The Linear Sigma Model coupled to quarks to locate the Critical End Point

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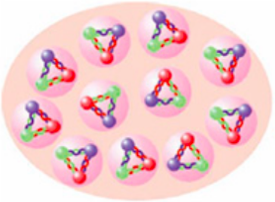
Critical Point and Onset of Deconfinement, Stony Brook, NY,
August 10 2017.

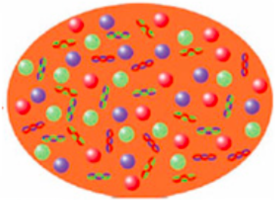
Outline.

- Chiral Symmetry restoration/Deconfinement
- The Linear Sigma Model.
- Effective Potential.
  - 1-loop correction
  - Ring diagrams/screening effects
- Results.
- Ongoing.

Int.J.Mod.Phys. A31 (2016) no.36, 1650199; Phys.Rev. D92 (2015) no.9, 096011; Nucl.Phys. B897 (2015) 77-86; Phys.Rev. D84 (2011) 056004

We are interested on both faces of the strongly interacting matter and how this transits between the two phases.

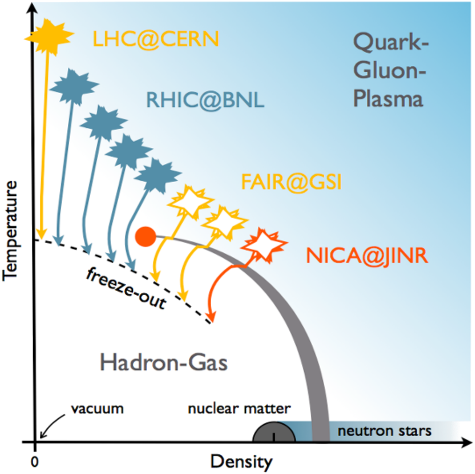




Hadrons
Low energies
(Non perturbative regime)

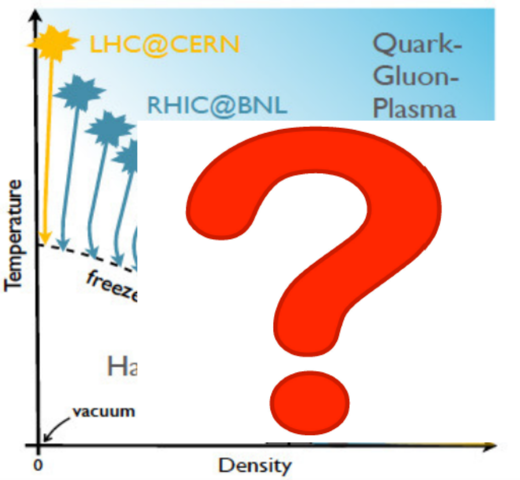
Quarks and gluons
Extreme conditions
(pQCD)

QCD phase diagram



- Experiments and astronomical objects provide information.
- Chiral symmetry.
- Confinement/deconfinement
- Critical End Point.

However



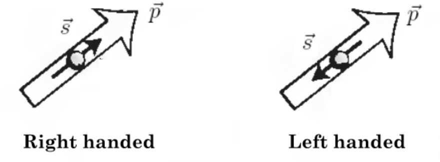
We have reliable information at low densities.

Chiral Symmetry.

QCD with massless quarks: Chiral symmetry.

- $\mathcal{L}^0_{QCD} = \overline{\psi}(x) i\gamma_\mu \partial^\mu \psi + \mathcal{L}_{quark-gluon} + \mathcal{L}_{glue}$
- $\psi = \psi_R + \psi_L$

$$\begin{aligned} \psi_R &= \frac{1}{2}(1+\gamma_5)\psi, \\ \psi_L &= \frac{1}{2}(1-\gamma_5)\psi \end{aligned}$$



