# Color screening in 2+1 flavor QCD

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in collaboration with
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(**TUMQCD** collaboration)

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**TUM-EFT 81/16**; PRD 93 114502 (2016); arXiv:1601:08001 (2016)

## Color screening in 2+1 flavor QCD

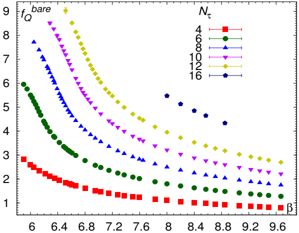
- Overview & Introduction
- Correlators of Polyakov loops
- Comparison to weak coupling
- Summary

# What is new about the TUMQCD lattices?

$a(\beta)N_\tau = 1/T(\beta, N_\tau)$

$T \in [135, 1935]$ MeV with at least four $N_\tau$:

**Continuum limit** with **realistic** quark masses




TUMQCD collaboration, **TUM-EFT 81/16**

$f_Q^{\mathrm{bare}} = -\log \langle P \rangle_T$

Two different volumes and two quark masses:

Controlled **finite volume** and **quark mass** dependence

- $N_\tau = 4 - 12$: ∼30 ens. each, $5.9 \leq \beta \leq 9.67$, $a = 0.0085 - 0.25$ fm.
- **HISQ/Tree** action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$; taste breaking much reduced.
- Ensembles: $\frac{N_\sigma}{N_\tau} = 4$, $m_l = \frac{m_s}{20} \Leftrightarrow m_\pi = 161$ MeV; $\beta \leq 7.825$, $a \geq 0.04$ fm most from A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]
- All $N_\tau$, $m_l = \frac{m_s}{5}$: $3 - 5$ ensembles each, $1 - 10 \times 10^4$ TU each, $7.03 \leq \beta \leq 8.4$, $a = 0.025 - 0.083$ fm; $T = 0$ lattices available.
- $r_1$ scale for $\beta > 8.4$ from non-perturbative $\beta$ function **PRD 90** 094503 (2014)

## Polyakov loops and free energies of static quark states

- The *Polyakov loop* $L$ is the gauge-invariant expectation value of the traced propagator of a static quark ($P$) and related to its **free energy**: $L(T) = \langle P \rangle_T = \langle \mathrm{Tr}\, S_Q(x,x) \rangle_T = e^{-F^{\mathrm{b}}_Q/T}$. $L$ needs renormalization.

  A. M. Polyakov, **PL 72B** (1978); L. McLerran, B. Svetitsky, **PRD 24** (1981)

- The *Polyakov loop correlator* is related to *singlet* & *octet free energies*

$$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-F^{\mathrm{b}}_S/T} + \frac{8}{9} e^{-F^{\mathrm{b}}_A/T} = \frac{1}{9} C_S(r,T) + \frac{8}{9} C_A(r,T).$$

  S. Nadkarni, **PRD 33, 34** (1986)

- Meaning of **gauge-dependent** *singlet* & *octet free energies* is unclear.
- $C_P$ is also related to the **gauge-invariant potentials** $V_{S,A}$ of **pNRQCD**

$$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-V^{\mathrm{b}}_S/T} + \frac{8}{9} L^{\mathrm{b}}_A\, e^{-V^{\mathrm{b}}_A/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$

  N. Brambilla et al., **PRD 82** (2010)

## Renormalization of free energies

- **Singlet free energy** and **potential** appear to be related for $rm_D \sim 1$:

$$F_S(r,T) = -C_F\alpha_s\left[\frac{e^{-r\,m_D}}{r} + m_D\right] + \mathcal{O}(g^4) = V_S(r) + \mathcal{O}(g^3).$$

N. Brambilla et al., **PRD 82** (2010)

$\Rightarrow$ $F_S$ and $V_S$ share the same renormalization $2C_Q$, which depends on $T$ only through the lattice spacing: $V_S = V_S^b + 2C_Q \Rightarrow F_S = F_S^b + 2C_Q$.

- Use $V_S$ at $T=0$: fix $r_1$ scale & determine $2C_Q$ using **static energy**.

A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]

- Cluster decomposition theorem: $F_{Q\bar{Q}} = F_S = 2F_Q$ for $r \gg 1/T$.

