Cumulant ratios of net-baryon number fluctuations at small values of the baryon chemical potential

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BNL-Bi-CCNU Collaboration:

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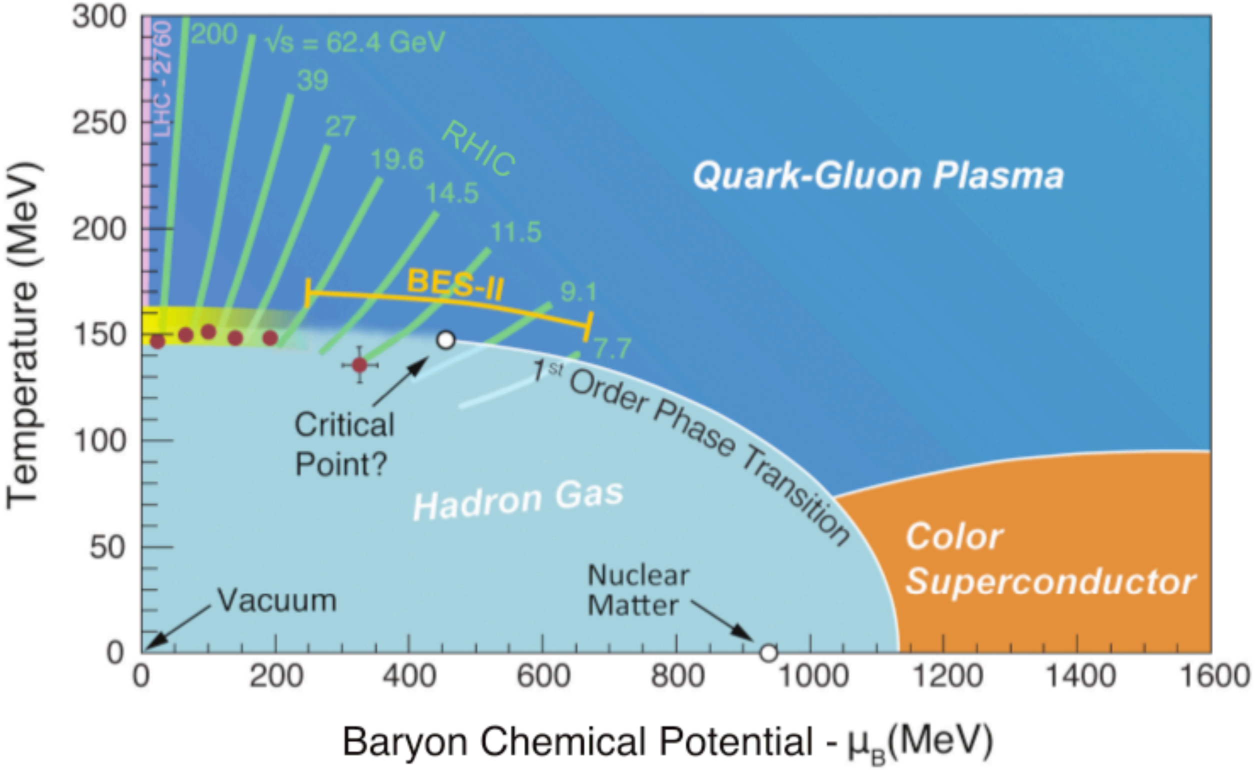
Motivation

- Understand and calculate QCD equilibrium thermodynamic quantities at nonzero temperature **and** baryon number densities from first principles

⟶ is there evidence for a QCD critical point?

⟶ can we understand the BES results of conserved charge fluctuations?

⟶ what is the validity range of the HRG model?

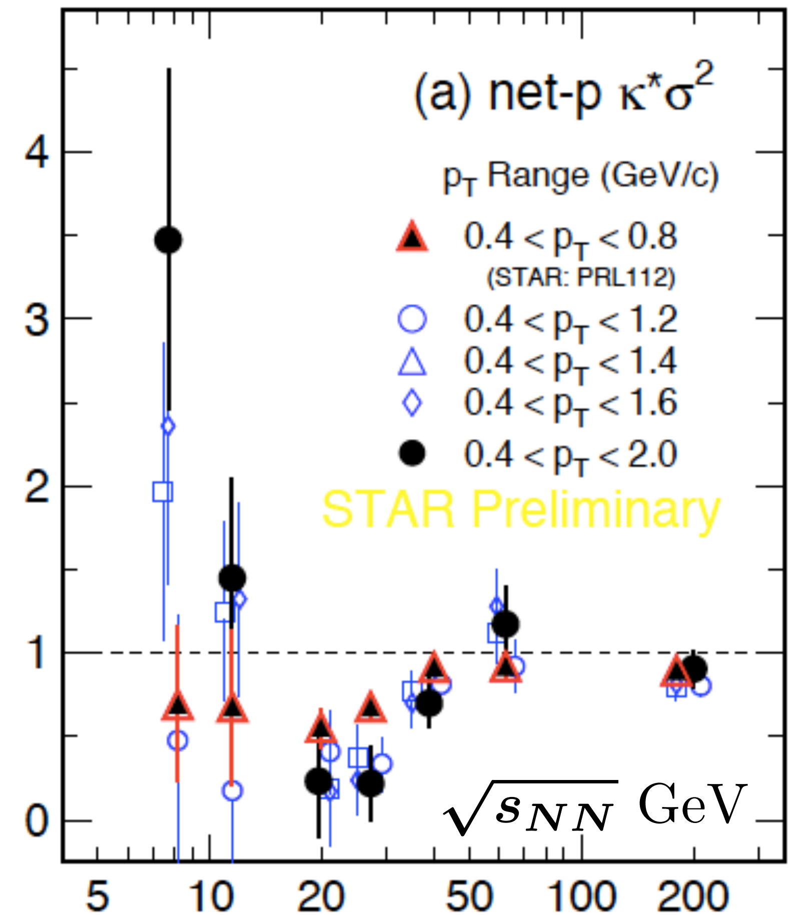


[NSAC 2015 Long Range Plan for Nuclear Physics]

Plan

The whole discussion and all results are based on Taylor expansion coefficients of the pressure of QCD, obtained at vanishing chemical potentials:

- Formulation of the method and status of ongoing calculations
- The radius of convergence and the QCD critical point
- Freeze-out and lines of constant physics
- Skewness and kurtosis of the net-proton number fluctuations at freeze-out



[X. Luo, CPOD'14]

X. Luo, CPOD'14

Taylor expansion in μ/T (methodology)

$$\frac{p(\vec{\mu},T)}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

with $\chi_{i,j,k}^{BQS}(T) = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z(\vec{\mu},T)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\vec{\mu}=0}$ and $\hat{\mu} = \mu/T$

Calculate all Taylor expansions coefficients of the QCD grand canonical partition function in terms of three chemical potential (μ_B, μ_Q, μ_S) up to a given order

→ flexible framework, study

- strangeness neutral matter (heavy ions)
- strangeness rich matter (quark starts?)
- electrically charged matter

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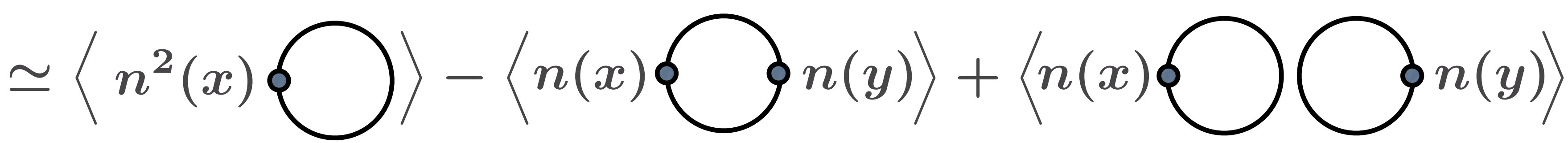
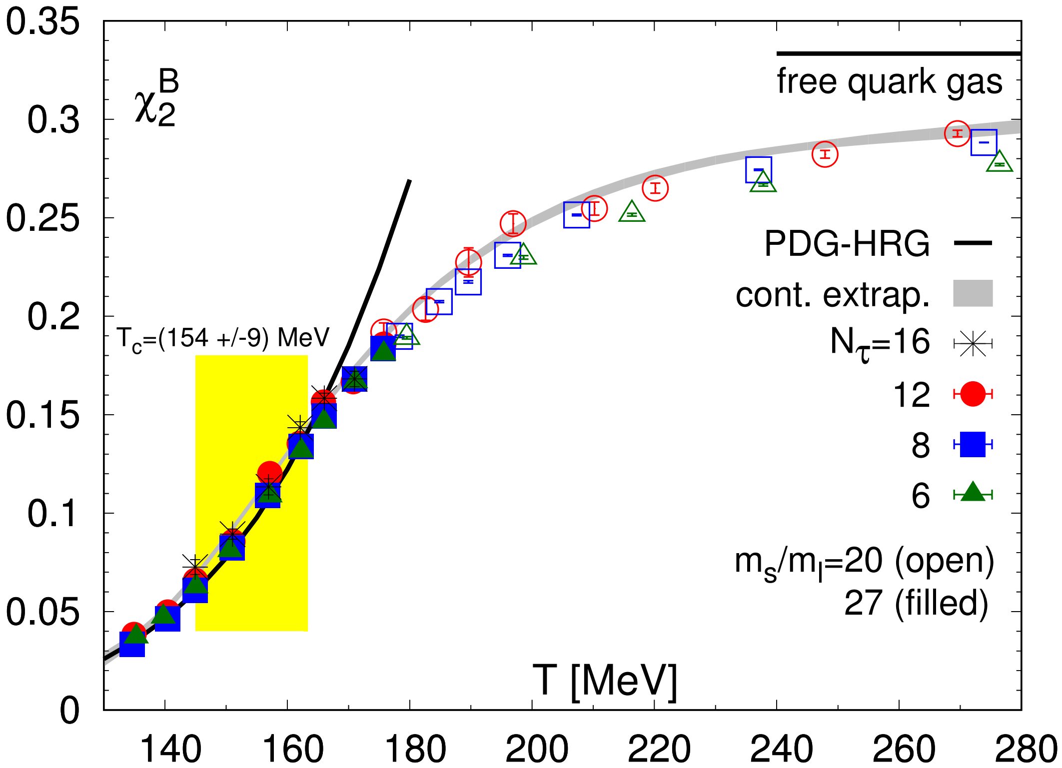
Example:

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1}M''\right]\right\rangle - \left\langle \mathrm{Tr}\left[M^{-1}M'M^{-1}M'\right]\right\rangle + \left\langle \mathrm{Tr}\left[M^{-1}M'\right]^2\right\rangle$$



Taylor expansion in μ/T (status)

$$\frac{p(T,\mu_B)-p(T,0)}{T^4}=\frac{\chi_2^B(T)}{2}\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B}{\chi_2^B}\hat{\mu}_B^4+\cdots\right)$$



- $\mathcal{O}\left(\mu^2\right)$: ✓

[BNL-Bi-CCNU, PRD 95 (2017), 054504]

similar results obtained by Budapest-Wuppertal

[Gunter et al., EPJ Web Conf 137 (2017) 07008]

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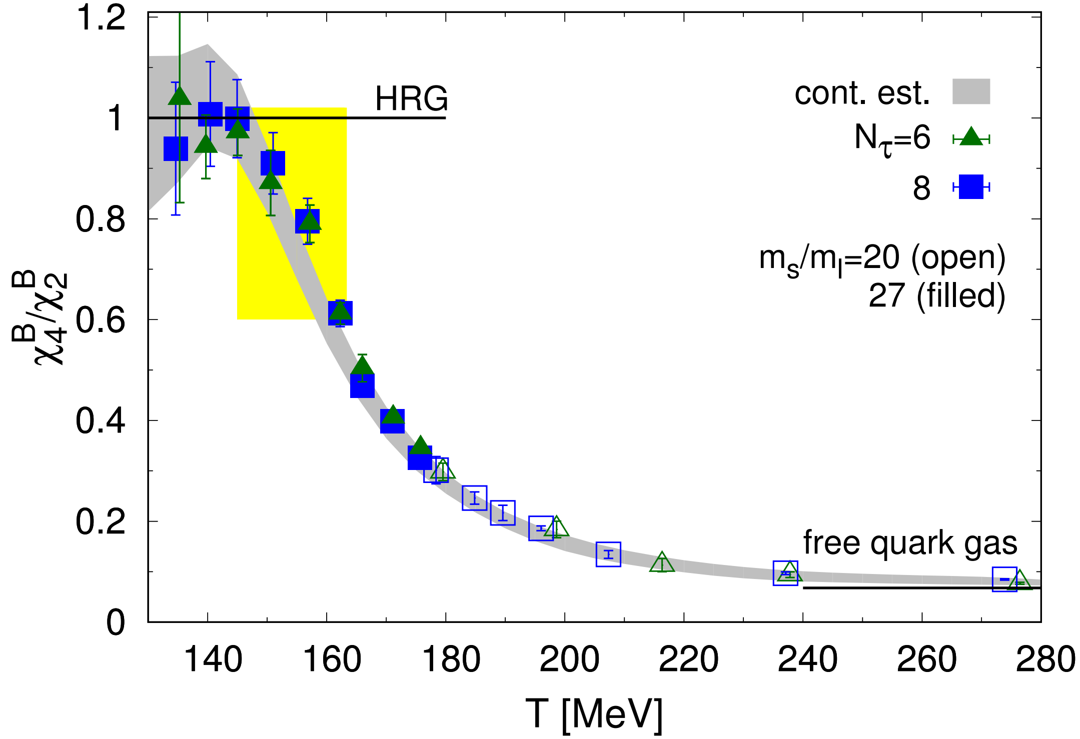

- $\mathcal{O}\left(\mu^2\right)$: ✓
- $\mathcal{O}\left(\mu^4\right)$: ✓

[BNL-Bi-CCNU, PRD 95 (2017), 054504]

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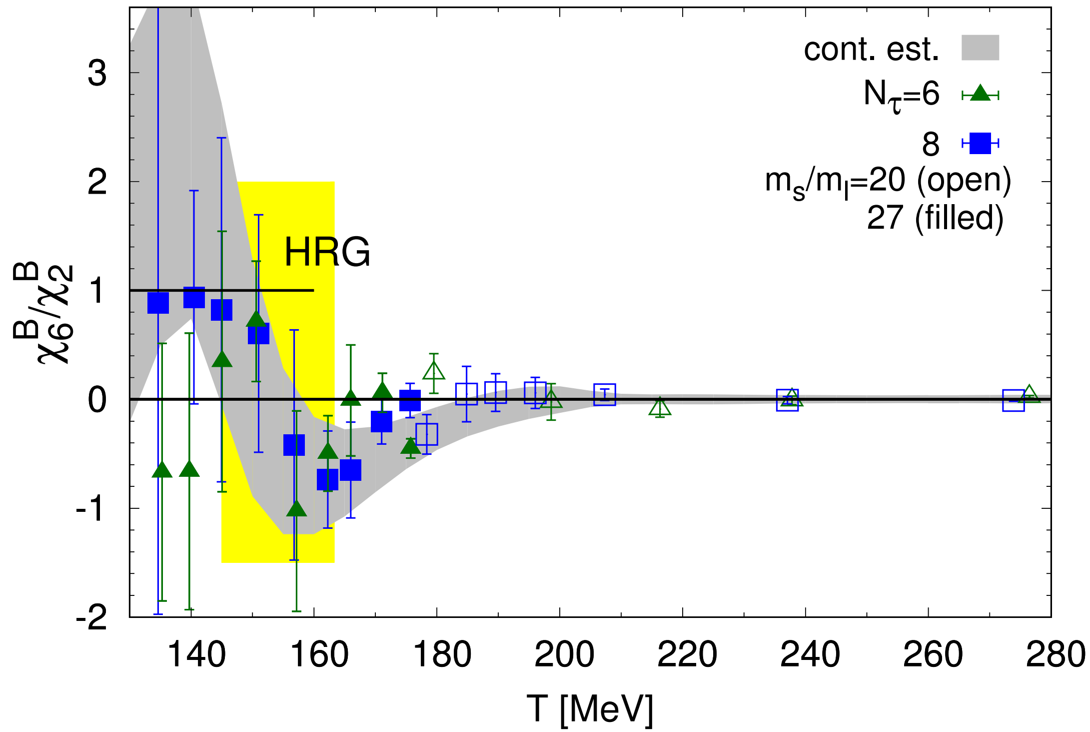

- $\mathcal{O}\left(\mu^2\right)$: ✓
- $\mathcal{O}\left(\mu^4\right)$: ✓
- $\mathcal{O}\left(\mu^6\right)$: still large stat. errors, need consolidations

