Transport coefficients of hot and dense quark matter in Polyakov loop quark meson model

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August 8, 2017

CPOD-2017, Stony Brook

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Outline

- **Introduction**
- **Boltzmann equation and transport coefficients**
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  - **Polyakov loop Quark meson model**
  - Thermodynamics
  - Relaxation time estimation
- **Results for transport coefficients**
- **Summary and Outlook**

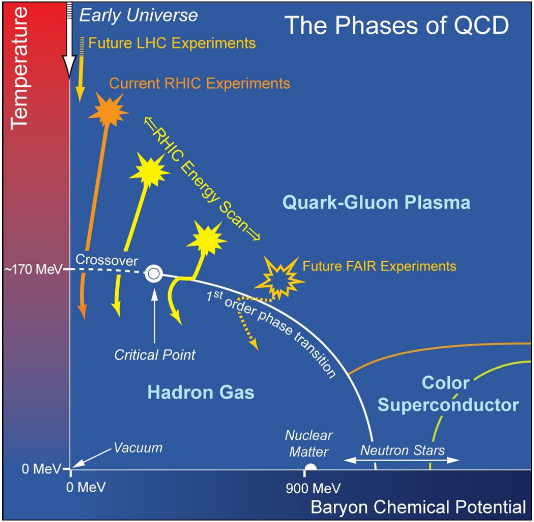
Introduction

- Transport properties of hot/dense matter are important for heavy ion collision (HIC), cosmology and important for near equillibrium evolution of any thermodynamic system
- The most studied transport coefficient is perhaps shear viscosity η. In HIC spatial anisotrpy of colliding nuclei gets converted to momentum anisotropy throgh a hydro evoln. The equllibriation is decided by η. ($\frac{\eta}{s} \sim \frac{1}{4\pi}$, the KSS bound)
- The bulk viscosity ζ - thought earlier to be not important for HIC hydro evolution. Argument: $\zeta \sim (\epsilon - 3p)/T^4$ that vanishes for ideal gas. However, lattice simulation $\Rightarrow$ large $(\epsilon - 3p)/T^4$ near T_c. This,in turn, can give rise to different physical effects (Cavitation).

Introduction -Contd. ···

- The temperature and chemical potential dependence of transport coefficients may reveal the location of phase transition
- In principle, one can estimate transport coefficients using Kubo formulation but, QCD is strongly coupled for the energies accessible at HIC. Lattice QCD simulation is numerically challanging and also has problems in doing simulations at finite baryon densities.
- We shall approach the problem here within Boltzmann kinetic equation within relaxation time approximation
- The dynamics of chiral symmetry breaking in QCD is with medium dependent masses are included in the formulation.
- Most earlier calculations are performed at zero baryon density ρ_B. Including finite density effects are relevent for upcoming HIC experiments, BES(Brookhaven), CBM at (GSI, Darmstadt), (NICA at Dubna).

QCD phase diagram and HIC



Boltzmann Equation

Boltzmann equation describes the evolution of particle distribution function

$$\frac{df_a}{dt} = \frac{\partial f^a}{\partial t} + \frac{p^i}{E_a}\frac{\partial f_a}{\partial x^i} - \frac{\partial E_a}{\partial x^i}\frac{\partial f_a}{\partial p^i} = C^a$$

To estimate viscosity coefficients, consider small departure from equllibrium

$$f_a = f_a^0 + \delta f_a$$

The equllibrium distribution function

$$f_a^0 = \frac{1}{\exp \beta(u_\alpha p^\alpha \mp \mu) + 1}$$

The collision term C^a involve scattering processes.
Relaxation time approximation⇒ in C^a, particle 'a' is out of equllibrium while all other particles are in equllibrium.

$$\frac{df_a}{dt} = \frac{p^\mu}{E_a}\frac{\partial f_a^0}{\partial x^\mu} - \frac{M}{E_a}\frac{\partial M}{\partial x^i}\frac{\partial f_a^0}{\partial p^i} = -\frac{\delta f_a}{\tau_a}$$

$T^{\mu\nu}$, J_μ and transport coefficients

Energy dependent relaxation time

$$\tau_a(E_a)^{-1} = \int d\Gamma_c d\Gamma_d d\Gamma_b f_b^0 W(ab, cd)$$

$$W(ab, cd) = \frac{(2\pi)^4 \delta^4(p_a + p_b - p_c - p_d)}{2E_a 2E_b 2E_c 2E_d} |M|^2$$

LHS of Boltzmann Eqn.$\Rightarrow$

$$\partial_\mu f_0^a = -f_0^a(1 \mp f_0^a)\partial_\mu \left(\beta(E^a - \mu - \mathbf{p}\cdot\mathbf{u}\right)$$

Boltzmann Eq. relates non equllibrium part of distribution function to variation in fluid velocity, temperature and chemical potential.