$\Rightarrow$ renormalize as $F_{Q\bar{Q}} = F_{Q\bar{Q}}^b + 2C_Q$ and $F_Q = F_Q^b + C_Q$. $\to$ **PRD 93 114502 (2016)**

Beyond $C_Q(\beta)$ from $T=0$ lattices – use **direct renormalization** of $F_Q$

$\Rightarrow$ Infer unknown $C_Q(\beta)$ from known $C_Q(\beta^{\mathrm{ref}})$ using different $N_\tau, N_\tau^{\mathrm{ref}}$

$$C_Q(\beta) = \left\{C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau)\right\}$$

$\to$ S. Gupta et al., **PRD 77** 034503 (2008)

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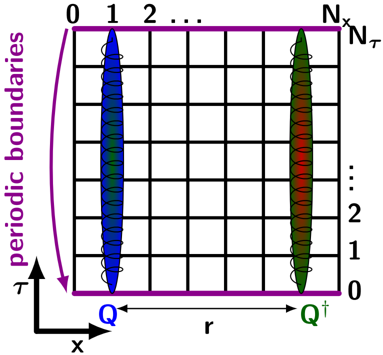
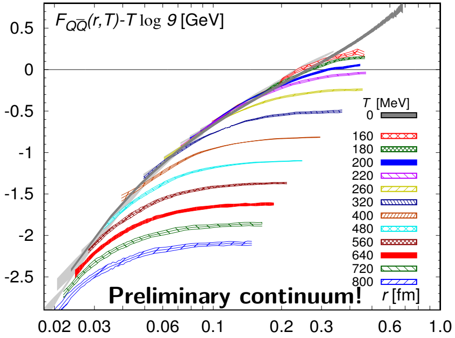
$$C_Q(\beta) = \left\{ C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau) + \Delta_{N_\tau, N_\tau^{\mathrm{ref}}} \right\}$$ $\rightarrow$ **PRD 93 114502 (2016)**

**Color screening for a static quark-antiquark pair**



## Polyakov loop correlator and $Q\bar{Q}$ free energy

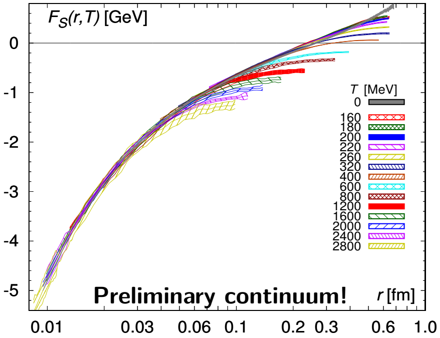



- **Free energy of a $Q\bar{Q}$ pair**, $F_{Q\bar{Q}}$, is also called *color-averaged potential*:

$$C_P^{\text{ren}}(r,T) = \langle P(0)P^\dagger(\boldsymbol{r})\rangle_T^{\text{ren}} = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9}e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9}e^{-\frac{F_A(r,T)}{T}}.$$

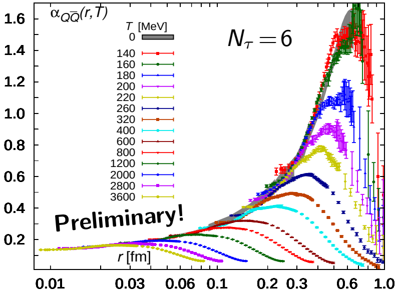
- $F_{Q\bar{Q}} - T\log 9$ is close to the $T=0$ **static energy** $V_S$ for very small $rT$.

## Singlet free energy in Coulomb gauge




- **Singlet free energy:** $C_S^{\mathrm{ren}}(r,T) = \frac{1}{3}\left\langle \sum_{a=1}^{3} W_a(0) W_a^\dagger(\boldsymbol{r}) \right\rangle_T^{\mathrm{ren}} = e^{-F_S(r,T)/T}$
- Wilson line correlator requires explicit **gauge fixing** (Coulomb gauge)
- $F_S$ is numerically close to the $T=0$ **static energy** $V_S$ for $rT \lesssim 0.3$.