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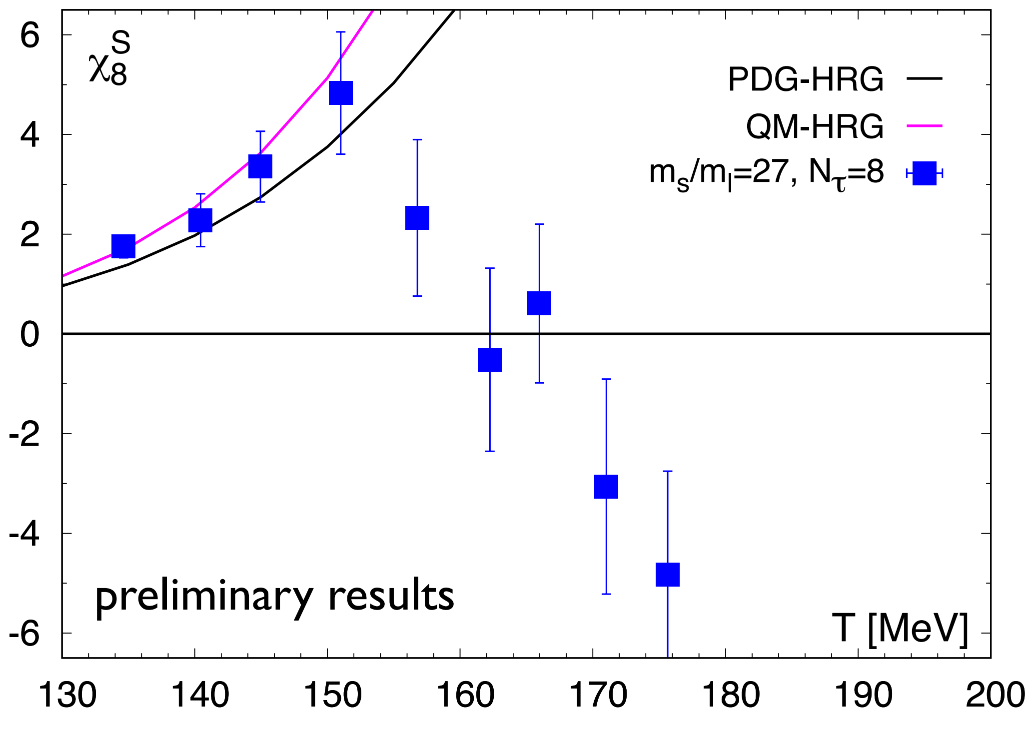
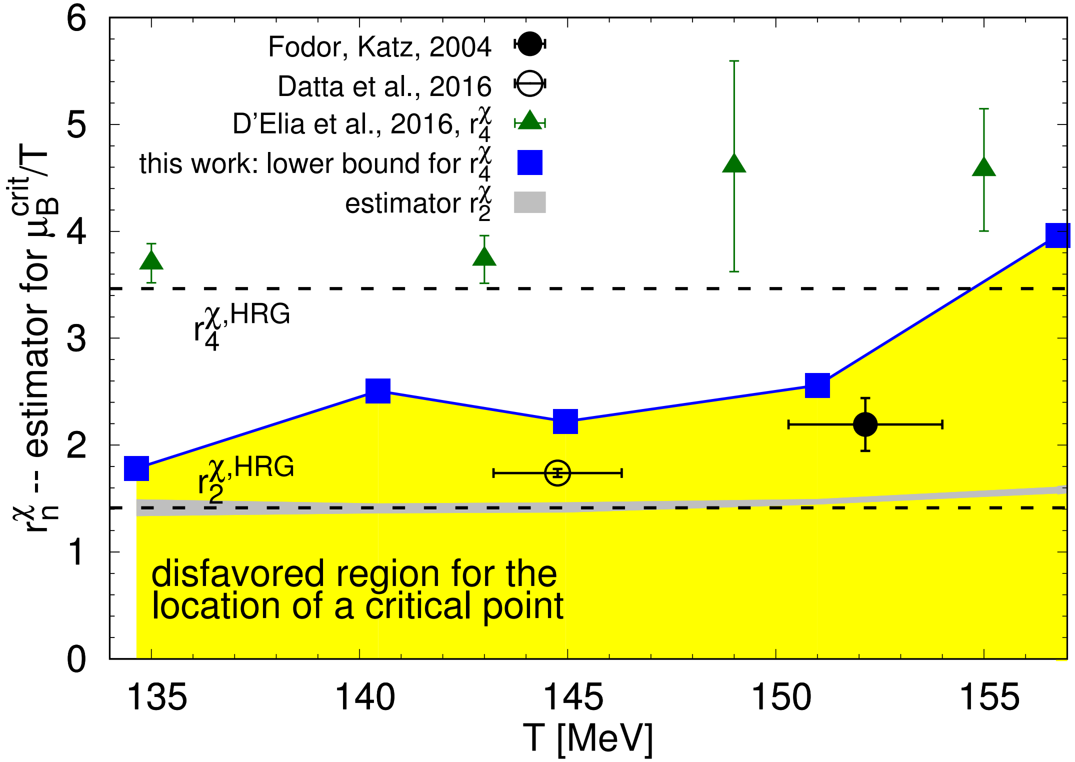


- $\mathcal{O}(\mu^2)$: ✓
- $\mathcal{O}(\mu^4)$: ✓
- $\mathcal{O}(\mu^6)$: still large stat. errors, need consolidations
- $\mathcal{O}(\mu^8)$: work in progress, some coefficients might require further tuning of algorithms
- $\mathcal{O}(\mu^{10})$: will require new strategies, many ideas to pursue

Estimating the radius of convergence

$$\frac{p(T,\mu_B)-p(T,0)}{T^4} = \frac{\chi_2^B(T)}{2}\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B}{\chi_2^B}\hat{\mu}_B^4+\cdots\right)$$



[BNL-Bi-CCNU, PRD 95 (2017), 054504]

possible definitions of estimators:

$$r_{2n}^P = \left|\frac{(2n+2)(2n+1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^{1/2}$$

$$r_{2n}^\chi = \left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^{1/2}$$

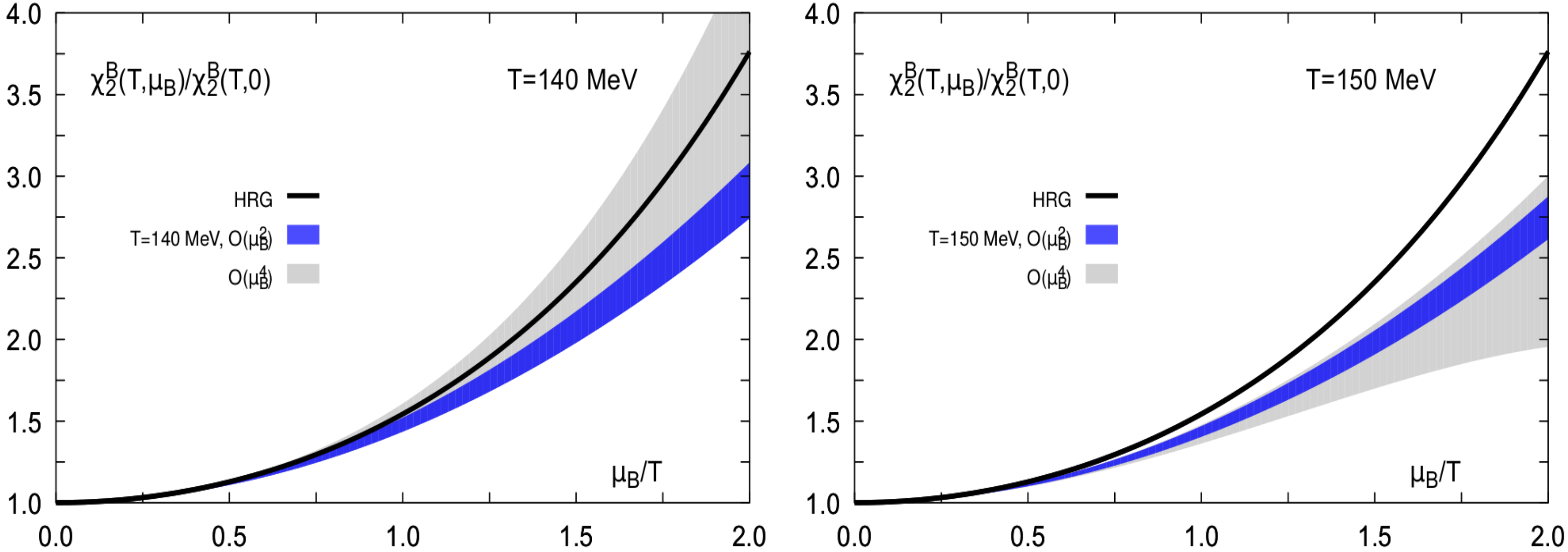
true radius of convergence:

$$\rho(T) = \lim_{n\to\infty} r_{2n}^P(T) = \lim_{n\to\infty} r_{2n}^\chi(T)$$

- the radius of convergence only corresponds to a critical point if all expansion coefficients are positive
- HRG: all ratios $\chi_{2n}^B/\chi_{2n+2}^B$ are unity.