Distribution function is related to the energy momentum tensor and the quark current

$$T^{\mu\nu} = \sum_a \int d\Gamma_a p^\mu p^\nu f_a + g^{\mu\nu} U(\sigma); \quad d\Gamma_a = \nu_a \frac{d\mathbf{p}}{(2\pi)^3}$$

$$J^\mu = \sum_a t_a \int d\Gamma_a \frac{p^\mu}{E_a} f_a$$

ζ, η,λ contd.$\cdots$

Change in nonequllibrium part $\Rightarrow$

$$\delta T^{ij} = \sum_a \int d\Gamma^a \frac{p^i p^j}{TE_a} \tau_a f_a^0 (1 - f_a^0) q_a(p, \beta, \mu)$$

$$\delta J^i = \sum_a t_a \int d\Gamma_a \frac{p^i}{E_a} \tau_a f_a^0 (1 - f_a^0) \left(t_a - \frac{nE_a}{\epsilon + p} \right) p^j \partial_j \left(\frac{\mu}{T} \right)$$

The non equllibrium contribution related to the velocity gradients can be reorganised as

$$q^a = Q^a \partial_i u_i - \frac{p^i p^j}{2E_a} W_{ij}$$

;

$$W_{ij} = \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k$$

Shear and bulk viscosities are defined through the dissipative part

$$\Delta T^{ij} = -\zeta \delta^{ij} \partial_k u_k - \eta W_{ij}$$

Thermal conductivity is defined through the dissipative part of the current

$$\Delta J_i = \lambda \left(\frac{nT}{w} \right)^2 \partial_i \left(\frac{\mu}{T} \right)$$

ζ, η,λ contd.$\cdots$

$$\eta = \frac{1}{15T}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a}\left(\tau_a f_a^0(1-f_a^0) + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\right)$$

$$\zeta = -\frac{1}{3T}\sum_a \int d\Gamma_a \frac{\mathbf{p}_a^2}{E_a}\left(\tau_a f_a^0(1-f_a^0)Q_a + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\bar{Q}_a\right)$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a t_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} f_a^0(1-f_a^0)\tau_a \left(t^a - \frac{nE_a}{w}\right)$$

In the bulk viscosity coefficient, the coefficient Q^a depends upon the equation of state

$$Q_a = -\left[\frac{\mathbf{p}_a^2}{3E_a} + \left(\frac{\partial P}{\partial n}\right)_\epsilon \left(\frac{\partial E}{\partial \mu} - 1\right) - \left(\frac{\partial P}{\partial \epsilon}\right)_n \left(E_a - T\frac{\partial E_a}{\partial T} - \mu\frac{\partial E_a}{\partial \mu}\right).\right]$$

ζ contd.

However, Q_a has to be supplemented by the conditions $u_\mu \delta J^\mu = 0$ and $u_\mu \delta T^{\mu\nu} u_\nu = 0$ corresponding to baryon number and energy momentum conservation. Within the relaxation time approximation, these Landau-Lifshitz conditions reduce to

$$\sum_a t_a \langle \tau_a Q_a \rangle = 0, \quad \sum_a \langle \tau_a E_a Q_a \rangle = 0$$

$$\langle \phi_a(p) \rangle = \int d\Gamma_a [\phi_a(p) f_a^0 (1 - f_a^0)]$$

If Landau Lifshitz conditions are not satisfied, replace

$$\tau_a Q_a \to \tau_a Q_a + \alpha t_a + \beta E_a$$

The unknown coefficients to be determined from the baryon number and energy momentum conservation equation. The expression for bulk viscosity consistent with the Landau Lifshitz condition is then given as

$$\zeta = -\frac{1}{T} \sum_a \langle (\tau_a Q_a + \alpha t_a + \beta E_a) \frac{\mathbf{p}^2}{3E_a} \rangle$$

Albright,Kapusta, Phys Rev C93,014903, 2016; Deb etal,Phys. Rev D,

η,ζ,λ contd.

The expressions for the transport coefficients become simpler when one realises that for ideal hydrodynamics the entropy per baryon (σ) is constant.

