Functional renormalization group study of the Quark-Meson model with ω meson

Defu Hou

Central China Normal University, Wuhan

Hui Zhang, Toru Kojo, Bin Qin

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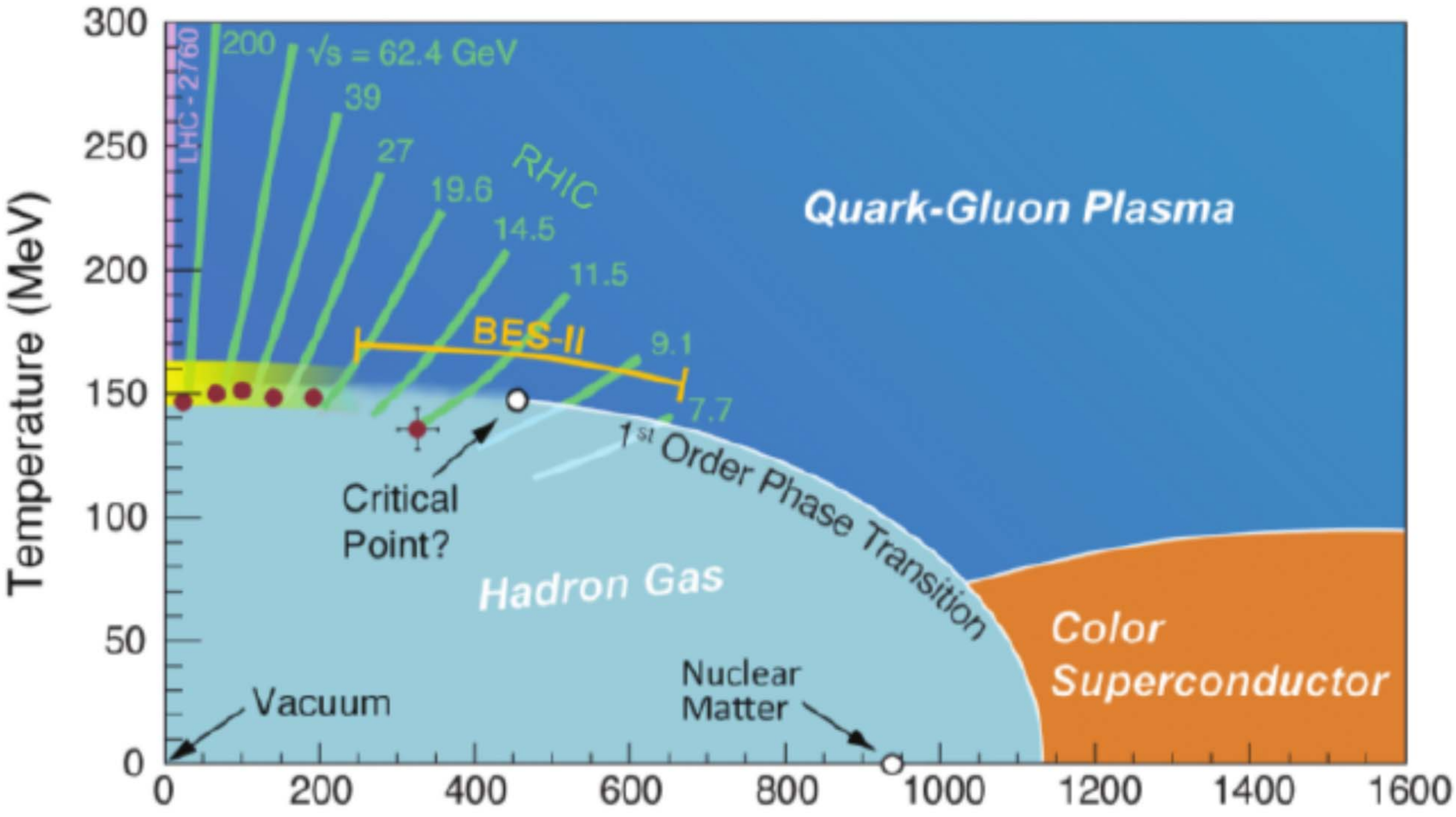
Outlines

1. Motivations
2. The Quark-Meson Model with ω Meson (MF & FRG)
3. Numerical Results
4. Summary and Outlooks

QCD Phase diagram



Motivation

Many interesting phenomena in QCD
lie in the strongly-coupled region.

Non-perturbative methods

Lattice: problematic with finite chemical potentioal, sign problem...

AdS/CFT: Notable success in RHIC physics, Viscosity, Jet quenching…

DSE: See the talk by Fisher

FRG: no sign problem

FRG– A variety of applications

The Functional Renormalization Group (FRG) method can be applied to variety of physical systems.

