Spectral Functions from the Functional Renormalization Group

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EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Outline

- in-medium properties of parity partners
- the Functional Renormalization Group
- consistent spectral functions
- vector mesons in the FRG

Functional Renormalization Group

partition function: (scalar field $\phi(x)$)

$$Z[j] = e^{W[j]} = \int [\mathcal{D}\phi] \; e^{-S[\phi] + \int d^4x \; \phi(x) j(x)}$$

effective action: (Legendre transform of W)

$$\Gamma[\varphi] = -W[j] + \int d^4x \; \varphi(x) j(x); \quad \varphi(x) \equiv \langle \phi(x) \rangle$$

stationarity condition and thermodynamic potential:

$$\left. \frac{\delta \Gamma[\varphi]}{\delta \varphi} \right|_{\varphi = \varphi_0} = 0; \quad \rightarrow \quad \Omega(T;\mu_i) = \frac{T}{V} \Gamma[\varphi_0]$$

Wilsonian coarse graining:

$$\phi(x) = \phi_{q \le k}(x) + \phi_{q > k}(x)$$

$$\rightarrow \quad Z[j] = \int [\mathcal{D}\phi_{q \le k}] \underbrace{\int [\mathcal{D}\phi_{q > k}] \; e^{-S[\phi] + \int d^4x \; \phi j}}_{= Z_k[j]}; \quad \lim_{k \to 0} Z_k[j] = Z[j]$$

Functional Renormalization Group

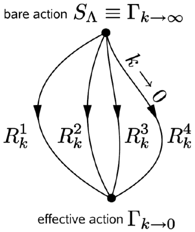
flow equation for Γ_k including bosons and fermions:

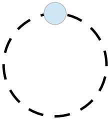
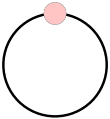
$$\partial_k \Gamma_k[\varphi, \psi] = \mathrm{Tr} \int_q \left(\frac{1}{2} G_{\varphi,k}(q) R_{\varphi,k}(q) - G_{\psi,k}(q) R_{\psi,k}(q) \right)$$

$$G_{\varphi,k}(q) = \left[\Gamma_k^{(2)}[\varphi] + R_{\varphi,k}(q) \right]^{-1}$$

$$G_{\psi,k}(q) = \left[\Gamma_k^{(2)}[\psi] + R_{\psi,k}(q) \right]^{-1}$$

$$\Gamma_k^{(2)}[\varphi] = \frac{\delta^2 \Gamma_k[\varphi, \psi]}{\delta \varphi^2}; \quad \Gamma_k^{(2)}[\psi] = \frac{\delta^2 \Gamma_k[\varphi, \psi]}{\delta \psi \delta \bar{\psi}}$$



$\partial_k \Gamma_k \;=\; \frac{1}{2}$  $-$ 

Quark-meson model

- effective low-energy model for QCD ($N_f = 2$)
- pion and sigma fields $\phi = \sigma + i\vec{\tau}\vec{\pi}$ and quarks $\bar{q}, q$
- describes spontaneous and explicit chiral symmetry breaking

Scale-dependent effective action: (gradient expansion)

$$\begin{aligned}\Gamma_k[q,\bar{q},\phi] &= \int_x \Big\{ \bar{q} Z_{q,k} \left(\partial\!\!\!/ - \mu\gamma_0\right) q + h_{q,k} \bar{q} \left(\sigma + i\vec{\tau}\vec{\pi}\gamma_5\right) q \\ &+ \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2 + \frac{1}{8} Y_{\phi,k} \left(\partial_\mu \phi^2\right)^2 + U_k(\phi^2) - c\sigma + \cdots \Big\}\end{aligned}$$

$$\int_x \equiv \int_0^{1/T} dx_0 \int_V d^3x$$

'Local potential approximation': (only U_k flows)

