Multi moment cancellation of participant fluctuations - MMCP method

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- The only experimentally controllable way to probe the **QCD phase diagram** is by studying interactions of **different system size** nuclei at various **energies**.
- **Participant fluctuations** is one of the **main background** effects in such study.
- It is the number of nucleons, N_P, that interacted inelastically and produced other particles during the collision.
- These fluctuations may hide the fluctuations from other sources.

There are several popular ways of reducing participant fluctuations:

(i) the selection of as **narrow centrality bins** as possible

(ii) the Centrality Bin Width Correction procedure (**CBWC**) (STAR, Luo (2011)),

(iii) the use of **strongly intensive** quantities (Gazdzicki, Mrowczynski (1992), Gorenstein, Gazdzicki (2011)),

We propose a different approach - to **cancel participant fluctuations** in a **combination** of several **high** fluctuation **moments** (V.B., Mackowiak-Pawlowska 1705.01110).

Fluctuation measures

A **multiplicity distribution**, $P(N)$, can be characterized by **central moments**, μ_n,

$$m_n = \sum_N (N - \langle N \rangle)^n P(N) , \qquad \text{where} \quad \langle N^n \rangle = \sum_N N^n P(N) .$$

They are related to **cumulants**, κ_n,

$$\kappa_2 = m_2 , \qquad \kappa_3 = m_3 , \qquad \kappa_4 = m_4 - 3m_2^2 , \qquad \ldots ,$$

and **susceptibilities**, χ_n,

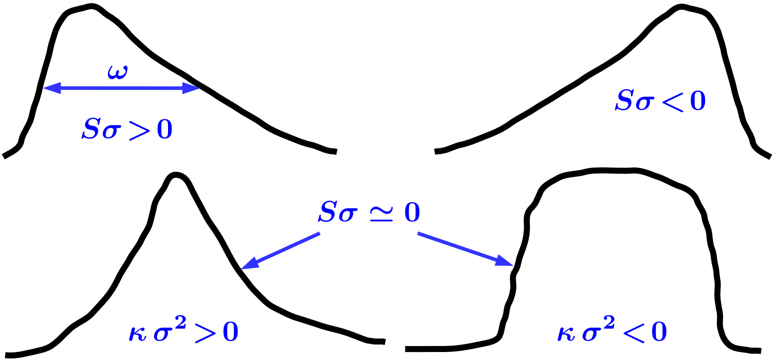
$$\chi_n = \frac{\partial^n (\mathcal{P}/T^4)}{\partial(\mu/T)^n} = \frac{\kappa_n}{V\,T^3} , \qquad \chi_{n,k} = \frac{\chi_n}{\chi_k} = \frac{\kappa_n}{\kappa_k} ,$$

where $\mathcal{P}$ is pressure, T - temperature, μ - chemical potential, and V - volume.

Frequently used cumulant ratios - **scaled variance**, normalized **skewness** and normalized **kurtosis** - are:

$$\omega = \frac{\kappa_2}{\langle N \rangle} = \frac{\sigma^2}{\langle N \rangle} , \qquad S\,\sigma = \frac{\kappa_3}{\kappa_2} , \qquad \kappa\,\sigma^2 = \frac{\kappa_4}{\kappa_2} ,$$

where $\sigma = \sqrt{\langle N^2 \rangle - \langle N \rangle^2} = \sqrt{\kappa_2}$ is **standard deviation**.



Poisson distribution: $\omega = S\sigma = \kappa\sigma^2 = 1$

Normal (Gauss) distribution: ω - **free** parameter, $S\sigma = \kappa\sigma^2 = 0$

Log-normal distribution: $\omega \sim \langle N \rangle,\ S\sigma \sim \langle N \rangle,\ \kappa\sigma^2 \sim \langle N \rangle^2$

- The approach `**just take negative binomial** (Poisson, Gauss...)' is **not working**, because it imposes a certain relation between moments, which might not exist.

