Effective Thermal Models from the Lattice

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Outline

Why an effective field theory?

First observation

Lattice computations show rise in P/T^4 and E/T^4 in the region around T_c.

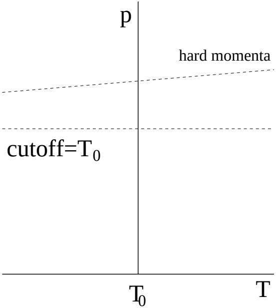
Is this due to excitation of many hadrons: resonance gas? Is there a dual description in terms of quarks and gluons? Is this a matter of energy scales?

Another observation

Lattice shows clear evidence of quarks and gluons in correlation functions just above T_c.

Since QCD has a cross over, could one see signals of quarks a little below T_c? Could an effective quark theory be quantitatively correct even below T_c? Danger: effective theory need not be simple. An EFT-style construction, keeping all possible operators at any given mass dimension?

Entangled cutoff and couplings



$p \simeq T$ domain of short distance physics. Want to describe long distance physics at $T \simeq T_0$, ie, choose $p \ll T_0$. But T_0 is a dummy variable. If T_0 is changed, then the couplings can be changed to compensate: running of couplings in the EFT.

Weinberg, Wilson

Outline

Global symmetries of QCD at finite temperature

Approximate chiral symmetry $SU_L(N_f) \times SU_R(N_f)$ of QCD acts globally on quark fields. Softly broken even in the limit when the symmetry is exact. Explicitly broken by quark masses. What N_f is appropriate for the EFT?

Scale anomaly of QCD generates an intrinsic scale: Λ_{QCD}. When quark masses $m \gg \Lambda_{QCD}$, inappropriate for EFT. As a result, $N_f = 2$ or 3. We choose $N_f = 2$ as a first exercise.

Lorentz invariance is broken at finite temperature: spatial invariances (rotation, P) remain, T remains, CP symmetry remains. Temporal and spatial components of vectors distinguished. Theory is still relativistic ($p \gg m$), *i.e.*, Lorentz covariant. The counting of mass dimensions is unchanged.

Grossman+, Yaffe+

The effective Lagrangian up to dimension 6

$$
\begin{aligned}
L \;=\;& d^0 + d^3\, T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4 \overline{\psi}\partial\!\!\!/_i\psi + \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \\
&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 \\
&+ \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 + \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2 \\
&+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&+ \frac{d^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
&+ \frac{d^{69}}{T_0^2}\left[(\overline{\psi}iS_{i4}\psi)^2 + (\overline{\psi}S_{ij}\tau^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi}iS_{i4}\tau^a\psi)^2 + (\overline{\psi}S_{ij}\psi)^2\right]
\end{aligned}
$$

The mean field approximation

Mean field approximations: a field operator is replaced by a number: condensate. Here we can write

$$\psi_\alpha \overline{\psi}_\beta = \delta_{\alpha\beta}\langle\overline{\psi}\psi\rangle.$$

Then doing the Fierz transformations, all terms reduce to a single combination of couplings

$$\lambda = (\mathcal{N}+2)d^{61} - 2d^{62} - d^{63} + d^{64} + d^{65} - d^{66} + d^{69} - d^{60}.$$

Using this, MFT similar to ordinary NJL. But three couplings: d^3, d^4, λ. Match correlators to fix couplings. A “renormalization scale” also enters through regulating divergent integrals in $\overline{\mathrm{MS}}$. Couplings depend on the scale T_0 and the ratio M/T_0.

The gap equation

The pressure is dominated by the UV modes: not captured in any EFT. A good EFT gets: the IR-dominated part of the free energy, critical points, singular parts near it. But also T_c, through the gap equation, with solution

$$\Sigma = -\frac{2\lambda}{T_0^2}\frac{\partial I_0}{\partial m}.$$

Well known chiral critical point at $T_c(\lambda, d^4)$ when $d^3 = 0$. Extend to finite μ. Then we find

$$\frac{T_c^2(\mu)}{T_c^2(0)} = 1 - K\frac{\mu^2}{T_c^2(0)},$$

where $K = 3/\pi^2$. Parameter free prediction of MFT, to be compared with lattice $K \simeq 0.01$–0.05. Lattice may need extrapolation to $d^3 = 0$ and MFT may need correction.