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1-f_a^0)$$

$$\zeta = \frac{1}{9T}\sum_a \int d\Gamma_a \frac{\tau_a f_a^0(1-f_a^0)}{E_a^2}\left[\mathbf{p}^2 + 3v_n^2 T^2 E_a \frac{\partial}{\partial T}\left(\frac{E_a-\mu_a}{T}\right)_\sigma\right]^2$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2\sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2}\tau_a f_a(1-f_a)\left(t_a - \frac{nE_a}{w}\right)^2$$

- Transport coefficients are nonnegative as they must be.
- It is important to include the Landau-Liftshitz conditions to obtain the above results.

Note- Transport coefficients of the system is sum of the contributions of each species to the same. On the otherhand, relaxation time of a given species is added inversely arising from scatterring of the given species with all other speciies with which it scatters.

η,ζ,λ contd.

- Knowing the equation of state and other thermodynamic quantities like velocity of sound etc. and the relaxation time one can estimate the viscosity coefficient.
- This thermodynamics and estimation of relaxation time is done within the Polyakov loop extended quark meson coupling (PQM) model.

Polyakovloop quark meson model

PQM model : captures features of chiral symmetry breaking and confinement properties of strong interaction.

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu D^\mu - m_0)\psi + (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi) - U_\chi - U_\phi$$

$$U_\chi = \frac{\lambda}{4}\left(\sigma^2 + \pi^2 - v^2\right) - C\sigma$$

$$\phi = \frac{1}{N_c}\left\langle Tr_c P e^{i\int_0^\beta dx_0 A_0(x_0,\mathbf{x})}\right\rangle$$

Polyakov gauge : A_0- time independent
Functional form of U: Choose the form so as to reproduce pure gauge lattice simulation results for thermodynamics

$$U(\phi,\bar{\phi}) = T^4\left[-\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{2}(\phi^3\bar{\phi}^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2\right]$$

Thermodynamics:PQM model

Thermodynamic potential (negative of pressure)

$$\begin{aligned}\Omega(T,\mu) &= -2N_f T\int d\mathbf{p}\Big[\ln\left(1+3(\phi+\bar{\phi}e^{-\beta\omega_-})e^{-\beta\omega_-}+e^{-3\beta\omega_-}\right)\\ &+ \ln\left(1+3(\phi+\bar{\phi}e^{-\beta\omega_+})e^{-\beta\omega_+}+e^{-3\beta\omega_+}\right)\Big]+U_\chi+U_P\end{aligned}$$

The quark excitation energies are $\omega_\mp = \sqrt{\mathbf{p}^2+m_q^2}\mp\mu$ and the constituent quark masses are dependent on the mean fields as $m_q^2 = g^2(\sigma^2+\pi^2)$.

The mean fields are obtained by extremization of Ω with respect to σ,ϕ,$\bar{\phi}$ and π.

Meson masses are determined by the curvature of Ω at the global minimum:

$$M_\sigma^2 = \frac{\partial^2\Omega}{\partial\sigma^2} \quad M_\pi^2 = \frac{\partial^2\Omega}{\partial\pi^2}$$

Scavanius,Mocsy,Mishustin,Rischke Phys Rev C64,045202, 2001

Meson masses ; Order parameters· · ·

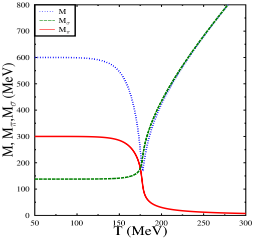


Fig. 1-a

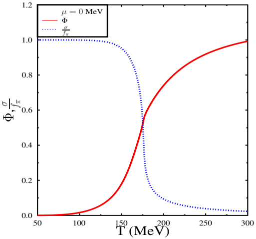


Fig. 1-b

Figure: Temperature dependence of the masses of constituent quark (M), and pions (M_π) and sigma mesons (M_σ) for $\mu = 0$ (Fig1-a) and the order parameters σ and ϕ as a function of temperature for $\mu = 0$ MeV.

PQM Thermodynamics contd.···

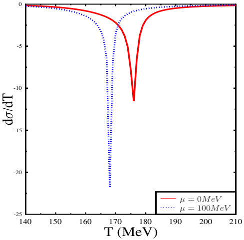


Fig. 2-a

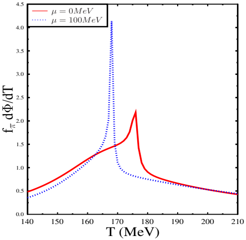


Fig. 2-b

Figure: Temperature derivative of the chiral order parameter (fig 2 a) and Polyakov loop order parameter.

