Velocity-Inferred Hamiltonian Networks: Symplectic Dynamics from Position-Only Observations

Ruichen Xu*, Claire Yu†, Zongyu Wu†, Siyao Wang‡, Luoyao Chen§, Georgios Kementzidis*, Haochun Wang*, Yuefan Deng¶

*Department of Applied Mathematics and Statistics, Stony Brook University, Stony Brook, NY, †Germantown Friends School, Philadelphia, PA, ‡Department of Statistics, University of California, Davis, §New York University, USA, ¶

Corresponding author: yuefan.deng@stonybrook.edu

Introduction

Hamiltonian Neural Networks (HNNs) require positions q and momenta p, limiting their use when only positional data is available [1]. Because momentum and velocity are mathematically invertible, the HNN can be reformulated as a *Velocity-Inferred Hamiltonian Neural Network (VI-HNN)*, which replaces the need for explicit momentum data with inferred velocity.

Methods

We evaluate the VI-HNN model on the spring–mass, pendulum, two-body, and three-body problems.

$$\mathcal{H}(q,p) \mapsto \widetilde{\mathcal{H}}(q,\dot{q})$$

We define the HNN as a feedforward network [2], trained to satisfy Hamilton's equations.

$$\widetilde{\mathcal{H}}_\theta : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$$

Synthetic datasets of noisy position data

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Estimated velocities using the midpoint rule

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Reformulated Hamiltonian with v, trained neural network $\widetilde{\mathcal{H}}_\theta$

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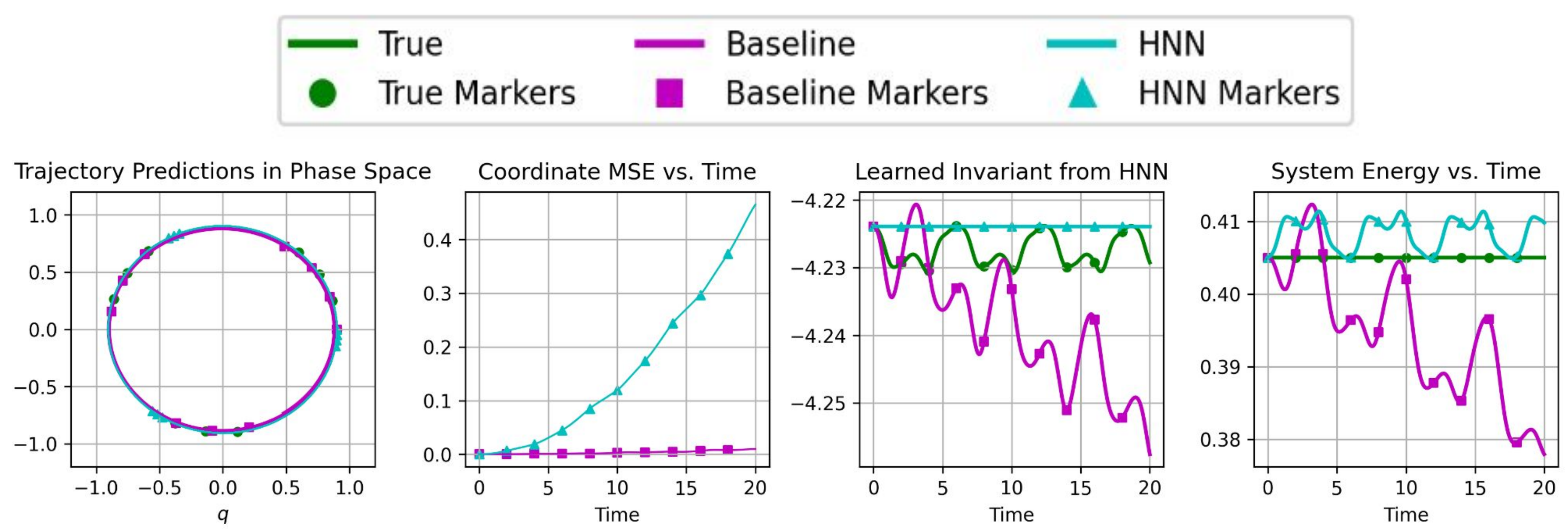
Assessment via trajectory reconstruction & energy conservation