Approximate symmetry

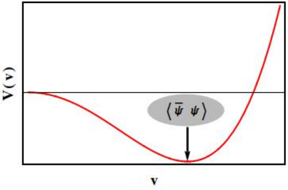
Symmetries of $\mathcal{L}_{QCD}$

- $m_s = m_u = m_d = 0$
  - $SU(3)_V \bigotimes SU(3)_A$
    $(\partial_\mu V^\mu \propto (m_s - m_{ud}) \qquad \partial_\mu A^\mu \propto (m_s + m_{ud}))$
- $m_s \neq 0$ and $m_u = m_d = 0$ (1^o symmetry broken)
  - $SU(2)_V \bigotimes SU(2)_A$
    $(\partial_\mu V^\mu \propto (m_d - m_u) \qquad \partial_\mu A^\mu \propto (m_d + m_u))$
- $m_s \neq 0$ and $m_u = m_d \neq 0$ (2^o symmetry broken)
  - $SU(2)_V$
    $(\partial_\mu V^\mu \propto (m_d - m_u))$
- $m_u \neq m_d \neq 0$ (3^o symmetry broken)
  - $U(1)$

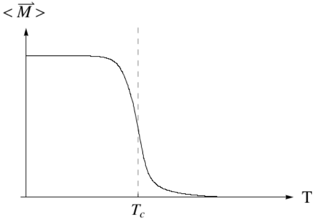
Spontaneously symmetry breaking

- Spontaneous breaking of the chiral symmetry.

$$Q_j^A|0\rangle \neq 0 \;\Leftrightarrow\; \langle\overline{\psi}\psi\rangle \neq 0$$



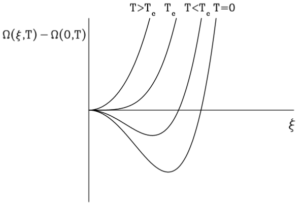
- Order parameter.



In QCD

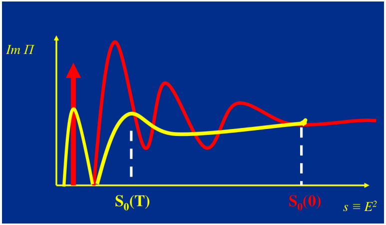
- High temperature $T > T_c$: symmetry is restored
- $\langle\overrightarrow{\mathcal{M}}\rangle \longleftrightarrow$ Chiral quark condensate $\langle\overline{q}q\rangle$

Chiral symmetry restoration.



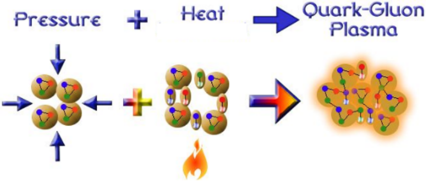
J. I. Kapusta, *Finite-Temperature Field Theory*

(Cambridge Univ. Press, Cambridge 1989).



C. A. Dominguez

where $\Omega(\xi, T) \to V^{eff}(v, T)$ is the effective potential and ImΠ the hadronic spectral function.



QCD phase transition.

- Effective approach, using the Linear Sigma Model coupled to quarks.
- Focusing in the chiral symmetry restoration phenomena.
- Compute the effective potential beyond the mean field approximation ($T \neq 0$ and $\mu \neq 0$).
- Find the pseudo-critical temperature for low values of quark chemical potential.
- Construct the effective QCD phase diagram.

Linear Sigma Model coupled to quarks.

- Effective model for low-energy QCD.
- Renormalizable theory.
- Implement ideas of chiral symmetry ($SU(2)_L \times SU(2)_R \rightarrow O(4)$).
- Effects of quarks and mesons on the chiral phase transition.
- Spontaneous symmetry breaking $O(4) \rightarrow O(3)$.

Linear Sigma Model coupled to quarks.

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi,$$

where ψ is an $SU(2)$ isospin doublet, $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ is an isospin triplet and σ is an isospin singlet.

To allow for spontaneous symmetry breaking

$$\sigma \to \sigma + v,$$

v can later be identified as the order parameter of the theory.

Linear Sigma Model coupled to quarks.

After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] \underline{- \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2}$$

$$- \frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}] \underline{- \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2}$$

$$+ i\bar{\psi}\gamma^\mu D_\mu\psi \underline{- gv\bar{\psi}\psi} + \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4$$

$$- \frac{\lambda}{4}[(\sigma^2 + \pi_0^2)^2 + 4\pi^+\pi^-(\sigma^2 + \pi_0^2 + \pi^+\pi^-)]$$

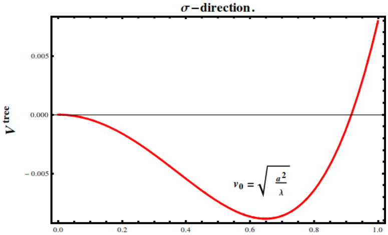
$$- g\hat{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2,$$

$$m_\pi^2 = \lambda v^2 - a^2,$$

$$m_f = gv.$$



Effective potential.