Outline

Fluctuations

Replace MFT ansatz by

$$\overline{\psi}_\alpha \psi_\beta = \langle \overline{\psi}\psi \rangle U_{\alpha\beta} \qquad \text{where} \qquad U = \exp\left(\frac{i\pi\gamma_5}{f}\right).$$

The meaning of the scale f is to be explored. The pion fields, π, are fluctuations around the MFT. Introduce this into the MFT quadratic Lagrangian, integrate over the fermions, and find the EFT for pions. Result in the form

$$L_f = c^2 T_0^2 \pi^2 + \frac{1}{2}(\partial_0 \pi)^2 + \frac{c^4}{2}(\nabla\pi)^2 + c^{41}\pi^4 + \cdots$$

Finite T GMOR obtained. **Klevansky (review)**. Similar pion theory used before, using intuition from hydrodynamics. **Son, Stephanov** Notational convenience: $u = \sqrt{c^4}$ and $m_\pi = \sqrt{c^2}\,T_0$.

New: microscopic derivation. Systematic extension to higher order terms in pion EFT possible. Investigating extension to $\mu > 0$.

Magnitudes of terms

Typical scales are

$$\pi \ll f \qquad m_\pi \ll f, \qquad p_0 \simeq f, \qquad p \simeq uf$$

Dimension-4 terms then have a hierarchy:

$$c\pi^4 \simeq \frac{m_\pi^2}{f^2}\pi^4 \simeq m_\pi^2\pi^2\left(\frac{\pi^2}{f^2}\right) \ll c' p^2\pi^2 \simeq u^2 f^2\pi^2.$$

Dimension-6 terms also have a similar hierarchy:

$$\begin{aligned}
c\frac{\pi^6}{f^2} &\simeq \frac{m_\pi^2}{f^2}\frac{\pi^6}{f^2} \ll m_\pi^2\pi^2\left(\frac{\pi^4}{f^4}\right). \\
c' p^2\frac{\pi^4}{f^2} &\simeq u^2 f^2\pi^2\left(\frac{\pi^2}{f^2}\right) \gg c\frac{\pi^6}{f^2}. \\
c'' p^4\frac{\pi^2}{f^2} &\simeq u^4 f^4\left(\frac{\pi^2}{f^2}\right) \gg c' p^2\frac{\pi^4}{f^2}.
\end{aligned}$$

At dimension-$2n$ terms of the form $(\nabla^{n-1}\pi)^2$ are dominant.

Some remarks

f/T is of order unity and insensitive to temperature. So, the domain of validity of the pion EFT could be similar to that of the fermion EFT.

$c^4 \simeq (1-t)$ near $t=1$, and vanishes for $t>1$. When $c^4=0$, equation of motion gives no propagating mode. Critical properties require higher dimensional terms, and resummations. Left for later.

We choose T_0 to be T_c in the chiral limit. Then λ is fixed by the solution of the gap equation. However, T_c/T_0 depends on the quark mass: defined by peak in chiral susceptibility.

Need to fix two parameters of the Fermion EFT (d^3 and d^4) by fixing two parameters of the pion EFT. Rest of the parameters are predictions!