1. Strong interaction Powlowski, Wambach
2. electroweak phase transition Reuter & Wetterich, NPB(1993)
3. effective models in nuclear physics Drews & Weise, (2013-2015)
4. condensed matter systems
   e.g. Hubbard model, liquid He 4 ,
   frustrated magnets,superconductivity . . .
5. ultra-cold atoms Diehl, Gies, Pawlowski & Wetterichb (2007)
6. quantum gravity Wetterich, PRD92, 083507 (2015)

Applications of FRG in HEP

1. Quark-Meson Model:
B. J. Schaefer and J. Wambach (2004), (2008)
T. K. Herbst, J. M. Pawlowski and B. J. Schaefer, (2011)

2. Quark-Meson Model + vector (ρ) & axial-vector (a1):
F. Rennecke (2015)
J. Eser, M. Grahl, & D. H. Rischke (2015)
C. Jung, F. Rennecke, R. A. Tripolt, L. V. Smekal & J. Wambach (2017)

3. Walecka type nucleon-σ-ω model:
M. Drews, T. Hell, B. Klein & W. Weise (2013)
M. Drews and W. Weise(2014), (2015)

In the quark model context, the mean field of the ω meson is known to have the significant impact on the phase boundary and the location of the critical end point (CEP). K. Fukushima, Phys. Rev. D 77, 114028 (2008);
So

Quark-Meson Model + vector (ω) this work

The Model

$$\mathcal{L} = \bar{\psi}\Big[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu - \gamma_0\mu\Big]\psi$$
$$+ \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- U(\sigma,\boldsymbol{\pi},\omega)$$

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad \psi = (u,d)^T$$

The potential for σ, π, and ω is

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For chiral limit

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2)^2 + H\sigma - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For explicit SB

Mean-Field

For T=0 & μ=0, the MF potential is

$$U_{\rm MF}(\sigma,\omega_0)=\frac{\lambda}{4}(\sigma^2-f_\pi^2)^2-\frac{m_v^2}{2}\omega_0^2,$$

For chiral limit

$$U_{\rm MF}(\sigma,\omega_0)=\frac{\lambda}{4}(\sigma^2-\nu^2)^2+H\sigma-\frac{m_v^2}{2}\omega_0^2,$$

For explicit Chiral SB.

For T≠0 & μ≠0, the MF potential is

$$\Omega_{\rm MF}=\Omega_{\bar\psi\psi}+U_{\rm MF}(\sigma,\omega_0)$$

$$\Omega_{\bar\psi\psi}=-\nu_q T\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\left\{\ln[1+e^{-(E_q-\mu_{\rm eff})/T}]+\ln[1+e^{-(E_q+\mu_{\rm eff})/T}]\right\}$$

$$m_{\rm eff}=g_s\sigma,\qquad \mu_{\rm eff}=\mu-g_v\omega_0.$$

f_π=93MeV, m_π=138MeV, λ=20, g_s=3.3 O. Scavenius, A. Mocsy, I. N. Mishustin & D. H. Rischke, Phys. Rev. C 64, 045202 (2001)

Mean-Field

The gap equation for ω_0 ,

$$\left.\frac{\partial\Omega_{\rm MF}}{\partial\omega_0}\right|_{\omega_0=\bar{\omega}_0(\sigma)} = 0. \quad \rightarrow \quad \bar{\omega}_0 = \frac{g_v}{m_v^2} n(T, \mu - g_v\bar{\omega}_0),$$

where the quark density n is determined by

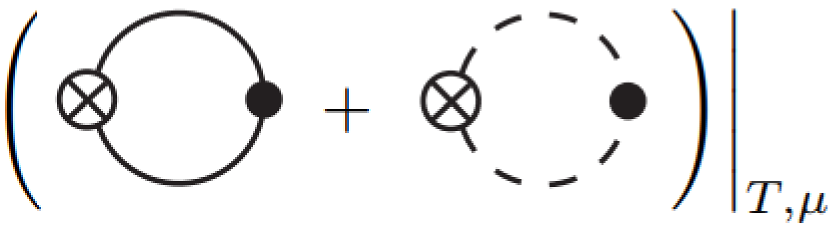
$$n(T, \mu - g_v\omega_0) = -\frac{\partial}{\partial\mu}\Omega_{\bar{\psi}\psi}(T, \mu - g_v\omega_0)$$

FRG flow equation

- For continuum field theory
- Non-perturbative
- (known) microscopic laws → complex macroscopic phenomena
- Flow from classical action S[φ] to effective action Γ[φ]
- Scale dependent effective action $\Gamma_k[\varphi]$

Wetterich, PLB301, 90 (1993).