Independent participant model

The only assumptions are that participants are **identical** and **independent**. Then mean multiplicity N is the sum of contributions from N_{P} participants,

$$N = n_1 + n_2 + \ldots + n_{N_{\mathrm{P}}}, \qquad \text{and} \quad \langle N \rangle = \langle N_{\mathrm{P}} \rangle \langle n_A \rangle ,$$

where ***identical*** and ***independent*** means that, $\langle n_i \rangle = \langle n_j \rangle = \langle n_1 \rangle = \langle n_A \rangle$, and $\langle n_i\, n_j \ldots n_k \rangle = \langle n_A \rangle^k$. Using multinoial theorem

$$\left(n_1 + n_2 + \ldots + n_{N_{\mathrm{P}}}\right)^k = \sum_{k_1, k_2, \ldots k_{N_{\mathrm{P}}}} \frac{k!}{k_1! k_2! \ldots k_{N_{\mathrm{P}}}!} n_1^{k_1} n_2^{k_2} \ldots n_{N_{\mathrm{P}}}^{k_{N_{\mathrm{P}}}} \, \delta\left(k - \sum_{i=1}^{N_{\mathrm{P}}} k_i\right),$$

where δ is the Kronecker delta, one can obtain arbitrarily high moments, e.g.

$$\omega = \omega_A + \langle n_A \rangle \, \omega_{\mathrm{P}} ,$$

$$S\,\sigma = \frac{\omega_{\mathrm{A}} \, S_{\mathrm{A}} \, \sigma_{\mathrm{A}} + \langle n_{\mathrm{A}} \rangle \, \omega_{\mathrm{P}} \left[3\, \omega_{\mathrm{A}} + \langle n_{\mathrm{A}} \rangle \, S_{\mathrm{P}} \, \sigma_{\mathrm{P}} \right]}{\omega_{\mathrm{A}} + \langle n_{\mathrm{A}} \rangle \, \omega_{\mathrm{P}}} ,$$

$$\kappa\,\sigma^2 = \frac{\omega_{\mathrm{A}} \, \kappa_{\mathrm{A}} \, \sigma_{\mathrm{A}}^2 + \langle n_{\mathrm{A}} \rangle \, \omega_{\mathrm{P}} \left[\langle n_{\mathrm{A}} \rangle^2 \, \kappa_{\mathrm{P}} \, \sigma_{\mathrm{P}}^2 + \omega_{\mathrm{A}} \left(3\, \omega_{\mathrm{A}} + 4\, S_{\mathrm{A}} \, \sigma_{\mathrm{A}} + 6 \langle n_{\mathrm{A}} \rangle \, S_{\mathrm{P}} \, \sigma_{\mathrm{P}} \right) \right]}{\omega_{\mathrm{A}} + \langle n_{\mathrm{A}} \rangle \, \omega_{\mathrm{P}}} ,$$

red - what we would like to measure, **black** - what we measure, **blue** - participant fluctuations (V.B. 1606.05358, Skokov, Friman, Redlich (2013), Braun-Munzinger, Rustamov, Stachel (2017))

- A moment of a rank n is a function of all lower moments for both participants, $\langle N_{\mathrm{P}}^{n}\rangle$, and a source, $\langle n_{\mathrm{A}}^{n}\rangle$,

$$\langle N^{n}\rangle ~=~ \mathcal{F}\left(\langle n_{\mathrm{A}}^{1}\rangle,\langle n_{\mathrm{A}}^{2}\rangle,\ldots\langle n_{\mathrm{A}}^{n}\rangle,~\langle N_{\mathrm{P}}^{1}\rangle,\langle N_{\mathrm{P}}^{2}\rangle,\ldots\langle N_{\mathrm{P}}^{n}\rangle\right)~.$$

  Therefore, one has only n **measures**, but $2n$ **unknowns** for their description.