Estimating the radius of convergence

baryon number fluctuations as function of $\hat{\mu}_B$:



- agreement with HRG starts to deteriorate for T > 150 MeV
- no evidence for enhanced net-baryon number fluctuations (for $\mu_B/T \leq 2$, T > 135 MeV)

Adapting the expansion to the HIC case

Apply conditions as in the HIC fireball

- strangeness neutrality: $\langle N_S \rangle = 0$
- isospin asymmetry: $\langle N_Q \rangle = r \langle N_B \rangle$

$r \approx 0.4$
for Au-Au
and Pb-Pb

expand in powers of μ_B, μ_Q, μ_S
solve for μ_Q, μ_S

$$\mu_Q(T,\mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3 + q_5(T)\hat{\mu}_B^5 + \cdots$$
$$\mu_S(T,\mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3 + s_5(T)\hat{\mu}_B^5 + \cdots$$

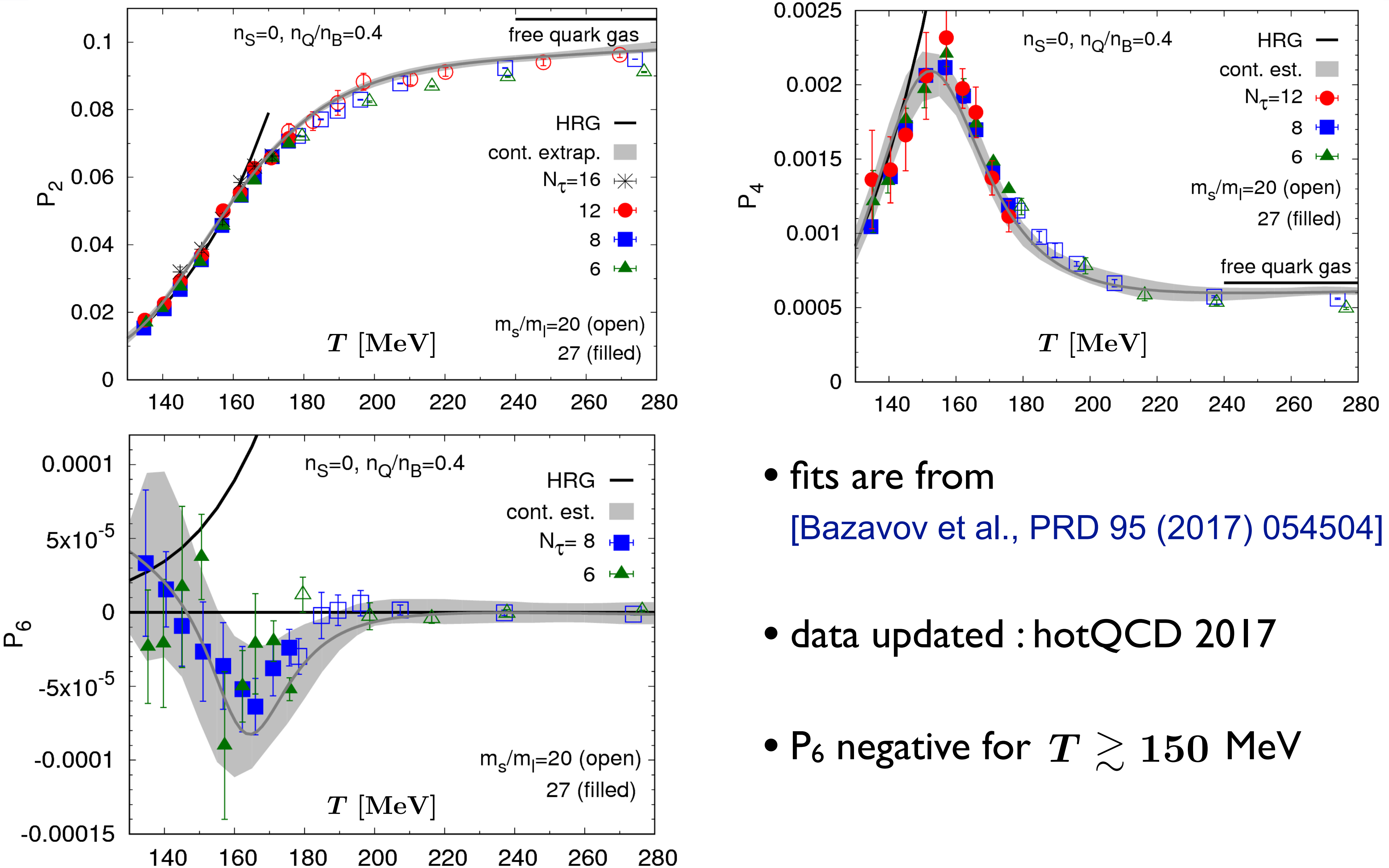
LO NLO NNLO

$\hat{\mu}_B = \mu_B/T$

define strangeness neutral coefficients p_n

$$\frac{\Delta p}{T^4} = \frac{1}{2}\chi_2^B\hat{\mu}_B^2 + \frac{1}{2}\chi_2^Q\hat{\mu}_Q^2 + \frac{1}{2}\chi_2^S\hat{\mu}_S^2 + \chi_{11}^{BQ}\hat{\mu}_B\hat{\mu}_Q + \chi_{11}^{BS}\hat{\mu}_B\hat{\mu}_S + \chi_{11}^{QS}\hat{\mu}_Q\hat{\mu}_S + \cdots$$
$$= \frac{1}{2}\underbrace{\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2 + 2\chi_{11}^{BQ}q_1 + 2\chi_{11}^{BS}s_1 + 2\chi_{11}^{QS}q_1 s_1\right)}_{p_2}\hat{\mu}_B^2 + \cdots$$

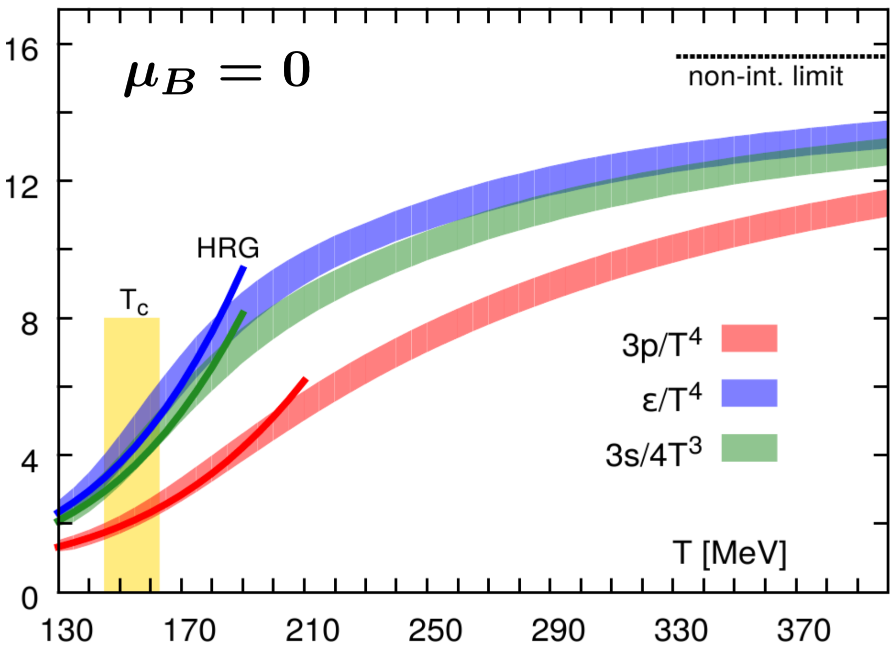
The strangeness neutral coefficients (r=0.4)

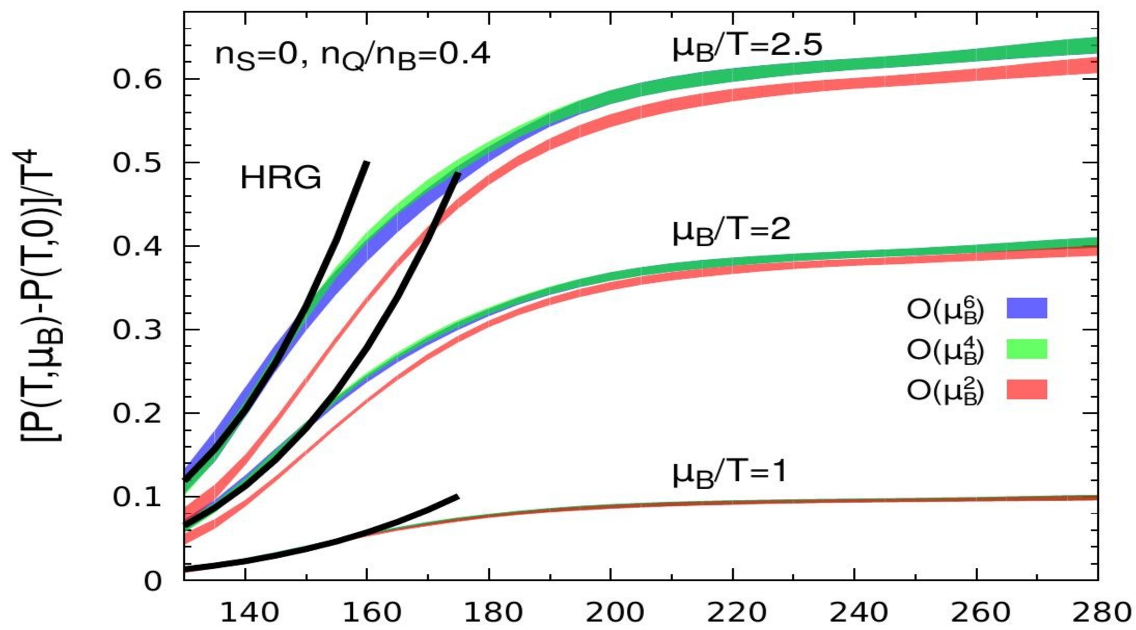


- fits are from
  [Bazavov et al., PRD 95 (2017) 054504]
- data updated : hotQCD 2017
- P_6 negative for $T \gtrsim 150$ MeV