- Mean field approximation (The first quantum and thermal correction).
  - Boson and fermion fields.
  - Imaginary time formalism.

$$V_b^{(1)} = T \sum_{i=\sigma,\vec{\pi}} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \ln D^{-1/2},$$

$$V_f^{(1)} = -T \sum_{i=u,d} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \mathsf{Tr} \ln S,$$

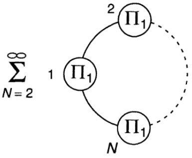
where the thermal boson and fermion propagators are given by

$$D = \frac{1}{k^2 + m_b^2 + \omega_n^2},$$

$$S = \frac{\not{k} + m_f}{k^2 + m_f^2 + (\omega_n - i\mu)^2}.$$

Beyond mean field.

Next term in the perturbative series is the ring diagrams (Dolan & Jackiw, Phys. Rev. D**12** 3320 (1974)).



M. Le Bellac, *Thermal Field Theory* (Cambridge Univ. Press, Cambridge 2000).

Screening properties of the plasma.

$$V^{ring} = \frac{1}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln[1 + \Pi(m_b)\Delta(\omega_n, k; m_b^2)]$$

with Π the self-energy

High temperature approximation.

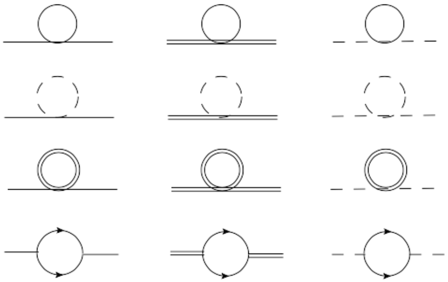
1-loop

$$V^{(b,1+R)} = \sum_{i=\sigma,\pi^0} \left\{ \frac{m_i^4}{64\pi^2} \left[\ln\left(\frac{m_i^2}{2a^2}\right) - 2\gamma_E + 1 \right] + \frac{m_i^4}{64\pi^2} \ln\left(\frac{(4\pi T)^2}{m_i^2}\right) - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T m_i^3}{12\pi} + \frac{T m_i^3}{12\pi} - \frac{T}{12\pi}(m_i^2 + \Pi)^{3/2} \right\},$$

$$V^{(f,1)} = -N_c \sum_{f=u,d} \left[\frac{m_f^4}{16\pi^2} \ln\left(\frac{m_f^2}{2a^2}\right) + \frac{m_f^4}{16\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{m_f^2}\right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T}\right) + \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T}\right) \right] + 8 m_f^2 T^2 [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] - 32 T^4 [Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \right]$$

Effective potential.

$$
\begin{aligned}
V^{(eff)} &= -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{i=\sigma,\pi^0}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right]\right. \\
&- \left.\frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi}(m_i^2 + \Pi)^{3/2}\right\} \\
&- N_c \sum_{f=u,d}\left[\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T}\right)\right.\right. \\
&+ \left.\psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T}\right)\right] + 8m_f^2T^2[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] \\
&- \left. 32T^4[Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})]\right]
\end{aligned}
$$



with the self-energy

$$\Pi = \lambda \frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2}[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

Pseudo-critical temperature

The criterion to find the temperature where the chiral symmetry is restored, is the following

$$\left.\frac{d^2V^{(eff)}}{dv^2}\right|_{v=0} = 0,$$

it means

$$curvature = mass^2,$$

and this is only valid when the restoration of the chiral symmetry is a second order phase transition.

Parameter space.

