Rapidity dependence of proton cumulants

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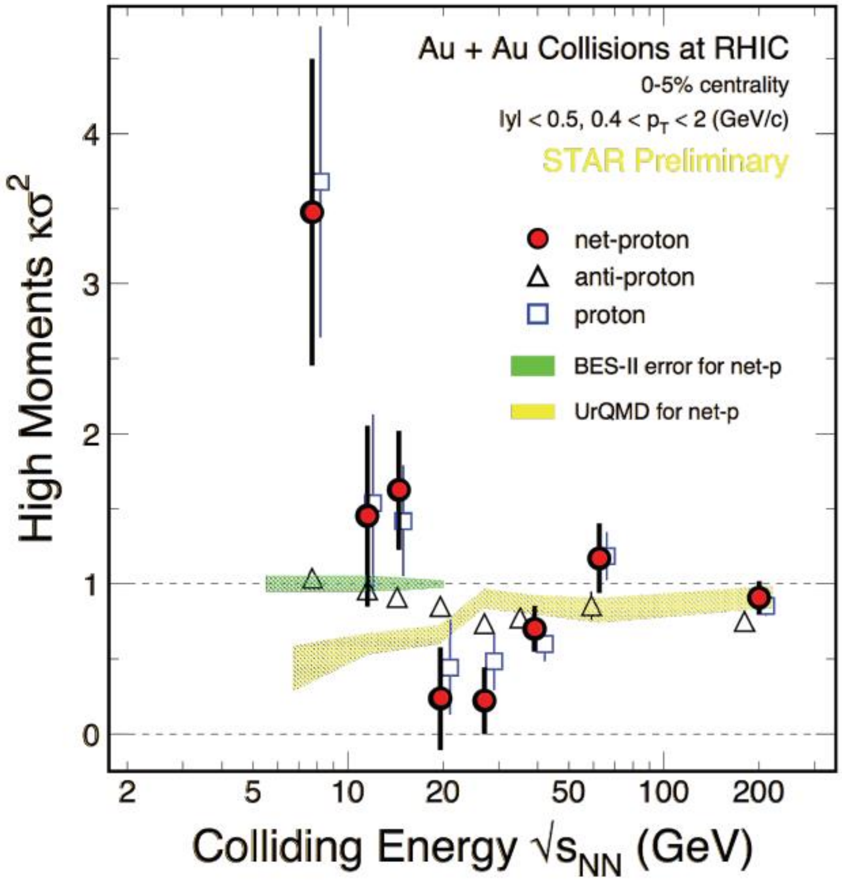
STAR results
comments
possible clusters
rapidity dependence
conclusions

AB, V. Koch, N. Strodthoff , 1607.07375
AB, V. Koch, V. Skokov, 1612.05128
AB, V. Koch, 1707.02640

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

K_4/K_2

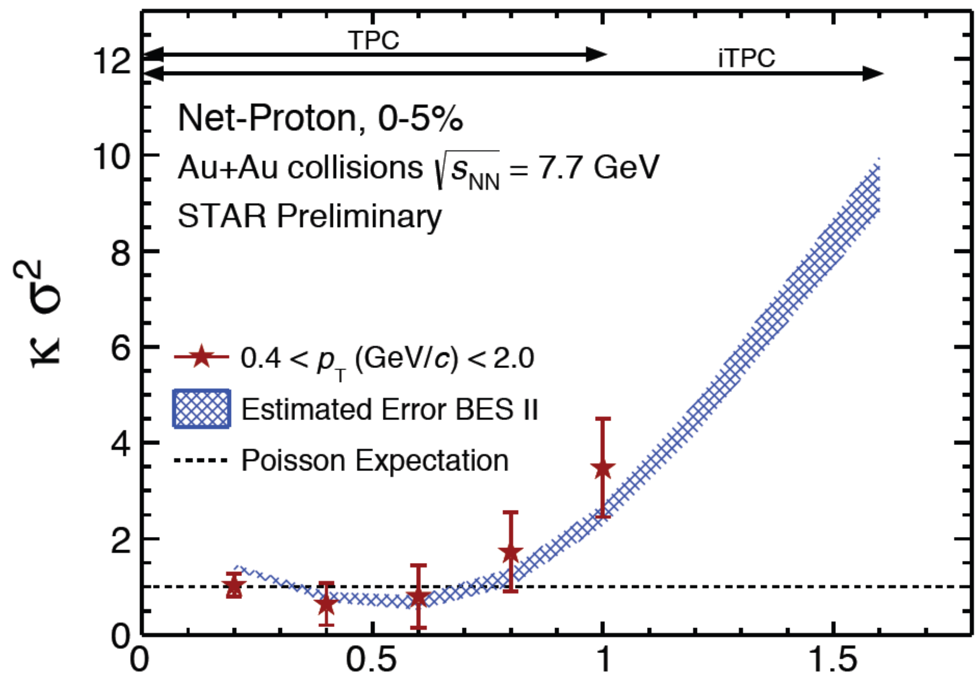
Is proton signal at 7.7 GeV large?
Is $K_4/K_2 \approx 1$ for anti-protons at 7.7 GeV boring?
Can we directly compare different energies?

STAR Preliminary at 7.7 GeV

my notation

K_4/K_2

X.Luo, N.Xu, 1701.02105



Δy (rapidity window size)

$-(\Delta y)/2 < y < (\Delta y)/2$

Is this dependence expected?

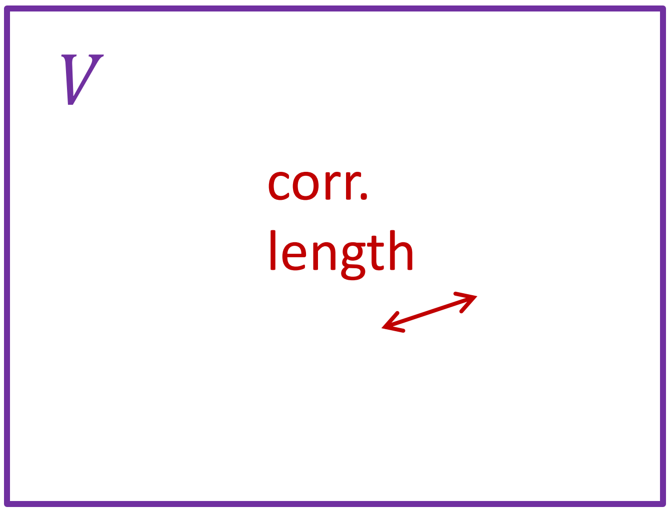
Is it somehow related to the QCD phase diagram?

General remarks:

"Cumulant ratios do not depend on volume" but depend on volume fluctuation

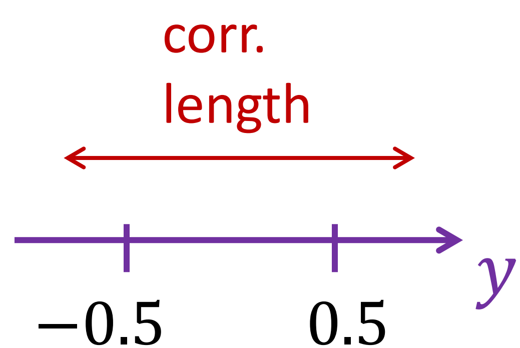
It is true if a correlation length is much smaller than the system size

real coordinate space



Here this condition is satisfied

momentum rapidity space



Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on rapidity "volume"

Cumulants are not optimal

$$K_2 = \langle (\delta N)^2 \rangle \qquad \delta N = N - \langle N \rangle$$

N – number of protons

we neglect anti-protons, good at low energies

$$K_3 = \langle (\delta N)^3 \rangle$$

$$K_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

$$K_n = \langle N \rangle + physics[2, \ldots, n]$$

physics = two-, three-, *n*-particle correlation functions

for Poisson distribution $K_n = \langle N \rangle$, $(physics = 0)$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix
correlation functions
of different orders

For example:

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

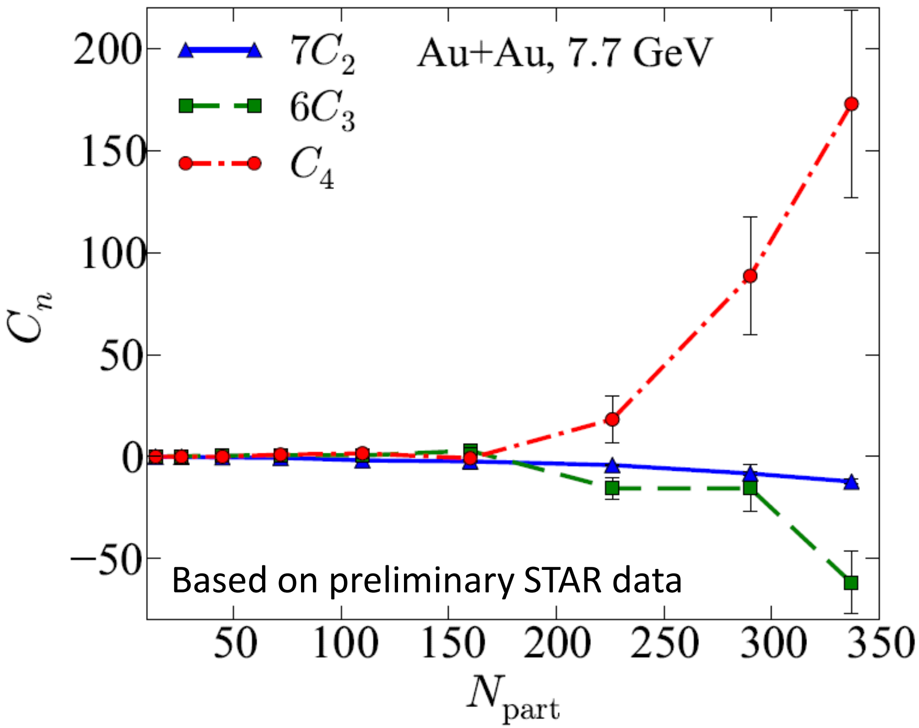
integrated
correlation function

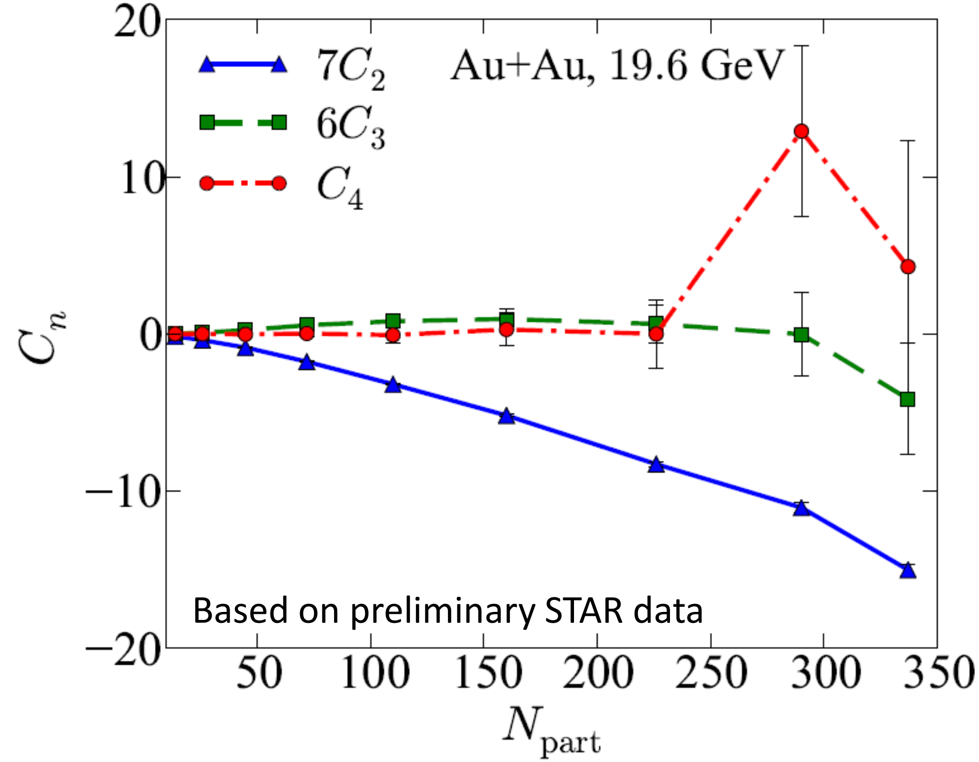
See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) no.3, 034915
AB, V. Koch, N. Strodthoff , PRC 95 (2017) no.5, 054906

Using preliminary STAR data we obtain C_n

central signal at 7.7 GeV is driven by large 4-particle correlations

central signal at 19.6 GeV is driven by 2-particle correlations





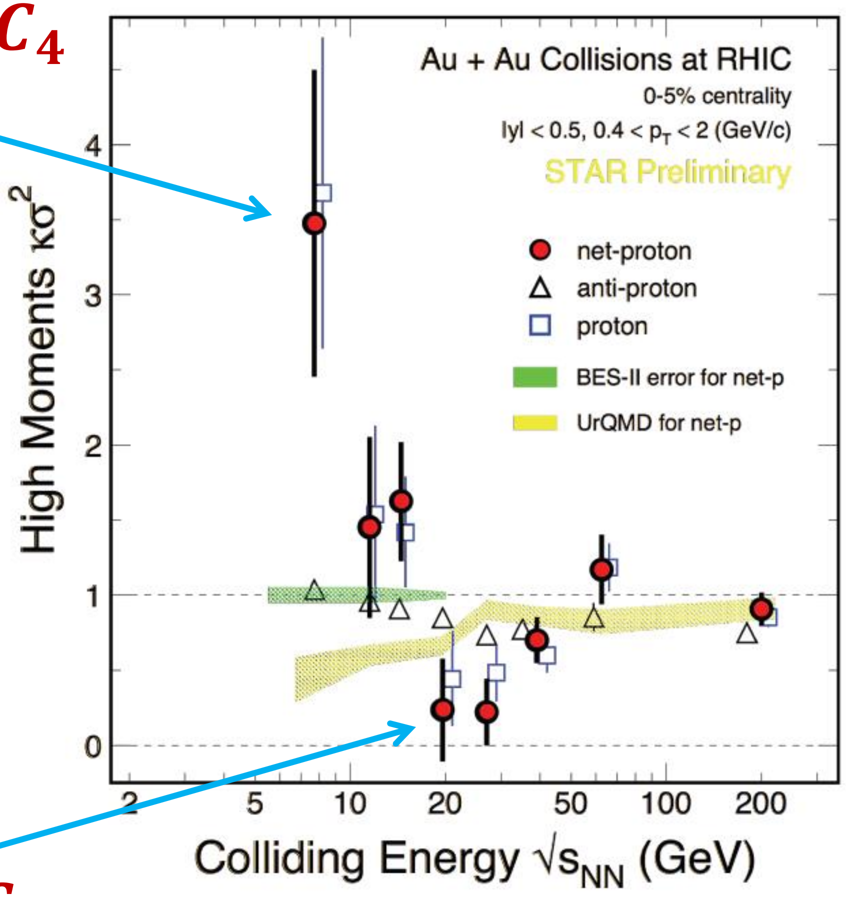
$C_4(7.7) \sim \mathbf{170}$

C_4 and $6C_3$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) no.5, 054906

STAR Preliminary
X.Luo, N.Xu, 1701.02105

here we see C_4



and here C_2

e.g., baryon conservation

AB, V. Koch, V. Skokov, 1612.05128

Let's put the STAR numbers in perspective.