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$



FRG flow equation

scale-dependent effective potential can be expressed by replacing the potential U with the scale-dependent one U_k

$$\Gamma_k = \int \mathrm{d}^4x \, \mathcal{L}|_{U \to U_k}$$

Due to the chiral symmetry, the potential U depends on σ and π only through the chiral invariant

$$\phi^2 \equiv \sigma^2 + \pi^2$$

Starting with some ultraviolet (UV) potentials U_Λ as our initial conditions, we integrate fluctuations and obtain the scale dependent U_k , which is artificially separated into the ω-independent and dependent terms,

$$U_k = U_k^\phi + U_k^\omega$$

where the function form of U_k^ϕ will be determined without assuming any specific forms, while the potential for the ω-field we keep using the same form

$$U_k^\omega = -\tfrac{1}{2} m_v^2 \omega_{0,k}^2$$

FRG flow equation

3d-analogue of the optimized regulator

$$R_{k,B}(\boldsymbol{p}) = (k^2 - \boldsymbol{p}^2)\theta(k^2 - \boldsymbol{p}^2),$$

$$R_{k,F}(\boldsymbol{p}) = -\boldsymbol{p}\cdot\boldsymbol{\gamma}\left(\sqrt{\frac{k^2}{\boldsymbol{p}^2}} - 1\right)\theta(k^2 - \boldsymbol{p}^2),$$

the flow equation for the potential U_k^ϕ can be obtained as Schaefer & Wambach NPA 2005

$$\partial_k U_k^\phi(T,\mu) = \frac{k^4}{12\pi^2}\left\{\frac{3[1+2n_{\rm B}(E_\pi)]}{E_\pi} + \frac{1+2n_{\rm B}(E_\sigma)}{E_\sigma} - \frac{2\nu_q\left[1 - n_{\rm F}(E_q,\mu_{\rm eff}^k) - n_{\rm F}(E_q,-\mu_{\rm eff}^k)\right]}{E_q}\right\}$$

with single-particle energies are

$$E_\pi = \sqrt{k^2 + 2U_k'}\,,$$

$$E_\sigma = \sqrt{k^2 + 2U_k' + 4\phi^2 U_k''}\,,$$

$$E_q = \sqrt{k^2 + g_s^2\phi^2}$$

The boson and fermion occupation numbers are

$$n_B(E) = \frac{1}{e^{\beta E} - 1}$$

$$n_F(E,\mu) = \frac{1}{e^{\beta(E-\mu)} + 1}$$

FRG flow equation

In principle, the flow equation should be solved in the φ and ω_0 directions. But neglecting the excitation of ω, the flow equation of ω0 fields can be computed for a given value of φ, like the Gauss law constraint in gauge theories. At each momentum scale k, we determine $\omega_{0,k}$ by solving

$$\frac{\partial U_k}{\partial \omega_{0,k}} = 0 \qquad \longrightarrow \qquad \partial_k\, \omega_{0,k} = -\frac{2 g_v\, k^4}{\pi^2 m_v^2 E_k} \frac{\partial}{\partial \mu} \left(n_{\mathrm{F}}(E_k, \mu_{\mathrm{eff}}^k) + n_{\mathrm{F}}(E_k, -\mu_{\mathrm{eff}}^k) \right)$$

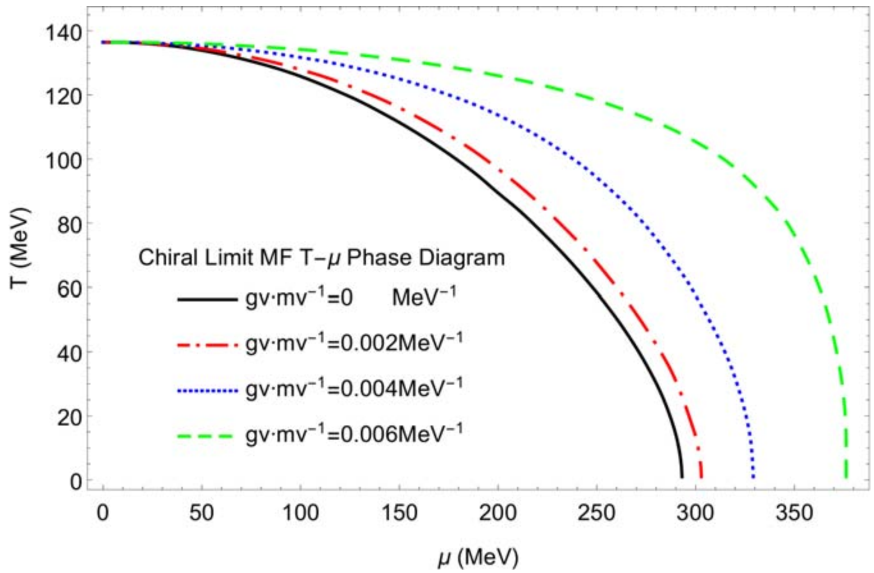
Initial condition:
Λ=500MeV, B. J. Schaefer & J. Wambach, Nucl. Phys. A 757, (2005) 479-492

$$U_\Lambda^\phi = \frac{\lambda}{4}\phi^4 \qquad\qquad \omega_{0,\Lambda}(\phi) = 0$$