- Four parameters.
  - Two coupling constants λ and g, the critical temperature T_c^0 at $\mu = 0$, and the parameter a.
- Boson thermal masses

$$m_\sigma^2(T) = 3\lambda v^2 - a^2 + \frac{\lambda T^2}{2} + \frac{N_f N_c g^2 T^2}{6},$$

$$m_\pi^2(T) = \lambda v^2 - a^2 + \frac{\lambda T^2}{2} + \frac{N_f N_c g^2 T^2}{6}.$$

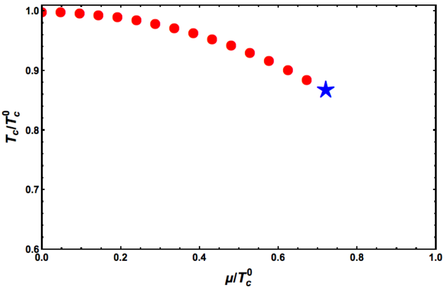
- At the phase transition with $\mu = 0$, the curvature of V^{eff} vanishes for $v = 0$

$$\frac{a}{T_c^0} = \sqrt{\frac{\lambda}{2} + \frac{N_f N_c g^2}{6}}.$$

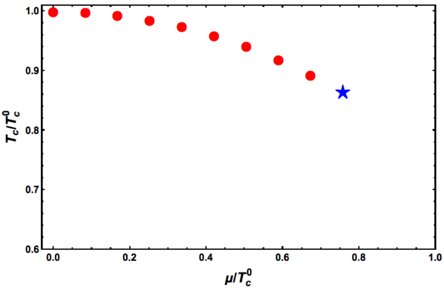
- From the vacuum boson masses, we can fix the value of a

$$a = \sqrt{\frac{m_\sigma^2 - 3m_\pi^2}{2}}.$$

Results



$T_c^0 = 174$ MeV, $\lambda = 0.40$ and $g = 0.63$.



$T_c^0 = 200$ MeV, $\lambda = 0.36$ and $g = 0.51$.

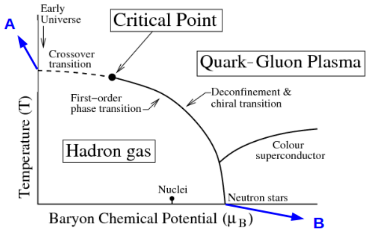
Since, we are in the high temperature approximation, we are not able to describe the region where $\mu \geq T$. However, we reach the CEP.

Conclusions.

- Working in the LSMq, CEP is located in the region found by mathematical extensions of lattice analyses.
- We computed the effective potential and included plasma screening effects through the boson's self energy.
- We explore the phase diagram in the high temperature approximation and for low values of μ where the phase transition within the model is second order.
- We used as an input the temperature at $\mu = 0$ and determined a from the vacuum values of m_σ and m_π.
- We found the CEP at $\{T_{\text{CEP}} = 0.86T_c, \mu_{\text{CEP}} = 0.72T_c\}$ with $T_c = 174$ MeV, $\lambda = 0.40$ and $g = 0.63$, and at $\{T_{\text{CEP}} = 0.86T_c, \mu_{\text{CEP}} = 0.76T_c\}$ with $T_c = 200$ MeV, $\lambda = 0.40$ and $g = 0.49$.

Ongoing

- Now, we are able to fix the values of λ and g, therefore a unique transition line is found without any ambiguity.



$$(A)\ m_\sigma^2(T_c, \mu = 0) = 3\lambda v^2 - a^2 + \Pi(Tc, \mu = 0) = 0\text{MeV}^2$$
$$(B)\ m_\sigma^2(T = 0, \mu_c) = 3\lambda v^2 - a^2 + \Pi(T = 0, \mu_c) = (600\ \text{MeV})^2$$

- We relax the condition $T > a$, such that we can explore a better description of the QCD phase diagram.

Ongoing

Set of parameters:

- $m_\sigma = 600$ MeV
- $m_\pi = 137$ MeV
- $m_q^b = 5$ MeV
- $m_q^D = 300$ MeV
- $a = 388.07$ MeV
- $N_c = 3$
- $N_f = 2$
- $T_c^{LQCD} = 170$ MeV

- Full computation of the self-energy $\Pi(T,\mu) = \Pi_b(T) + \Pi_f(T,\mu)$, i.e. T, μ and m_i without restrictions among them.
- Full thermal contribution to the effective potential (numerical calculation).

Many Thanks!!!

- See you all there!!!

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