Two-point functions

Pion correlator:

$$C_\pi(q) = \left[m_\pi^2 + q_4^2 + c^4\mathbf{q}^2\right]^{-1} \xrightarrow{q_4\to 0} \frac{1}{c^4}\left[\mathbf{q}^2 + m_\pi^2/c^4\right]^{-1}$$

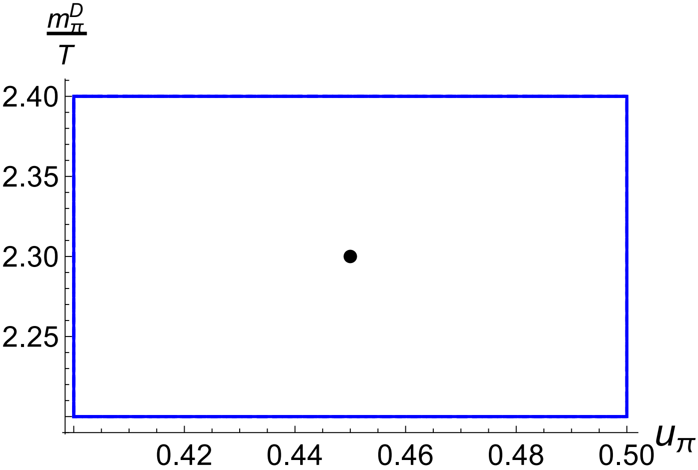
This means that the screening mass $m_\pi^D = m_\pi/u$.

Axial current correlator:

$$C_{A_4}(q) = (2f)^2 q_4^2 \; C_\pi(q) \qquad C_{A_i}(q) = (2fc^4)^2\mathbf{q}_i^2 \; C_\pi(q)$$

The parameters of the quadratic part of the pion EFT can be extracted from C_π and C_{A_μ} in many ways. Some of these can be used as checks.

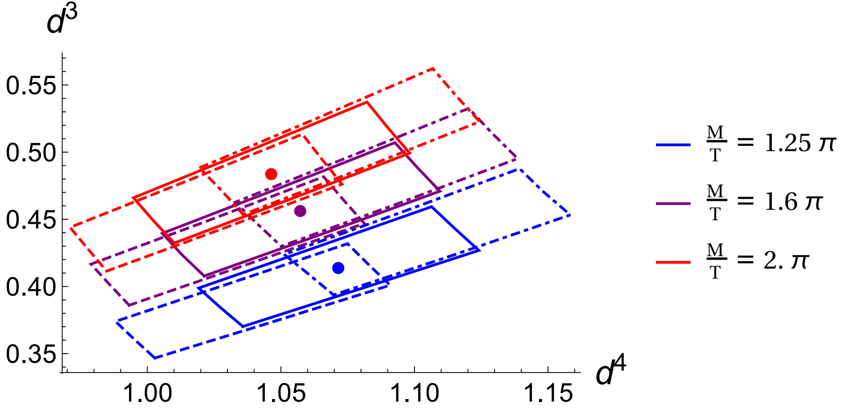
Lattice measurements



Improved Wilson fermions, $N_f = 2$. Data set C1, $T/T_c = 0.84$

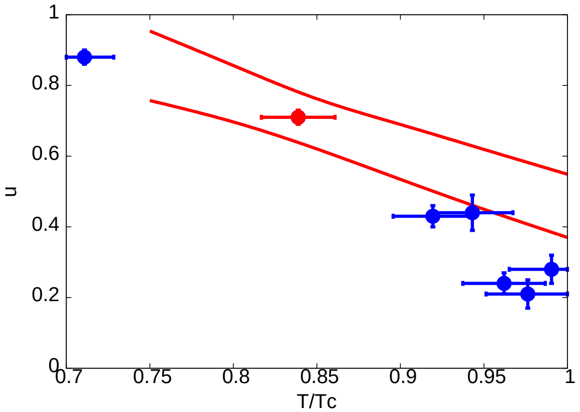
Brandt, Francis, Meyer, Robaina arxiv/1406.5602

Extracted fermion parameters



Three boxes of each colour for central and 1σ error bounds for T_c.

Quality of fit



Couplings of the fermion EFT fixed at one T. Running of pion couplings with T then prediction: compared with lattice measurements, where available.