The equation of state for $\mu_B > 0$

$$\frac{P(T,\mu_B)-P(T,0)}{T^4} = P_2(T)\left(\frac{\mu_B}{T}\right)^2 + P_4(T)\left(\frac{\mu_B}{T}\right)^4 + P_6(T)\left(\frac{\mu_B}{T}\right)^6 + \cdots$$





- (20-30)% contribution to the total pressure at $\mu_B/T = 2$

⇒ The 6th-order EoS is well controlled for $\mu_B/T \leq 2$ or equivalently $\sqrt{s_{NN}} \geq 19.6 GeV$

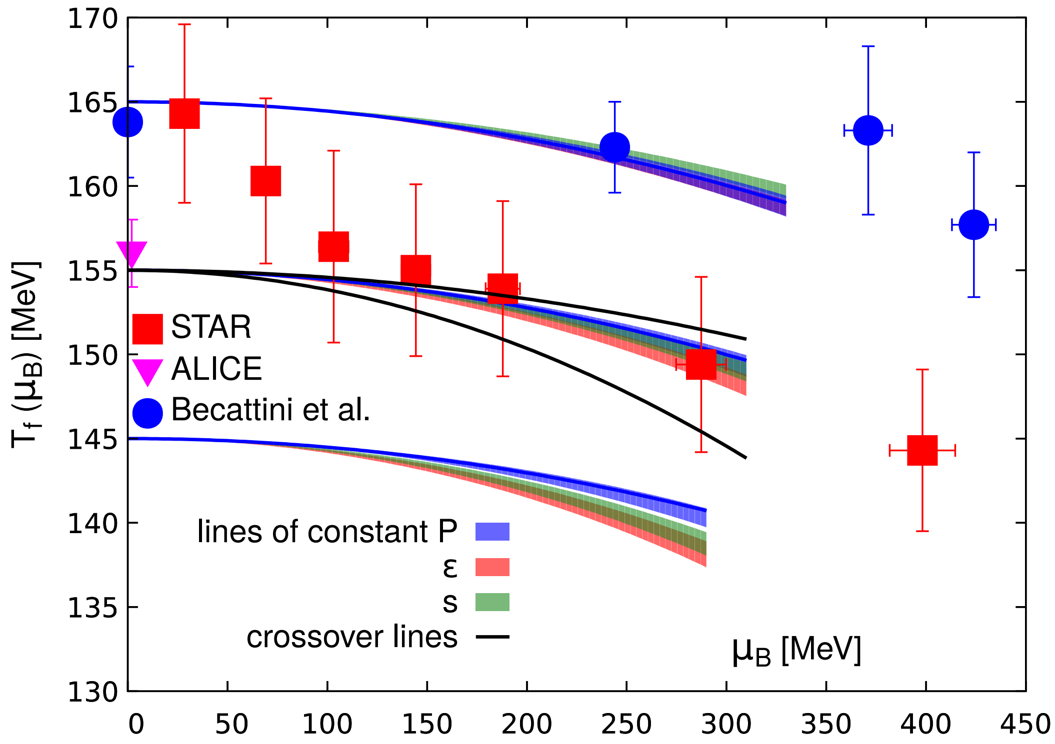
Lines of “constant physics”

- assume parametrization of line of constant observable f, with $f \in \{P, \epsilon, s\}$, i.e. pressure, energy density or entropy. f is even function of μ_B:

$$T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T}\right)^4\right)$$

$$T_c = 154(9)\ \mathrm{MeV}$$

[hotQCD, PRD 90 (2014) 094503]



[Bazavov et al., PRD 95 (2017) 054504]

- obtained curvatures are similar to the curvature of the pseudo-critical (the latter is not yet determined very well)

$$0.0064 \leq \kappa_2^P \leq 0.0101$$

- compare to freeze-out data from STAR and ALICE: ***where does hadronization set in?***
- note: physics changes rapidly in the interval 145 MeV < T < 165 MeV

Cumulant ratios (definition)

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

- consider cumulant ratios to eliminate the freeze-out volume

Lattice		**Experiment**
$R_{12}^X(T,\mu_B) \equiv \frac{\chi_1^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\frac{M_X}{\sigma_X^2}$
$R_{32}^X(T,\mu_B) \equiv \frac{\chi_3^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$S_X \sigma_X$
$R_{42}^X(T,\mu_B) \equiv \frac{\chi_4^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\kappa_X \sigma_X^2$

$M :=$ mean
$\sigma^2 :=$ variance
$S :=$ skewness
$\kappa :=$ kurtosis

Cumulant ratios (expansion)

- expand cumulant ratios in $\hat{\mu}_B = \mu_B/T$

$$R_{12}^X(T,\mu_B) = r_{12}^{X,1}\hat{\mu}_B + r_{12}^{X,3}\hat{\mu}_B^3 + \cdots$$

$$R_{32}^X(T,\mu_B) = r_{32}^{X,1}\hat{\mu}_B + r_{32}^{X,3}\hat{\mu}_B^3 + \cdots$$

$$R_{42}^X(T,\mu_B) = r_{42}^{X,0} + r_{42}^{X,2}\hat{\mu}_B^3 + \cdots$$

$\Rightarrow$ ratios are either even or odd in $\hat{\mu}_B$

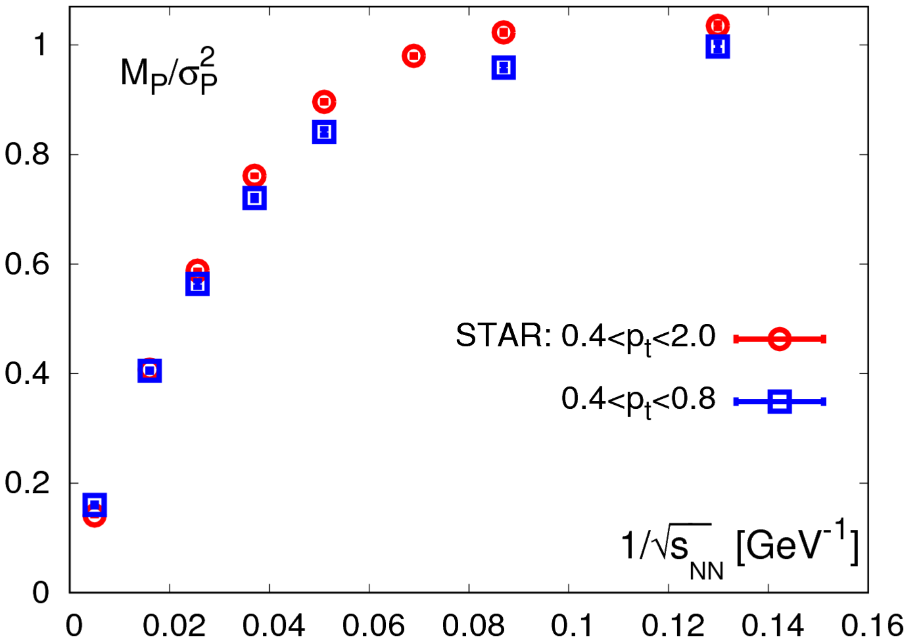
- expand cumulant ratios along a line of constant physics $T_f(\mu_B)$

$\Rightarrow$ need to take into account the change in temperature

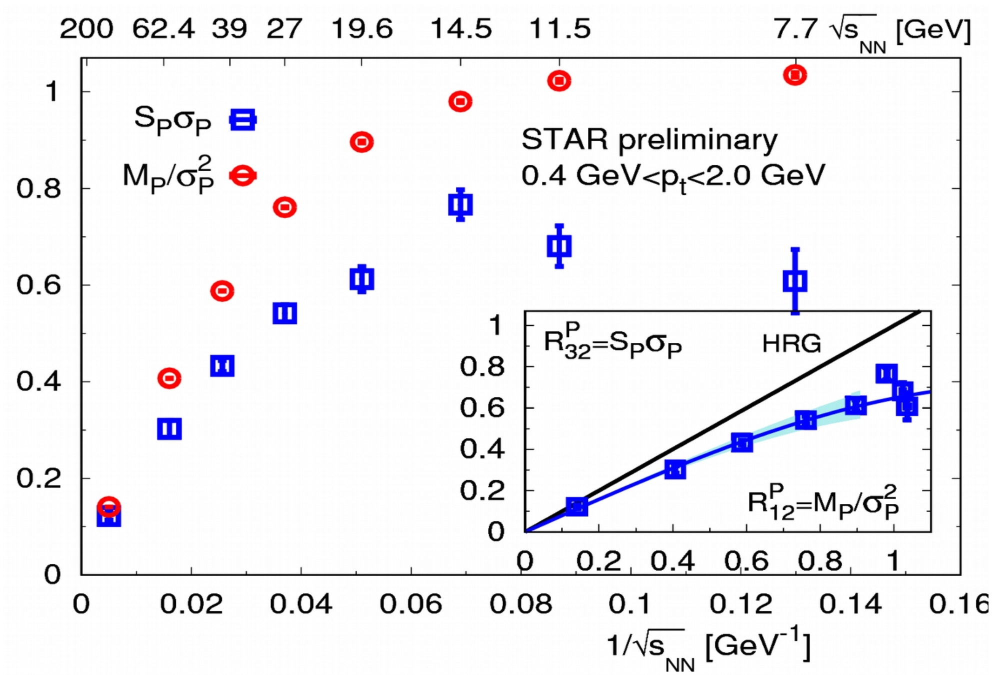
$$r_{nm}^{X,k} \rightarrow r_{nm}^{X,k} - \kappa_2^f T_0 \left.\frac{\mathrm{d}r_{nm}^{X,k-2}}{\mathrm{d}T}\right|_{T=T_0} \qquad \text{(for } k \geq 2\text{)}$$