$T_c \simeq 176$ MeV $(\mu = 0)$; critical point $(T_c, \mu_c) = (165, 163)$ MeV

PQM Thermodynamics contd.· · ·

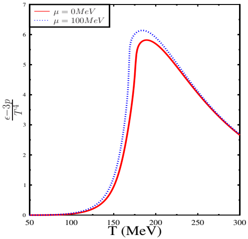


Fig. 3-a

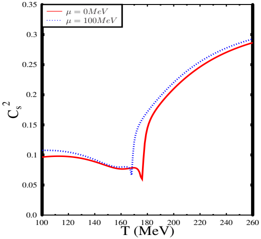


Fig. 3-b

Figure: Temperature dependence of the scaled trace anomaly $\frac{\epsilon - 3p}{T^4}$ (Fig 3a) and square of velocity of sound.

Conformal symmetry is broken maximally at the critical temperature

Estimating the relaxation time: meson scatterring

Energy depenedent relaxation time $\tau_a(E_a)$ for a scatterring process $a, b \rightarrow c, d$ ($d\Gamma_i = \dfrac{d\mathbf{p}}{(2\pi)^3}$)

$$\tau_a^{-1}(E_a) = \omega(E_a) = \sum_b \int d\Gamma_b f_b^0 W_{ab}(s)$$

$$W_{ab}(s) = \frac{1}{1+\delta_{ab}} \int d\Gamma_c d\Gamma_d (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d)|M|^2(1 \pm f_c) + (1 \pm f_d)$$

Meson scatterrings:

$$M_{\sigma,\sigma\rightarrow\sigma,\sigma} = -6\lambda - 36\lambda^2 {f_\pi}^2 \left(\frac{1}{s - {m_\sigma}^2} + \frac{1}{t - {m_\pi}^2} + \frac{1}{u - {m_\pi}^2}\right)$$

$$M_{\boldsymbol{\pi},\sigma\rightarrow\boldsymbol{\pi},\sigma} = -2\lambda - 4\lambda^2 {f_\pi}^2 \left(\frac{3}{t - {m_\sigma}^2} + \frac{1}{u - {m_\pi}^2} + \frac{1}{s - {m_\pi}^2}\right)$$

$$M_{\boldsymbol{\pi},\boldsymbol{\pi}\rightarrow\boldsymbol{\pi},\boldsymbol{\pi}} = -2\lambda \left(\frac{s - {m_\pi}^2}{s - {m_\sigma}^2}\delta_{\mathrm{ab}}\delta_{\mathrm{cd}} + \frac{t - {m_\pi}^2}{t - {m_\sigma}^2}\delta_{\mathrm{ac}}\delta_{\mathrm{bd}} + \frac{u - {m_\pi}^2}{u - {m_\sigma}^2}\delta_{\mathrm{ad}}\delta_{\mathrm{bc}}\right)$$

$$M_{\boldsymbol{\pi},\boldsymbol{\pi}\rightarrow\sigma,\sigma} = -6\lambda - 4\lambda^2 {f_\pi}^2 \left(\frac{3}{s - {m_\sigma}^2} + \frac{1}{t - {m_\pi}^2} + \frac{1}{u - {m_\pi}^2}\right)$$

Poles in s and u channels in the propagators– approximate by taking $s, t, u \rightarrow \infty$ limit.

Estimating viscosities: meson scatterrings

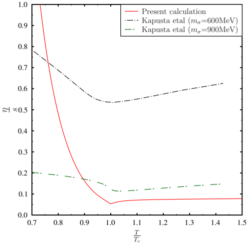


Fig. 4-a

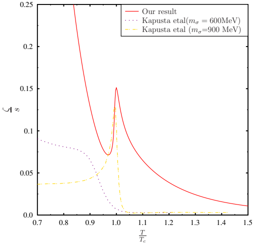


Fig. b-b

Figure: Shear viscosity to entropy ratio (Fig4-a) and bulkviscosity to entropy ratio (Fig4 b)

Chakrabarti and Kapusta Phys Rev C83, 014906 (2011)

Estimating viscosity coefficients-quark scatterrings

- For two flavors we consider the following possible scatterings through meson exchanges.

$$u\bar{u} \to u\bar{u}, \quad u\bar{d} \to u\bar{d}, \quad u\bar{u} \to d\bar{d},$$

$$uu \to uu, \quad ud \to ud, \quad \bar{u}\bar{u} \to \bar{u}\bar{u},$$

$$\bar{u}\bar{d} \to \bar{u}\bar{d}, \quad d\bar{d} \to d\bar{d}, \quad d\bar{d} \to u\bar{u},$$

$$d\bar{u} \to d\bar{u}, \quad dd \to dd, \quad \bar{d}\bar{d} \to \bar{d}\bar{d},$$

- Using i-spin symmetry, charge conjugation symmetry as well as the crossing symmetry to relate the matrix element square for the above 12 processes reduce to evaluating only two independent matrix elements $u\bar{u} \to u\bar{u}$ and $u\bar{d} \to u\bar{d}$
- Dominant contribution comes from propagation of pion and sigma mode in the s-channel.
- The temperature dependence of π and σ modes play an important role in these cross section evaluation.

