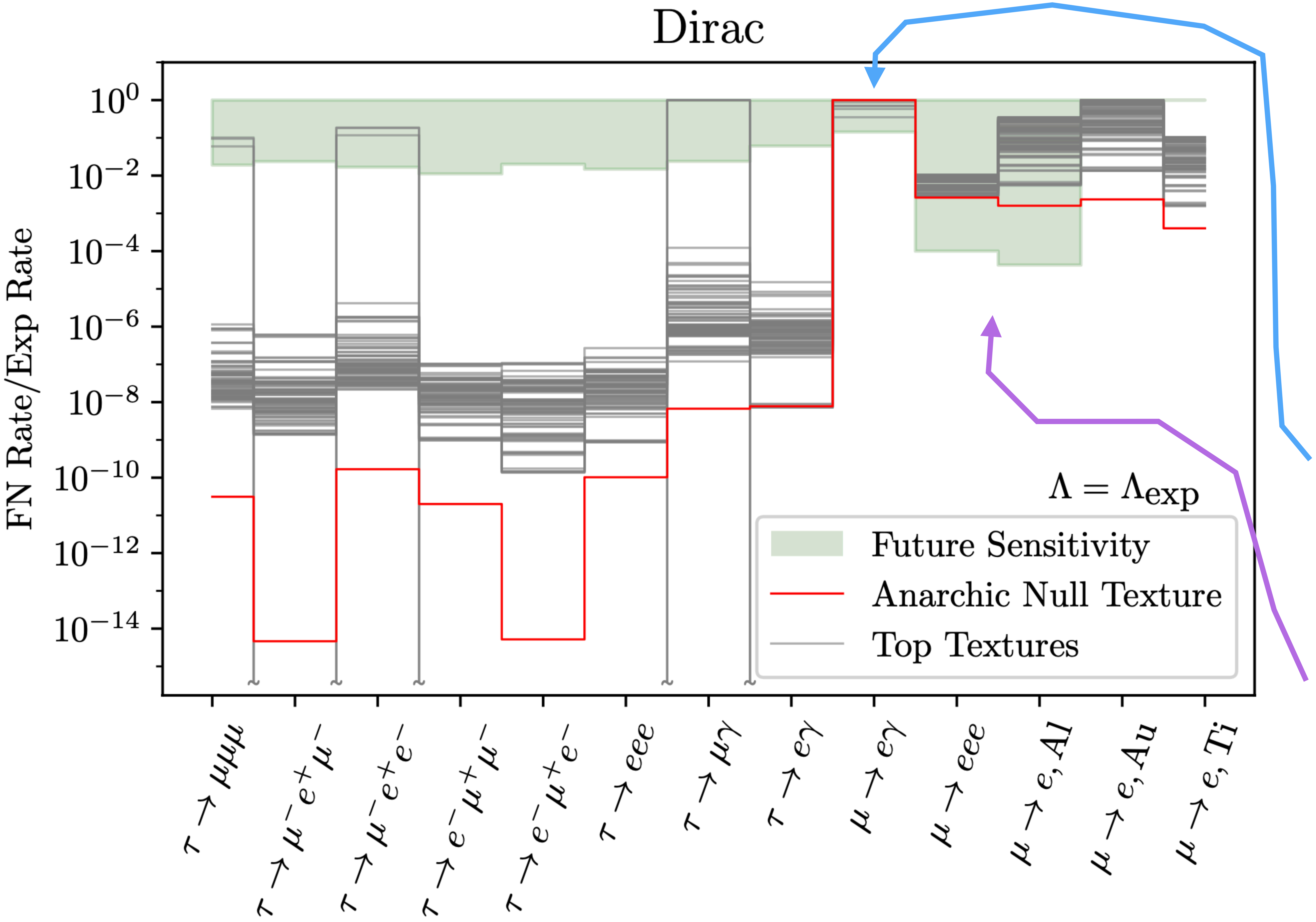


Consider 100 most SM-like textures, compare to flavour-anarchic null texture.

Most stringent constraint on Λ_F (almost) always from $\mu \to e\gamma$.

Let Λ_F saturate constraint $\Rightarrow$ best-case predictions for other observables.