Cumulant ratios from STAR

$R_{12}^P = M_P/\sigma_P^2$



$R_{32}^P = S_P\sigma_P$



Observations:

- R_{12}^P is monotonically rising in $1/\sqrt{s_{NN}}$ (relation is invertable)
- $R_{32}^P < R_{12}^P$ (HRG: $R_{32}^P = R_{12}^P$)

Key idea for the comparison of STAR data with lattice QCD:

STAR: replace $\sqrt{s_{NN}}$ by R_{12}^P

LQCD: replace $\hat{\mu}_B$ by R_{12}^B

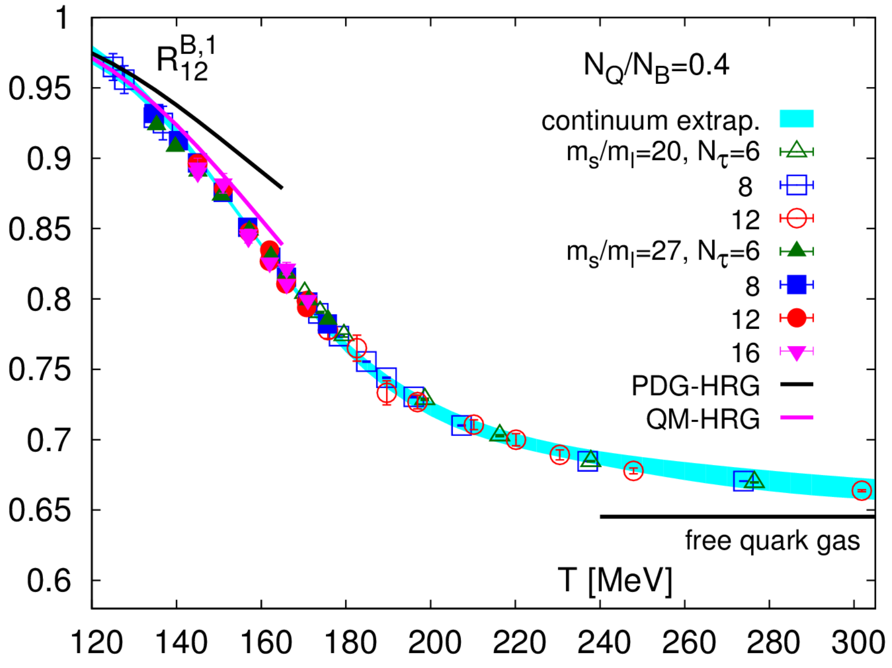
Cumulant ratios: aim for a comparison

trick: express all ratios as function of $R_{12}^{B/P}$

$\Rightarrow$ eliminates the need to first determine μ_B from the STAR data

- caution: RHIC measures net-proton and not net-baryon number fluctuations

  $\Rightarrow$ comparison is subject to systematic errors



[Bazavov et al., PRD 93 (2016) 014512]

HRG:

$$R_{12}^P = \tanh(\hat\mu_B + \hat\mu_Q) = \hat\mu_B + \mathcal{O}(\hat\mu_B^3)$$

no $\hat\mu_S$ -dependence,
neglect $\hat\mu_Q$ -dependence

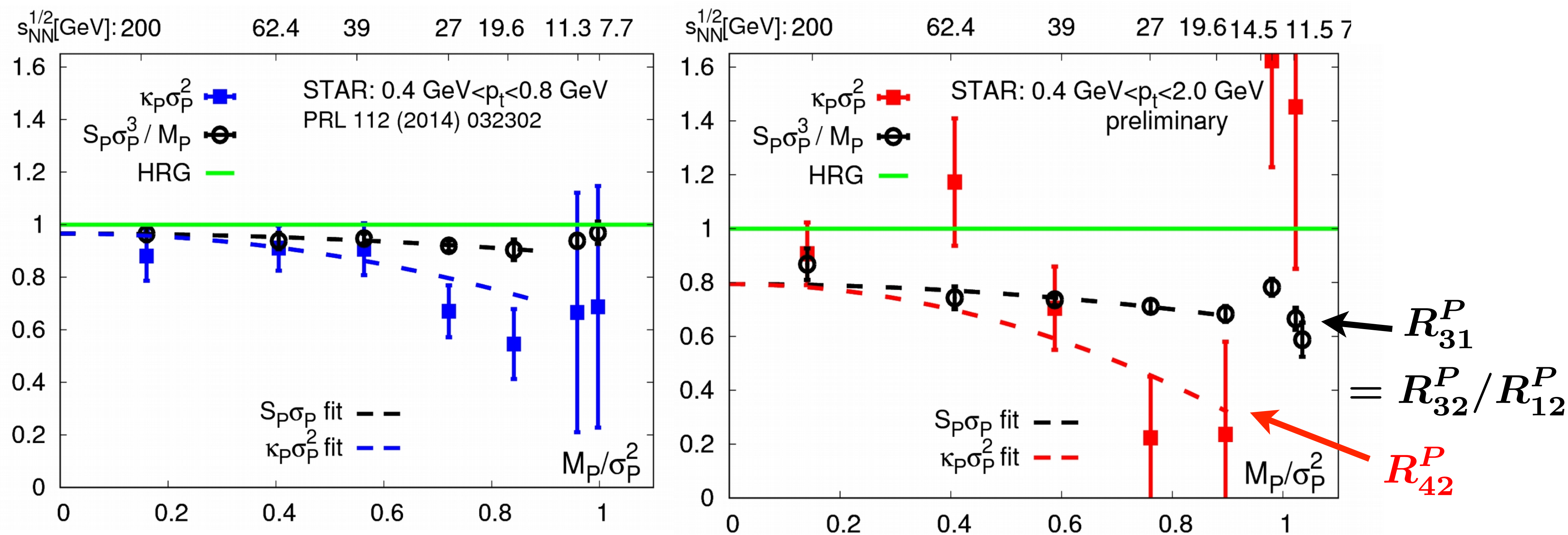
QCD:

$$R_{12}^B = r_{12}^{B,1}\hat\mu_B + \mathcal{O}(\hat\mu_B^3)$$

(strangeness neutral, r=0.4)

$\Rightarrow$ to leading order: $R_{12}^B / R_{12}^P = r_{12}^{B,1}$

Cumulant ratios from STAR



Observations:

- At $R_{12}^P = 0$: $R_{31}^P \simeq R_{42}^P$ ②
- Slope: $3r_{31}^{P,2} \simeq r_{42}^{P,2}$ ③

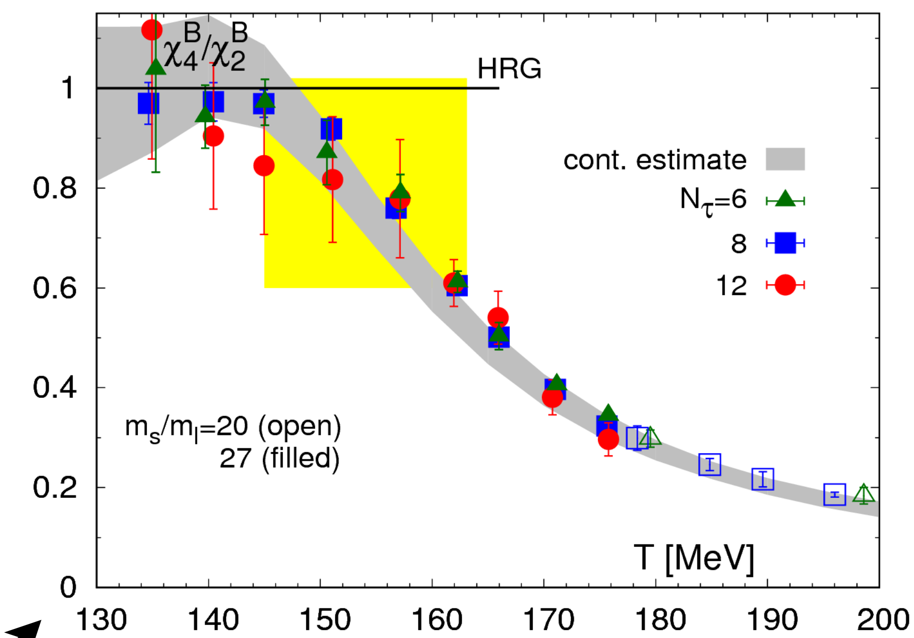
Cumulant ratios: insights from the Taylor expansion

expanding ratios of baryon number fluctuations at $\mu_Q = \mu_S = 0$:

$$R_{12}^B = \frac{M_B}{\sigma_B^2} = \hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$$

$$R_{32}^B = S_B \sigma_B = \frac{\chi_4^B}{\chi_2^B} \hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$$

$$R_{42}^B = S_B \sigma_B = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\hat{\mu}_B^2)$$



we obtain the (exact) relations:

1. $R_{32} < R_{12} \Leftrightarrow \frac{\chi_4^B}{\chi_2^B} < 1$
2. $r_{31}^{B,0} = r_{42}^{B,0}$
3. $3r_{31}^{B,2} = r_{42}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$

$\Rightarrow R_{31}^B$ and R_{42}^B are closely related

Cumulant ratios: insights from the Taylor expansion

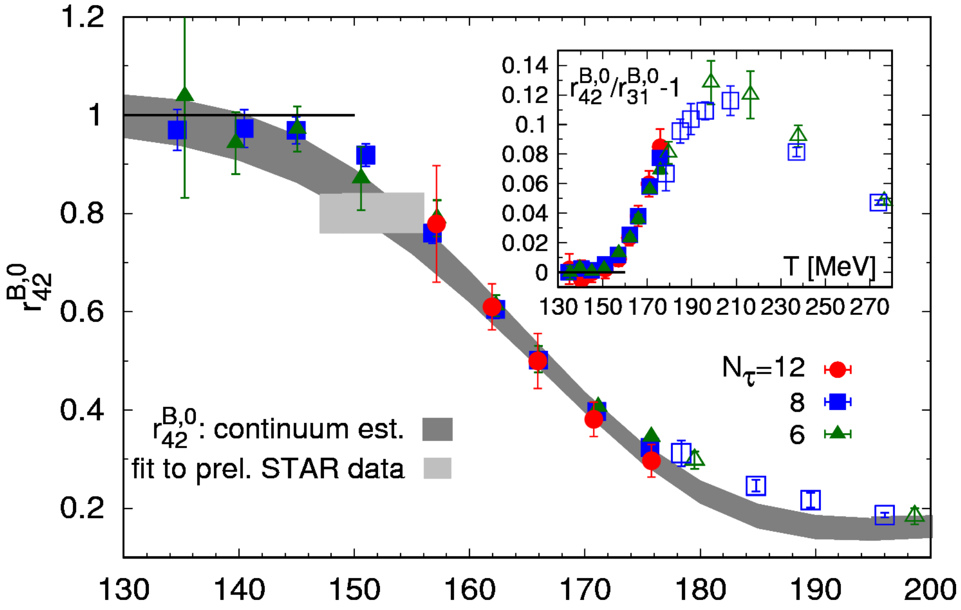
expanding ratios of baryon number fluctuations at $\boxed{n_S = 0,\ n_Q/n_B = 0.4}$:

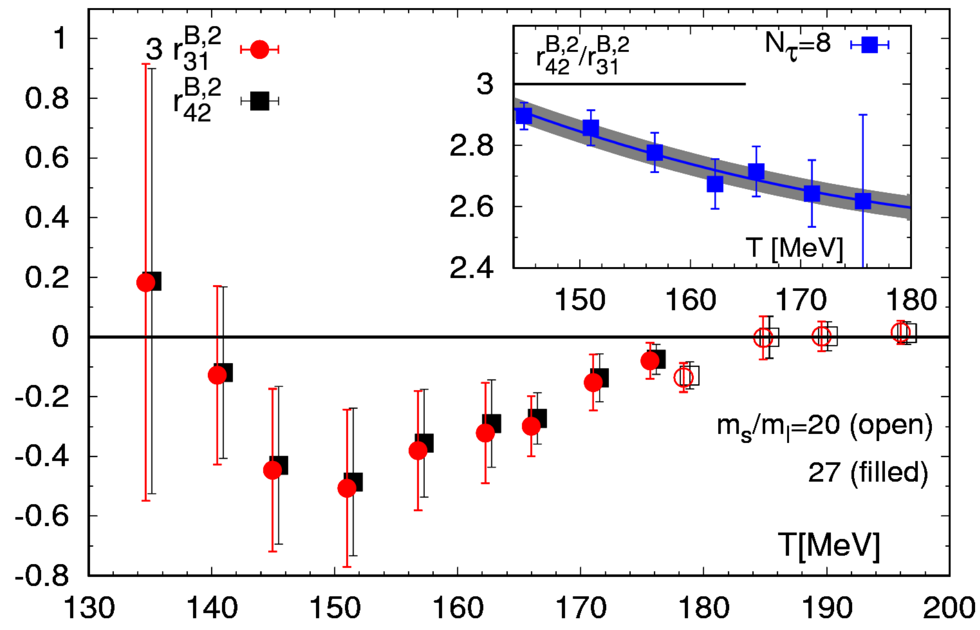
$$R_{31}^B(T,\mu_B) = r_{31}^{B,0} + r_{31}^{B,2}\left(R_{12}^B\right)^2 + \cdots$$
$$R_{42}^B(T,\mu_B) = r_{42}^{B,0} + r_{42}^{B,2}\left(R_{12}^B\right)^2 + \cdots$$

we obtain numerically:

② $r_{31}^{B,0} \simeq r_{42}^{B,0}$

③ $3r_{31}^{B,2} \simeq r_{42}^{B,2}$





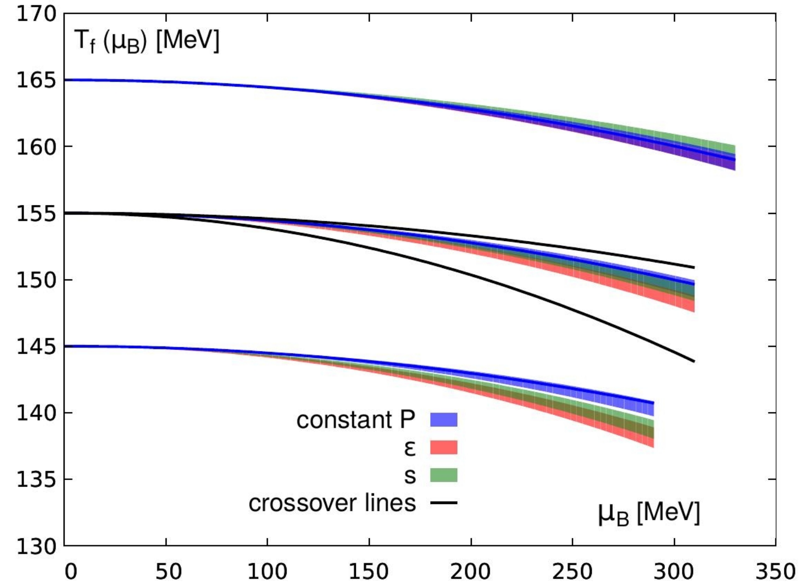
[Bazavov et al. (hotQCD), in preparation]

Cumulant ratios: insights from the Taylor expansion

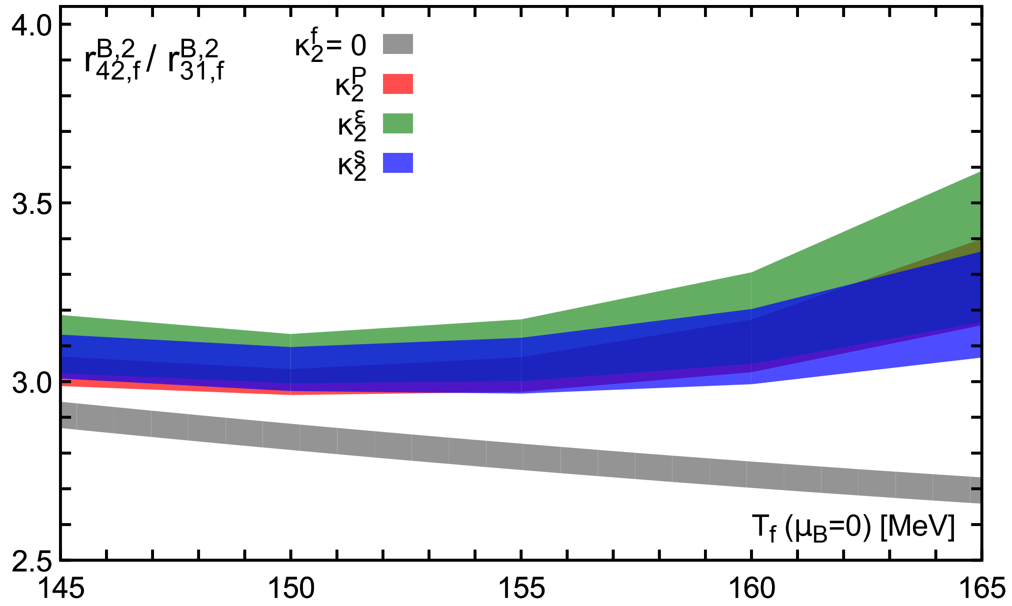
expanding ratios of baryon number fluctuations along lines of constant physics:

$$r_{31}^{B,2} \rightarrow r_{31,f}^{B,2} \equiv r_{31}^{B,2} - \kappa_2^f T_0 \left.\frac{\mathrm{d}r_{31}^{B,0}}{\mathrm{d}T}\right|_{T=T_0}$$

$$r_{42}^{B,2} \rightarrow r_{42,f}^{B,2} \equiv r_{42}^{B,2} - \kappa_2^f T_0 \left.\frac{\mathrm{d}r_{42}^{B,0}}{\mathrm{d}T}\right|_{T=T_0}$$