Results

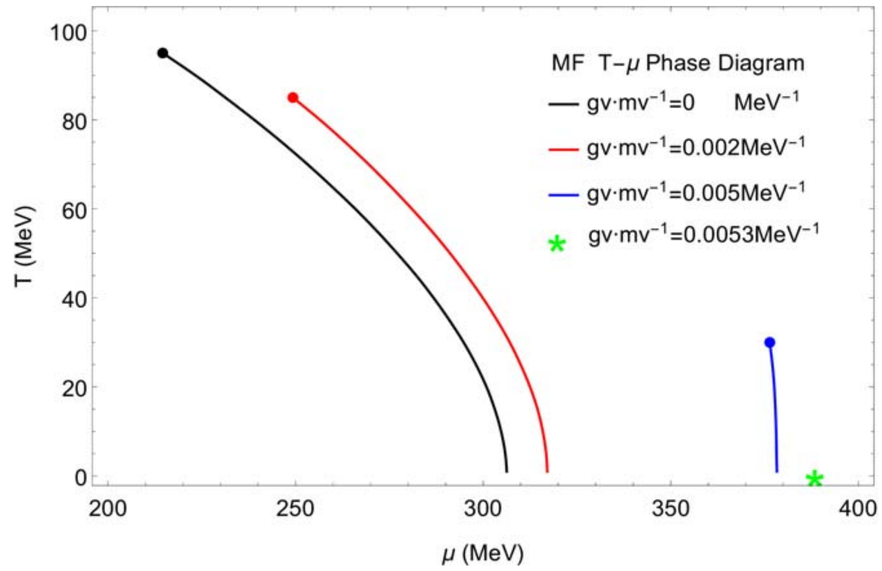
Mean-Field

Chiral limit



The MF phase diagram for 2-flavor massless QCD for chiral limit. The phase boundary is the first order PT

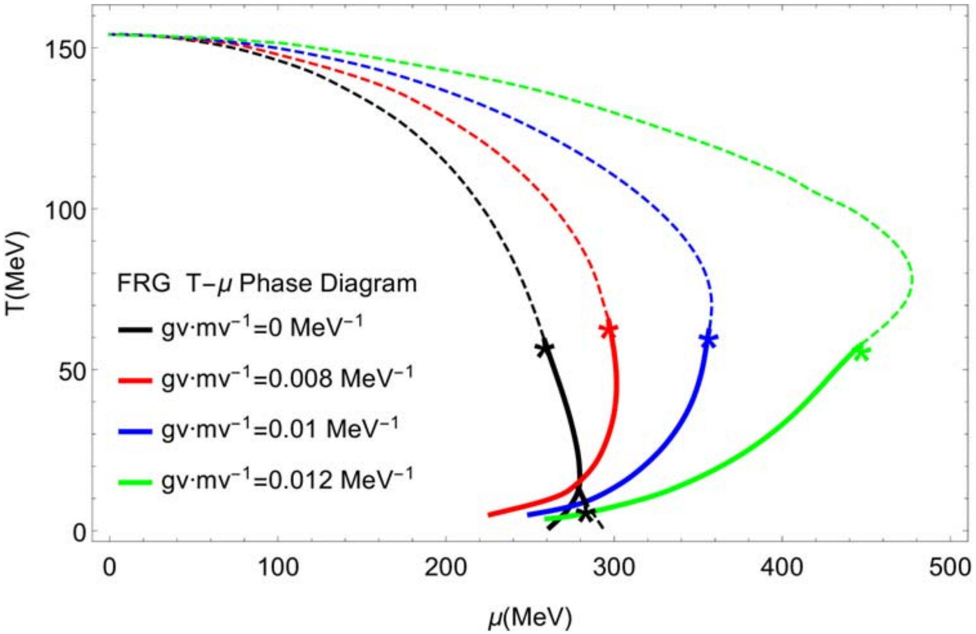
Explicit chiral symmetry breaking



The mean-field phase diagram for the explicit SB case. Solid lines show 1st order. Dots show the CEP, star shows the vanishing of the CEP. be consistent with Fukushima (2008)

