Quark-hadron matter in the critical region

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1. Introduction: Beth-Uhlenbeck (BU) and Generalized BU
2. GBU from Φ-derivable approach: 2-loop approximation
3. GBU/BU EoS for quark-hadron matter in (P)NJL-type models
4. Application: "horn" effect for K+/π+ and critical endpoint
5. Conclusions & Outlook

CPOD 2017, Stony Brook University, August 10, 2017

JINR DUBNA
MEPhI
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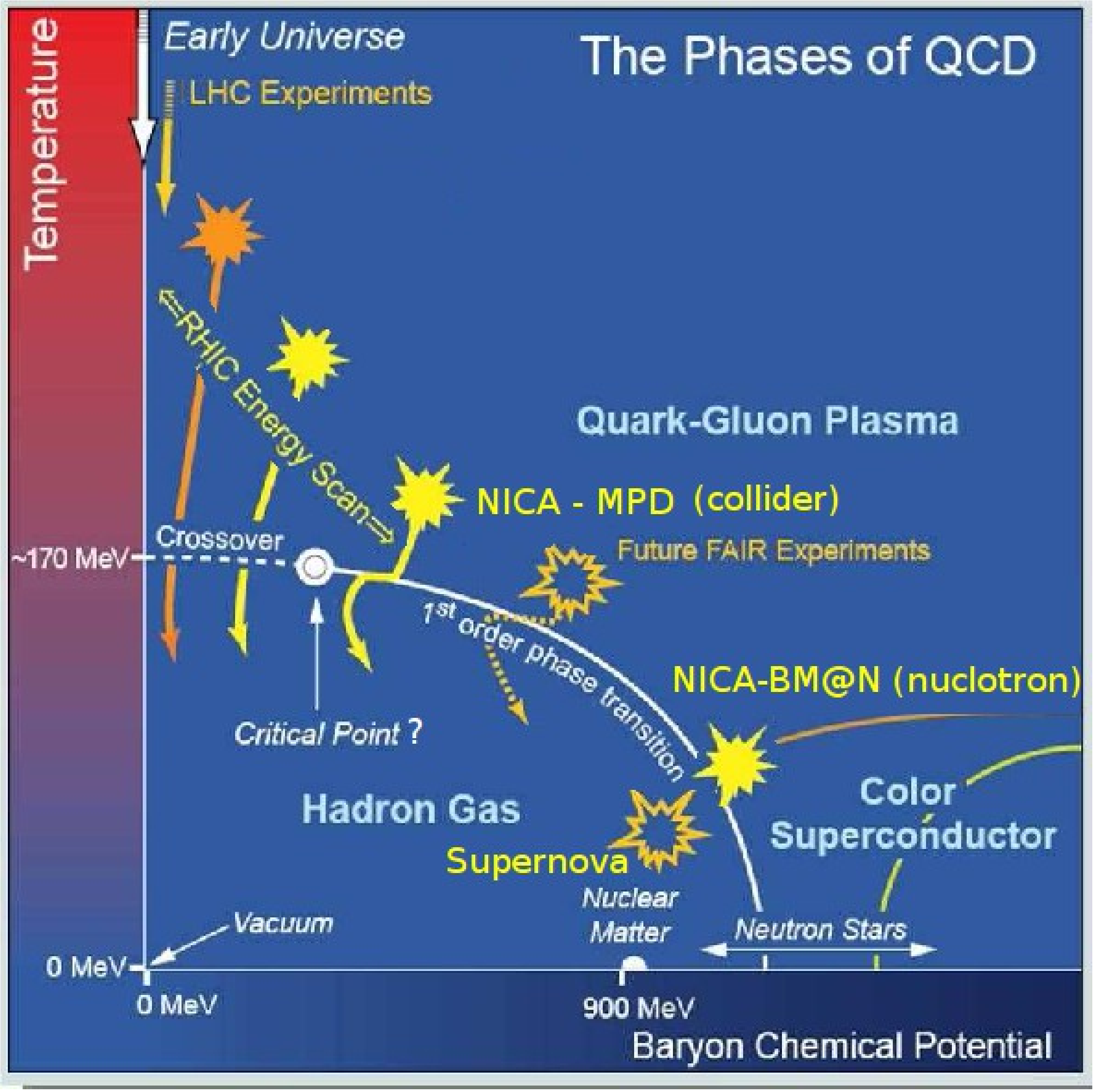
COST European Cooperation in Science and Technology
Uniwersytet Wrocławski
RSF Russian Science Foundation

The Goal: Theory of the QCD Phase Diagram



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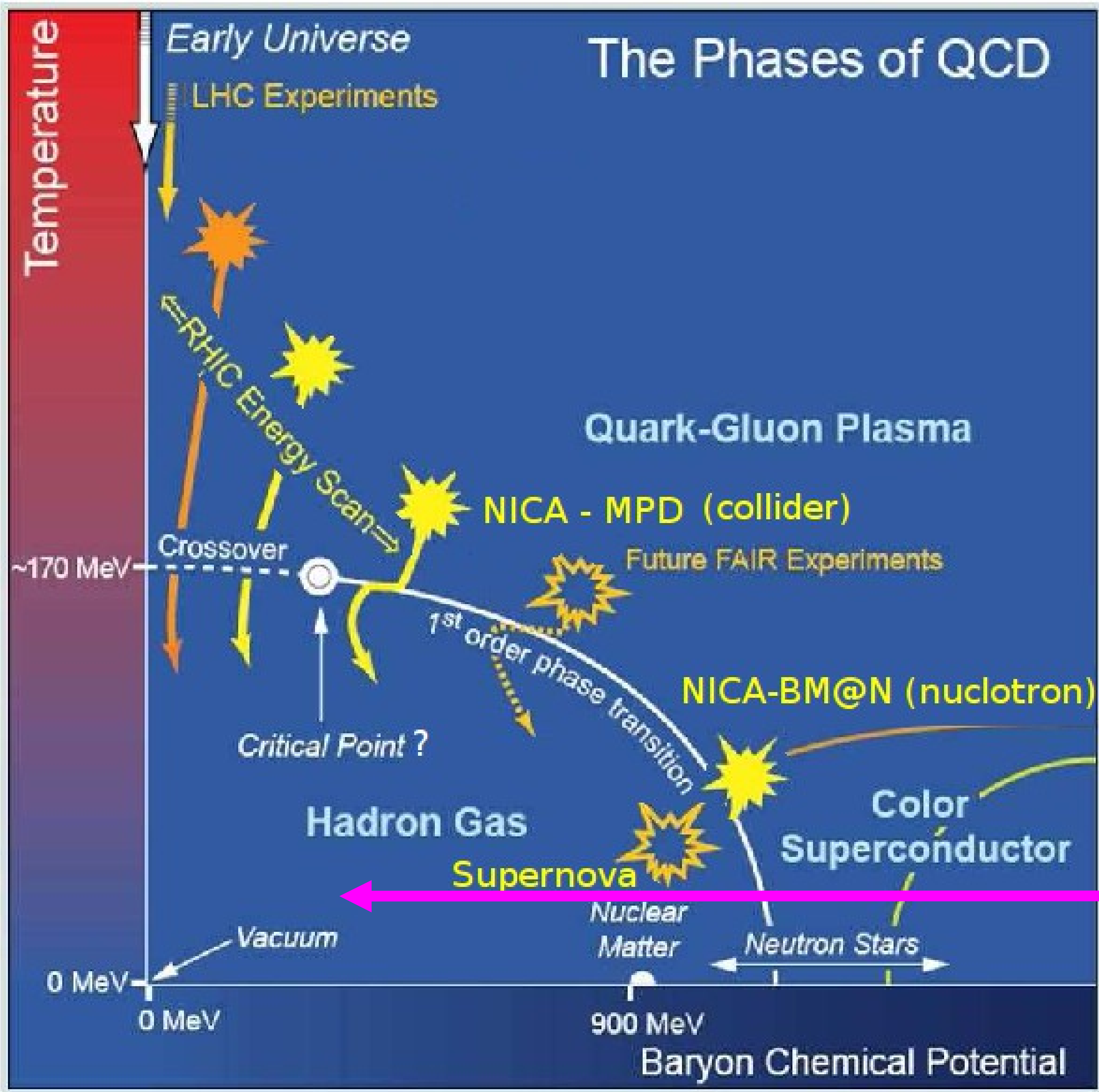
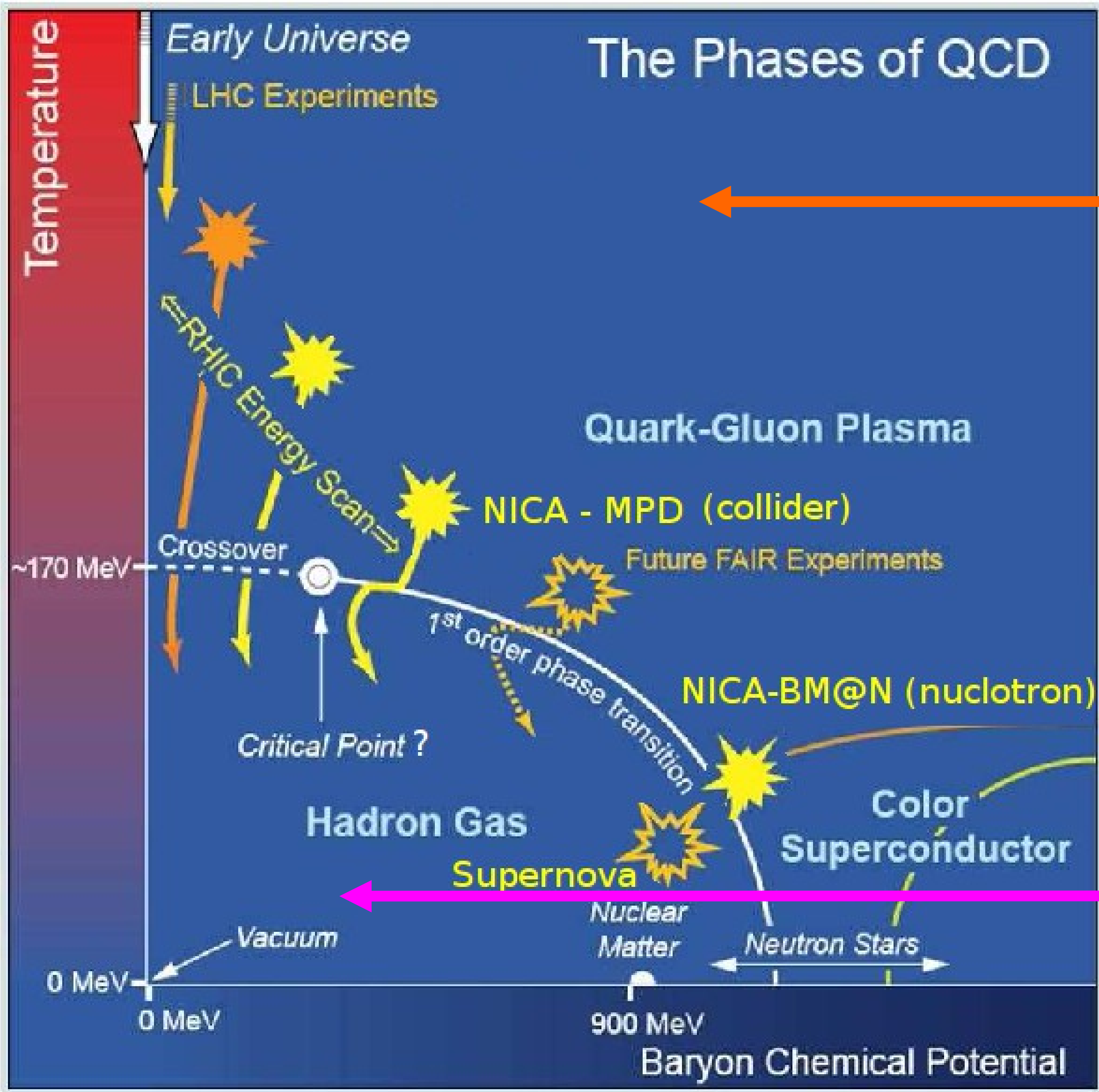


Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

The Goal: Theory of the QCD Phase Diagram



Perturbative QCD

Approximately selfconsistent HTL resummation
($T > 2.5$ Tc , $\mu > 1500$ MeV)

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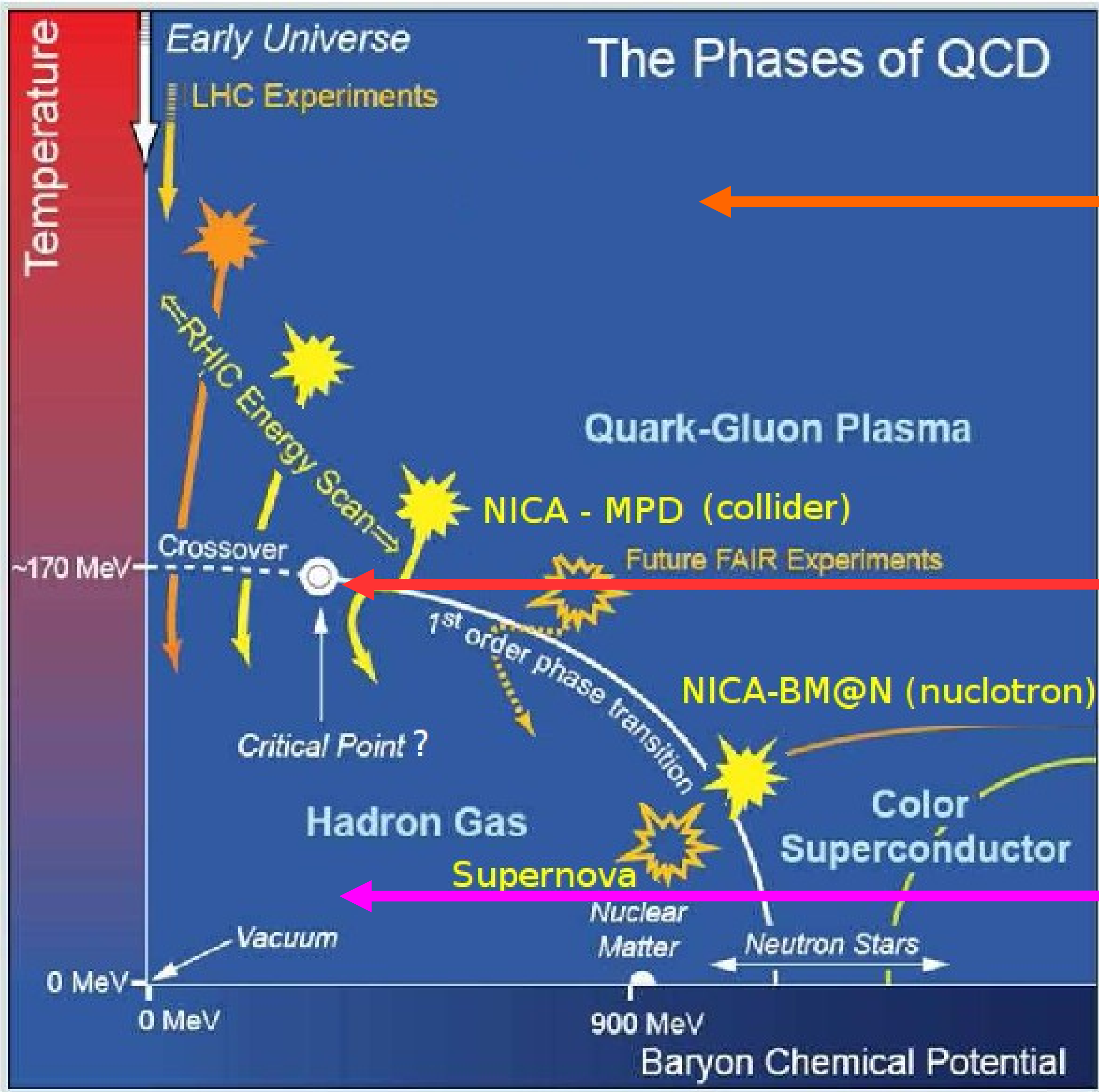


Perturbative QCD

Approximately selfconsistent HTL resummation
(T > 2.5 Tc , μ > 1500 MeV)

QCD Phase transition(s)

Mott dissociation of hadrons, Deconfinement, χSR

Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

Introduction: Beth-Uhlenbeck vs. Generalized BU

Beth-Uhlenbeck: 2nd virial coefficient B(T)

$$pV = NkT\left(1 + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \dots\right)$$

BU for virial expansion of density:

$$n(\mu, T) = n_{\text{free}}(\mu, T) + 2n_{\text{corr}}(\mu, T)$$

$$n_{\text{free}}(\mu, T) = 4\int \frac{d^3p}{h^3} e^{-(p^2/2m-\mu)/T} = \frac{4}{\lambda^3} e^{\mu/T}$$

$$n_{\text{corr}}(\mu, T) = \int \frac{d^3\mathbf{P}}{h^3} e^{-(P^2/4m-2\mu)/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E)$$

$$= \frac{2^{3/2}}{\lambda^3} e^{2\mu/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E).$$

Density of states: bound and scattering part

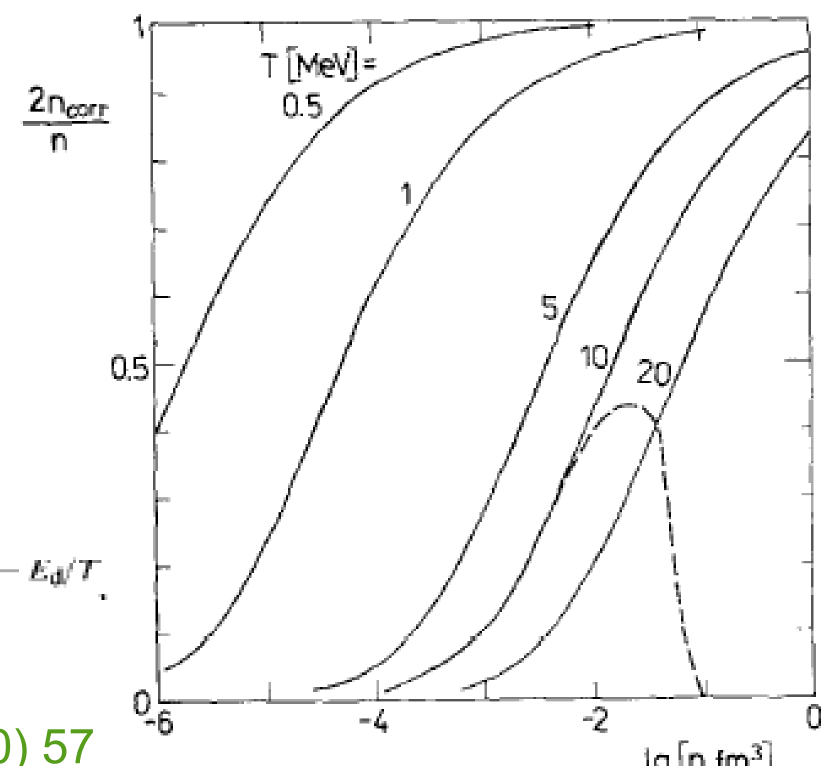
$$D(E) = \sum_\alpha c_\alpha \left[\pi\delta(E - E_\alpha) + \frac{d}{dE}\delta_\alpha(E)\right],$$

Example: Deuterons in nuclear matter

$$n = n_{\text{free}} + 2n_{\text{free}}^2 I(T)$$

$$I(T) = \lambda^3 \frac{2^{1/2}}{8}\left[3(e^{-E_d/T} - 1) + \int_0^{\infty} \frac{dE}{\pi T} e^{-E/T} \sum_\alpha c_\alpha \delta_\alpha(E)\right].$$

For T<<E$_d$: $n = n_{\text{free}} + 2n_{\text{deut}}\,, \quad n_{\text{deut}} = n_{\text{free}}^2 \lambda^3 3\, \frac{2^{1/2}}{8} e^{-E_d/T}.$



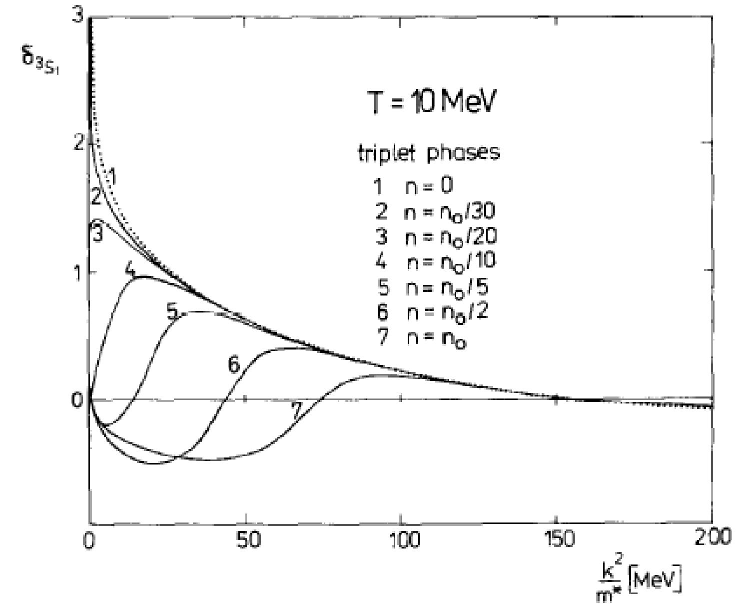
E. Beth and G.E. Uhlenbeck, Physica IV (1937) 915;
S. Schmidt, G. Roepke, H. Schulz, Ann. Phys. 202 (1990) 57

Introduction: Beth-Uhlenbeck vs. Generalized BU

Thermodynamic Greens function approach:

$$n(1,\mu_1,T)=\int\frac{dE}{2\pi}f_1(E)\,A(1,E)$$

$$A(1,E)=\frac{2\Sigma_I(1,E-i0)}{(E-E(1)-\Sigma_R(1,E))^2+\Sigma_I(1,E-i0)^2}\;\approx\;\frac{2\pi\,\delta(E-e(1))}{1-((d/dz)\,\Sigma_R(1,z))|_{z=e(1)+i0}}-2\Sigma_I(1,E+i0)\frac{d}{dE}\frac{\mathrm{P}}{E-e(1)}$$