CLFV with Dirac Neutrinos



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$\mu \to eee$ and μ-conversion have future sensitivity, with significant spread between different charge assignments!

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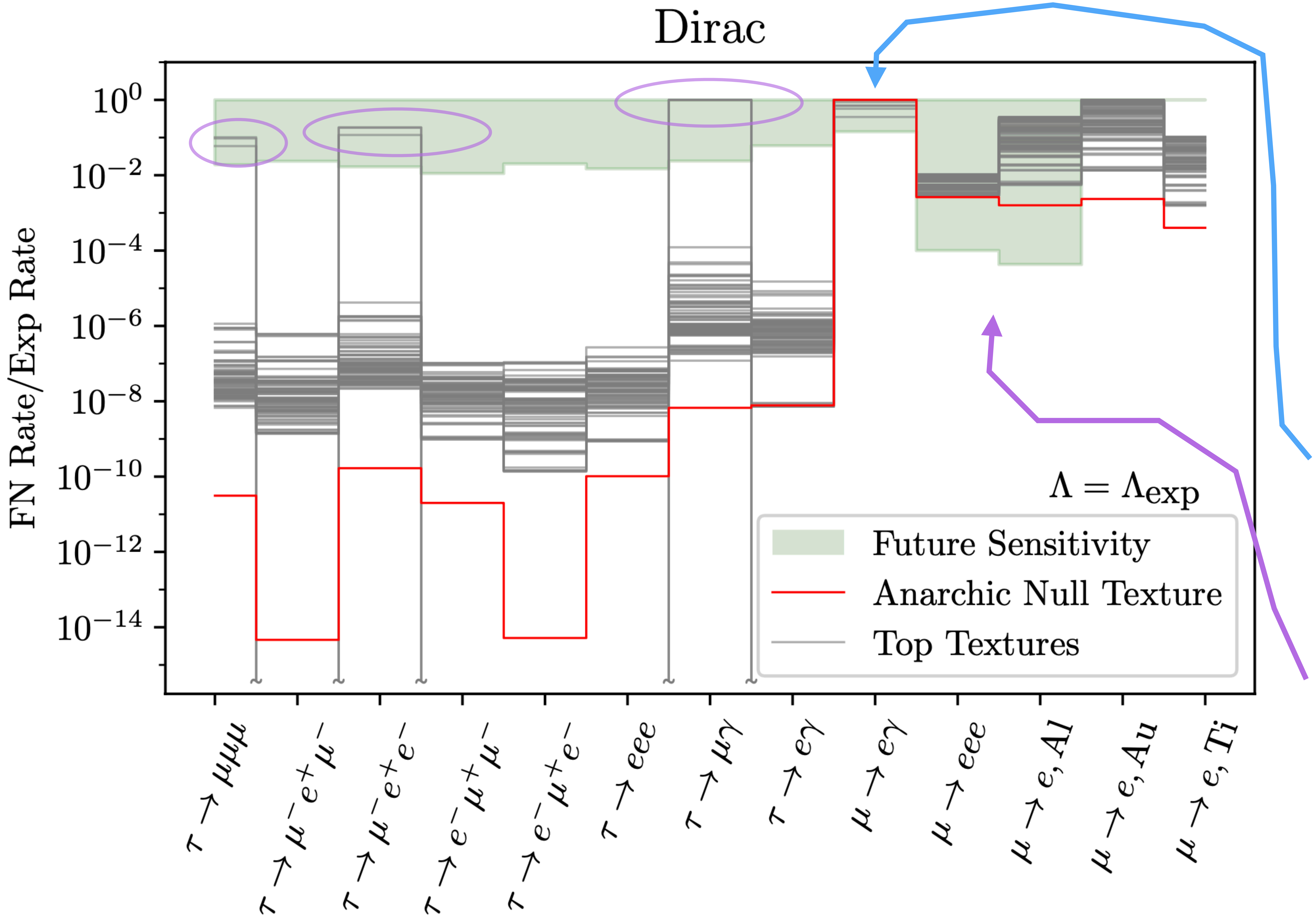