Results

FRG



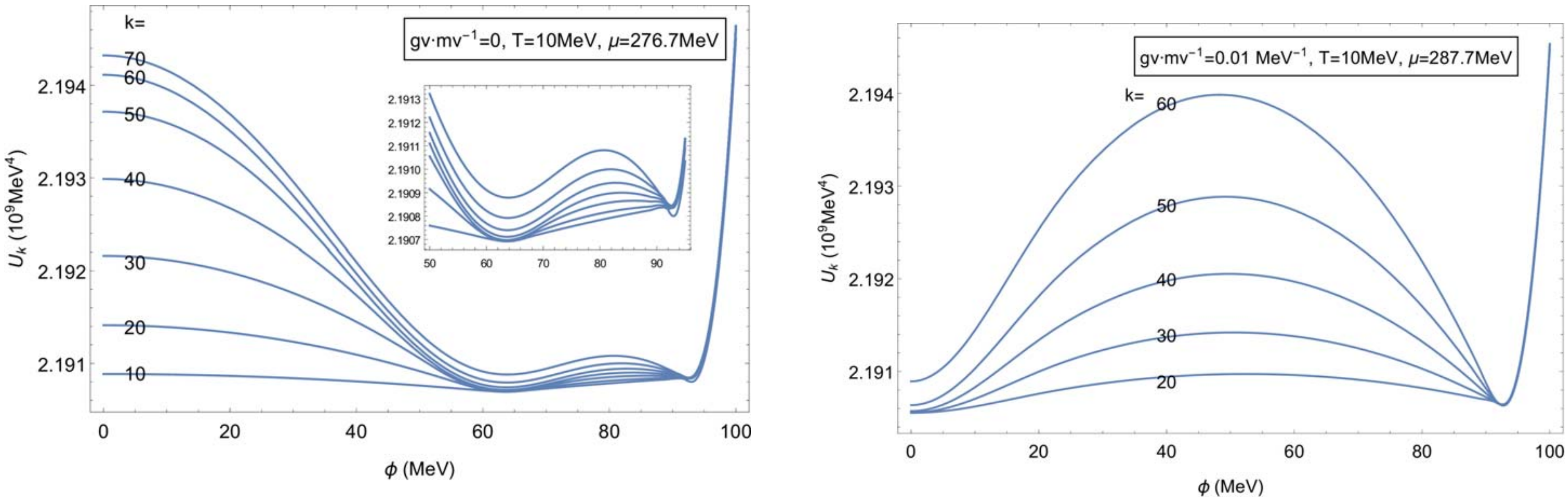
Dashed (solid) lines show 2nd (1st) order phase transition. Stars show the tri-critical end point (TCP)

(i) At high T the fluctuations turn the 1st order line in the MF into 2nd order, yielding the TCP

(ii) While the critical μ of the TCP is sensitive to the vector coupling, its critical T is similar

(iii) At high T, gv shifts the phase boundaries to higher μ as in the MF, but the curves strongly bend back toward lower T irrespective to the value of g_v and approach one another.

FRG



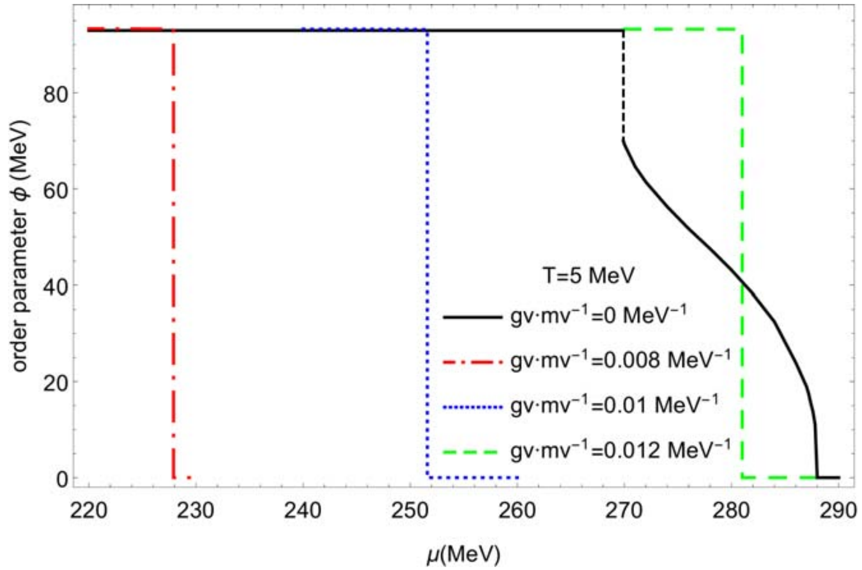
The scale evolution of the EP $\Gamma_k(\varphi)$ at low T. (left) g_v/m_v=0, T=10 MeV & μ=276.7 MeV; (right) g_v/m_v=0.01 MeV^−1, T=10 MeV & μ=287.7 MeV.