References: *[1] O. Azencot, O. Vantzos, and M. Ovsjanikov, Symplectic neural networks in continuous and discrete time, ACM Transactions on Graphics, 41 (2022), pp. 1–14, https://doi.org/10.1145/3514354.*
[2] S. Greydanus, M. Dzamba, and J. Yosinski, Hamiltonian neural networks, in Advances in Neural Information Processing Systems, vol. 32, 2019.

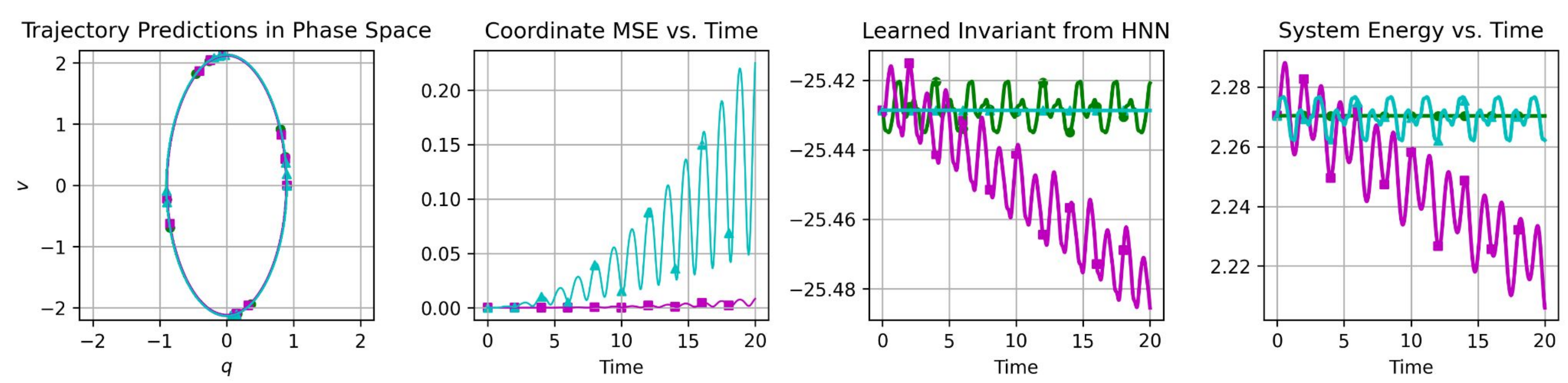
Results

System	Model	Train Loss	Test Loss	Energy
Spring–Mass	Baseline	5.32 ± 0.16	5.46 ± 0.18	0.30 ± 0.04
	HNN	4.25 ± 0.133	4.09 ± 0.14	0.030 ± 0.0015
Simple Pendulum	Baseline	20.58 ± 0.40	20.58 ± 0.40	438 ± 164
	HNN	20.9 ± 0.40	20.7 ± 0.42	187.79 ± 39.20
Two-Body†	Baseline	1.09 ± 0.036	1.03 ± 0.067	1685.0 ± 77.0
	HNN	0.73 ± 0.038	0.71 ± 0.069	1.56 ± 0.56
Three-Body	Baseline	3.79 ± 0.13	3.97 ± 0.25	95.73 ± 34
	HNN	4.53 ± 0.22	4.94 ± 0.47	0.59 ± 0.14

Figure 1: Performance of Baseline and HNN models.