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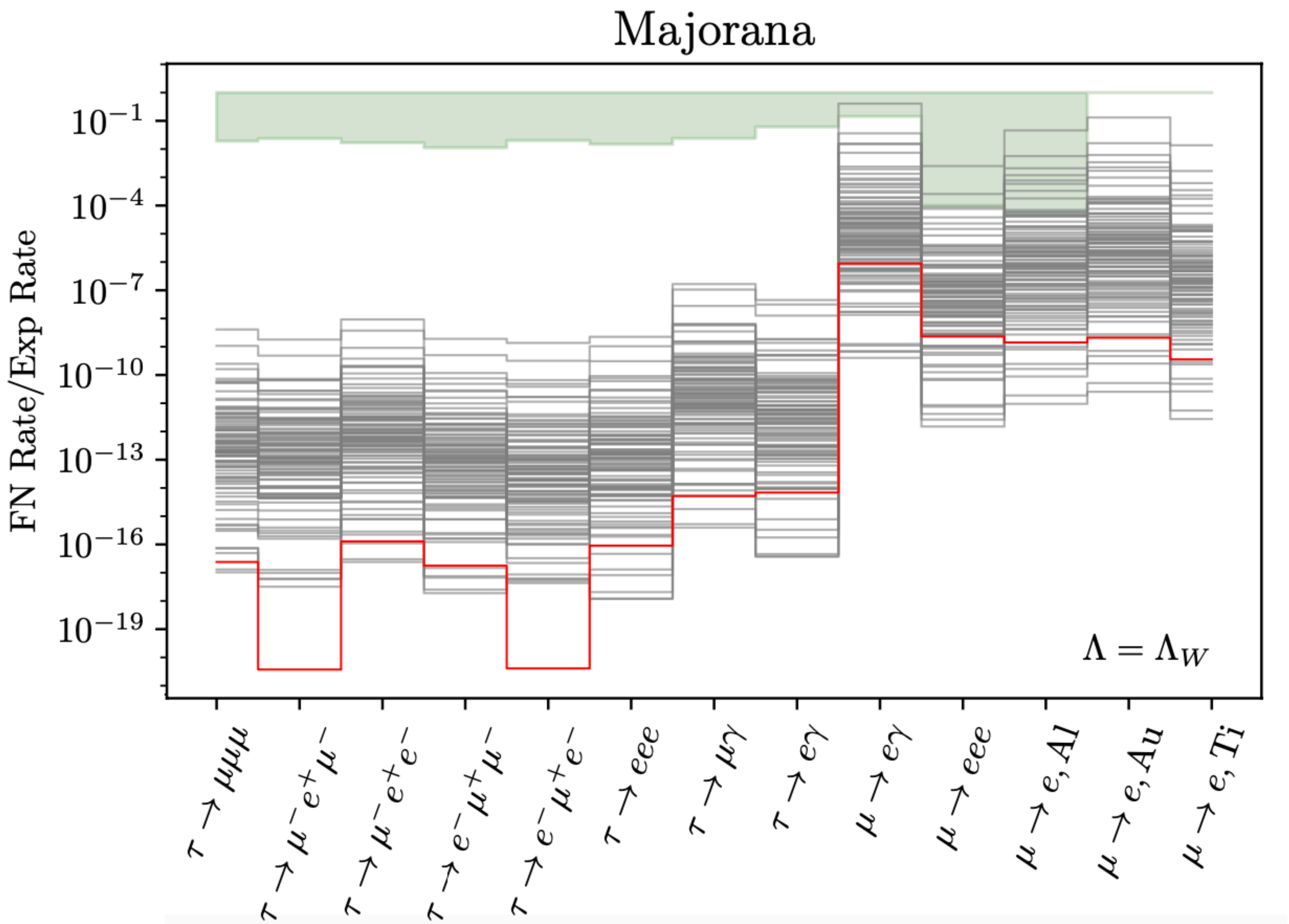