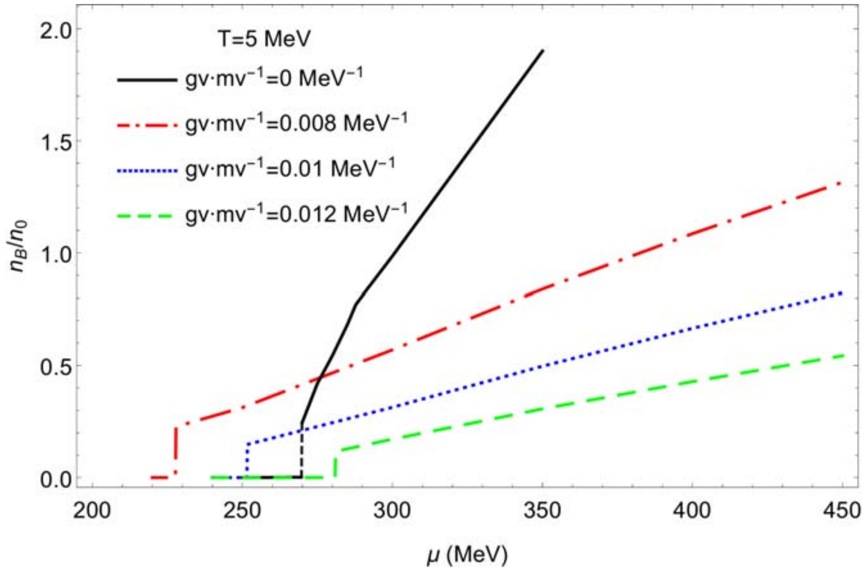
The fluctuations erase the barrier between two local minima in the MF potential.

At finite vector coupling, the essential features remain the same as the g_v=0 case; the flucts. do not modify the potential around φ≃93 MeV, the potential around φ≃0 is strongly affected.

FRG

Order parameter and baryon density



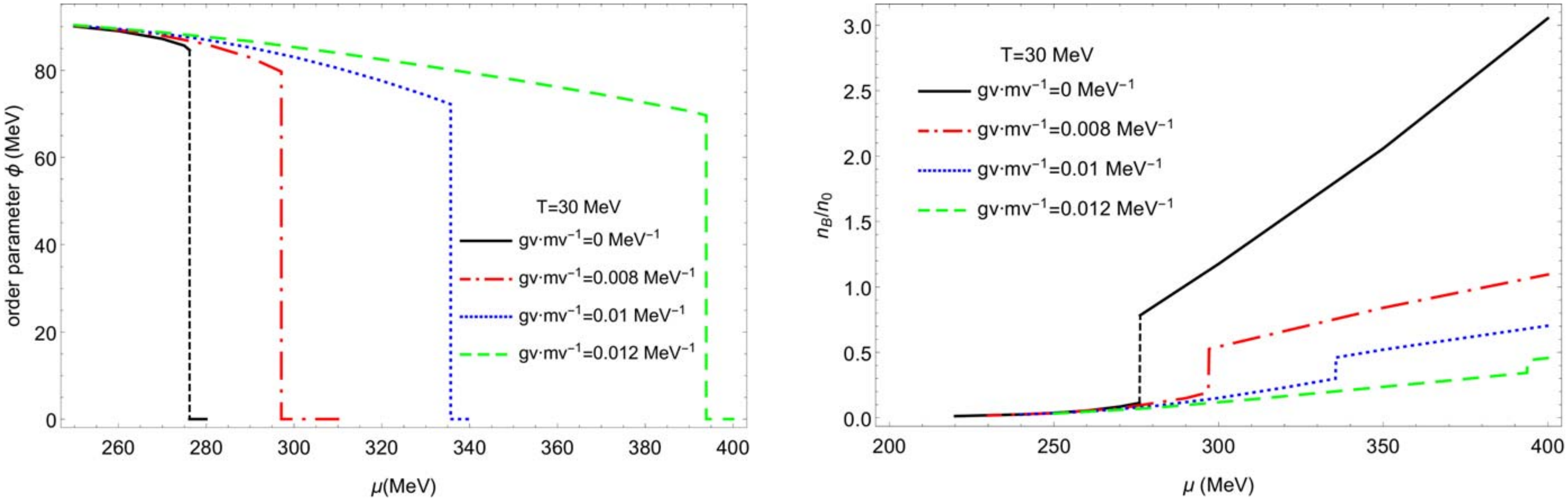


The order parameter φ and baryon density as a function of μ for T=5 MeV from the FRG with different gv. The g_v=0 case has 1st and 2nd phase transitions.