Density formula
(free and corr. Quasiparticles):

$$n(\mu,T)=n_{\rm free}(\mu,T)+2n_{\rm corr}(\mu,T),$$

$$\Sigma(1,z_\nu)=T\sum_2\sum_{z_{\nu'}}[T(1212,z_\nu+z_{\nu'})-{\rm ex}]\,G(2,z_{\nu'}).$$

$$n_{\rm corr}(\mu,T)=\int\frac{dE}{2\pi}g(E)\,F(E)$$

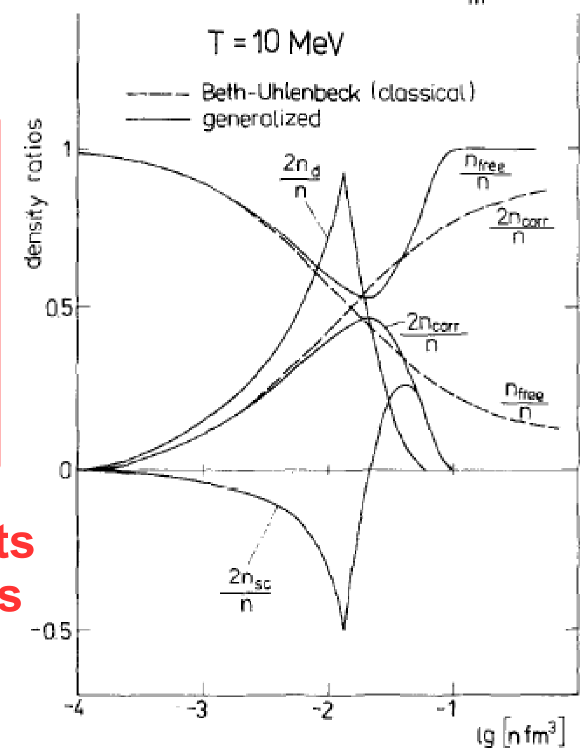
$$F(E)=F_{\rm deut}(E)+\frac{2}{4\pi}\sum_\alpha c_\alpha F_\alpha(E),$$

$$F_{\rm deut}(E)=6\sum_{K>K^{\rm Mott}}\pi\delta(E-E_b(K,\mu,T)).$$

$$F(E)=\sum_{12}[1-f(e(1))-f(e(2))]\cdot\left[(T_I(1212,E+i0)-{\rm ex})\frac{d}{dE}\frac{\mathrm{P}}{e(1)+e(2)-E}-\pi\delta(E-e(1)-e(2))\frac{d}{dE}(T_R(1212,E+i0)-{\rm ex})\right].$$

$$F_\alpha(E)=8\pi\sum_K\sin^2\delta_\alpha(E,K,\mu,T)\frac{d}{dE}\delta_\alpha(E,K,\mu,T).$$

The $\sin^2\delta$ term accounts for quasiparticle effects



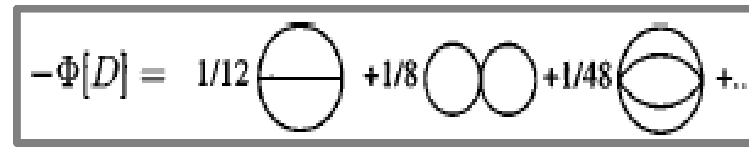
S. Schmidt, G. Röpke, H. Schulz, Ann. Phys. 202 (1990) 57

Φ-derivable approach, 2-loop approximation

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

Skeleton expansion for thermodynamic potential and entropy

$$\beta\Omega[D] = -\log Z = \frac{1}{2}\mathrm{Tr}\log D^{-1} - \frac{1}{2}\mathrm{Tr}\,\Pi D + \Phi[D]$$



Inv. Temp: 1/T — trace in conf. Space — self-energy related to D

Dyson equation: $D^{-1} = D_0^{-1} + \Pi$ Free propagator Do is known

Essential property of Ω[D] is Stationarity under variation of D: δ Ω[D] / δ D = 0

This implies δ Φ[D] / δ D = 1/2 Π

Physical propagator and selfenergy are defined self-consistently !

Self-consistent approximations are defined by the choice of Φ

→ **Φ – derivable theories**

G. Baym, Phys. Rev. 127 (1962) 1391; Vanderheyden & Baym; J. Stat. Phys. 93, 843 (1998)

Approximately selfconsistent thermodynamics

Matsubara summation:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} n(\omega)[\mathrm{Im}\log(-\omega^2+k^2+\Pi) - \mathrm{Im}\,\Pi D] \; + T\Phi[D]/V$$

Analytic properties:

$$D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0,k)}{k_0-\omega}, \qquad \mathrm{Im}\,D(\omega,k) \equiv \mathrm{Im}\,D(\omega+i\epsilon,k) = \frac{\rho(\omega,k)}{2}.$$

Thermodynamics from entropy density: $\mathcal{S} = -\partial(\Omega/V)/\partial T$

$$\mathcal{S} = -\int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\log D^{-1}(\omega,k) \; + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\,\Pi(\omega,k) \mathrm{Re}\,D(\omega,k) + \mathcal{S}'$$

$$\mathcal{S}' \equiv -\left.\frac{\partial(T\Phi/V)}{\partial T}\right|_D + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Re}\,\Pi\,\mathrm{Im}\,D \longrightarrow 0$$

for two-loop skeleton diagrams

Loosely speaking: S' accounts for residual interactions of "independent quasiparticles"

d/dω [Im log D^{-1} + ImΠ ReD] = 2 Im [D ImΠ (d/dω D*) ImΠ] = 2 $\sin^2\delta$ dδ/dω , for D = |D|$e^{i\delta}$

D. B., G. Röpke, G. Baym, in preparation (2017)

Φ-derivable Q-M-D PNJL model, 2-loop approximation

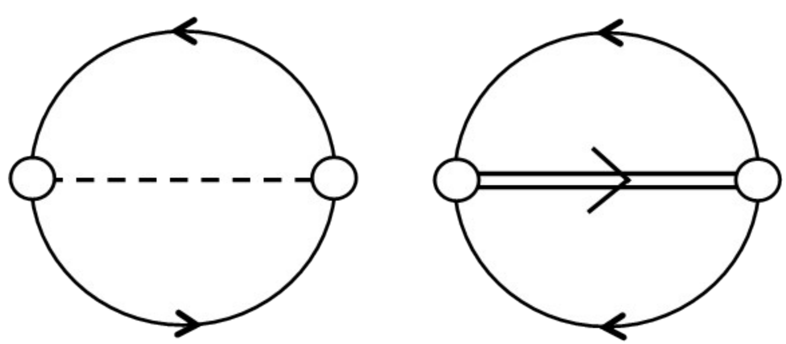
$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i\,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i\Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \ ,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \ , \qquad \frac{\delta\Omega}{\delta S_i} = 0 \ , \quad \text{if} \quad \Pi_i = \frac{\delta\Phi}{\delta S_i} \ .$$

$$\Omega \ = \ \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}f_i(\omega)\,\mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\,S_i\,\mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} \ = \ \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}f_i(\omega)\,\mathrm{Tr}\left\{\left[\mathrm{Im}\,S_i\,\mathrm{Re}\,\Pi_i\right]\right\} \ ,$$

$\Phi =$



$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \ .$$

Derivation from (P)NJL model in 1/Nc approximation: D.B., D. Ebert: NPB 921 (2017) 753

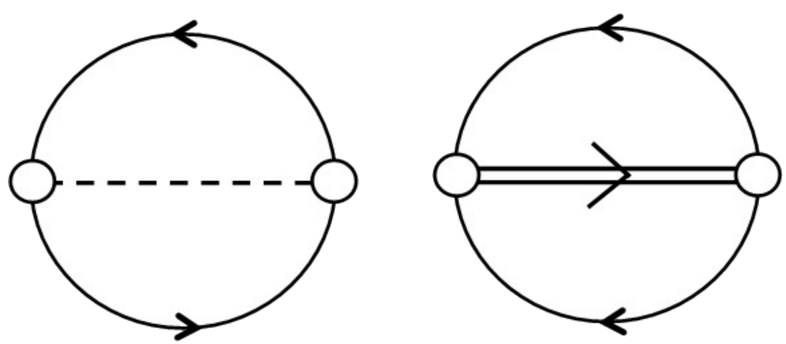
Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i\Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \,,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \,, \qquad \frac{\delta\Omega}{\delta S_i} = 0 \,, \quad \text{if} \quad \Pi_i = \frac{\delta\Phi}{\delta S_i} \,.$$

$$\Omega = \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}f_i(\omega)\,\mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\,S_i\,\mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} = \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}f_i(\omega)\,\mathrm{Tr}\left\{\left[\mathrm{Im}\,S_i\,\mathrm{Re}\,\Pi_i\right]\right\} \,,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \,.$$

Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\left(\mathrm{Im}\ln S^{-1}\right)' = -\,\mathrm{Im}\left(S\Pi'\right) = \underbrace{S'_R\Pi_I - S_I\Pi'_R}_{2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I)} - \underbrace{\left(\Pi_I S'_R + S_R\Pi'_I\right)}_{(\Pi_I S_R)'} \;,$$

Use optical theorems ...

$$S\Pi_I \;=\; \sin\delta\, \mathrm{e}^{\mathrm{i}\delta} \;,\quad S^{*\prime}\Pi_I = -\mathrm{i}\delta' \sin\delta\, \mathrm{e}^{-\mathrm{i}\delta} \;,\quad 2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I) = -2\delta' \sin^2\delta \;.$$

Generalized Beth-Uhlenbeck EoS

$$\Omega \;=\; -\sum_{i=Q,M,D} d_i \int \frac{\mathrm{d}^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, T\ln[1-\mathrm{e}^{-(\omega-\mu_i)/T}]\, \sin^2\delta_i(\omega,\mathbf{q}) \frac{\partial\delta_i(\omega,\mathbf{q})}{\partial\omega}$$

Effect of the sin^2 term ... example: Breit-Wigner ...

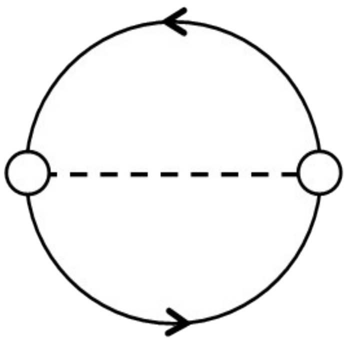
$$\delta_i(\omega) = -\,\arctan\left[\frac{\omega_i\Gamma_i}{\omega^2-\omega_i^2}\right] \;,\quad \frac{\partial\delta_i(\omega)}{\partial\omega} = \frac{2\omega\omega_i\Gamma_i}{(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2} \;,$$

$$\sin^2\delta_i(\omega)\frac{\partial\delta_i(\omega)}{\partial\omega} \;=\; \frac{2\omega(\omega_i\Gamma_i)^3}{[(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2]^2} \;.$$

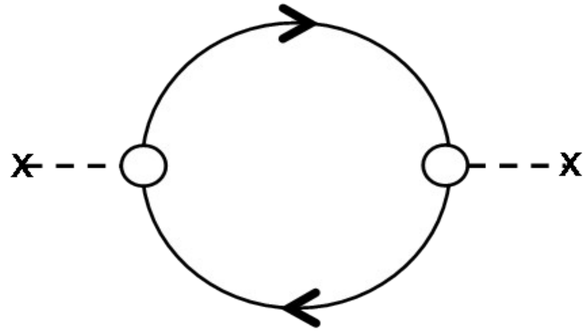
“Squared Lorentzian” ...
Vanderheyden & Baym (1998)
Morozov & Röpke (2009)

Example: Mesons in quark matter

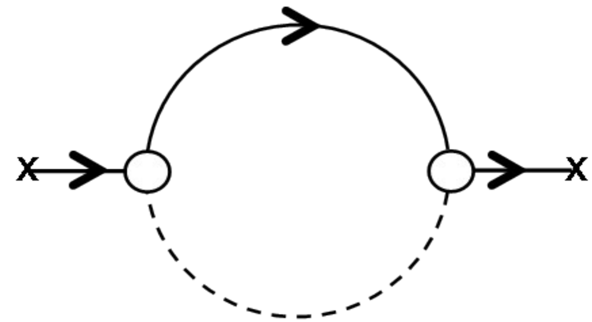
Φ-functional



Meson selfenergy (RPA)



Quark selfenergy



$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1} \mathrm{e}^{-i\delta_M(q,\omega)} \, , \quad \delta_M(q,\omega) = \arctan(\Im T_M / \Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M \, , \qquad \sigma_{\mathrm{MF}} = 2N_f N_c G_S \int \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} [1 - f_-(E_p) - f_+(E_p)] \, ,$$

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3p}{(2\pi)^3} \left[E_p + T \ln\left(1 + \mathrm{e}^{-(E_p - \Sigma_+ - \mu)/T}\right) + T \ln\left(1 + \mathrm{e}^{-(E_p + \Sigma_- + \mu)/T}\right) \right] \, ,$$