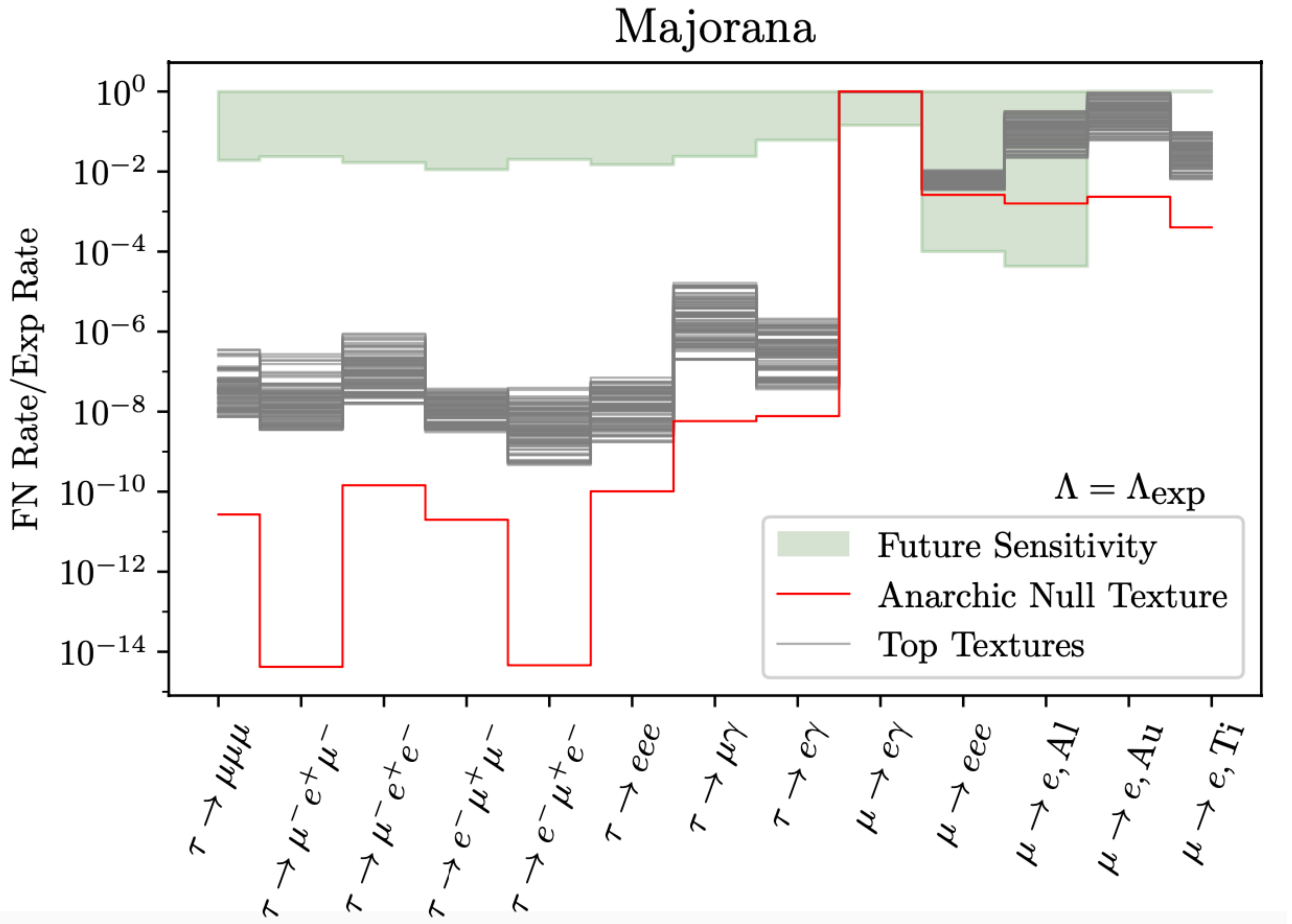
CLFV with Majorana Neutrinos