- **Strongly intensive** measures **require** two types of values, e.g. pions and kaons,

$$\langle N_{\mathrm{A}}^{n}\rangle ~=~ \mathcal{F}\left(\langle n_{\mathrm{A}}^{1}\rangle,\langle n_{\mathrm{A}}^{2}\rangle,\ldots\langle n_{\mathrm{A}}^{n}\rangle,~\langle N_{\mathrm{P_A}}^{1}\rangle,\langle N_{\mathrm{P_A}}^{2}\rangle,\ldots\langle N_{\mathrm{P_A}}^{n}\rangle\right)~,$$
$$\langle N_{\mathrm{B}}^{n}\rangle ~=~ \mathcal{F}\left(\langle n_{\mathrm{B}}^{1}\rangle,\langle n_{\mathrm{B}}^{2}\rangle,\ldots\langle n_{\mathrm{B}}^{n}\rangle,~\langle N_{\mathrm{P_B}}^{1}\rangle,\langle N_{\mathrm{P_B}}^{2}\rangle,\ldots\langle N_{\mathrm{P_B}}^{n}\rangle\right)~.$$

  and the **assumption** that all corresponding participant fluctuations moments are the same $\langle N_{\mathrm{P_A}}^{n}\rangle = \langle N_{\mathrm{P_B}}^{n}\rangle = \langle N_{\mathrm{P}}^{n}\rangle$, which gives $3n$ **unknowns** for $2n$ **measured** values.

- **Wounded nucleon** model gives all $\langle N_{\mathrm{P}}^{n}\rangle$, but it is **not working** - **participants are not protons**. The new SPS data of the **NA49** and **NA61/SHINE** show that

$$\omega_{\mathrm{p+p}} > \omega_{\mathrm{Ar+Sc}}~,~\omega_{\mathrm{Pb+Pb}} \qquad \text{at SPS (Rybczynski (2013), Aduszkiewicz (2015), Seryakov (2017))}$$

  i.e. ω_{P} can be negative, which is forbidden by definition. At higher energies the wounded nucleon model clearly contradicts the data, because

$$\omega_{\mathrm{p+p}} \gg \omega_{\mathrm{Pb+Pb}} \qquad \text{at LHC (V.B. 1606.05358)}$$

- **Suppose** that **current** experimental **methods** are **effective enough** to make scaled variance for the fluctuations from a source close to the measured fluctuations,

$$\omega \simeq \omega_A\,, \qquad \text{then} \quad \alpha = \frac{\omega - \omega_A}{\omega_A} = \langle n_A \rangle \frac{\omega_P}{\omega_A} \ll 1 \quad \text{is a small parameter}$$

$$S\,\sigma \simeq S_A\,\sigma_A\,(1-\alpha) + \alpha\,[\,3\,\omega_A + \langle n_A\rangle\,S_P\,\sigma_P\,]\,,$$

$$\kappa\,\sigma^2 \simeq \kappa_A\,\sigma_A^2\,(1-\alpha) + \alpha\,\Big[\,\langle n_A\rangle^2\,\kappa_P\,\sigma_P^2 + \omega_A\,(\,3\,\omega_A + 4\,S_A\,\sigma_A + 6\,\langle n_A\rangle\,S_P\,\sigma_P\,)\,\Big]$$

- ω_A competes with $\langle n_A\rangle\,S_P\,\sigma_P$ and $\langle n_A\rangle^2\,\kappa_P\,\sigma_P^2$, **assume** that their ratio is also small,

$$\beta = \langle n_A\rangle\,\frac{S_P\,\sigma_P}{\omega_A} \ll 1\,, \qquad \gamma = \langle n_A\rangle^2\,\frac{\kappa_P\,\sigma_P^2}{\omega_A^2} \ll 1\,,$$

The $\alpha,\ \beta,\ \gamma \ll 1$ is the **mathematical meaning** of the **'small participant fluctuations'**. One can decrease participant fluctuations by **decreasing bin width**, or by decreasing $\langle n_A\rangle$, choosing **rare particles**, or **net charges** for analysis (Braun-Munzinger, Rustamov, Stachel (2017)). Then

$$\omega = \omega_A\,(1+\alpha)\,,$$

$$S\,\sigma \simeq S_A\,\sigma_A\,(1-\alpha) + 3\,\alpha\,\omega_A\,,$$

$$\kappa\,\sigma^2 \simeq \kappa_A\,\sigma_A^2\,(1-\alpha) + 3\,\alpha\,\omega_A^2\left[\,1 + \frac{4}{3}\,\frac{S_A\,\sigma_A}{\omega_A}\,\right], \qquad \alpha,\ \beta,\ \gamma \ll 1\,,$$