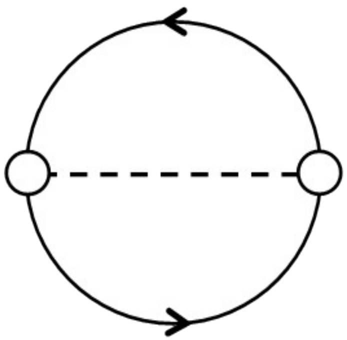
$$\Omega_M = d_M \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln\left[1 - \mathrm{e}^{-\omega/T}\right] 2\sin^2 \delta_M(k,\omega) \, \frac{\delta_M(k,\omega)}{d\omega} \right\} \, ,$$

$$\Sigma_M(0,p_0) = d_M \int \frac{d^4q}{(2\pi)^4} \pi \rho_M(\mathbf{q},q_0) \left\{ \frac{(\gamma_0 + m/E_q)[1 + g(q_0) - f_-(E_q)]}{q_0 - p_0 + E_q - \mu - i\eta} + \frac{(\gamma_0 - m/E_q)[g(q_0) + f_+(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} \right\} \, ,$$

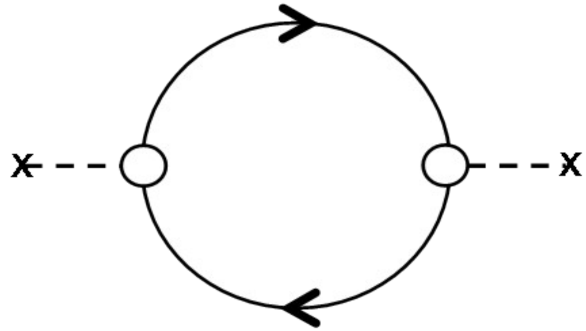
D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228
D. Blaschke, PoS Baldin ISHEPXXII (2015) 113; arxiv:1502.06279

Example: Mesons in quark matter

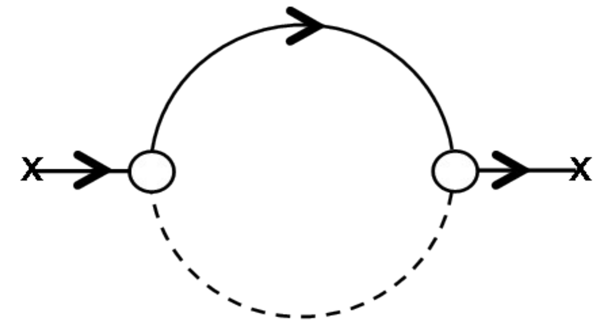
Φ-functional



Meson selfenergy (RPA)



Quark selfenergy



$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1} \mathrm{e}^{-i\delta_M(q,\omega)} , \quad \delta_M(q,\omega) = \arctan(\Im T_M / \Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M , \qquad \sigma_{\mathrm{MF}} = 2N_f N_c G_S \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}[1 - f_-(E_p) - f_+(E_p)] ,$$

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3p}{(2\pi)^3}\left[E_p + T\ln\left(1+\mathrm{e}^{-(E_p-\Sigma_+-\mu)/T}\right) + T\ln\left(1+\mathrm{e}^{-(E_p+\Sigma_-+\mu)/T}\right)\right] ,$$

$$\Omega_M = d_M \int \frac{d^3k}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left\{\omega + 2T\ln\left[1-\mathrm{e}^{-\omega/T}\right] \boxed{2\sin^2\delta_M(k,\omega)} \frac{\delta_M(k,\omega)}{d\omega}\right\} ,$$

new !

$$\Sigma_M(\mathbf{0},p_0) = d_M \int \frac{d^4q}{(2\pi)^4}\pi\rho_M(\mathbf{q},q_0)\left\{\frac{(\gamma_0+m/E_q)[1+g(q_0)-f_-(E_q)]}{q_0-p_0+E_q-\mu-i\eta} + \frac{(\gamma_0-m/E_q)[g(q_0)+f_+(E_q)]}{q_0-p_0-E_q-\mu-i\eta}\right\} ,$$

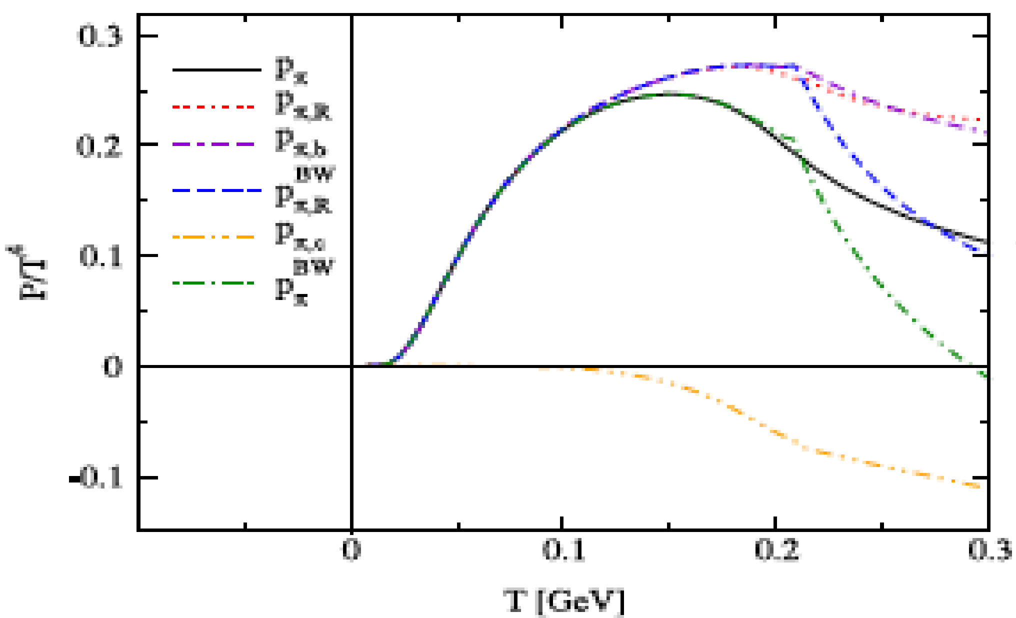
D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228
D. Blaschke, PoS Baldin ISHEPXXII (2015) 113; arxiv:1502.06279

Example: Mesons in quark matter (BU level)

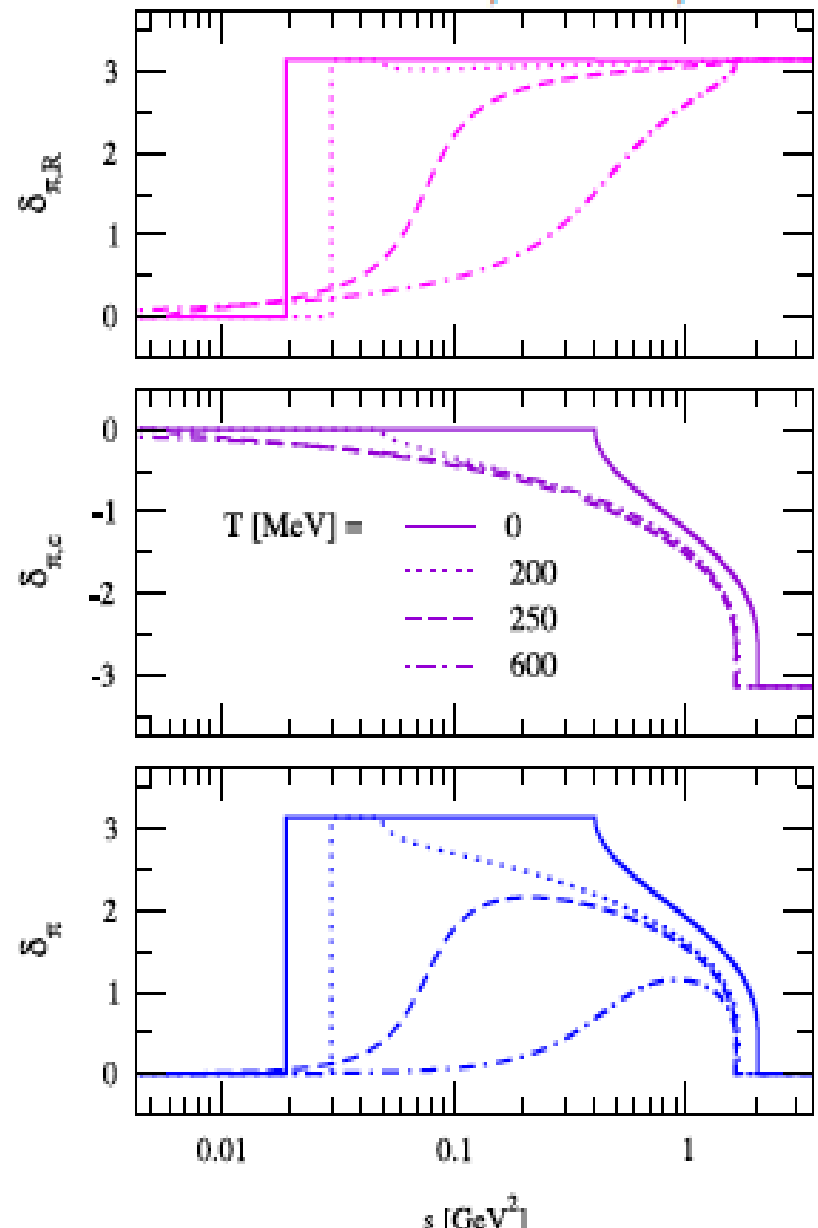
$$\Omega_X(T,\mu) = -d_X \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, n_X^-(\omega)\,\delta_X(\omega,\mathbf{q}),$$

$$\int_0^{\infty} d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega} = 0 = \underbrace{\int_0^{\omega_{\rm thr}(T)} d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega}}_{n_{B,X}(T)} + \underbrace{\frac{1}{\pi}\int_{\omega_{\rm thr}(T)}^{\infty} d\omega \frac{d\delta_X(\omega;T)}{d\omega}}_{\frac{1}{\pi}[\delta_X(\infty;T)-\delta_X(\omega_{\rm thr};T)]},$$

$$p_\pi(T) = -d_\pi T \int \frac{d^3q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{\pi} \ln\left(1-e^{-\omega/T}\right) \frac{d\delta_\pi(\omega,\mathbf{q})}{d\omega}$$