## Effective coupling: vacuum-like and screening regimes

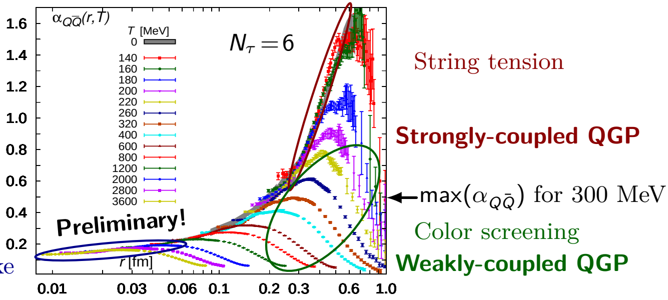



- **Effective coupling** $\alpha_{Q\bar{Q}}(r,T)$ is a proxy for the **force** between $Q$ and $\bar{Q}$.

$$\alpha_{Q\bar{Q}}(r,T) = \frac{r^2}{C_F}\frac{\partial E(r,T)}{\partial r}, \quad E = \{F_S(r,T), V_S(r)\}$$

- $\alpha_{Q\bar{Q}}$ clearly distinguishes different regimes at small and large $r$.

## Effective coupling: vacuum-like and screening regimes




| Vacuum-like regime | Screening regime | max($\alpha_{Q\bar{Q}}$) |
|---|---|---|
| $rT \lesssim 0.2$ | $rT \gtrsim 0.3$ | $r_{\max} T \sim 0.4$ |

- $r_{\max}$ defined through max($\alpha_{Q\bar{Q}}$), which is proxy for the **maximal force**.
- For $T \lesssim 300$ MeV: max($\alpha_{Q\bar{Q}}$)($T$) $\gtrsim 0.5$ – **strongly-coupled QGP**.

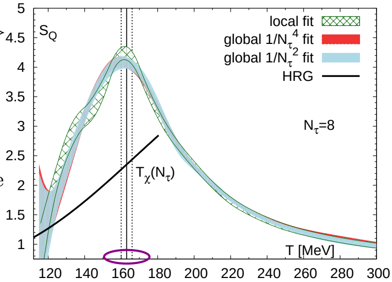
## $T_\chi$ from chiral observables vs $T_S$ from the peak of the entropy

$F_{Q\bar{Q}}(r \to \infty) = 2F_Q \Rightarrow$

**Static quark entropy**

$$S_Q = -\frac{dF_Q}{dT}$$

independent of volume and renormalization scheme (**continuum**)




Reminder: $aN_\tau = 1/T$
Discretization errors:
$T_c(N_\tau) = T_c + \mathcal{O}(1/N_\tau^2)$

Bazavov et al. [TUMQCD]
**PRD 93** 114502 (2016)

- The entropy peaks at $T_S = 153^{+6.5}_{-5}$ MeV in the continuum limit.
- $T_S(N_\tau) \simeq T_\chi(N_\tau)$ for any $N_\tau$ Bazavov et al., **PRD 93** 114502 (2016) [TUMQCD], suggests a **tight link between chiral symmetry and deconfinement**.

  e.g. as in glueball-sigma mixing scenarios, Y. Hatta, K. Fukushima **PRD 69** 097502 (2004).

N.b. $T_\chi$ defined via $O(2)$ scaling of $\chi_{m,l}$ ($O(4)$: $1-3.5$ MeV lower $T_\chi$ )

A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]

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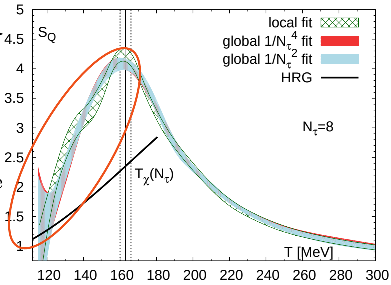


Reminder: $aN_\tau = 1/T$
Discretization errors:
$T_c(N_\tau) = T_c + \mathcal{O}(1/N_\tau^2)$

$\frac{dS_Q}{dT} > 0$ for $T < T_c$

Bazavov et al. [TUMQCD]
**PRD 93** 114502 (2016)

- *Hadron resonance gas* (HRG) is limited to only below $T \sim 125$ MeV.
  static HRG results from: A. Bazavov, P. Petreczky, **PRD 87**, 094505 (2013)
- $\frac{dS_Q}{dT} > 0$ for $T < T_c$: the number of bound states of bound states including a static quark increases faster than HRG predictions.
- Large number of **additional states** or **strong thermal modification of (low-lying) states** are needed already substantially below $T_c$.