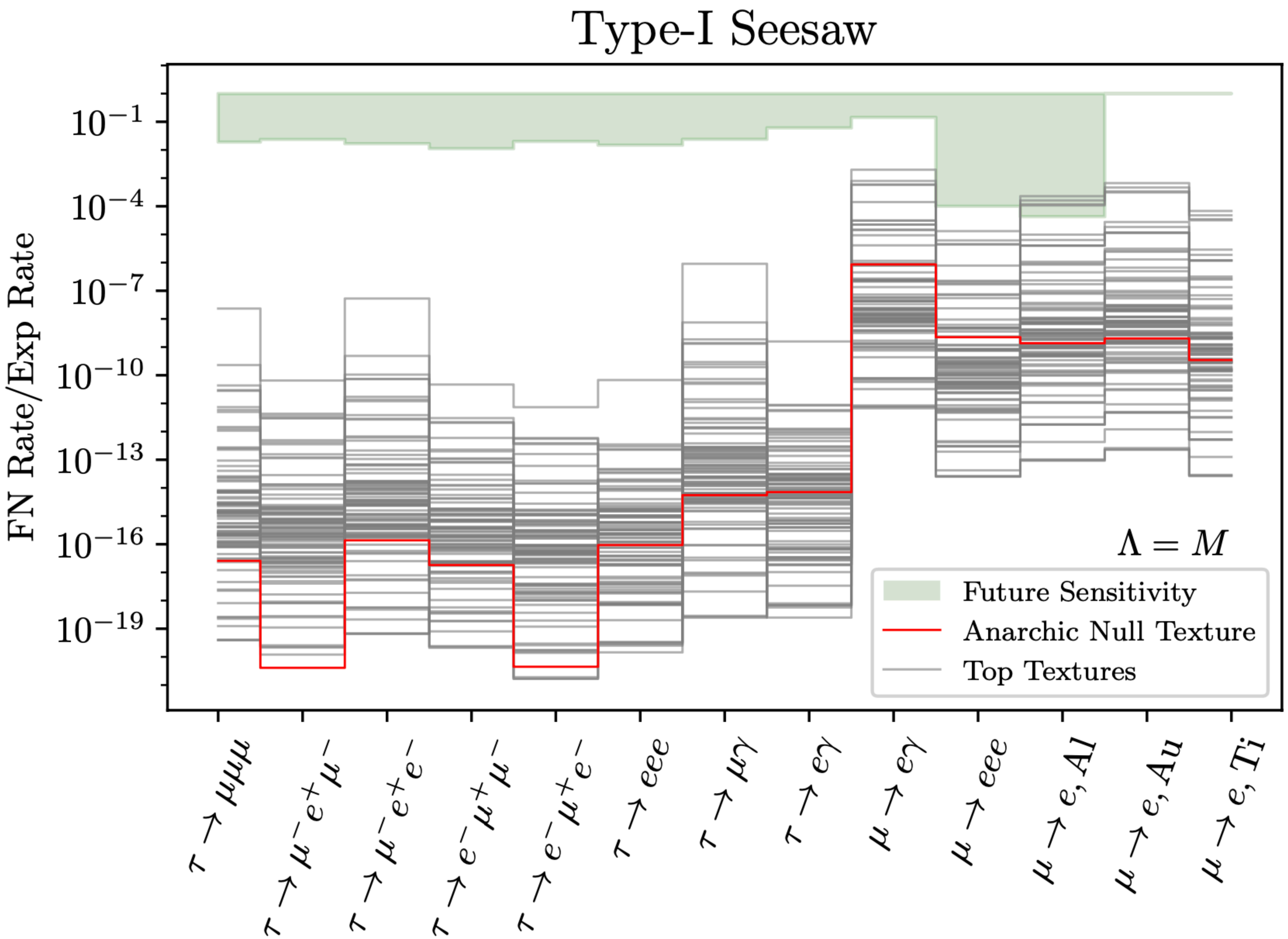


Here we fix $\Lambda_F = \Lambda_W$, thereby FIXING the flavour scale from $\sum_\nu m_\nu$ requirement.

Here we fix $\Lambda_F \neq \Lambda_W$, instead fixing it to $\mu \to e\gamma$ constraint as for Dirac.

Again $\mu \to e\gamma$, $\mu \to eee$ and μ conversion are the most promising observables.