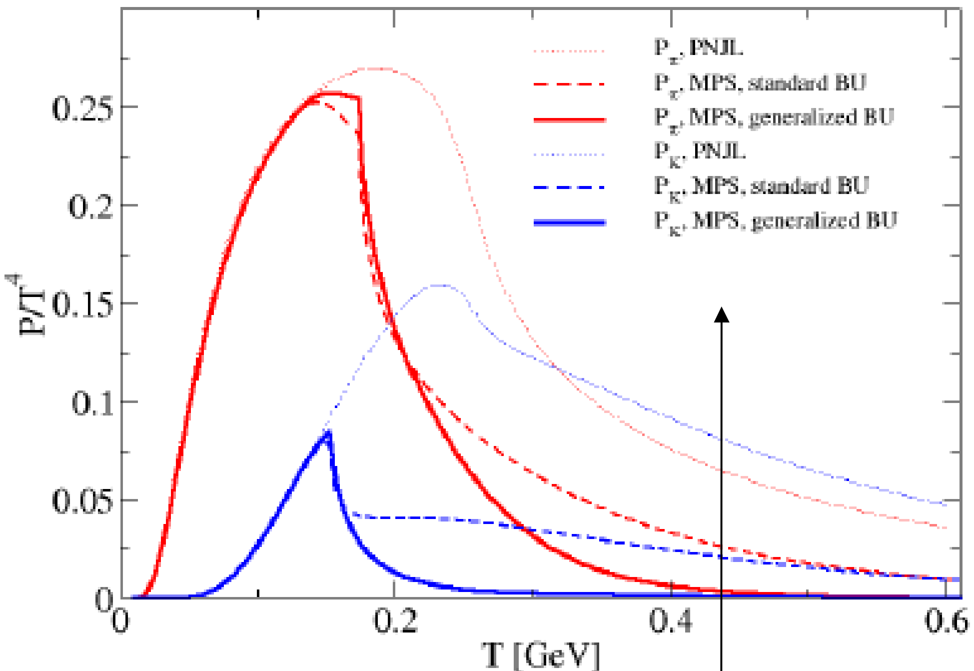


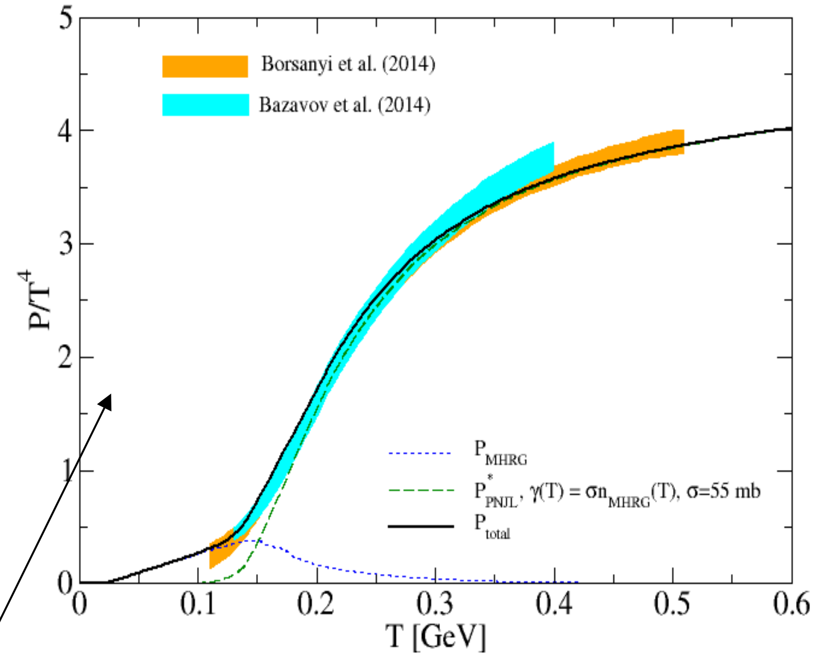
$$\delta_\pi = \delta_{\pi,c} + \delta_{\pi,R}$$

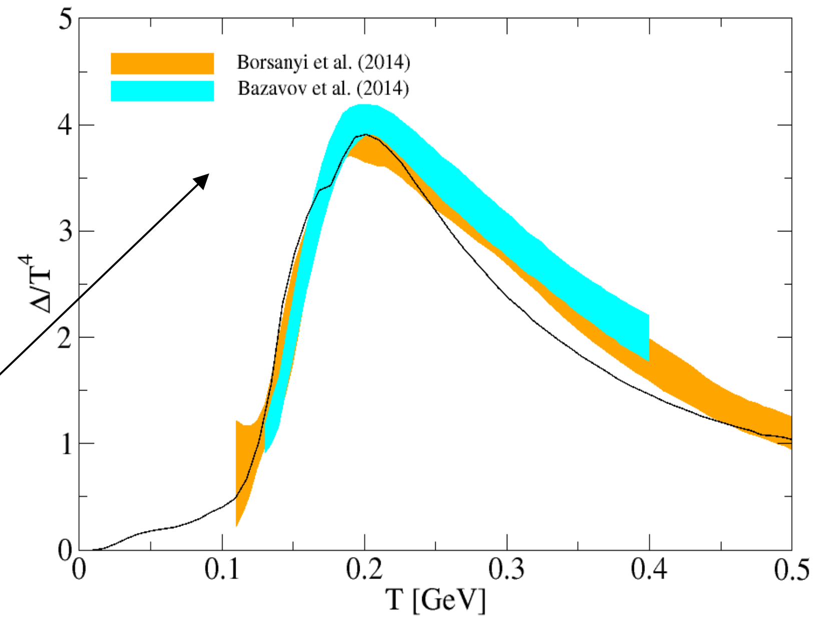


A. Wergieluk, D. B., A. Friesen, Yu. Kalinovsky: PPNLett. 10 (2013) 660
D. B., M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228

3. Mott HRG / PNJL – effective model (GBU level)







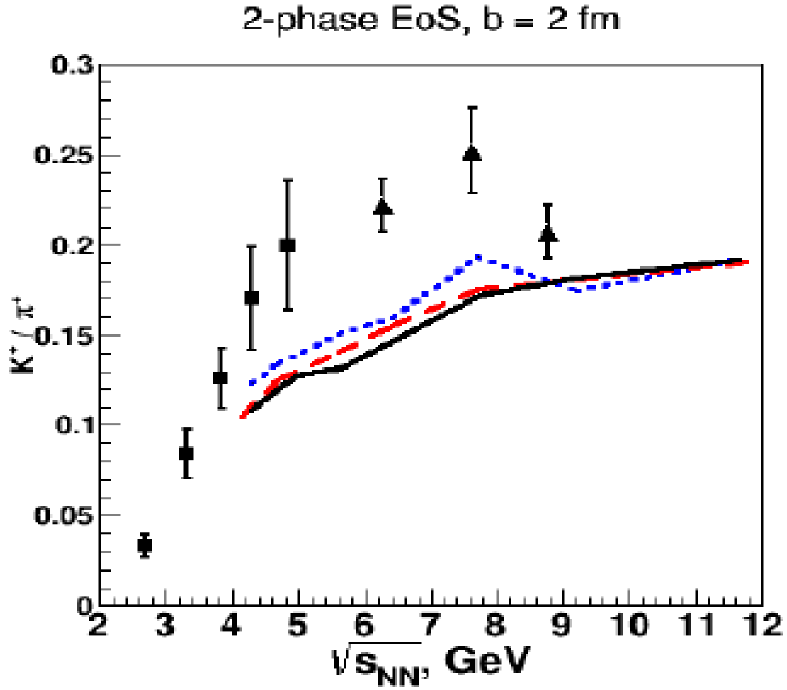
- Mott dissociation of hadrons (here pi, K) at the Chiral restoration temperature T_c = 153 MeV

- Asymptotic behaviour of quark-gluon Pressure can be adjusted with rescattering Parameter gamma

- Very good correspondence between lattice QCD Thermodynamics and improved MHRG/PNJL model; Hadronic and partonic contributions quantified

D.B., A. Dubinin, L. Turko, arxiv:1612.09556

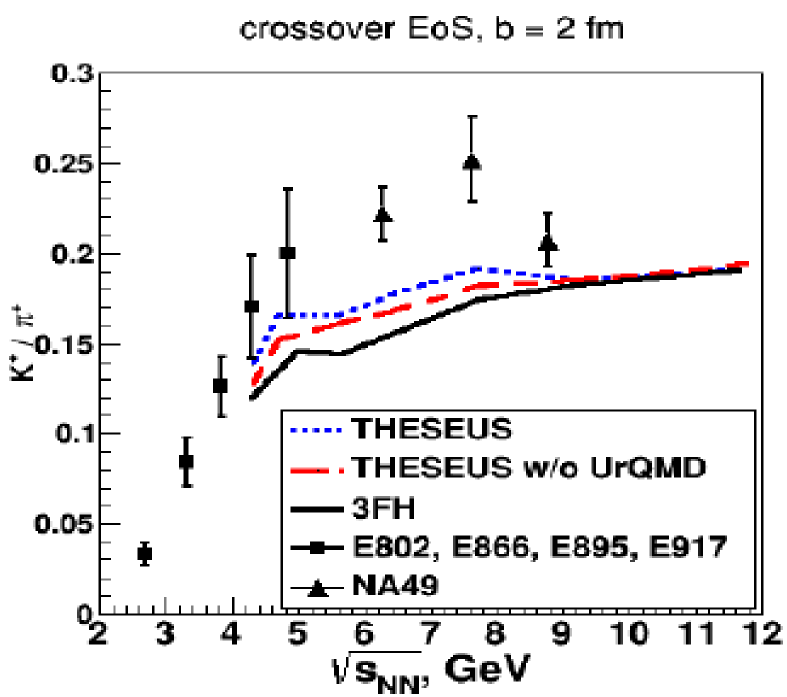
What about K+/π+ (Marek's horn) in THESEUS ?



THESEUS simulation reproduces 3FH result,
Thus it has the same discrepancy with experiment

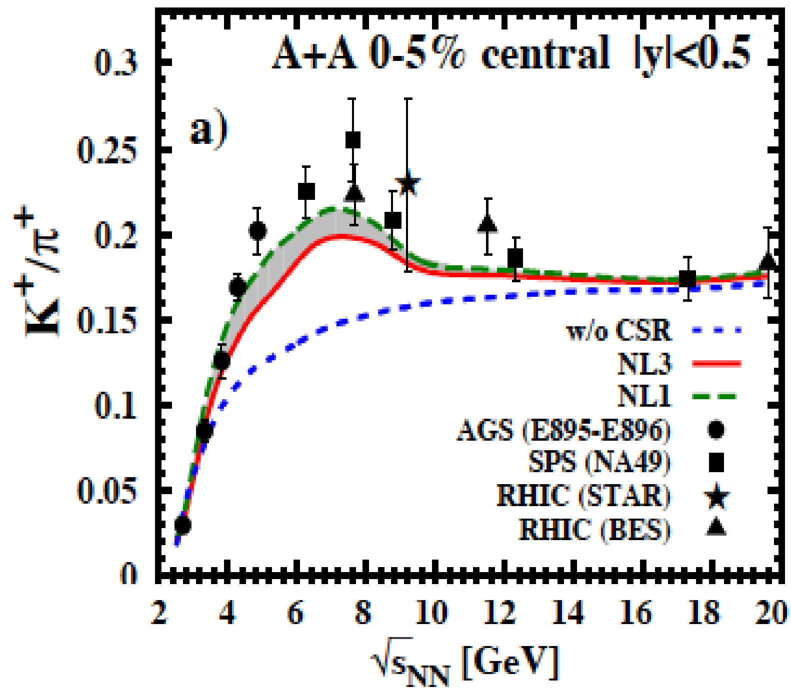
--> some key element still missing in the program

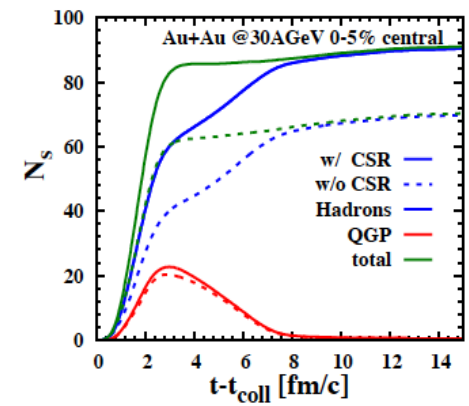
P. Batyuk, D.B., M. Bleicher et al., PRC 94, 044917



Recent new development in PHSD

"Chiral symmetry restoration in HIC at intermediate ..."
A. Palmese et al., arxiv: 1607.04073; PRC 94, 044912