## $T_\chi$ from chiral observables vs $T_S$ from the peak of the entropy

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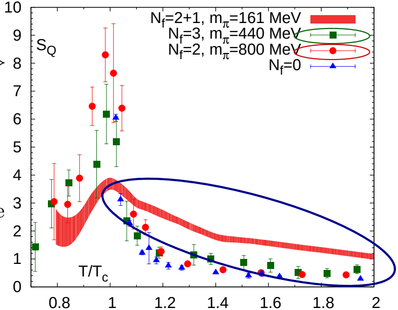



O. Kaczmarek, F. Zantow, hep-lat/0506019 (2005)

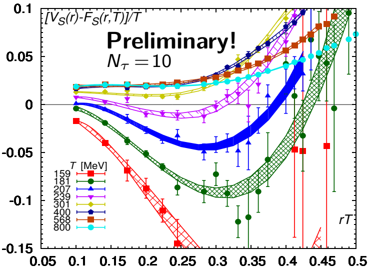
P. Petreczky, K. Petrov PRD 70 054503 (2004)

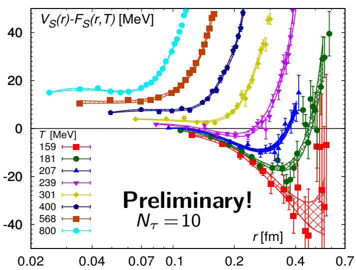
$\frac{dS_Q}{dT} < 0$ for $T > T_c$

Bazavov et al. [TUMQCD] PRD 93 114502 (2016)

- $\frac{dS_Q}{dT} < 0$ for $T > T_c$: the static quark interacts with the **medium only inside its Debye screening radius**, $r \sim 1/m_D \overset{T\to\infty}{\longrightarrow} 0$.
- Deconfinement and **onset of screening** are clearly defined via $S_Q(T_S) = 0$ in the QCD crossover scenario. **MPL A31** no.35, 1630040 (2016)
- The peak is broader and lower for smaller $m_{\text{sea}}$ or larger $N_f$.

## Static energy and singlet free energy (I) - discretization effects

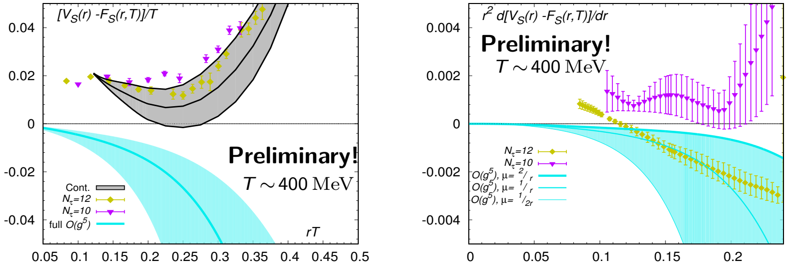



log scale!

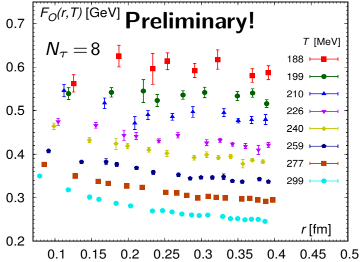
- *pNRQCD*: $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(g^6)$ M. Berwein et al., arXiv:1704.07266
- **Smooth $r$ dependence** due to cancellations in $V_S - F_S$ for $r/a < 3$.
- **Strong $N_\tau$ dependence** for $rT > 0.15$, but $N_\tau \geq 12 \sim$ continuum limit.
- $V_S - F_S \sim 0.02T$ for $rT \lesssim 0.1$ & $T > 300\,\text{MeV}$, mild $N_\tau$ dependence.
- $T$ independent for small $r$, then **sudden onset of medium effects**.