CLFV with Type-I Seesaw



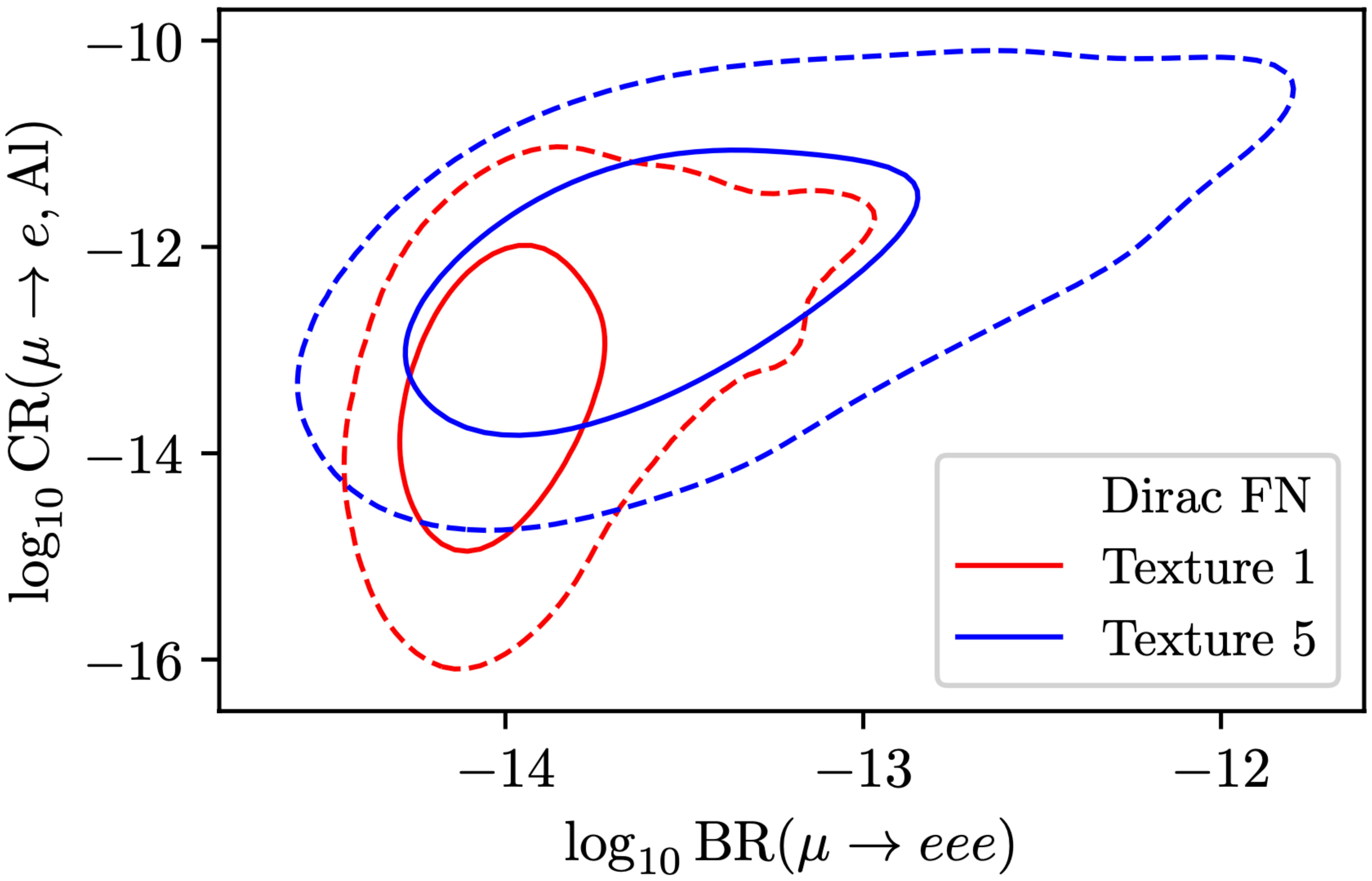
By our assumption that $M = \Lambda_F$, $\sum_\nu m_\nu$ fixes the flavour scale.

Detection prospects of CLFV are dim, some exceptions.

These concrete predictions strongly motivate significant improvement of muon conversion experimental sensitivity.

Significant possibility of detecting and discriminating between FN models.