Strange particle number increase by CSR

Mott dissociation of π and K in hot, dense quark matter

D. Blaschke, A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

Andrey Radzhabov in front of the University of Wroclaw

PNJL model for N_f=2+1 quark matter with π and K

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu + \hat{m}_0\right)q + G_S \sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}i\gamma_5\lambda^a q\right)^2\right] - \mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$$

$$\Pi^{M^a}_{ff'}(q_0,\mathbf{q}) = 2N_c T\sum_n \int \frac{d^3p}{(2\pi)^3}\mathrm{tr}_D\left[S_f(p_n,\mathbf{p})\Gamma^{M^a}_{ff'}S_{f'}(p_n+q_0,\mathbf{p}+\mathbf{q})\Gamma^{M^a}_{ff'}\right]$$

$$\Gamma^{P^a}_{ff'} = i\gamma_5\, T^a_{ff'}\ , \quad \Gamma^{S^a}_{ff'} = T^a_{ff'}\ , \quad T^a_{ff'} = \begin{cases} (\lambda_3)_{ff'}, \\ (\lambda_1 \pm i\lambda_2)_{ff'}/\sqrt{2}, \\ (\lambda_4 \pm i\lambda_5)_{ff'}/\sqrt{2}, \\ (\lambda_6 \pm i\lambda_7)_{ff'}/\sqrt{2}, \end{cases}$$

$$P^a = \pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0$$

$$\Pi^{P^a,S^a}_{ff'}(q_0+i\eta,\mathbf{0}) = 4\left\{I^f_1(T,\mu_f) + I^{f'}_1(T,\mu_{f'}) \mp \left[(q_0+\mu_{ff'})^2 - (m_f \mp m_{f'})^2\right] I^{ff'}_2(z,T,\mu_{ff'})\right\}$$

$$I^f_1(T,\mu_f) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f}\left(n^-_f - n^+_f\right),$$

$$I^{ff'}_2(z,T,\mu_{ff'}) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f E_{f'}}\left[\frac{E_{f'}}{(z-E_f-\mu_{ff'})^2 - E^2_{f'}}\,n^-_f - \frac{E_{f'}}{(z+E_f-\mu_{ff'})^2 - E^2_{f'}}\,n^+_f + \frac{E_f}{(z+E_{f'}-\mu_{ff'})^2 - E^2_f}\,n^-_{f'} - \frac{E_f}{(z-E_{f'}-\mu_{ff'})^2 - E^2_f}\,n^+_{f'}\right]$$

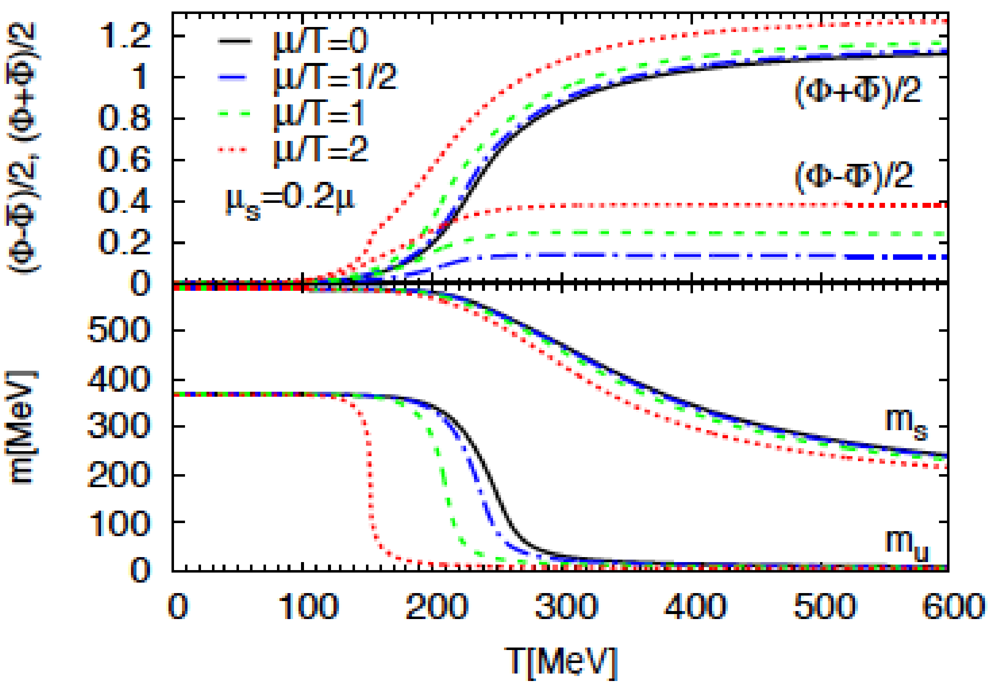
PNJL model for N_f=2+1 quark matter with π and K

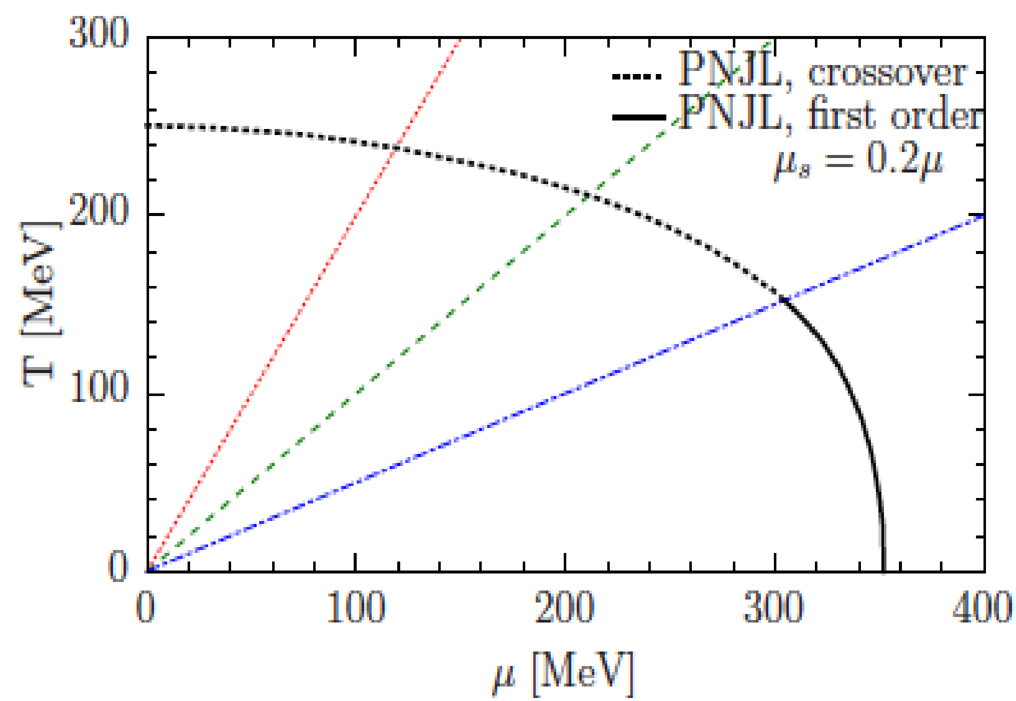
$$m_f = m_{0,f} + 16\, m_f G_S I_1^f(T,\mu), \quad \mathcal{P}^{M^a}_{ff'}(M_{M^a}+i\eta,\mathbf{0}) = 1 - 2G_S \Pi^{M^a}_{ff'}(M_{M^a}+i\eta,\mathbf{0}) = 0.$$

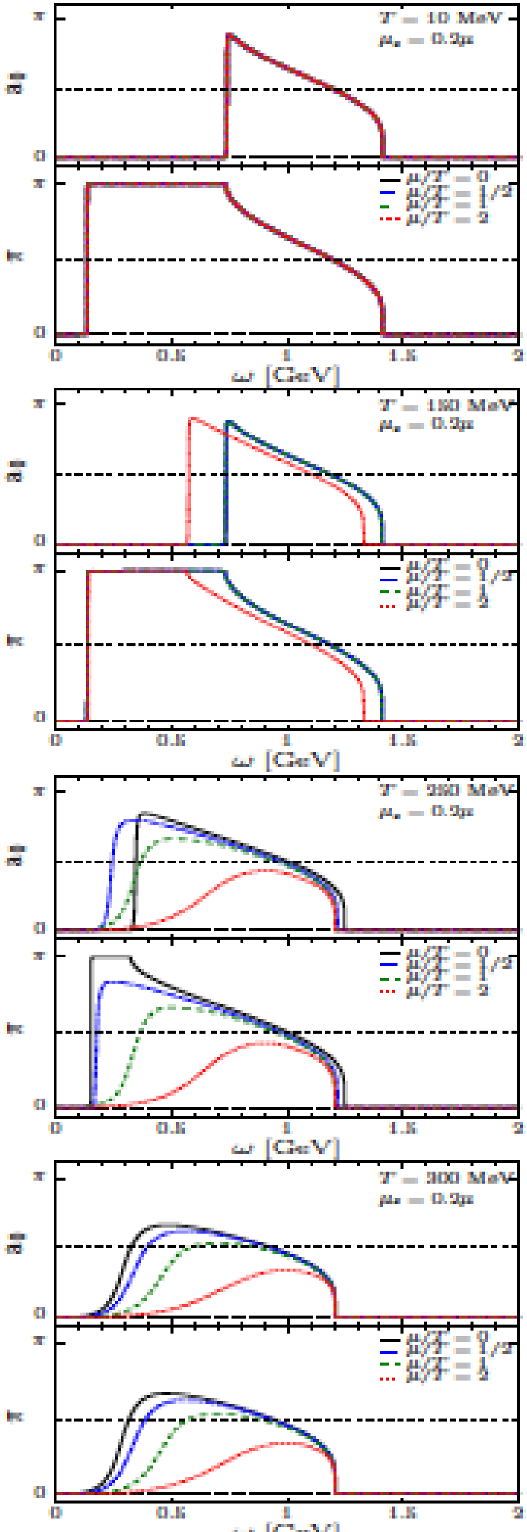
$$P_f \;=\; -\frac{(m_f - m_{0,f})^2}{8G} + \frac{N_c}{\pi^2}\int_0^\Lambda dp\, p^2\, E_f + \frac{N_c}{3\pi^2}\int_0^\infty \frac{dp\, p^4}{E_f}\left[f_\Phi^+(E_f) + f_\Phi^-(E_f)\right]$$

$$P_M \;=\; d_{\rm M}\int \frac{{\rm d}^3 q}{(2\pi)^3}\int_0^\infty \frac{{\rm d}\omega}{2\pi}\left\{ g(\omega-\mu_{\rm M}) + g(\omega+\mu_{\rm M})\right\}\delta_{\rm M}(\omega,\mathbf{q})$$

$$\delta_{\rm M}(\omega,\mathbf{q}) = -\arctan\left\{\frac{{\rm Im}\left(\mathcal{P}^M_{ff'}(\omega - i\eta,\mathbf{q})\right)}{{\rm Re}\left(\mathcal{P}^M_{ff'}(\omega + i\eta,\mathbf{q})\right)}\right\}$$







Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383
D.B., M. Buballa, A. Dubinin, G. Ropke, D. Zablocki, Ann. Phys. (2014)

Thermodynamics of resonances (M) via phase shifts

$$P_M = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{ds}{4\pi} \frac{1}{\sqrt{s+q^2}} \left\{ g(\sqrt{s+q^2} - \mu_M) \right\} \delta_M(\sqrt{s}; T, \mu)$$

Polyakov-loop Nambu – Jona-Lasinio model

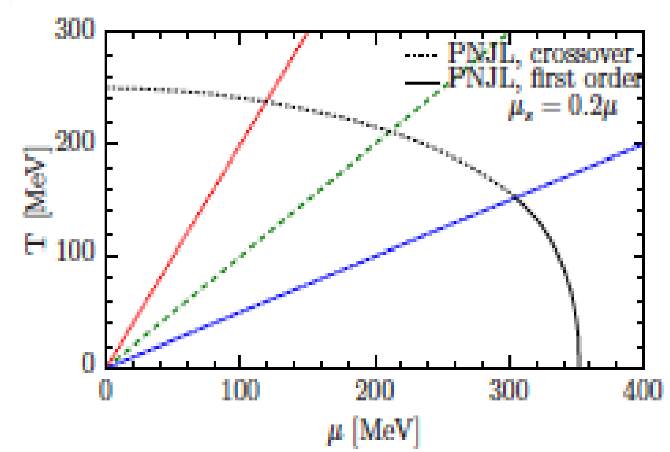
$$\Pi^{M^*}_{ff'}(q_0, \mathbf{q}) = 2N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{tr}_D \left[S_f(p_n, \mathbf{p}) \Gamma^{M^*}_{ff'} S_{f'}(p_n + q_0, \mathbf{p} + \mathbf{q}) \Gamma^{M^*}_{ff'} \right],$$