Suppose that we have clusters (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 4!/(4-k)!$$

↑ mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 5!/(5-k)!$$

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 120$$

and $\langle N_{\mathrm{cl}} \rangle \sim 1$

To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\mathrm{cl}} \rangle \sim 7$, it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + \boldsymbol{c_2}(y_1, y_2)]$$

reduced correlation function

e.g., does not depend on binomial efficiency

integrated reduced correlation function "coupling"

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2} = \frac{\boldsymbol{C_2}}{\langle N\rangle^2}$$

and the second order cumulant

$$K_2 = \langle N\rangle + \underbrace{\langle N\rangle^2 \boldsymbol{c_2}}_{\boldsymbol{C_2}}$$

Finally we obtain

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

For $c_n(y_1, \dots, y_n) = const$, K_n strongly depends on rapidity window size since $\langle N \rangle \sim \Delta y$

btw, K_n is strongly efficiency dependent through $\langle N \rangle$

At 7.7 GeV, $K_4/K_2 \sim \langle N \rangle^3 \sim (\Delta y)^3$

See Appendix of [AB, V. Koch, N. Strodthoff , 1607.07375] for net-proton K_n

When, e.g., $K_4/K_2 \approx 1$?

AB, V. Koch, 1707.02640

$$K_4 \approx \langle N \rangle \qquad K_2 \approx \langle N \rangle \qquad K_4/K_2 \approx 1$$

$$7\langle N \rangle^2 \boldsymbol{c_2} \ll \langle N \rangle$$

$$6\langle N \rangle^3 \boldsymbol{c_3} \ll \langle N \rangle$$

$$\langle N \rangle^4 \boldsymbol{c_4} \ll \langle N \rangle$$

STAR at 7 GeV:

$\boldsymbol{c_2} \sim -1 \cdot 10^{-3}$

$\boldsymbol{c_3} \sim -2 \cdot 10^{-4}$

$\boldsymbol{c_4} \sim +7 \cdot 10^{-5}$

$\langle N \rangle \approx 40$ protons

two obvious options:

$\boldsymbol{c_n} \approx 0$, that is we are close to Poisson distribution

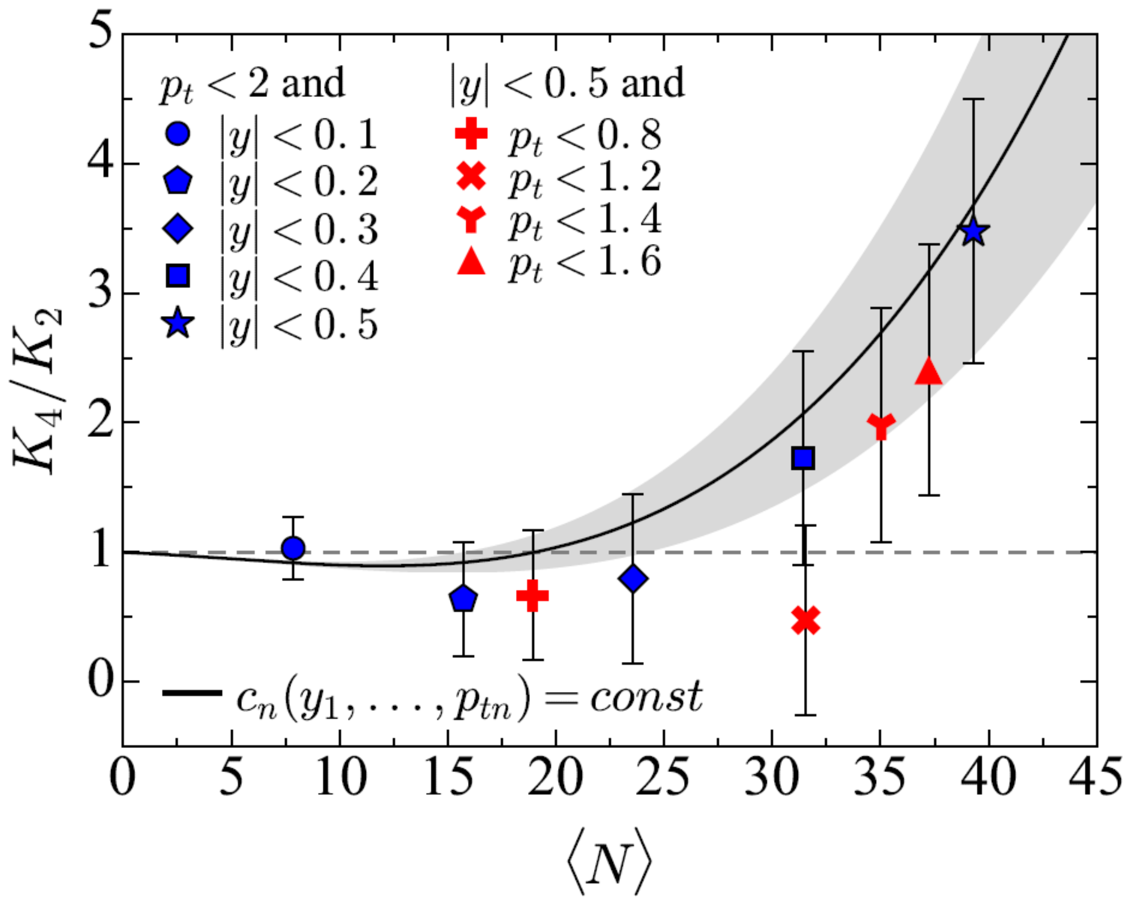
$\boldsymbol{c_n}$ is “large” but $\langle N \rangle$ is “small” $\longrightarrow$ anti-protons at 7.7 GeV ?

So for small $\langle N \rangle$ (rare particles, efficiency, acceptance) $K_4/K_2 \approx 1$

Let us start with

$$c_2 = \frac{\int \rho(y_1)\rho(y_2) c_2(y_1, y_2) dy_1 dy_2}{\int \rho(y_1)\rho(y_2) dy_1 dy_2}$$

$$c_n(y_1, p_{t1}, \dots, y_n, p_{tn}) = c_n^0 = const \quad \rightarrow \quad c_n = c_n^0$$