There are **three measured values**, ω, $S\sigma$, $\kappa\sigma^2$, and **four unknowns** α, ω_A, $S_A\sigma_A$, $\kappa_A\sigma_A^2$.
Therefore, it is only possible to express the result in powers of α:

$$\begin{aligned} \omega_A &\simeq \omega - \alpha\,\omega\,, \\ S_A\,\sigma_A &\simeq S\,\sigma + \alpha\,(S\,\sigma - 3\,\omega)\,, \\ \kappa_A\,\sigma_A^2 &\simeq \kappa\,\sigma^2 + \alpha\left(\kappa\,\sigma^2 - 4\,\omega\,S\,\sigma - 3\,\omega^2\right)\,, \qquad \alpha,\ \beta,\ \gamma \ll 1\,. \end{aligned}$$

red - what we would like to measure, **black** - what we measure, **blue** - participants

These equations are **also valid if** a **useful signal is larger** than fluctuations of **participants**, e.g., close to **QGP** phase transition, or near the **QCD critical point**. However, one can not solve it without further assumptions, if the signal is not dominating the background.

If the **QCD** or **CP signal is weak**, which seems to be the case, then one may assume that the interplay of **resonance decays** and other 'trivial' effects **dominate** in experimental **measurements**. This background has to be understood and filtered out.

It seems reasonable to assume that **'trivial' source is Gauss-like**, i.e. has symmetric multiplicity distribution

$$\delta = \frac{S_A\,\sigma_A}{\omega_A} \ll 1$$

Then

$$\omega = \omega_A (1+\alpha),$$
$$S\sigma \simeq 3\alpha\omega_A,$$
$$\kappa\sigma^2 \simeq \kappa_A\sigma_A^2(1-\alpha) + 3\alpha\omega_A{}^2, \qquad \alpha,\beta,\gamma,\delta \ll 1$$

and it is possible to **solve the problem**:

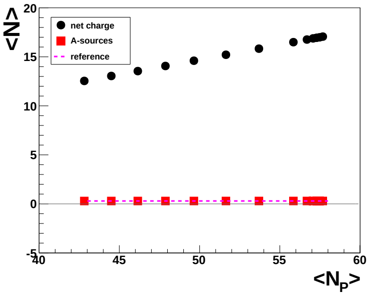
$$\alpha \simeq \frac{S\sigma}{3\omega - S\sigma},$$
$$\omega_A \simeq \omega - \frac{S\sigma}{3},$$
$$\kappa_A\sigma_A^2 \simeq \kappa\sigma^2 - \omega_A S\sigma, \qquad \alpha,\beta,\gamma,\delta \ll 1.$$

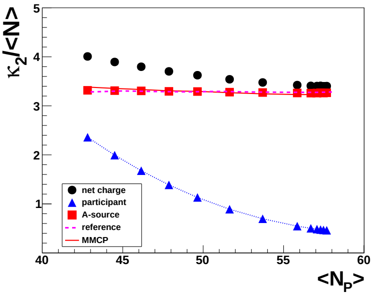
red - what we would like to measure, **black** - what we measure, **blue** - participants

These approximate equations **remove** fluctuations of **participants** and **obtain** the fluctuation of **sources** through ***measured*** values. This is the meaning of the **MMCP method**.

Test of the MMCP in EPOS – average multiplicity and scaled variance

Net electric **charge** in $^{40}_{18}Ar + ^{45}_{21}Sc$ at $p_{lab} =$ **150 GeV/c** and with $y^{CMS} > 0$ in centrality, **left** to **right**: **20%**, 17.5%, 15%, 12.5%, 10%, 7.5%, 5%, 2.5%, 1.5%, 1%, 0.75%, 0.5% and **0.2%**.