## Static energy and singlet free energy (II) – weak coupling



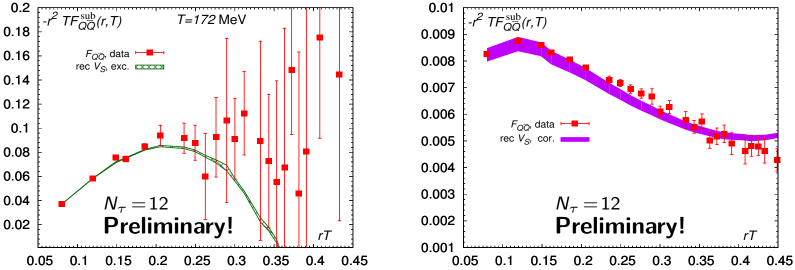

- *pNRQCD*: $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(\alpha_s^3)$ M. Berwein et al., arXiv:1704.07266
- $r$ independent term at $\propto \alpha_s^3$ allowed, can explain difference at small $r$?
- Constant term removed in $r$ derivative $\Rightarrow$ eff. coupling $\alpha_{Q\bar{Q}}[V_S - F_S]$.
- *pNRQCD* prediction works for $V_S - F_S$ in the range $rT \lesssim 0.25$.

## Color octet contribution in the Polyakov loop correlator (I)



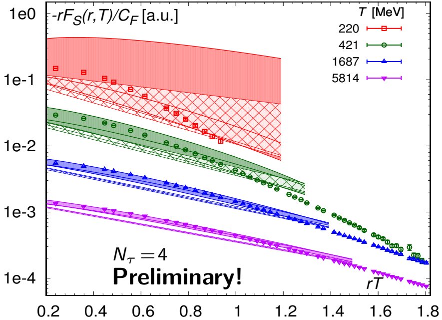

- *pNRQCD*: $C_P$ is given in terms of **potentials** $V_S$ **and** $V_A$ **at** $T=0$ and of the *adjoint Polyakov loop* $L_A$ at $T>0$ N. Brambilla et al., **PRD 82** (2010)
  $C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-V_S/T} + \frac{8}{9} L_A\ e^{-V_A/T} + \mathcal{O}(g^6)$ for $rT \ll 1$.
- *Gauge-invariant* decomposition of $C_P$ into **color singlet and octet** is defined assuming *weak coupling* – test if it works for lattice as well.
- **Color octet contribution**: define $e^{-F_O/T} \sim \frac{9}{8}\left(e^{-F_{Q\bar{Q}}(r,T)} - \frac{1}{9} e^{-V_S/T}\right)$
- As $F_O$ rapidly decreases for higher $T$, the **octet** becomes important.

## Color octet contribution in the Polyakov loop correlator (II)




- Low $T = 172\,\mathrm{MeV}$: **color singlet** $V_S$ is enough for reconstructing $C_P$ (no sensitivity to **color octet** due to large statistical errors).
- High $T = 666\,\mathrm{MeV}$: cancellation between **color singlet and octet** leads to $1/r^2$ behavior in $F_{Q\bar{Q}}$. We use *pNRQCD*, i.e. $\frac{1}{9}e^{-V_S/T} + \frac{8}{9}L_A\ e^{-V_A/T}$.
- We include **Casimir scaling violation**: $8V_A + V_S = 3\frac{\alpha_s^3}{r}[\frac{\pi^2}{4} - 3] + \mathcal{O}(\alpha_s^4)$.
- $F_{Q\bar{Q}}$ for $rT \lesssim 0.4$ can be understood in terms of **vacuum physics** only.