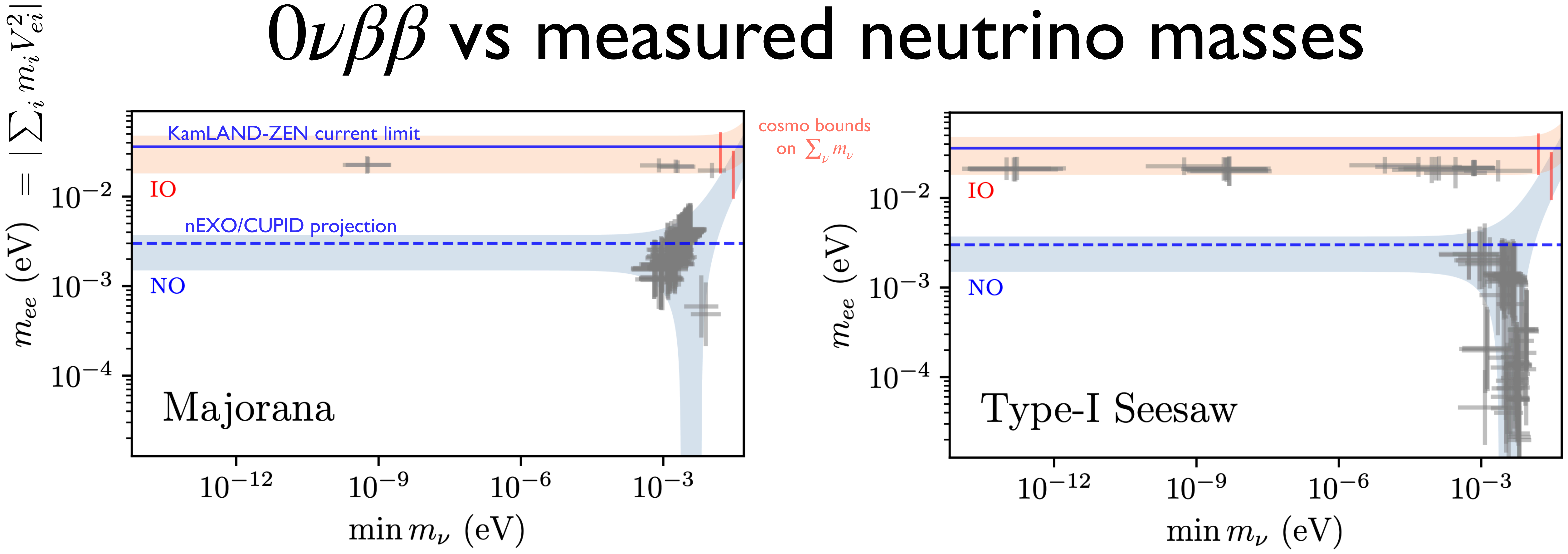
Discriminating between charge assignments



As for the quark case, simultaneous detection of different CLFV signals could strongly favour some charge assignments.

Different charge assignments not only predict different average signals, but also different correlations between signals.