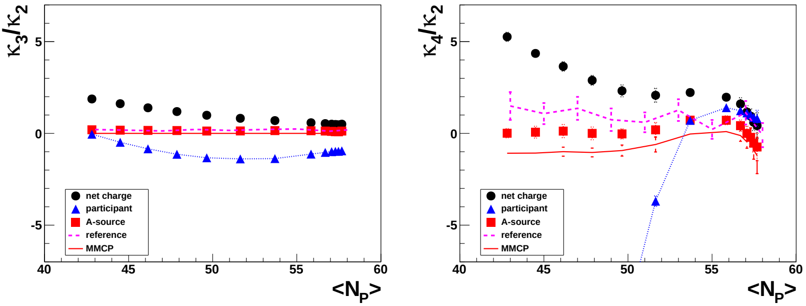




red - what we would like to measure, **black** - what we measure, **blue** - participants, **magenta** - the 'reference' is obtained selecting $\mathbf{N_P = const}$.

$\langle N_{\textbf{net charge}}\rangle / \langle N_{\mathbf{P}}^{max}\rangle = (18+21)/(40+45) \simeq 0.5$, condition $y^{CMS} > 0$ corresponds to **1/2** of created system, therefore, $\langle n_A \rangle \simeq 0.5 * 0.5 = 0.25 \simeq 0.3$.

- $\langle n_A \rangle$ and ω_A are **independent** of **centrality** in **EPOS**
- $\langle n_A \rangle$ **is smal, but** it's **fluctuations** give the **main contribution** $\omega \sim \omega_A \simeq 3.3 \gg \omega_P \gtrsim 0.5$
- ω_P is small, but it **exist**, even if **bin width** goes to **zero** (V.B., Mackowiak-Pawlowska 1705.01110)



red - what we would like to measure, **black** - what we measure, **blue** - participants, **magenta** - the 'reference' is obtained selecting $\mathbf{N_P = const}$.

- The **skewness of a source**, $S_A\,\sigma_A$, is also **independent** on centrality and is close to **zero**.
- The large values of the net charge $S\,\sigma$ are due to the ***second*** moment fluctuations of **participants**, $S_A\,\sigma_A \simeq 0.1 \ll S\,\sigma \simeq 3\,\langle n_A\rangle\,\omega_P > 0$, while $S_P\,\sigma_P \leq 0$, for all δc
- The **higher** the **order**, the **stronger** is the **dependence** on the **bin width** δc
- In very central collision in a fixed target experiment $\mathbf{N_P \simeq const}$, while in collider mode and in peripheral collisions $\mathbf{N_P \neq const}$. Therefore, **STAR**, **NA61** and **STAR fixed target** data may be **hard to compare** (V.B., Mackowiak-Pawlowska 1705.01110).

The **CBWC** procedure used by **STAR** means that a value X is measured in r sub-samples, and then summed up with the relative weights w_r of the sub-samples r,

$$X = \sum_r w_r X_r \,, \qquad w_r = n_r / \sum_r n_r \,,$$

where n_r is the number of events in the bin r.

bin width	n_r	w_r	$\langle N \rangle$	$\sigma^2/\langle N \rangle$	$S\sigma$	$\kappa\sigma^2$
0-1%	624827	$\simeq 0.2$	16.88(1)	3.405(3)	0.503(4)	1.2(3)
1-2%	626043	$\simeq 0.2$	16.36(1)	3.417(3)	0.603(4)	2.4(3)
2-3%	611242	$\simeq 0.2$	**15.83(1)**	**3.447(3)**	**0.609(4)**	**2.2(3)**
3-4%	665988	$\simeq 0.2$	15.32(1)	3.467(3)	0.660(4)	2.9(3)
4-5%	623110	$\simeq 0.2$	14.81(1)	3.486(3)	0.741(3)	2.3(3)
0-5%	3151210	1.0	**15.834(4)**	**3.442(1)**	**0.625(2)**	**2.2(1)**

The sub-bin values for the collected number of events n_r, the weight of the sub-bin w_r, the average net charge $\langle N \rangle$, scaled variance $\omega = \kappa_2/\langle N \rangle = \sigma^2/\langle N \rangle$, normalized skewness $S\sigma = \kappa_3/\kappa_2$, and normalized kurtosis $\kappa\sigma^2 = \kappa_4/\kappa_2$ (V.B., Mackowiak-Pawlowska 1705.01110).