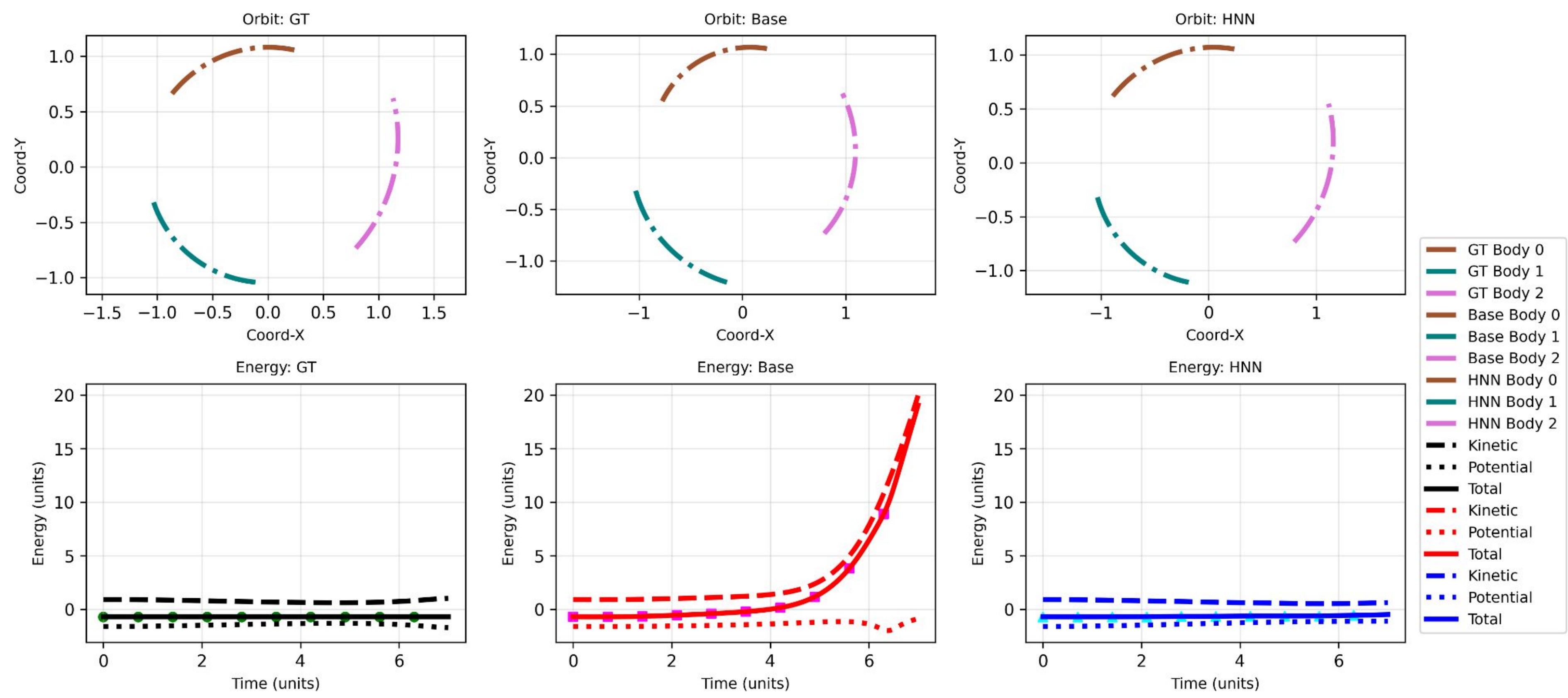
(a) Models on the spring-mass system. The baseline drifts from ground truth and diverges in MSE, while HNN preserves energy- like quantities.