The μ-dependence of the baryon density considerably deviates from ~ μ^3 behavior expected from the single particle contributions

FRG

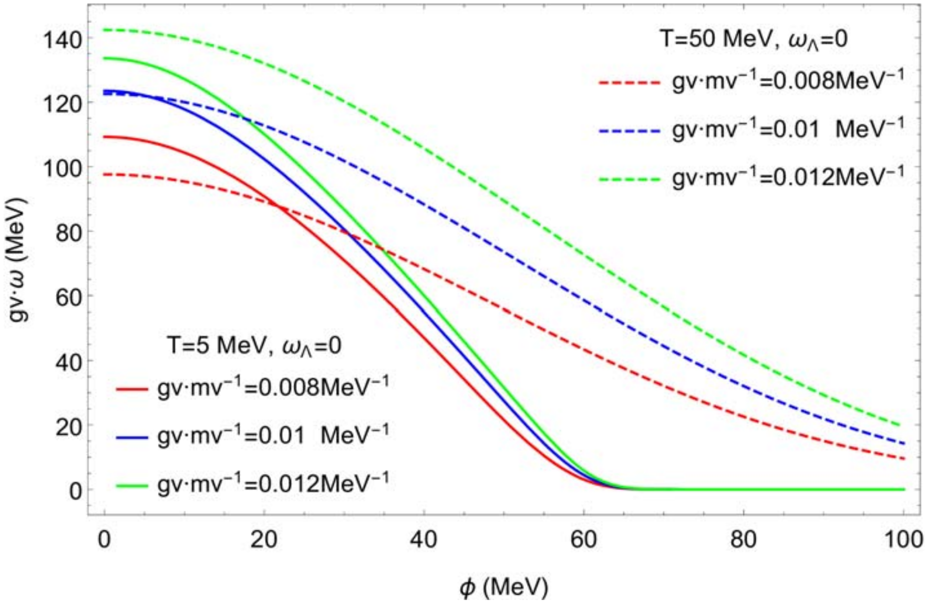
Order parameter and baryon density



The vacuum expectation value of order parameter φ and baryon density as a function of chemical potential μ for T=30 MeV from the FRG with different vector couplings.

FRG

Omega meson



Solutions $g_v \cdot \omega$ as a function of the chiral condensate σ for fixed T=50 MeV

Several other checks

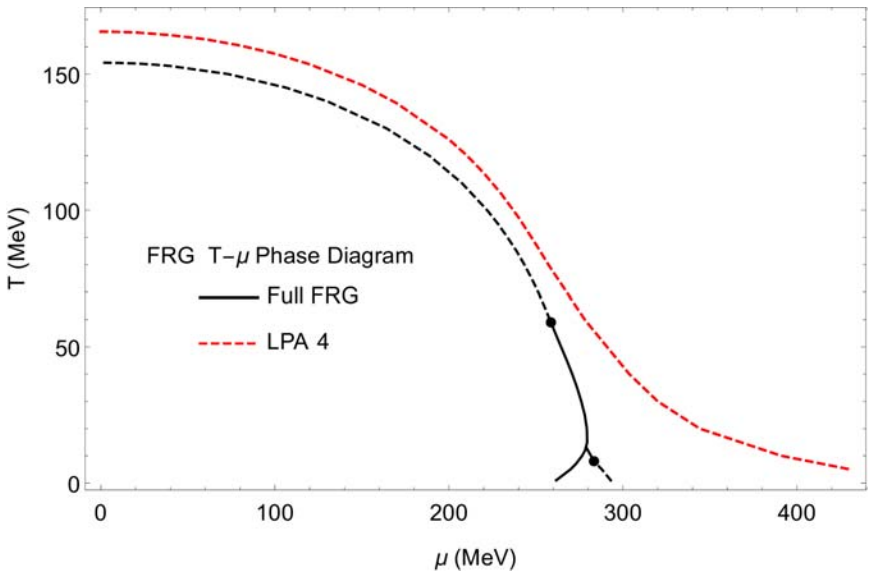
LPA 4 $U_k(\phi) = \frac{\lambda_k}{4}(\phi^2 - a_k)^2$