- The **CBWC reduces** statistical **uncertainty**.
- The **CBWC gives average measured** fluctuations over the selected sub-bins.

Comparison of different methods for 0%–5% and 0%–20% centrality

0 – 5%	CBWC	net charge	reference	**A-source**	**MMCP**
$\kappa_2/\langle N\rangle$	3.4	3.5	3.3	**3.3**	**3.2**
κ_3/κ_2	0.6	0.7	0.2	0.2	0
κ_4/κ_2	2.2(1)	2.2(2)	1.3(5)	0.7(2)	0.0(2)

0 – 20%	net charge	**A-source**	**MMCP**
$\kappa_2/\langle N\rangle$	4.0	**3.3**	**3.4**
κ_3/κ_2	1.9	0.2(1)	0
κ_4/κ_2	5.3(2)	0.0(2)	-1.1(2)

- The **MMCP coincides** within the uncertainty with the **A-source** for $\omega = \kappa_2/\langle N\rangle$ in the **0 – 5%** centrality, and deviates only for **2%** from A-source in the **0 – 20%** centrality.
- The **CBWC overestimates** $S\sigma = \kappa_3/\kappa_2$ and $\kappa\sigma^2 = \kappa_4/\kappa_2$ **three times**.
- The κ_3/κ_2 of sources is zero by definition in the **MMCP**. It **agrees** within three standard deviations with the **A-source** generated by EPOS.
- The κ_4/κ_2, in MMCP for 0-5% bin underestimates the A-source. This is the result of neglecting skewness of the participants $S_P\,\sigma_P$ (V.B., Mackowiak-Pawlowska 1705.01110).

Conclusions

- The **MMCP works well for** the **scaled variance**.
- The average number of particles produced by a source, $\langle n_{\mathbf{A}} \rangle$, and it's fluctuations of the second, $\omega_{\mathbf{A}}$, and the third order, $S_{\mathbf{A}}\,\sigma_{\mathbf{A}}$, **do not depend on centrality**.
- However, the fourth order fluctuations of a source, $\kappa_{\mathbf{A}}\,\sigma^2_{\mathbf{A}}$, **change non-monotonously** for the bin width smaller than **5%** in the range from -1 to $+1$. This effect **should be studied**, especially **for higher moments**.
- $S\,\sigma$ and $\kappa\,\sigma^2$ depend on the lower order fluctuations, which give the largest contribution to their values. The fluctuations from a source that one would like to access, $S_{\mathbf{A}}\,\sigma_{\mathbf{A}}$ and $\kappa_{\mathbf{A}}\,\sigma^2_{\mathbf{A}}$, **are almost zero** in the considered example:

$$S\,\sigma \;\simeq\; 3\,\langle n_{\mathbf{A}}\rangle\,\omega_{\mathbf{P}} \;>\; 0\,, \quad \kappa\,\sigma^2 \;\simeq\; 3\,\langle n_{\mathbf{A}}\rangle\,\omega_{\mathbf{A}}\,\omega_{\mathbf{P}} \;>\; 0\,, \quad S_{\mathbf{A}}\,\sigma_{\mathbf{A}} \;\simeq\; 0\,, \quad \kappa_{\mathbf{A}}\,\sigma^2_{\mathbf{A}} \;\simeq\; 0\,.$$

- The **CBWC** is **unable to remove participant fluctuations**. It gives the average of the 'total', i.e. non-processed fluctuations, which are dominated by participant fluctuations. Its results **depend** on the **width**, **weight**, and the **position** of the sub-bins, i.e. may give arbitrary result.
- **The same is true if one** does not mix the bins, but **selects a particular centrality bin** and increases statistics.
- The **fluctuations of participants** may **persist** even if the bin width approaches zero. Therefore, one should further develop the methods of their reliable exclusion.

Thank you!

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