## Confronting weak-coupling predictions in the screening regime (I)




Hashed bands: LO
Solid bands: NLO

Scale uncertainty $\mu = (1-4)\pi T$ due to resummation smaller for larger $T$

$\Lambda_{\mathrm{QCD}} = 320\,\mathrm{MeV}$

- $F_S(r,T)$ for $rm_D \sim 1$ at **NLO in EQCD** M. Laine et al., **JHEP 0703** 054 (2007)

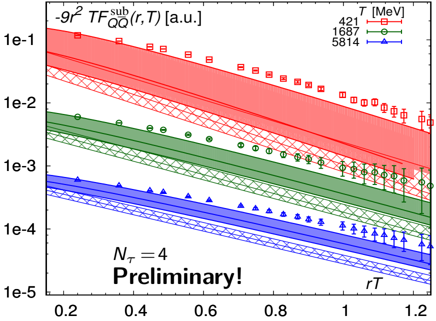
$$F_S^{\mathrm{sub}} = F_S - 2F_Q = -\frac{C_F\alpha_s e^{-rm_D}}{r}(1+\alpha_s[\delta Z_1(\mu) + rT\, f_1(rm_D)])$$

- $F_S^{\mathrm{sub}}$ on the lattice is **compatible with EQCD@NLO** up to $rT \sim 0.6$.

⇒ **Weakly-coupled EQCD** is reasonable in electric screening regime of $F_S$.

- For $rT > 0.8$: asymptotic screening is inherently non-perturbative.

## Confronting weak-coupling predictions in the screening regime (II)




Hashed bands: LO
Solid bands: NLO

Scale uncertainty $\mu = (1-4)\pi T$ due to resummation smaller for larger $T$

$\Lambda_{\text{QCD}} = 320\,\text{MeV}$

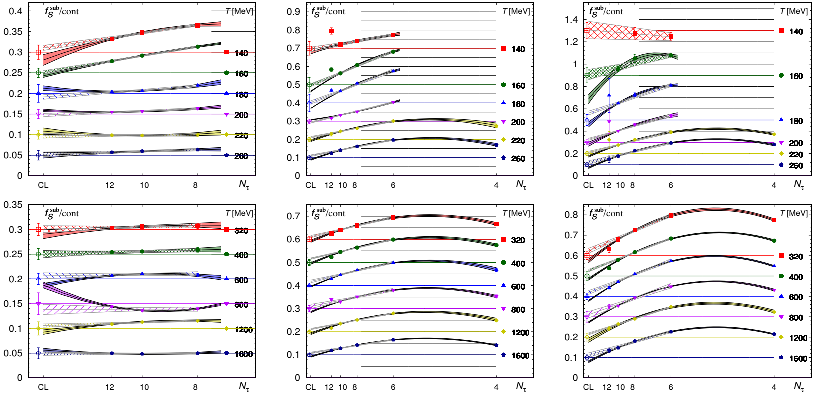
- *Singlet and octet cancel* at LO in $F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2}{9}\frac{e^{-2rm_D}}{r^2} + C_F\alpha_s m_D$.
- $F^{\text{sub}}_{Q\bar{Q}} = -\frac{\alpha_s^2}{9}\frac{e^{-2rm_D}}{r^2}(1+\alpha_s[\delta Z_1(\mu) + rT\, f_1(rm_D)])$ S. Nadkarni, **PRD 33** (1986)
- $F^{\text{sub}}_{Q\bar{Q}}$ on the lattice is **close ($\sim$ 10%) to EQCD@NLO** up to $r \sim 1/m_D$.

⇒ **Weakly-coupled EQCD** is reasonable in the electric screening regime of $F_{Q\bar{Q}}$, but non-perturbative (*chromo-magnetic*) effects are stronger .

- We study color screening and deconfinement using the renormalized Polyakov loop correlator and related observables.
- We identify in the entropy $S_Q = -\frac{dF_Q}{dT}$ crossover behavior at $T \sim T_c$.
- We extract $T_S = 153^{+6.5}_{-5}$ MeV from the entropy, in agreement with $T_\chi = 160(6)$ MeV (chiral susceptibilities, O(2) scaling fits, $\frac{m_l}{m_s} = \frac{1}{20}$).
- Continuum limit of static quark correlators in $N_f = 2{+}1$ QCD up to $T \sim 1.9$ GeV and down to $r \sim 0.01$ fm.
- **Color-singlet correlators** are **vacuum-like up to** $rT \ll 0.3$, exhibit *color-electric screening for* $rm_D \sim 1 \Leftrightarrow 0.3 \lesssim rT \lesssim 0.6$ *compatible with weak coupling* and change to *asymptotic screening for* $rT \gg 0.7$.
- The *Polyakov loop correlator* $C_P$ has a substantial *color adjoint contribution* for $T \gtrsim 200$ MeV. For $rT \lesssim 0.4$ weakly-coupled *pNRQCD* describes $C_P$ well in terms of $T = 0$ **potentials** and the *adjoint Polyakov loop* $L_A$.
- $C_P$ has a *color-electric screening regime* $rm_D \sim 1$. Non-perturbative effects (i.e. the *chromo-magnetic sector*) are much larger.