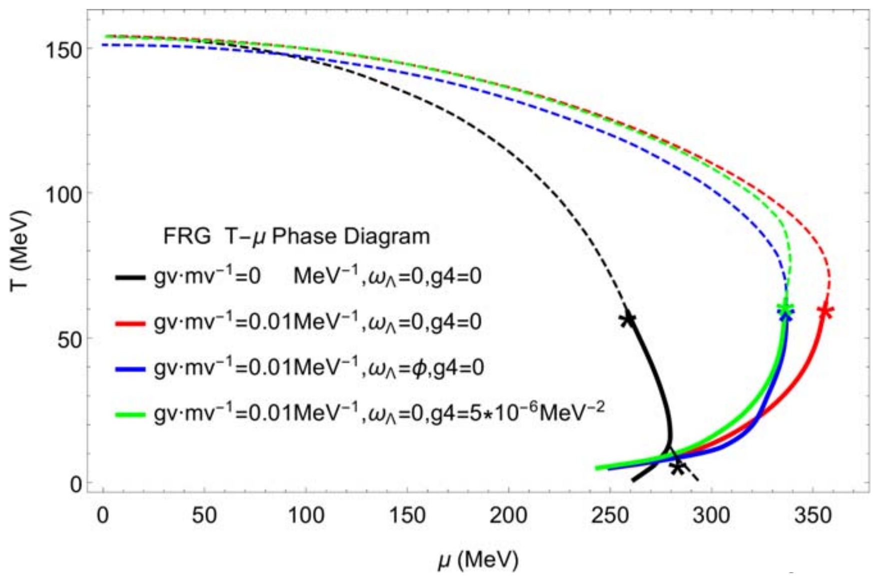
With g_4 & initial condition for ω

$$\omega_\Lambda = \phi$$

$$U_k(\omega) = -\frac{1}{2}m_v^2\omega_{0,k}^2 + \frac{1}{12}g_4 \cdot (g_v^2 m_v^2) \cdot \omega_{0,k}^4$$

With the quartic term, the overall structure such as the back bending behavior is not significantly affected.





Summary and outlook

1. We discuss the quark meson model with σ, π, and ω mesons at finite temperature and density using the FRG.

2. Without ω-fields, FRG calculations typically lead to the back bending behavior at low temperature phase boundary.

3. The low temperature first order phase transition in FRG is induced by fluctuations, rather than number density as in the MF case, so that the structure of the low temperature boundaries remains similar for different values of vector couplings.

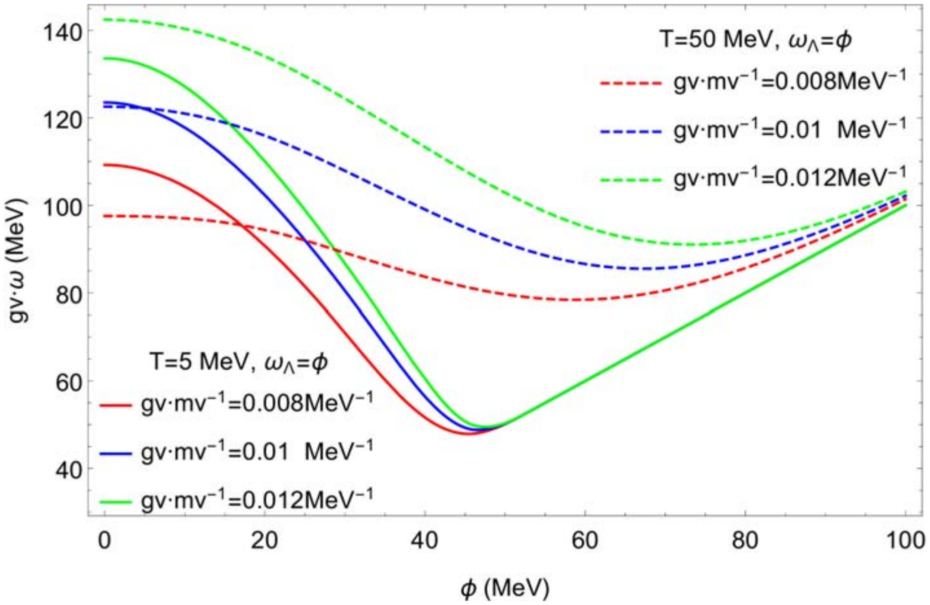
4. The effective potential at small φ is very sensitive to the infrared cutoff scale k. If we artificially stopped the integration before stabilizing the result, we would get very different phase boundaries

   1. Confinement
   2. Current quark mass / explicit chiral symmetry breaking

Thank you very much for your attention

FRG

Omega meson



Solutions $g_v \cdot \omega$ as a function of order parameter φ for fixed chemical potential T=50 MeV with different second order vector coupling constants for $\omega_\Lambda=\sigma$ and g4=0