Zhuang etal Phys Rev D51,3728, 1995

$\eta/s, \zeta/s$: QUARK SCATTERING

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1 - f_a^0)$$

With Polyakov loop dependent quark distribution functions,

$$f_a^0(1 - f_a^0) \rightarrow \left[\frac{\phi e^{-\beta\omega} + 4\bar{\phi}e^{-2\beta\omega} + 3e^{-3\beta\omega}}{1 + 3\phi e^{-\beta\omega} + 3\bar{\phi}e^{-2\beta\omega} + 3e^{-3\beta\omega}} - 3f_a^{02}\right]$$

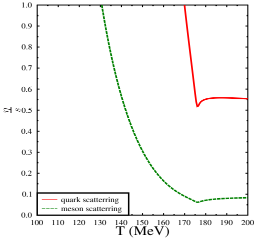


Fig. 5-a

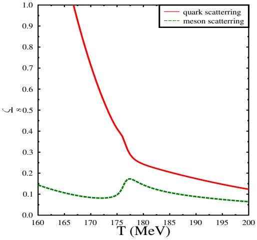


Fig. 5-b

Estimating the viscosities-quark meson scatterings

- Quark pion scattring: $(q, \pi \rightarrow q, \pi)$ Matrix element:

$$T_{ba} \sim \frac{g^2}{2}(\gamma^\mu q_{1\mu} + \gamma^\mu q_{2\mu}) \left[B^{+}\delta_{ab} + B^{(-)} i\epsilon_{abc}\tau^c\right]$$

$$B^{(+)} = \left[\frac{1}{u - m_q^2} - \frac{1}{s - m_q^2}\right], \quad B^{(-)} = -\left[\frac{1}{u - m_q^2} + \frac{1}{s - m_q^2}\right]$$

- Poles can arise in the u-channel
  Include in medium quark width

$$\Sigma = m\Sigma_0 + \gamma \cdot \mathbf{p}\Sigma_3 - \gamma_0 p_0 \Sigma_4$$

  Relevant imganinary part is included in the propagator.

$\eta/s,\zeta/s$: QUARK MESON SCATTERING

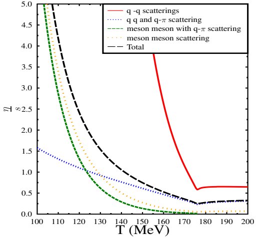


Fig. 6-a

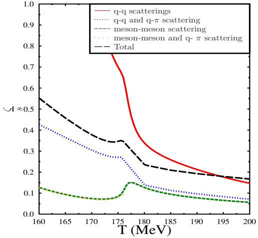


Fig. 6-b

Figure: Different contributions as well as the total contribution to $\frac{\eta}{s}$ (Fig 6 -a) and $\frac{\zeta}{s}$ (Fig. (6-b) as a function of temperature . ($\mu = 0$).

η/s,ζ/s: CONTD.···

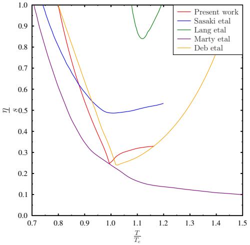


Fig. 8-a

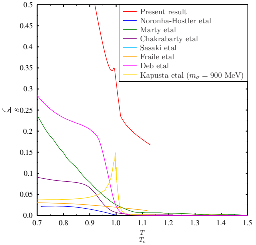


Fig. 8-b

Summary,conclusions and outlook

- We tried to derive the viscosity coefficients using Boltzmann kinetic equation with relaxation time approximation with mean field and medium dpendent masses.
- While η depends only on the behaviour of relaxation time and the medium dependent masses, ζ depends on other thermodynamic quantities and the equation of state.
- The deviation from equllibrium should be consistent the Landau Lifshitz conditions.
- The thermodynamics of hot and dense matter is estimated within PQM model.
- The transport coefficients are non negative in the relaxation time approximation which is a consequence of Landau-Liftshitz conditions of fit.
- Relaxation times are estimated using meson meson scattering, quark scatterring through meson exchange as well as quark meson scattering.
- Medium dependence of meson masses and widths affect the relaxation time and hence the transport coefficients.

Thank you