[Bazavov et al., PRD 95 (2017) 054504]

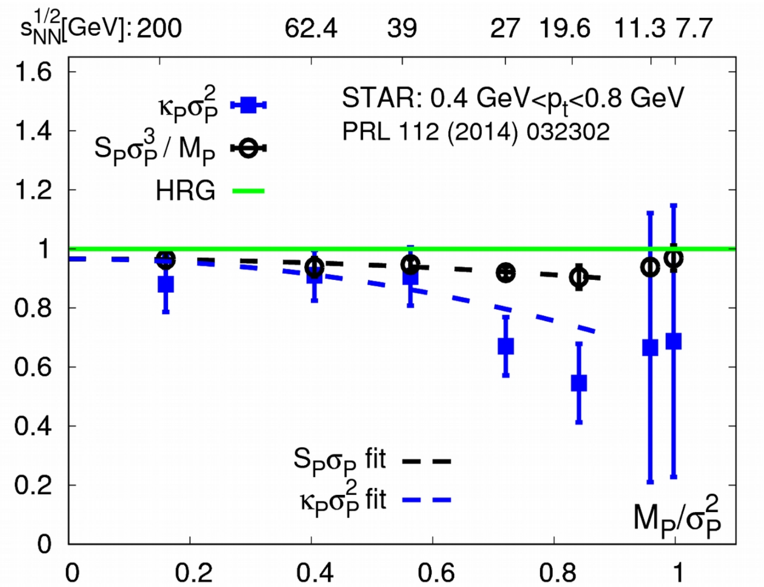


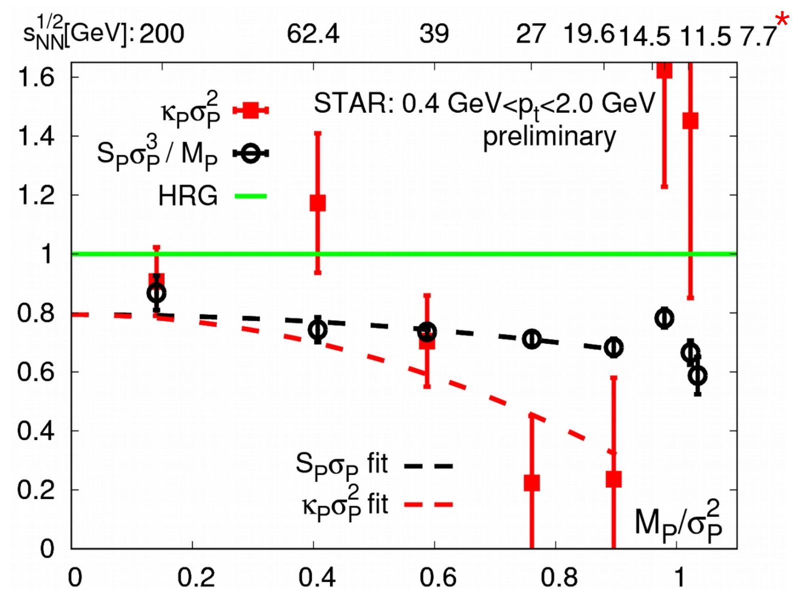
[Bazavov et al. (hotQCD), in preparation]

⇒ we find:

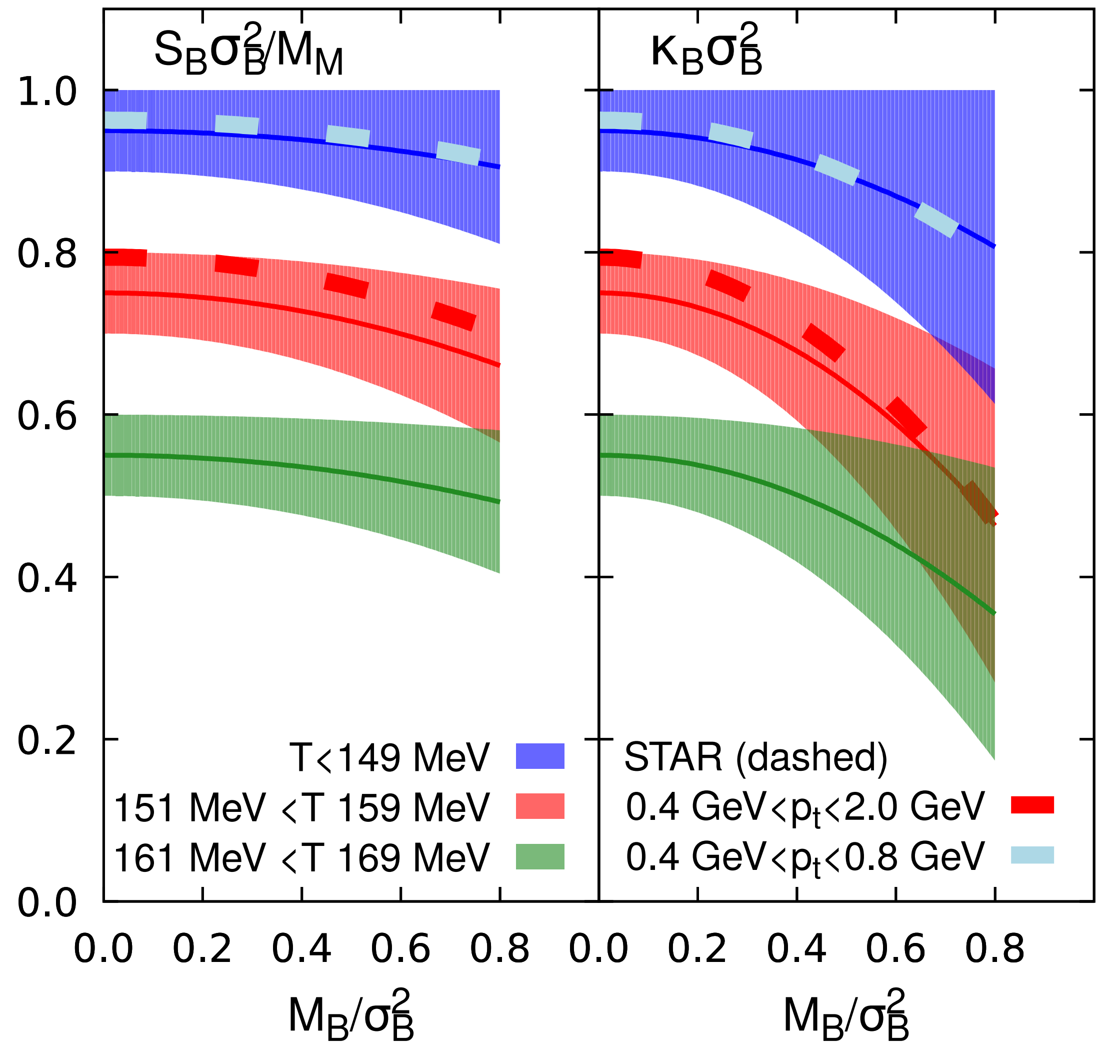
③ $r_{42,f}^{B,2}/r_{31,f}^{B,2} = 3 - 3.6$

Cumulant ratios: STAR vs. Lattice





(not yet continuum extrapolated!)



[Bazavov et al. (hotQCD), in preparation]

Disclaimer

need to understand further effects:

- non-equilibrium effects
  [Berdnikov, Rajagopal, hep-ph/9912274]
  [Mukherjee et al., arXiv:1506.00645, arXiv:1605.09341]

- proton vs. baryon number distributions
  [Kitazawa et al. arXiv:1205.3292, arXiv:1303.3338]

- acceptance and pt-cuts
  [Bzdak, Koch, arXiv:1206.4286]
  [Garg et al., arXiv:1304.7133]
  [Karsch, Morita, Redlich, arXiv:1508.02614]
  [Lin, Stephanov, arXiv:1512.09125]

- rapidity dependence
  [Bzdak, Koch, arXiv:1707.02640]

Summary

- The equation of state is very well determined for $\mu_B/T \leq 2$ or equivalently for $\sqrt{s_{NN}} \geq 19.6 GeV$
- A critical point at $\mu_B/T \leq 2$ is strongly disfavored in the temperature range $135\ \mathrm{MeV} < T < 155\ \mathrm{MeV}$ and its location at higher values of the temperature seems to be ruled out.
- A expansion in R^B_{12} is well suited for a comparison of RHIC-BES data and lattice QCD data
- The skewness and kurtosis ratios are strongly related R^B_{31}, R^B_{42}, lattice calculations reproduce quite well qualitative features observed in these ratios, and also in R^B_{32}. (Results are not yet continuum extrapolated.)
- Need to further increase the accuracy of the 6th and 8th-order expansion coefficients, in order to further constrain the EoS, the location of the critical point and the cumulant rations.