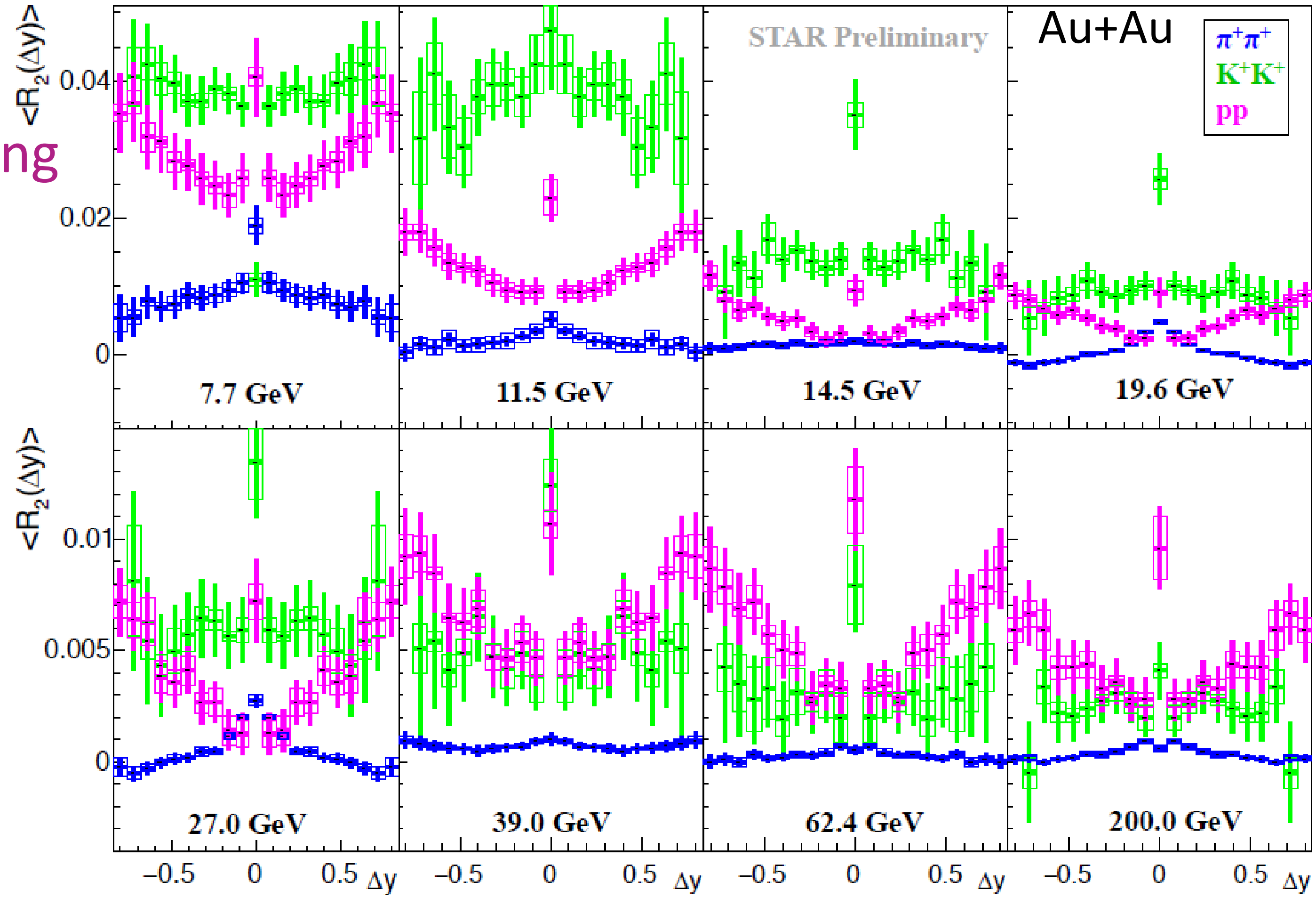


AB, V. Koch, 1707.02640

We do not understand basic baryon physics

S. Jowzaee, QM2017

stopping



production

$$R_2(y_1,y_2) = -1 + \frac{\langle \rho_2(y_1,y_2) \rangle}{\langle \rho_1(y_1) \rangle \langle \rho_1(y_2) \rangle}$$

← Same event pair distributions

← Mixed event

AB, V. Koch, 1707.02640

Repulsive vs attractive rapidity correlations

$$c_2(y_1, y_2) = c_2^0 + \underline{\gamma_2} \, (y_1 - y_2)^2$$

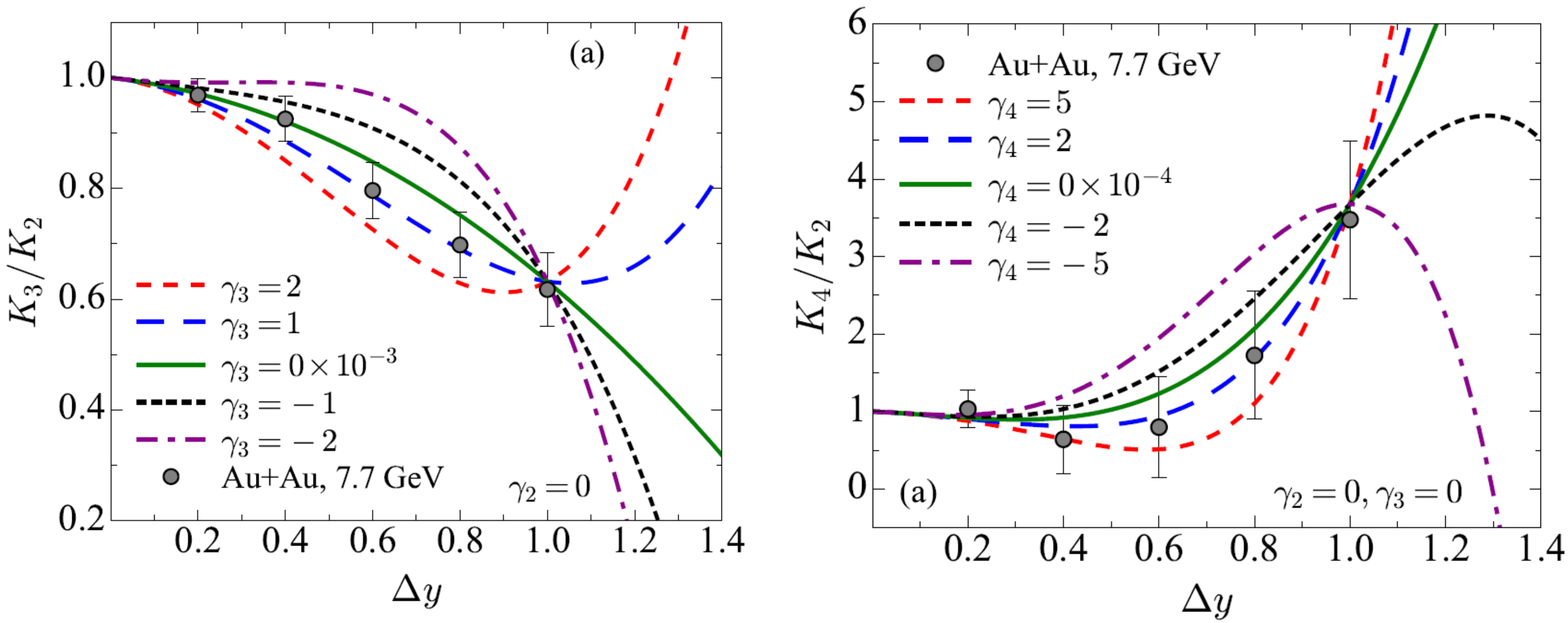
$$c_3(y_1, y_2, y_3) = c_3^0 + \underline{\gamma_3} \, \tfrac{1}{3} \left[(y_1 - y_2)^2 + (y_1 - y_3)^2 + (y_2 - y_3)^2 \right]$$

$$c_4(y_1, y_2, y_3, y_4) = c_4^0 + \underline{\gamma_4} \, \tfrac{1}{6} \left[(y_1 - y_2)^2 + (y_1 - y_3)^2 + (y_1 - y_4)^2 + (y_2 - y_3)^2 + (y_2 - y_4)^2 + (y_3 - y_4)^2 \right]$$

$\gamma_n > 0$ - rapidity “repulsion”

$\gamma_n < 0$ - rapidity “attraction”