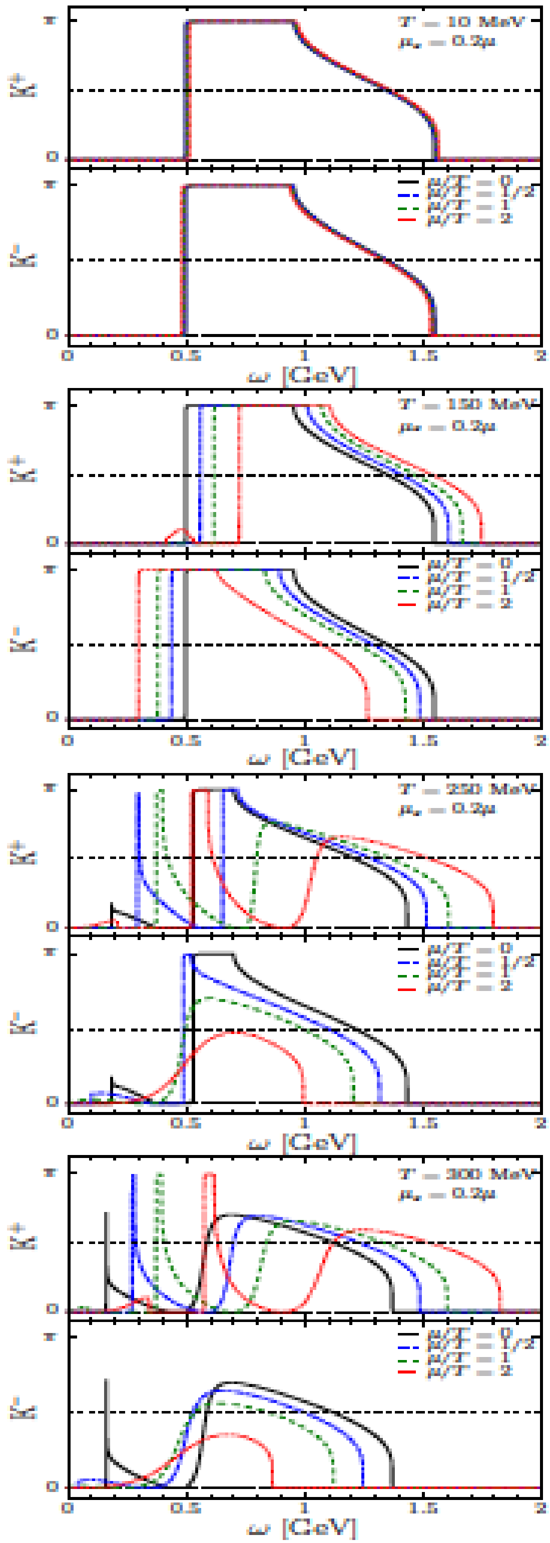
$$\mathcal{P}^{M^*}_{ff'}(M_{M^*} + i\eta, 0) = 1 - 2G_S \Pi^{M^*}_{ff'}(M_{M^*} + i\eta, 0)$$

$$\delta_M(\omega, \mathbf{q}) = -\arctan \left\{ \frac{\mathrm{Im}\left(\mathcal{P}^M_{ff'}(\omega - i\eta, \mathbf{q}) \right)}{\mathrm{Re}\left(\mathcal{P}^M_{ff'}(\omega + i\eta, \mathbf{q}) \right)} \right\}$$

Evaluation along trajectories μ/T=const in the phase diagram:

- Pion and a0 as partner states,
- Chiral symmetry restoration,
- Mott dissociation of bound states,
- Levinson theorem





Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

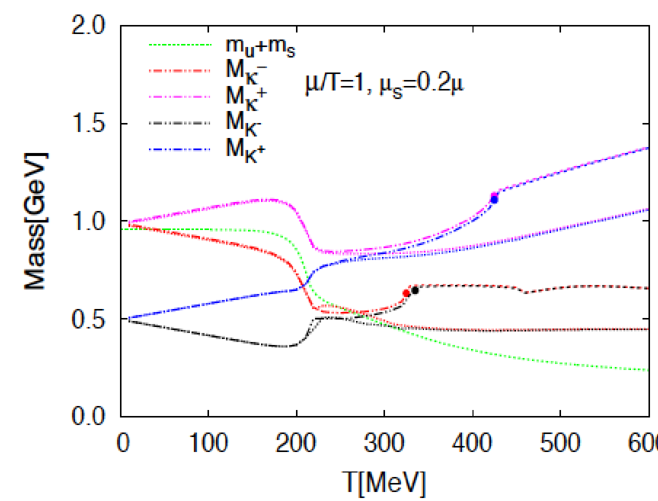
D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

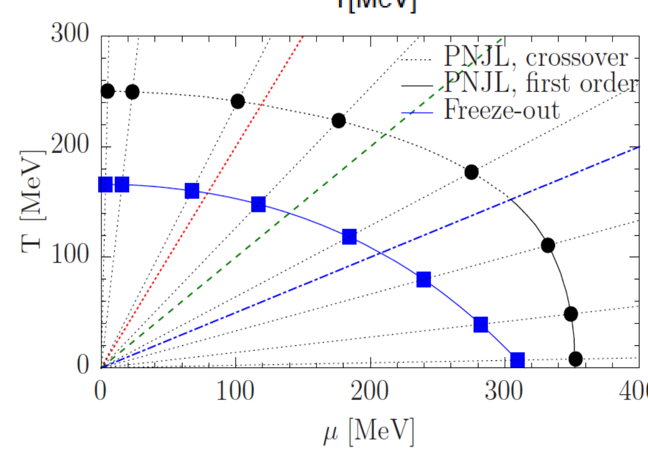
Polarization loop in Polyakov-loop Nambu – Jona-Lasinio model

$$\Pi^{P^a,S^a}_{ff'}(q_0+i\eta,\mathbf{0}) = 4\{I_1^f(T,\mu_f)+I_1^{f'}(T,\mu_{f'}) \mp \left[(q_0+\mu_{ff'})^2-(m_f\mp m_{f'})^2\right] I_2^{ff'}(z,T,\mu_{ff'})\}$$

$$I_1^f(T,\mu_f) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f}\left(n_f^- - n_f^+\right),$$

$$I_2^{ff'}(z,T,\mu_{ff'}) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_fE_{f'}} \left[\frac{E_{f'}}{(z-E_f-\mu_{ff'})^2-E_{f'}^2}\,n_f^- - \frac{E_{f'}}{(z+E_f-\mu_{ff'})^2-E_{f'}^2}\,n_f^+ + \frac{E_f}{(z+E_{f'}-\mu_{ff'})^2-E_f^2}\,n_{f'}^- - \frac{E_f}{(z-E_{f'}-\mu_{ff'})^2-E_f^2}\,n_{f'}^+\right]$$



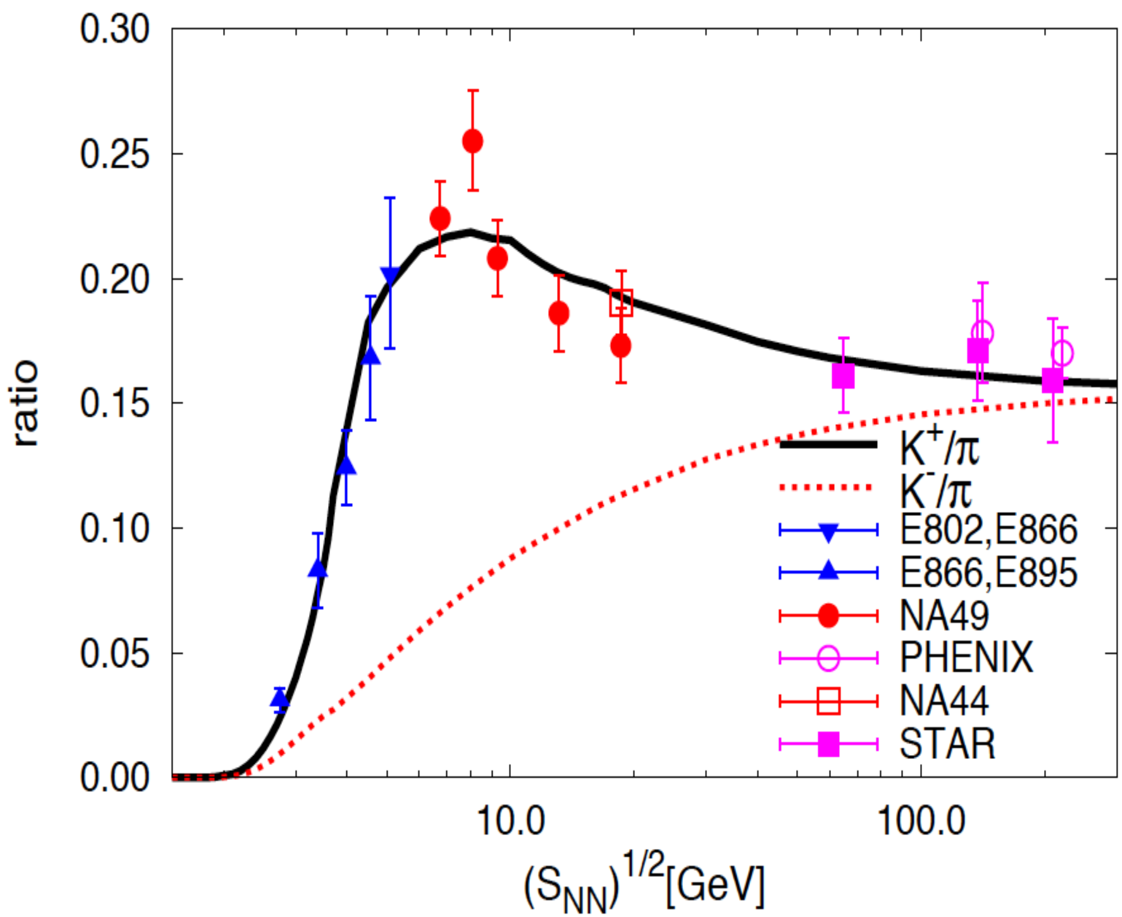


Anomalous low-mass mode for K+ in the dense medium !!

Mott dissociation of pions and kaons in Beth-Uhlenbeck: Explanation of the "horn" effect for K+/π+ in HIC?

Ratio of yields in BU approach defined via phase shifts:

$$\frac{n_{K\pm}}{n_{\pi\pm}} = \frac{\int dM \int d^3p \ (M/E) g_{K\pm}(E)[1+g_{K\pm}(E)]\delta_{K\pm}(M)}{\int dM \int d^3p \ (M/E) g_{\pi\pm}(E)[1+g_{\pi\pm}(E)]\delta_{\pi\pm}(M)}$$



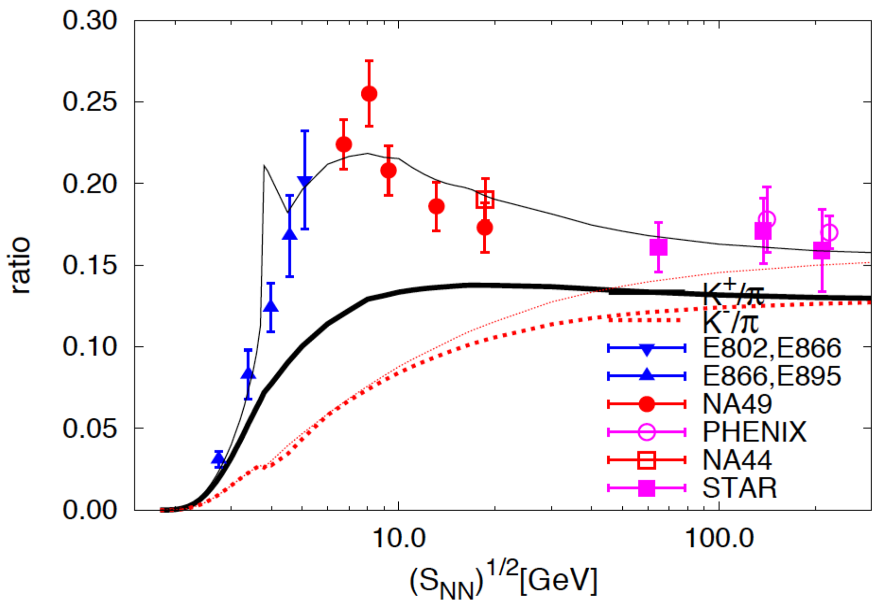
Evaluation along the freeze-out Curve parametrized by Cleymans et al.