$$U_\Lambda(\phi^2) = \vec{\pi}^2 + \sigma^2; \quad \langle \sigma \rangle = 0$$

Momentum flow of the effective action

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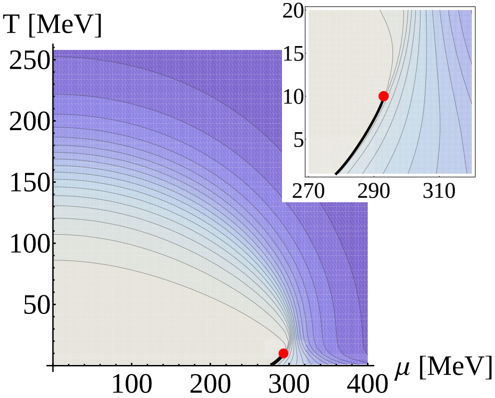
Phase diagram of the Quark-Meson Model

- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- vacuum: $\sigma_0 = 93.5$ MeV, $m_\pi = 138$ MeV, $m_\sigma = 509$ MeV, $m_q = 299$ MeV



In-medium spectral functions from the FRG

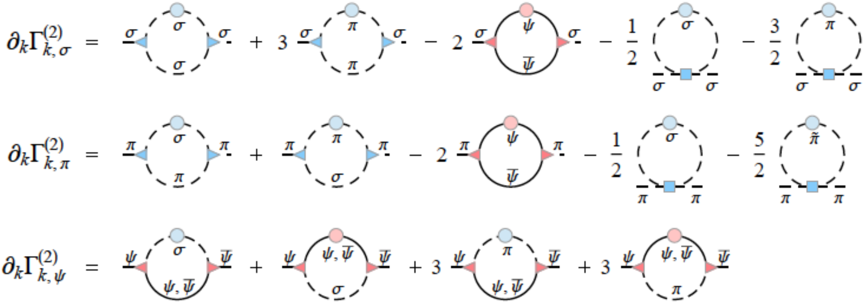
How are in-medium modifications of hadrons related to the change of the vacuum structure of QCD? (deconfinement and chiral symmetry restoration ..)

- **need to calculate equilibrium properties and spectral function on the same footing → FRG**
  - equilibrium FRG formulated and solved in Euclidean space-time
  - flow equations for Euclidan two-point functions:

$$\partial_k \Gamma_k^{(2)} = \frac{1}{2}\frac{\delta^2}{\delta\varphi^2}\mathrm{Tr_q}\left\{G_{\varphi,k}R_{\varphi,k})\right\} - \frac{\delta^2}{\delta\psi\bar{\psi}}\mathrm{Tr_q}\left\{G_{\psi,k}R_{\psi,k}\right\}$$

$$= \mathrm{Tr_q}\left\{\partial_k R_{\varphi,k}\left(G_{\varphi,k}\Gamma_k^{(3)}G_{\varphi,k}\Gamma_k^{(3)}G_{\varphi,k}\right)\right\} - \frac{1}{2}\mathrm{Tr_q}\left\{\partial_k R_{\varphi,k}\left(G_{\varphi,k}\Gamma_k^{(4)}G_{\varphi,k}\right)\right\} + \cdots$$

Flow equations for two-point functions



- mesonic vertices from scale-dependent effective potential, $U_k^{(3)}$, $U_k^{(4)}$
- quark-meson vertices are givn by $\Gamma^{(3)}_{\bar\psi\psi\sigma} = h, \quad \Gamma^{(3)}_{\bar\psi\psi\vec\pi} = ih\gamma^5\vec\tau$
- one-loop structure preserved
- thermodynamically consistent and symmetry preserving

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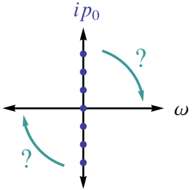
$$= \mathrm{Tr}_\mathrm{q}\left\{\partial_k R_{\varphi,k}\left(G_{\varphi,k}\Gamma_k^{(3)}G_{\varphi,k}\Gamma_k^{(3)}G_{\varphi,k}\right)\right\} - \frac{1}{2}\mathrm{Tr}_\mathrm{q}\left\{\partial_k R_{\varphi,k}\left(G_{\varphi,k}\Gamma_k^{(4)}G_{\varphi,k}\right)\right\} + \cdots$$