$0\nu\beta\beta$ vs measured neutrino masses

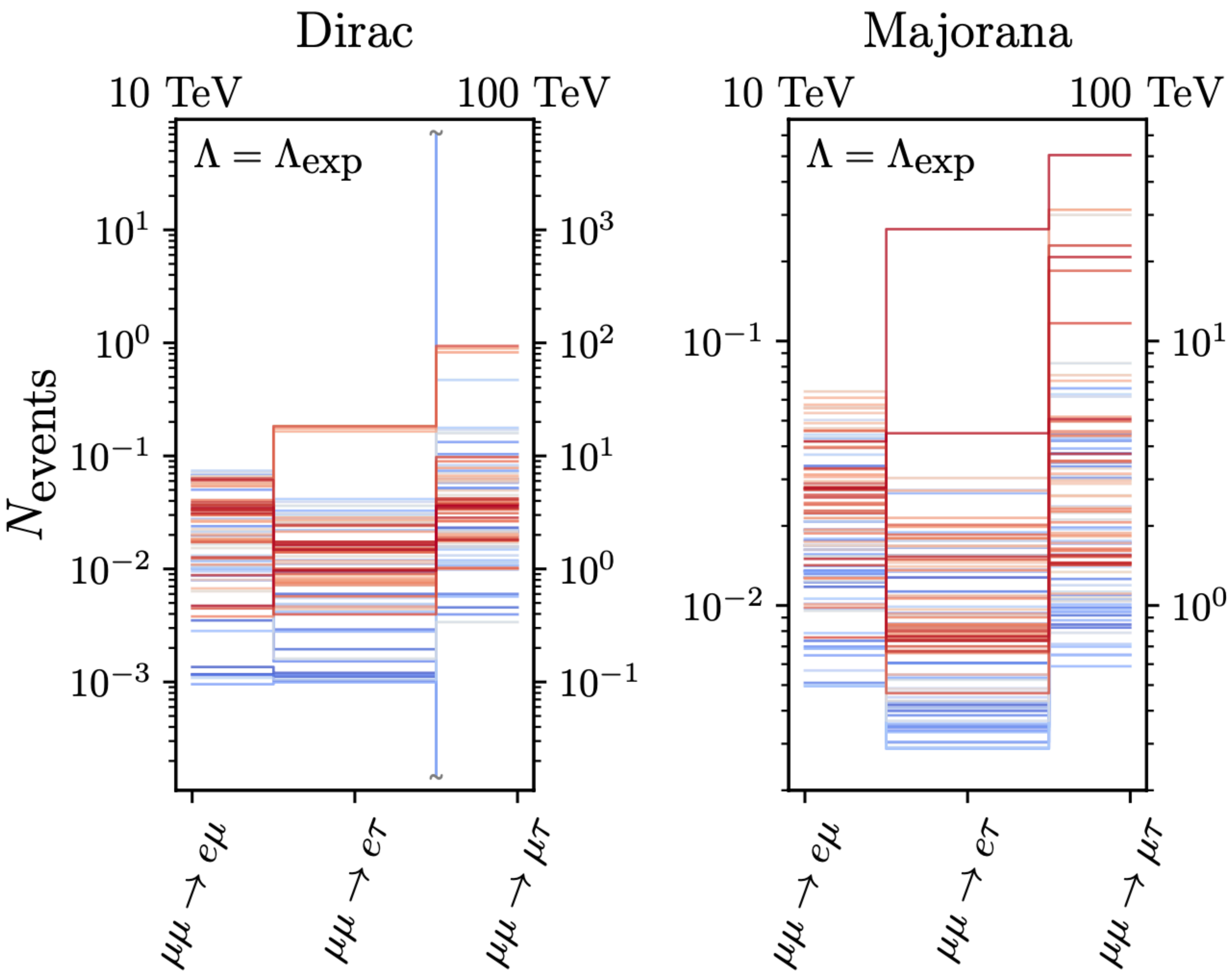


Consider 100 most SM-like charge assignments. Error bars show 1σ spread of predictions.

IO textures in both scenarios *just* avoid KamLAND-ZEN limits.
This is a prediction, not imposed by scan!

All SM-like Majorana textures either detectable at nEXO/CUPID or within ~ x10.
$\Rightarrow$**Could exclude entire FN Majorana scenario in future?**

Future Muon Collider (with $10\mathrm{ab}^{-1}$)



In both cases, set Λ_F to tightest current constraint.

Not very promising for "realistic" proposed $\sqrt{s}$.

Seesaw is even more impossible.

A bit disappointing, but useful information.

Upshot for the lepton sector

FN in the lepton sector predicts signals in $\mu \to e\gamma, \mu \to eee$ and μ conversion.

Strongly motivates new experiment for μ conversion in gold.

Different SM-like charge assignments (can) give distinguishably different correlated predictions for CLFV observables.

$0\nu\beta\beta$ experiments could in principle exclude FN Majorana scenario.

Surprisingly, muon colliders have no sensitivity to CLFV from FN in the lepton sector.

Conclusions and Outlook

Conclusions and Outlook

Our approach generates a truly global and general view of the Froggatt-Nielsen mechanism, and its experimental predictions.

Demonstrates how future measurements can exclude or detect entire types of FN models/mechanisms, and provides guidance on most motivated future experiments.

Lists of most SM-like FN charge assignments is useful to explicitly interrogate how the flavour problem may interact with other BSM mechanisms like origin of DM or solution of hierarchy problem (see Micah's visit in a few months).

Obvious generalizations: combine quark + lepton FN, 2HDM, flavon pheno.

See also 2412.19484 Ibe, Shirai, Watanabe

Our methodology can be applied to other flavour mechanisms. May eventually generate a truly general understanding of the Theory Space of the Flavour Problem.