# Continuum extrapolations: singlet free energy

$rT = 0.15$ $rT = 0.30$ $rT = 0.45$



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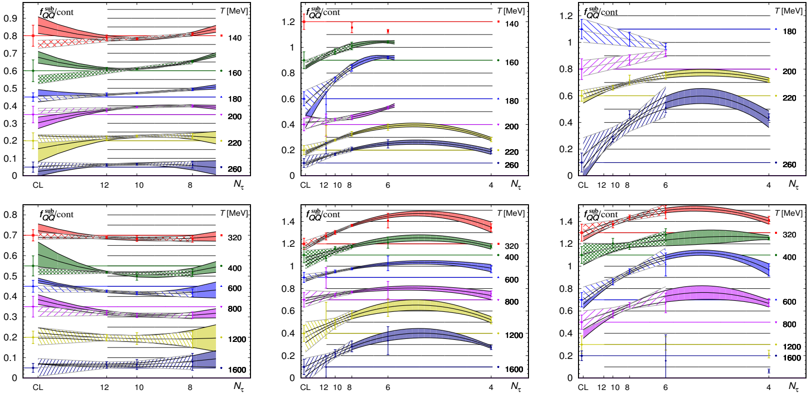

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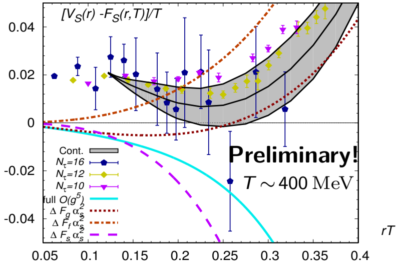
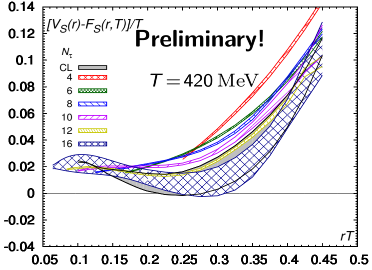



- *pNRQCD*: $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(g^6)$ M. Berwein et al., arXiv:1704.07266
  $V_S(r) - F_S(r,T) = (\Delta F_g + \Delta F_f + \Delta F_s)\,\alpha_s^2 T + \mathcal{O}(g^6)$ for $x = \frac{\pi}{3} rT \ll 1$,
  $$\underbrace{\Delta F_g = N_c C_F \left[-\tfrac{1}{3}x + 2x^2 - \tfrac{22}{25}x^3\right]}_{\text{gluons}},\quad \underbrace{\Delta F_f = N_f C_F \left[\tfrac{3}{2}x^2 - \tfrac{7}{10}x^3\right]}_{\text{fermions}},\quad \underbrace{\Delta F_s = -C_F \sqrt{\frac{2\alpha_s}{3\pi}\left[2N_c + N_f\right]^3}\,x^2}_{\text{Debye screening}}$$
- Cancellations between $\Delta F_g + \Delta F_f$ and $\Delta F_s \Rightarrow$ vacuum-like behavior.

## Static energy and singlet free energy (IV) - discretization effects




- *pNRQCD*: $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(g^6)$ M. Berwein et al., arXiv:1704.07266
- **Strong $N_\tau$ dependence** for $rT > 0.15$, but $N_\tau \geq 12 \sim$ continuum limit.
- $V_S - F_S \sim 0.02T$ for $rT \lesssim 0.1$ & $T > 300\,\mathrm{MeV}$, mild $N_\tau$ dependence.
- We estimate the systematic uncertainty of continuum extrapolation by using subsets of data and different assumptions about scaling behavior.