  - spectral functions are real-time quantities!
  - analytic continuation procedure is needed



Analytic continuation

- use periodicity of occupation numbers with discrete Euclidean energy $ip_0 = i2n\pi T$

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

- substitute p_0 by continuous real frequency ω

$$\Gamma^{(2),R}_{k,j}(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma^{(2),E}_{k,j}(p_0 = -i(\omega + i\epsilon),\vec{p}); \quad \text{for} \quad j = \pi,\sigma$$

- solve flow equations $\mathrm{Re}\,\partial_k\Gamma^{(2),R}_{k,j}$, $\mathrm{Im}\,\partial_k\Gamma^{(2),R}_{k,j}$ at global minimum of $U_{k\to 0}$

- finally obtain spectral functions as

$$\rho_j(\omega,\vec{p}) = \frac{1}{\pi}\frac{\mathrm{Im}\,\Gamma^{(2),R}_{k\to 0,j}(\omega,\vec{p})}{\left(\mathrm{Re}\,\Gamma^{(2),R}_{k\to 0,j}(\omega,\vec{p})\right)^2 + \left(\mathrm{Im}\,\Gamma^{(2),R}_{k\to 0,j}(\omega,\vec{p})\right)^2}$$

Flow of the σ and π spectral functions in vaccum

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σ- and π spectral function for finite T at $\mu = 0$

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σ spectral function with increasing T at $\mu = 0$

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π spectral function with increasing T at $\mu = 0$

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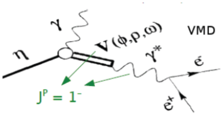
Modelling vector mesons

- Sakurai (1960): vector mesons as gauge bosons of local gauge symmetry $SU(2)$
  - → Electromagnetic-hadronic interaction via exchange of vector mesons
  - → Current field identity (CFI):

$$j^{\mu}_{\text{em}} = \frac{m_\rho^2}{g_\rho}\rho^\mu + \frac{m_\omega^2}{g_\omega}\omega^\mu + \frac{m_\phi^2}{g_\phi}\phi^\mu$$

⇒ **Vector Meson Dominance (VMD)**



[Berghaeuser, www.staff.uni-giessen.de (2016)]

- Lee and Nieh (1960s): Gauged linear sigma model, local gauge symmetry $SU(2)_L \times SU(2)_R$

⇒ $\boldsymbol{\rho}$ **meson** and chiral partner $\boldsymbol{a_1}$ **meson** as gauge bosons

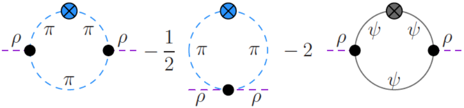
Gauged Liner Sigma Model with Quarks

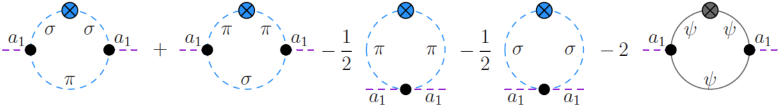
- Local gauge symmetry $SU(2)_L \times SU(2)_R$: Low-energy model of two-flavor QCD
- Ansatz for the effective average action $\Gamma_k \equiv \Gamma_k[\sigma, \pi, \rho, a_1, \psi, \bar{\psi}]$ (LPA):

$$\begin{aligned}\Gamma_k = \int d^4x \Big\{ & \bar{\psi} \left(\partial\!\!\!/ - \mu\gamma_0 + h_S \left(\sigma + \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5 \right) + \mathrm{i}h_V \left(\gamma_\mu \vec{\tau}\vec{\rho}^{\mu} + \gamma_\mu\gamma_5\vec{\tau}\vec{a}_1^{\mu} \right) \right) \psi + U_k(\phi^2) - c\sigma \\ & + \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{8}\mathrm{Tr}\left(\partial_\mu V_\nu - \partial_\nu V_\mu \right)^2 + \frac{1}{4} m_{V,k}^2 \mathrm{Tr} V_\mu V_\mu - \mathrm{i}g V_\mu \Phi \partial_\mu \Phi \\ & - \frac{1}{2} g^2 \left(V_\mu \Phi \right)^2 - \frac{\mathrm{i}}{2} g \mathrm{Tr} \partial_\mu V_\mu [V_\mu, V_\nu] - \frac{1}{4} g^2 \mathrm{Tr} V_\mu V_\nu [V_\mu, V_\nu] \Big\}\end{aligned}$$