It seems that rapidity repulsion ($\gamma_{3,4} > 0$) is favored

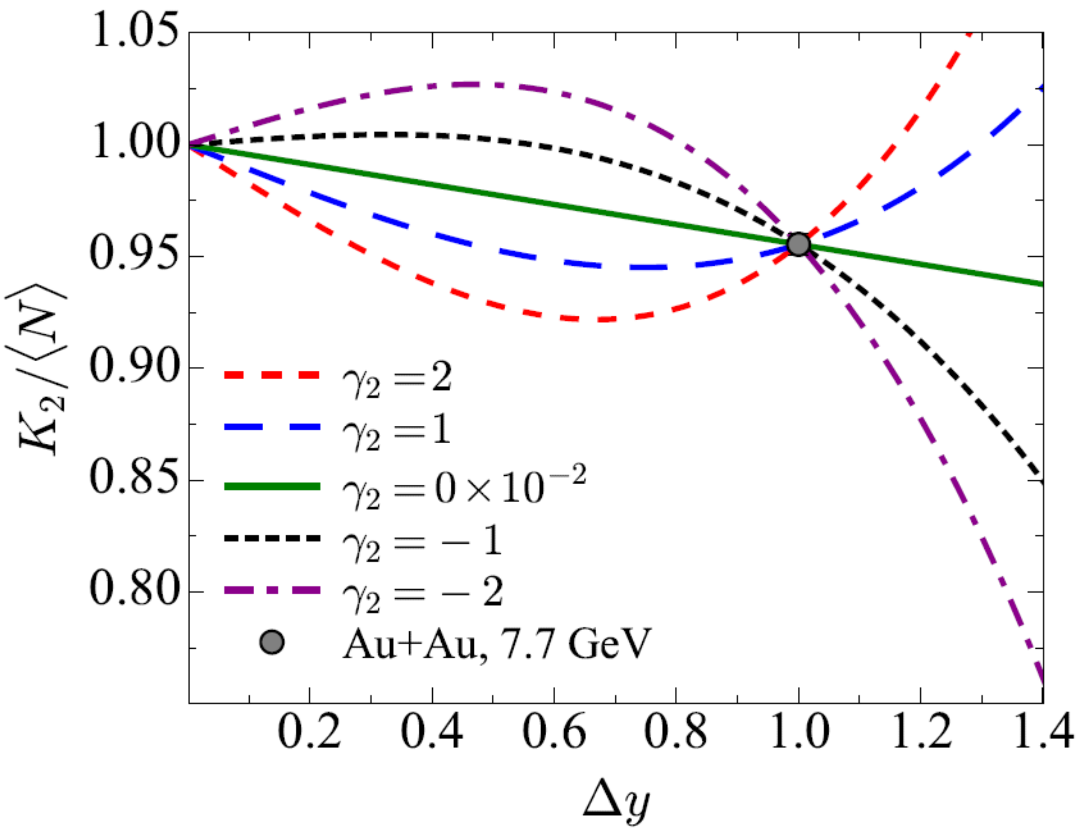


K_3/K_2 above $\Delta y > 1$ could even start growing

$\gamma_{3,4} < 0$ (attraction) seems to be excluded

Presence of proton clusters would naively result in $\gamma_{2,3,4} < 0$ ...

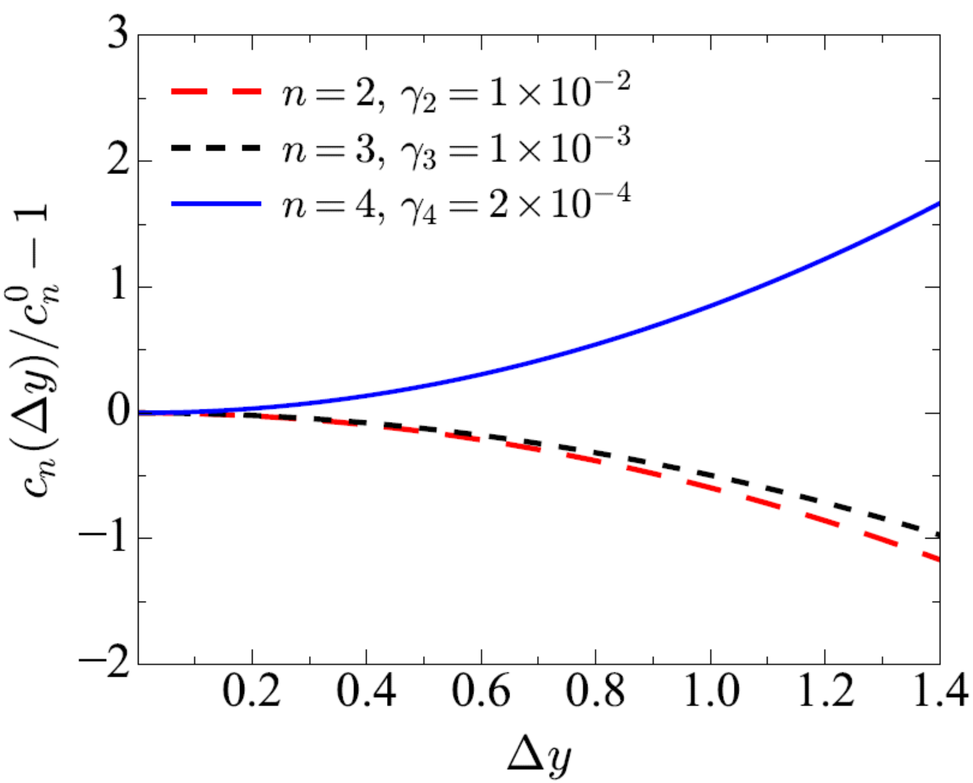
γ_2 is well visible in $K_2/\langle N \rangle$



We should study the integrated reduced correlation function

$$c_n(\Delta y) = \frac{C_n}{\langle N \rangle^n} = c_n^0 + \gamma_n \frac{1}{6}(\Delta y)^2$$

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$



Conclusions

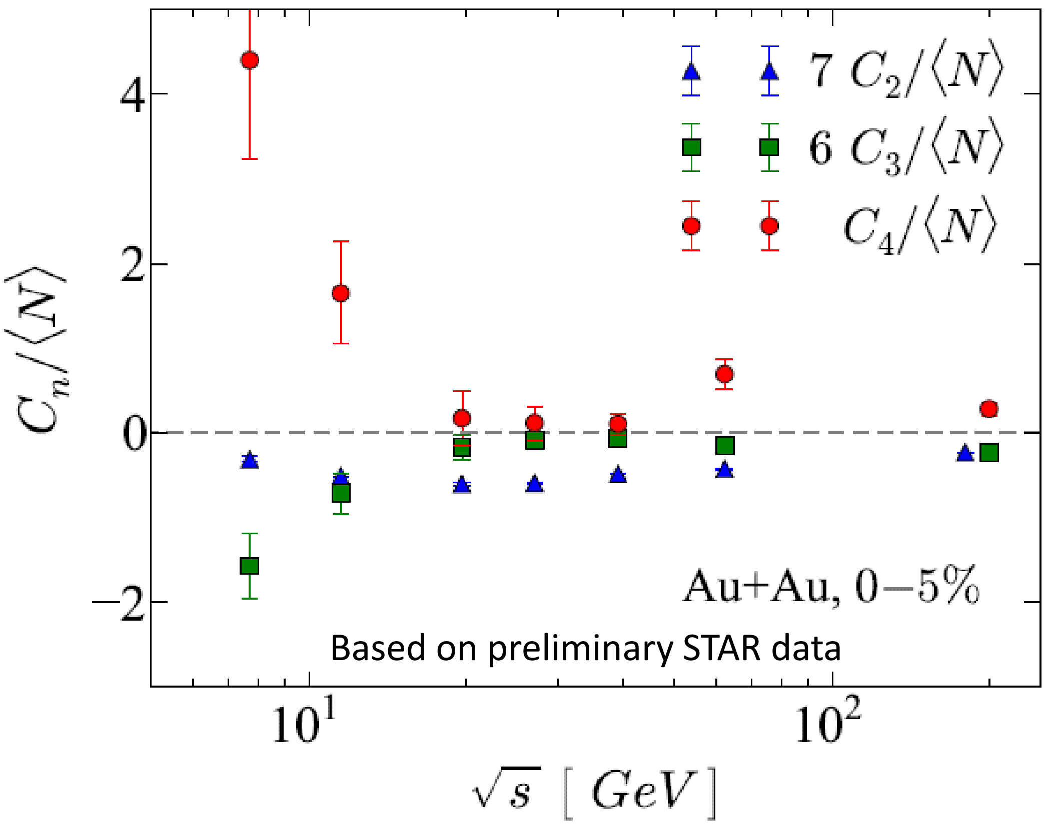
Four-proton correlation function at 7.7 GeV is surprisingly large.
We need a strong source of multi-proton correlation.
Proton clusters?

The STAR data at 7.7 GeV is consistent with constant correlation functions. A small multi-proton rapidity repulsion is slightly favored.

The cumulants are not the best choice.
The reduced correlation functions are much cleaner.