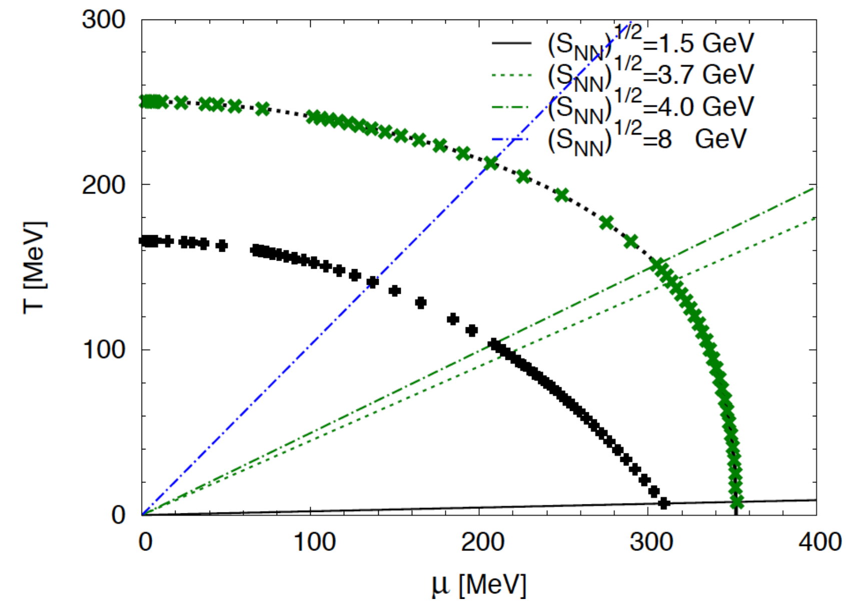
- enhancement for K+ due to anomalous in-medium bound state mode
- no such enhancement for K- or pions

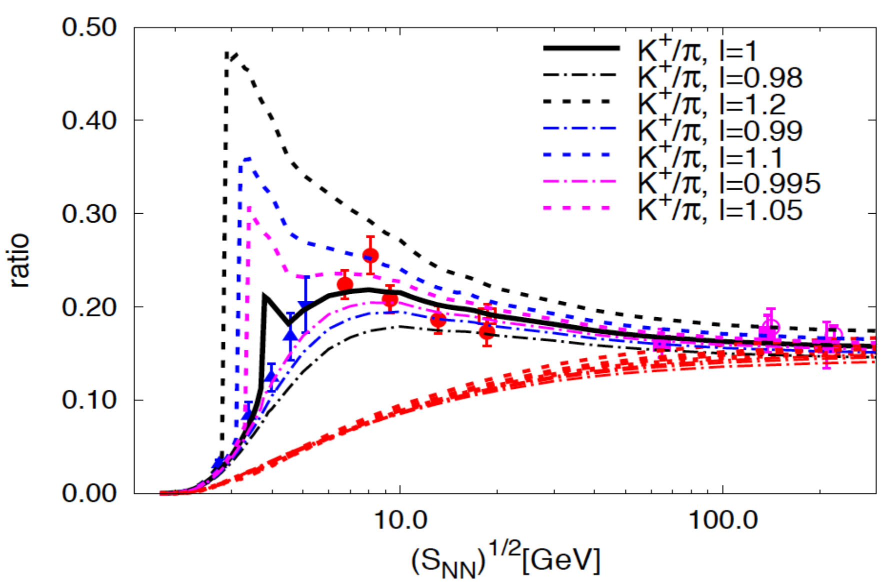
- explore the effect in thermal statistical models and in THESEUS ...

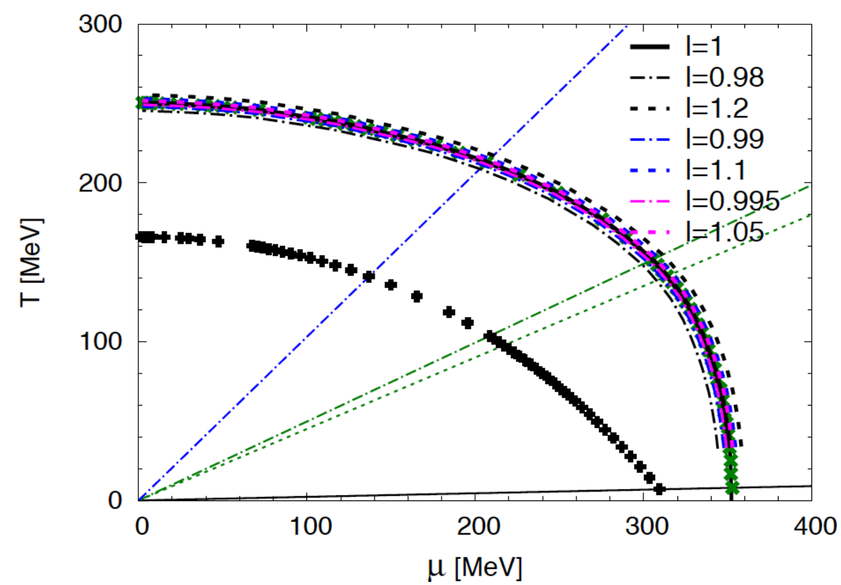
D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

3. “Tooth” on the “horn” due to anomalous K+; sign of CEP?





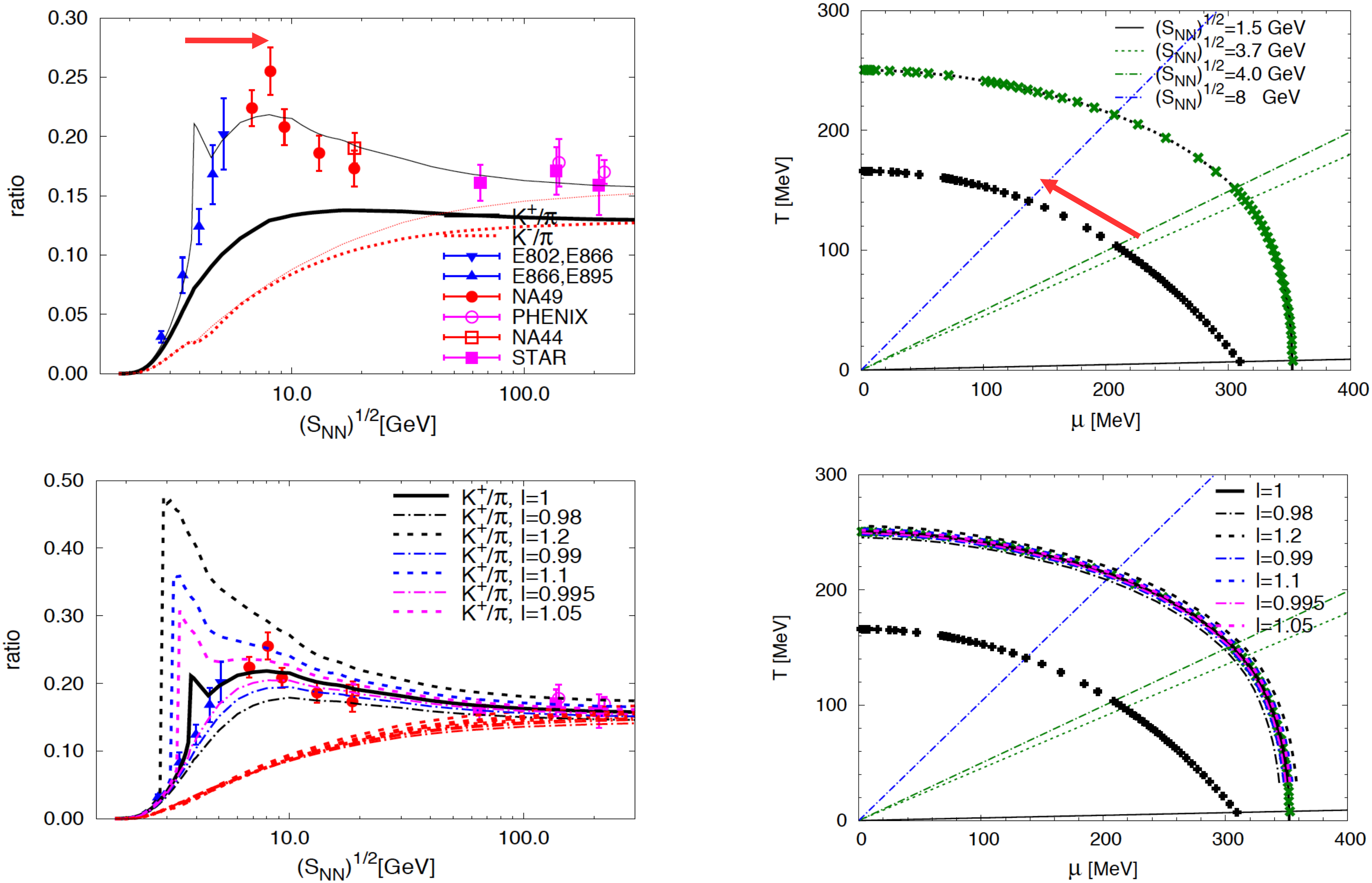




- enhancement for K+ due to anomalous in-medium bound state mode

D.B., A. Radzhabov, in prep. (2017)

3. “Tooth” on the “horn” due to anomalous K+; sign of CEP?



- “tooth” correlated to the CEP → indicator for CEP !!

D.B., A. Radzhabov, in prep. (2017)

Conclusions

- Φ-derivable approach with two-loop skeleton diagrams as Φ-functional corresponds to the Generalized Beth-Uhlenbeck approach
- BU/GBU approaches describe dissociation of bound states into a plasma of their constituents
- GBU corresponds to a “squared Breit-Wigner” spectral distribution which narrows the range of importance of correlations above the Mott transition as compared to the standard BU case Which would have a Breit-Wigner distribution.
- Positively charged kaons develop an “anomalous” bound state mode in the medium, enhancing the K+/pi+ ratio just for sqrt(s) ~ 8 GeV, at its peak (the “horn”)
- A sharp peak (“tooth”) on top of the the K+/pi+ ratio occurs when freeze-out happens on the plasma side of the chiral transition at the critical endpoint

Outlook

- Use the GBU approach for thermodynamics and particle abundances, consistently including a phase shift for quarks that arises from the backreaction of correlations on the quark sector
- Use a microscopic model that reproduced in this approach the lattice QCD pseudocritical temperature and has a critical endpoint at the freeze-out parameters for sqrt(s) ~ 8 GeV. Candidates: nonlocal PNJL model, hPNJL model, EPNJL model, RDF model, ...
- Compare the bahaviour of the chiral condensate and susceptibilities with lattice QCD, include more hadronic states ...

6th International Workshop on

Compact Stars in the QCD Phase Diagram VI

(Cosmic matter in heavy-ion collision laboratories?)

Dates:	**26.-29. September 2017**
Venue:	**Dubna, Russian Federation**
Organizers:	**D. Blaschke, H. Grigorian**
Website:	**http://www.quarknova.ca/CSQCD.html**
	http://theor.jinr.ru/meetings/2017 (t.b.u.)

Previous meetings:
Copenhagen (2001), Beijing (2009), Guaruya (2012), Prerow (2014), Gran Sasso (2016)

Topics:
- **QCD phase diagram for HIC vs. Astrophysics**
- **Quark deconfinement in HIC vs. supernovae, neutron stars and their mergers**
- **Strangeness in HIC and in compact stars**
- **...**

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