- Mesonic fields: $\phi \equiv (\vec{\pi}, \sigma)$ and $V_\mu \equiv \vec{\rho}_\mu \vec{T} + \vec{a}_{1,\mu} \vec{T}^5$
- Scale dependent quantities: $U_k, m_{k,V}^2$

Flow equations for two-point functions

$\partial_k \Gamma^{(2)}_{\rho,k} =$ 

$\partial_k \Gamma^{(2)}_{a_1,k} =$ 

- Neglect vector mesons inside the loops
- Vertices extracted from ansatz of the effective average action Γ_k
- Tadpole diagrams give ω-independent contributions

ρ- and a_1 spectral function for finite T at $\mu = 0$

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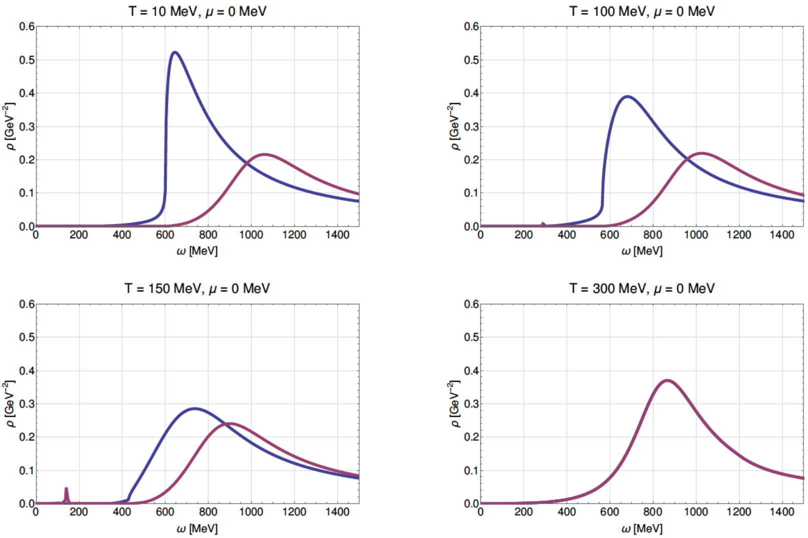
T-dependence of V-A spectral functions



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

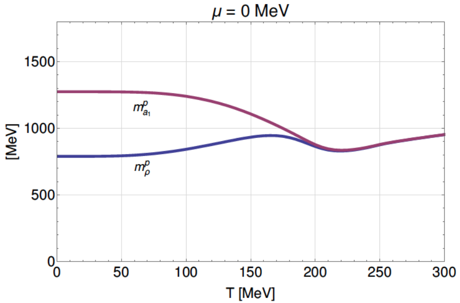
T-dependent pole masses

- Degeneration of ρ and a_1 spectral functions in chirally symmetric phase

- Broadening of spectral functions with increasing T

- Pole masses do not vary much, no dropping ρ mass

$\Rightarrow$ Consistent with broadening/melting-ρ-scenario



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

ρ spectral function with increasing T at $\mu = 0$

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Summary and Outlook

- analytically continued flow equations for scalar- and vector-meson two-point functions including parity partners within the FRG
- chiral order parameter and in-medium spectral functions obtained within the **same** theoretical framework
- complete degeneration of the $(\omega, \vec{p})$-dependent spectral functions of parity partners in the chirally restored phase
- degeneracy of ρ and a_1 spectral functions consistent with broading ρ-scenario

work in progress:

- improve truncation (e.g. include wave function renormalization
- improve phenomenology (e.g. include vetor mesons in side loops

aim: provide spectral functions for coarse-grained transport simulations to compute dilepton spectra