Backup

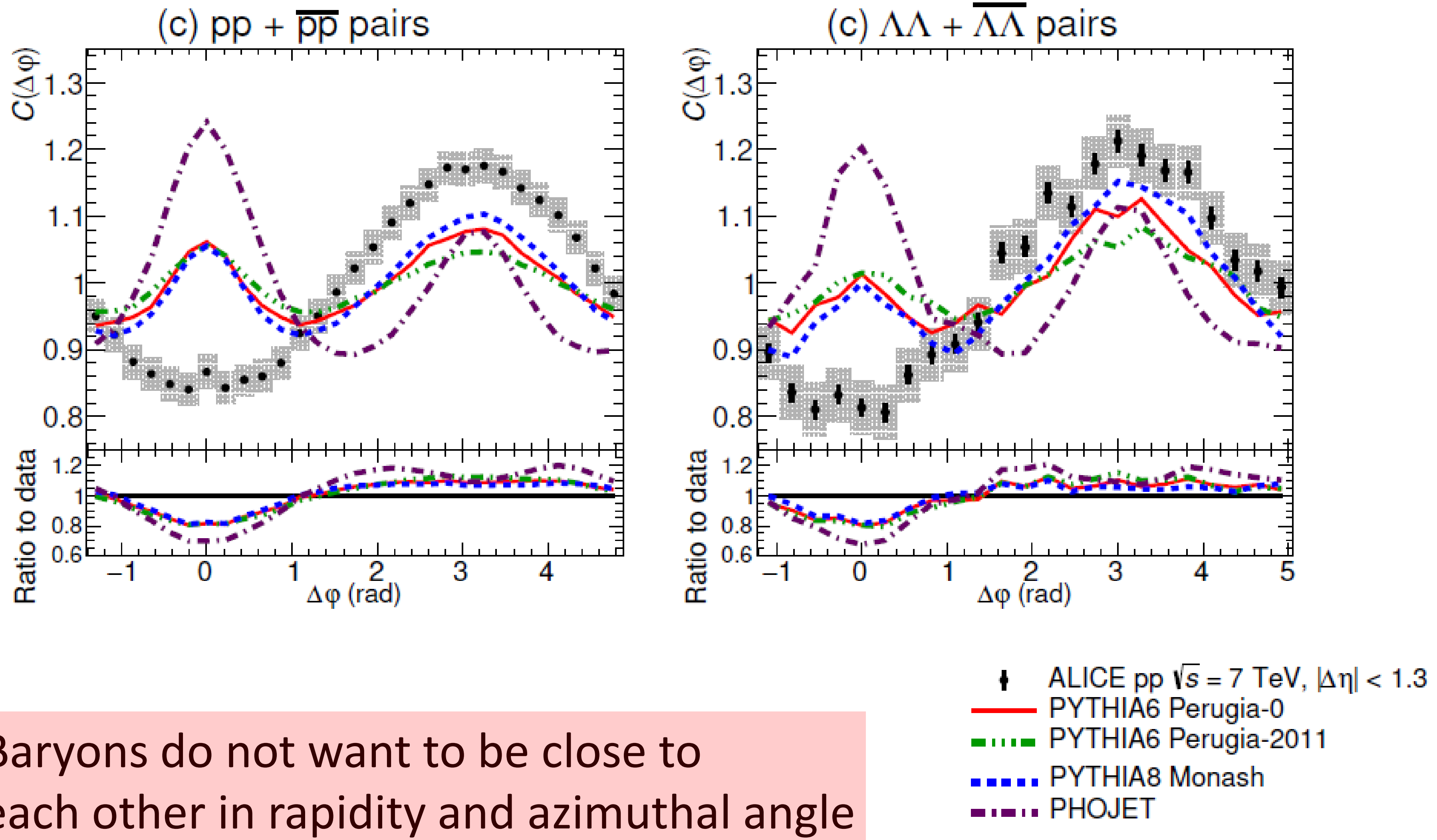
AB, V. Koch, N. Strodthoff , 1607.07375



C_4 at 62 GeV !

ALICE

ALICE, 1612.08975



Baryons do not want to be close to each other in rapidity and azimuthal angle

First seen by TPC/Two Gamma Collaboration in e+e- annihilation at 29-GeV, PRL 57, 3140 (1986).

Full acceptance

$N_{(b)}$

$N_{(a)}$

$$N_{(a)} + N_{(b)} = B = const.$$

baryon conservation

$$K_{2,(a)} = K_{2,(b)} \qquad K_{3,(a)} = -K_{3,(b)}$$

$$K_{4,(a)} = K_{4,(b)} \qquad K_{5,(a)} = -K_{5,(b)}$$

$$\frac{K_4}{K_2} \to 1, \quad \frac{K_3}{K_2} \to -1$$

for full acceptance

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

Rapidity dependence:

long-range correlation

$$c_n(y_1, \ldots, y_n) = c_n^0$$

$$c_n = c_n^0$$

short-range correlation

$$c_2(y_1, y_2) = c_2^0 \delta(y_1 - y_2)$$

$$c_2 \sim 1/(\Delta y)$$

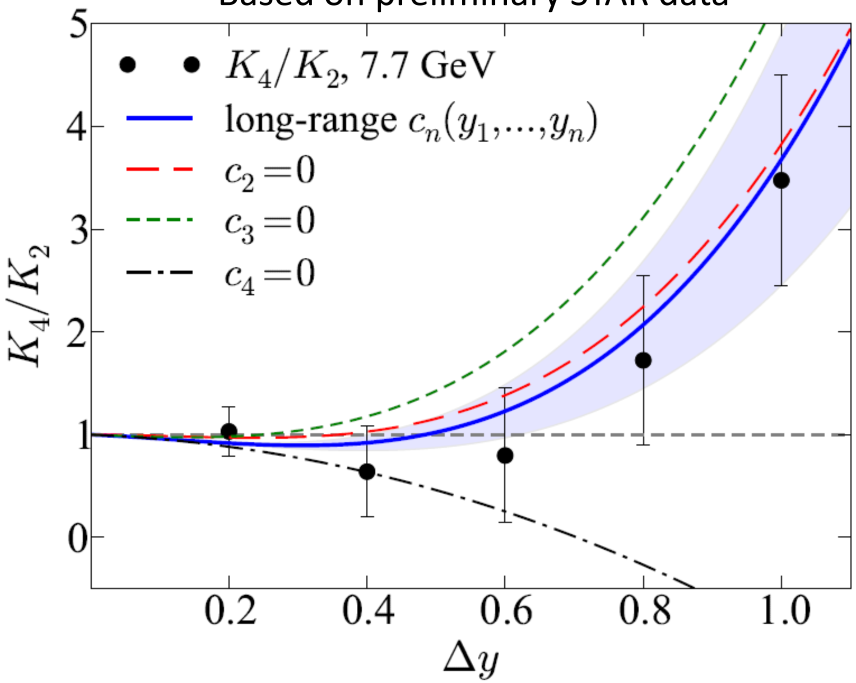
$$K_n \sim \Delta y$$

$$K_2 = \langle N \rangle + c_2^0 \langle N \rangle^2, \qquad \langle N \rangle \sim \Delta y$$

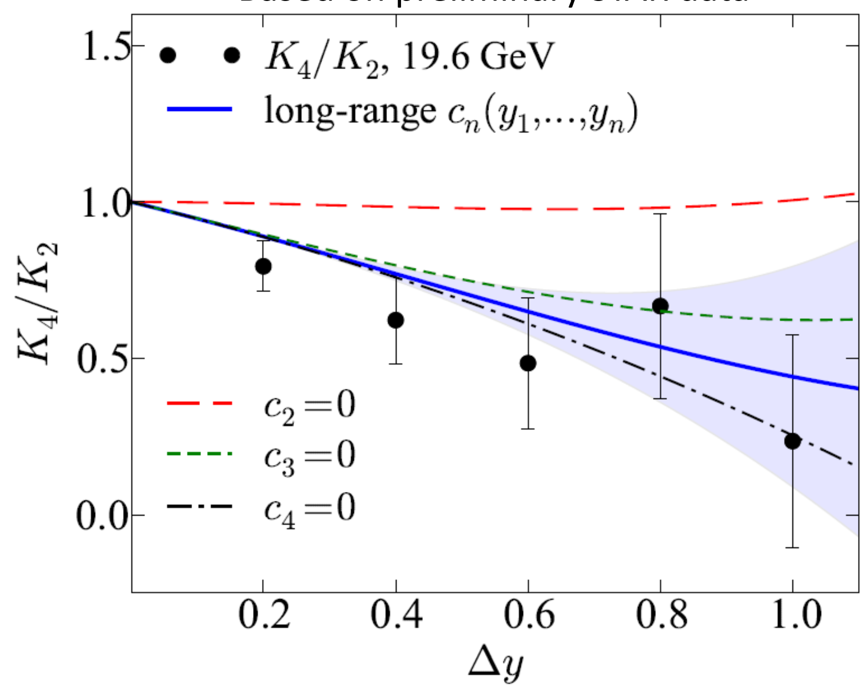
$$K_4 = \langle N \rangle + 7c_2^0 \langle N \rangle^2 + 6c_3^0 \langle N \rangle^3 + c_4^0 \langle N \rangle^4$$