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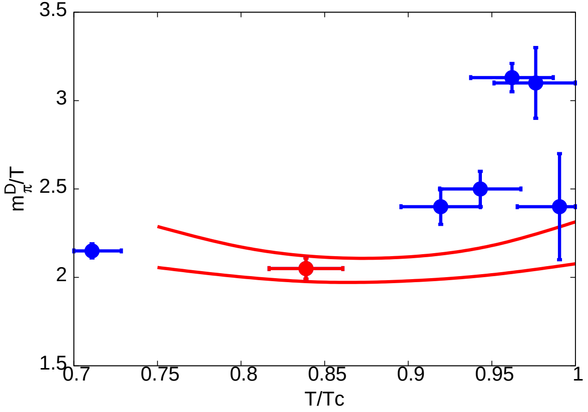

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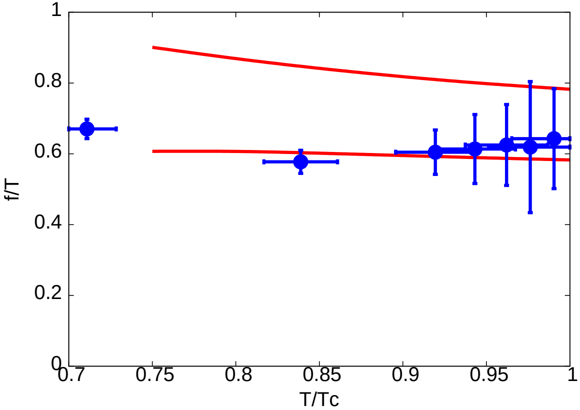

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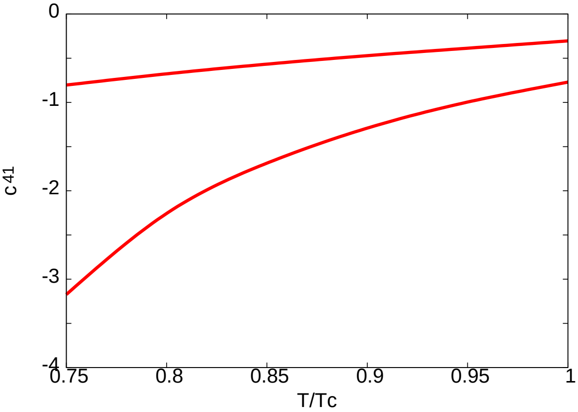
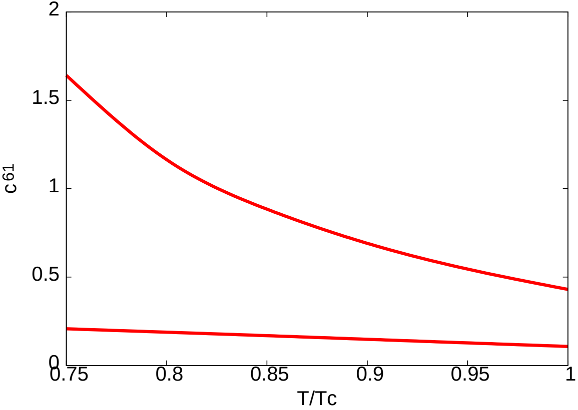
Prediction: quartic pion coupling



Error band in the prediction includes statistical errors in the input measurements, variation due to the setting of the RG scale, and lattice scale setting errors. The last contributes to about 50% of the width of the band at $T/T_c = 0.85$.

Prediction: dominant dimension-6 coupling



At smaller T/T_c the RG scale is the largest uncertainty, causing extreme sensitivity to input fermion couplings. This is because M begins to approach m_π. Expect better behaviour at small m_π.

Outline

Summary

1. Fermionic EFT (including V, AV, T terms) after Fierz transformations equivalent to pure NJL model in mean field approximation.
2. EFTs at finite temperature cannot give the full EOS, but may be used to estimate the singular part of free energy.
3. Corrections to mean-field of Fermionic EFT described by a pion EFT. Only two of the free parameters are independent, and can be traded for couplings in the Fermion EFT. All other couplings of the pion EFT related.
4. Surprisingly good description of lattice measurements obtained.
5. Highly predictive for lattice computations of n-point functions of PS operators just below chiral cross-over. Example: 4-point function predicted.