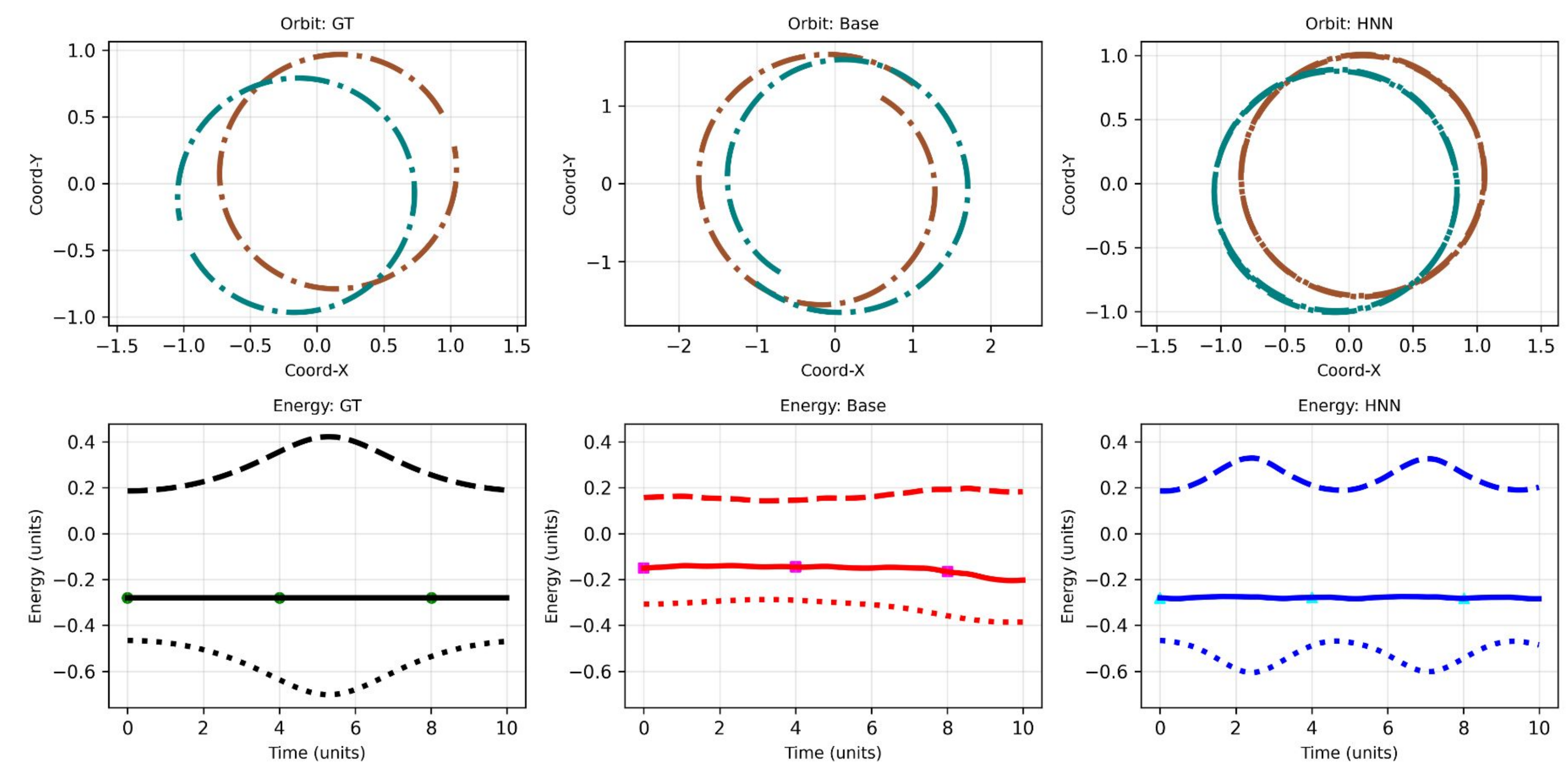
(b) Models on the pendulum system. HNN remains accurate and conserves energy-like quantities, unlike the diverging baseline.

Figure 2: Comparison of baseline models and HNNs.

Results



(c) Two-body problem. HNN accurately follows ground truth and conserves total energy, unlike the baseline model.



(d) Three-body problem. HNN approximates conserved dynamics, outperforming the baseline.

Across all tested systems, VI-HNN maintains long-term trajectory stability and near-exact energy conservation while relying solely on position observations, demonstrating its effectiveness as a robust extension of Hamiltonian learning methods to scenarios where momentum data is unavailable.

Future Work

Extend VI-HNN to systems with partially known mass matrices or unknown geometries, investigate robustness under high noise, sparse sampling, and high-dimensional systems.