Rapidity dependence consistent with long-range correlations

Based on preliminary STAR data



Based on preliminary STAR data

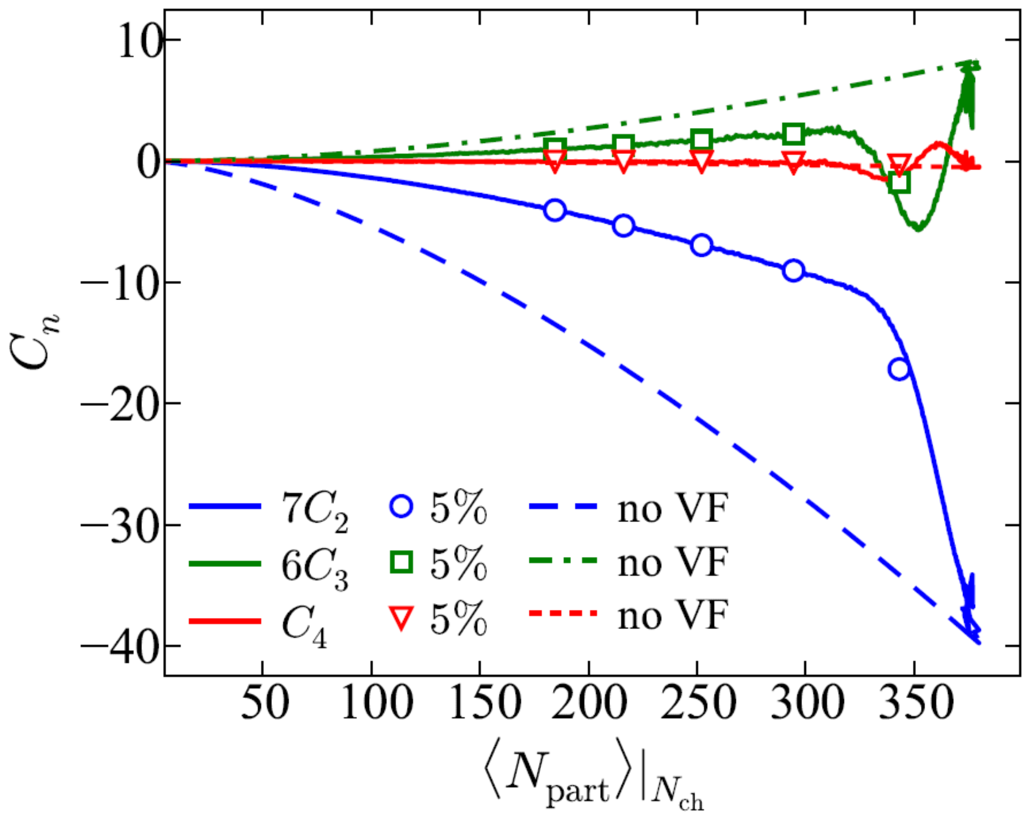


AB, V. Koch, N. Strodthoff , 1607.07375

Minimal model (MM) at low energies

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

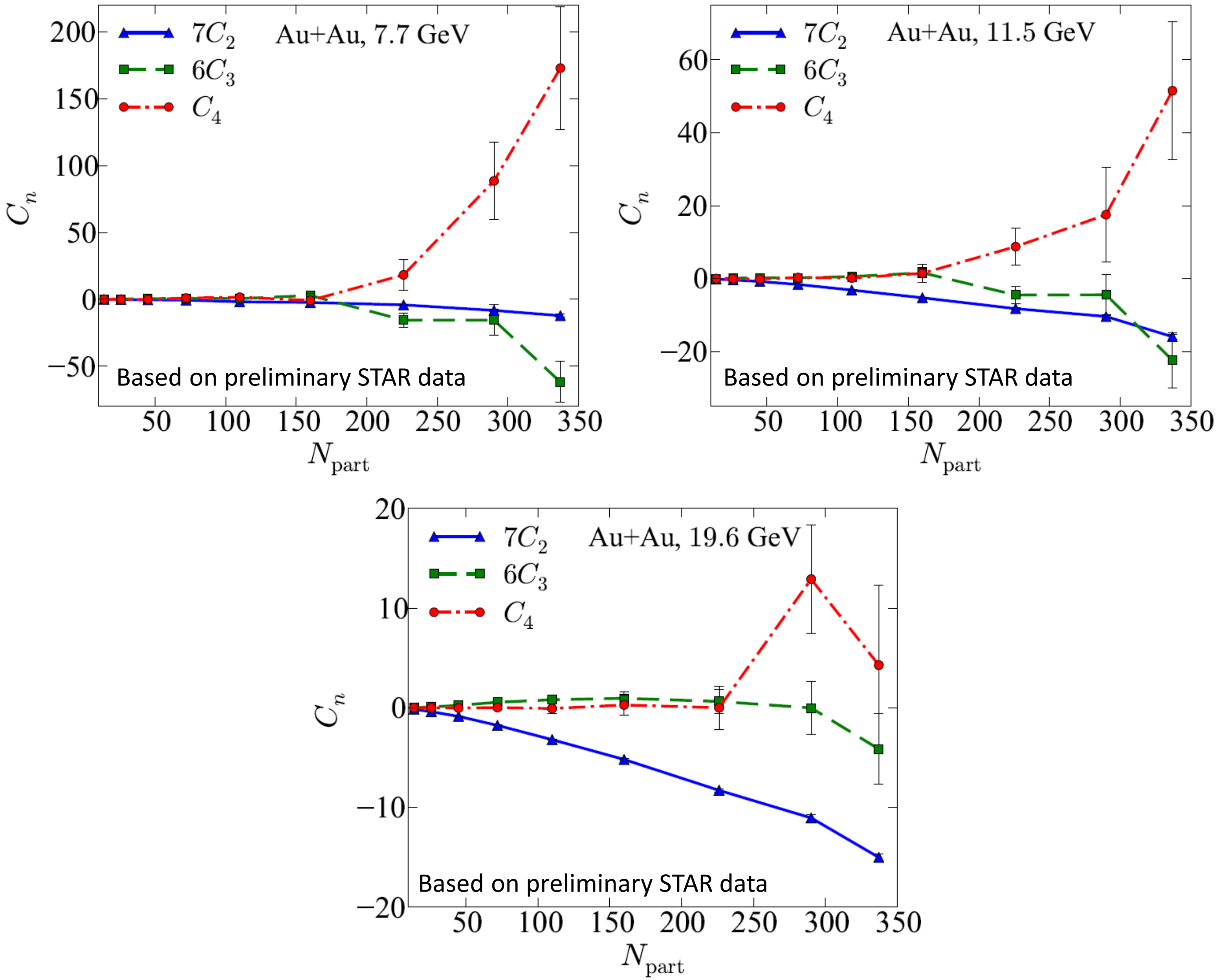
$6C_3 \sim -60$

$7C_2 \sim -15$

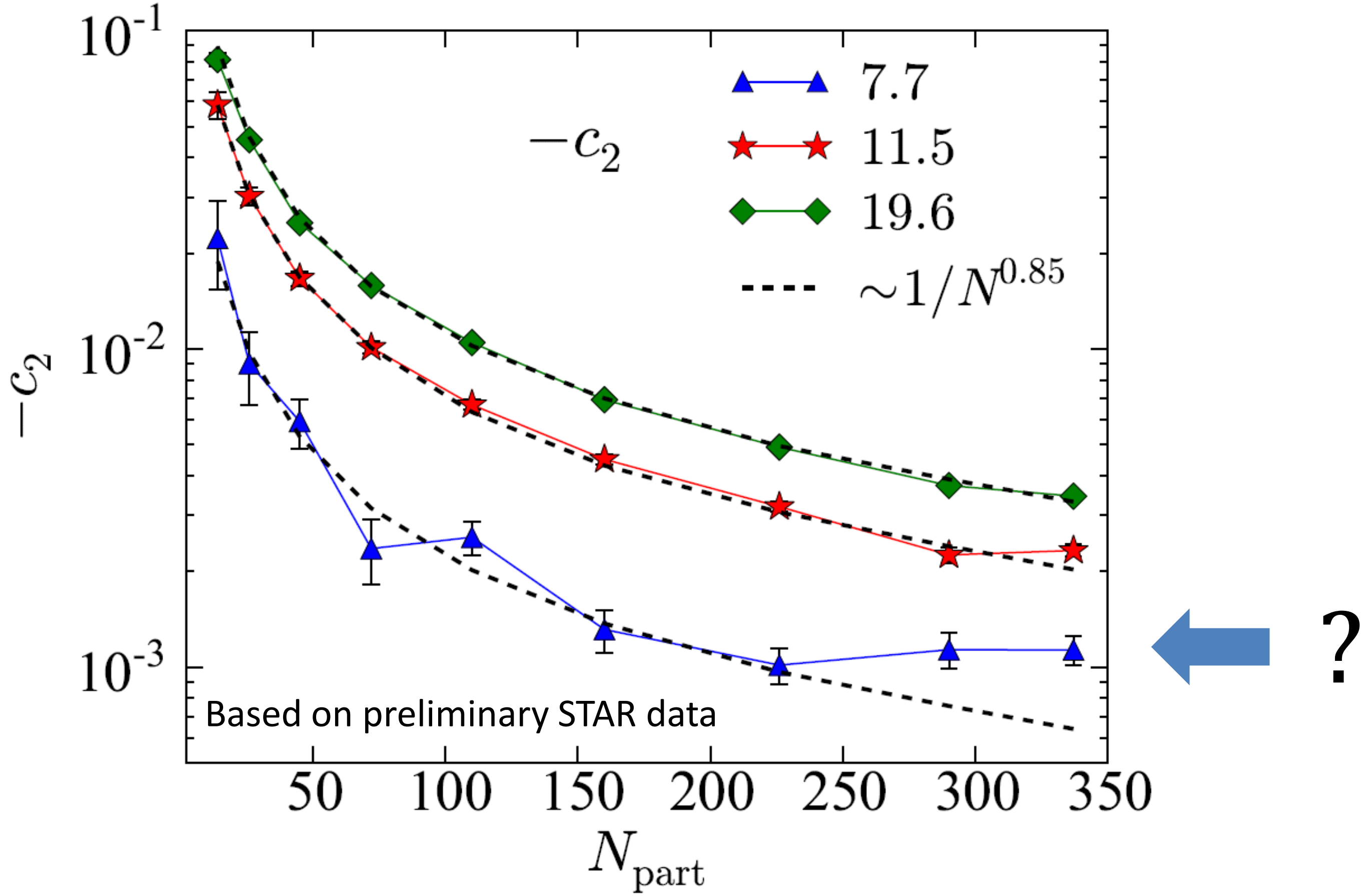
we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, 1612.05128

Comparison of 7.7, 11.5 and 19.6 GeV



results for c_2

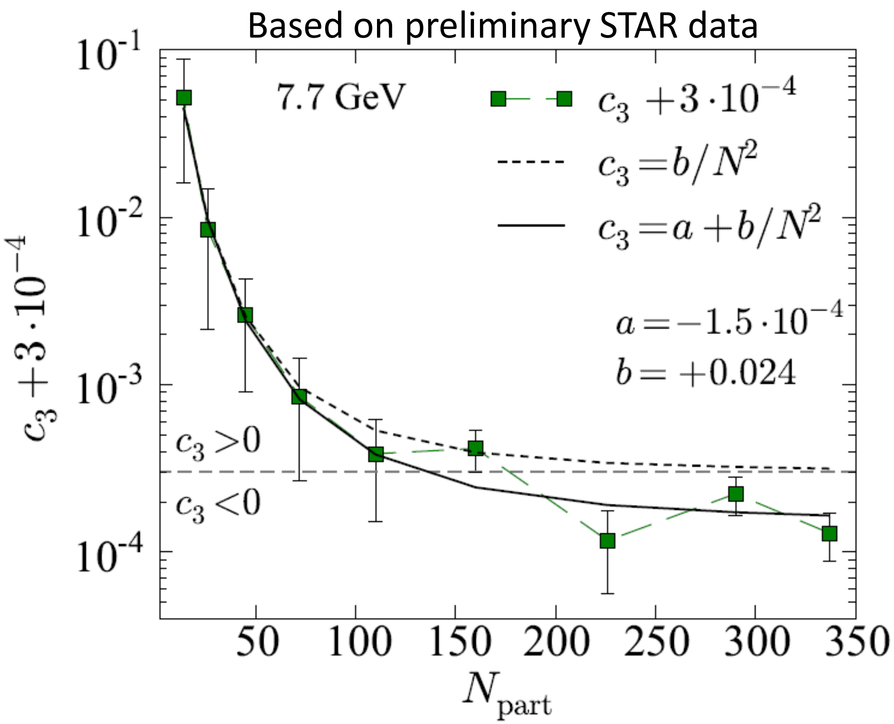


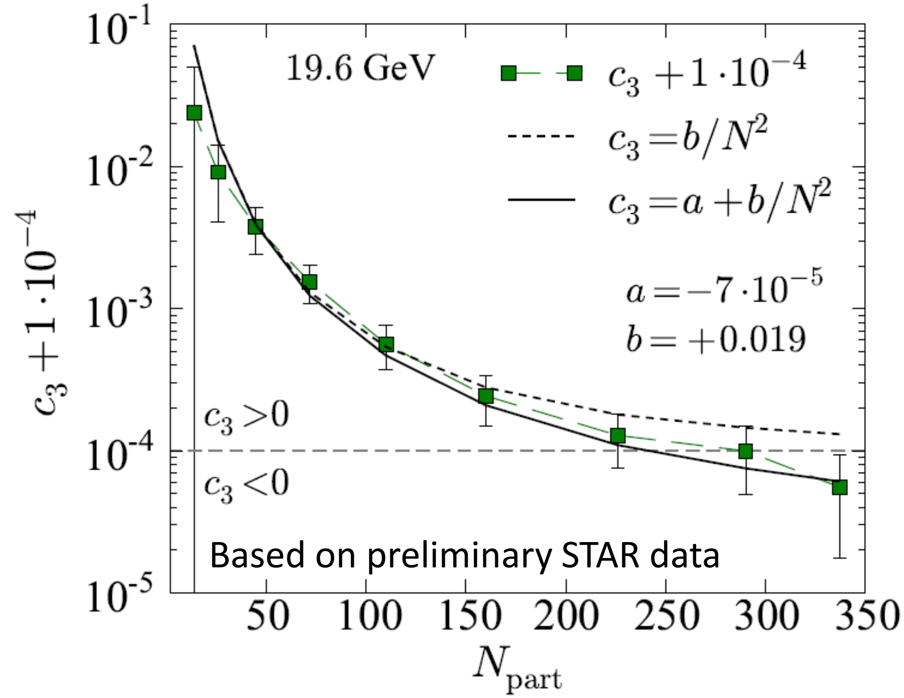
central 7 GeV points are somehow special

AB, V. Koch, N. Strodthoff , 1607.07375

Using preliminary STAR data we obtain c_3

AB, V. Koch, N. Strodthoff, 1607.07375

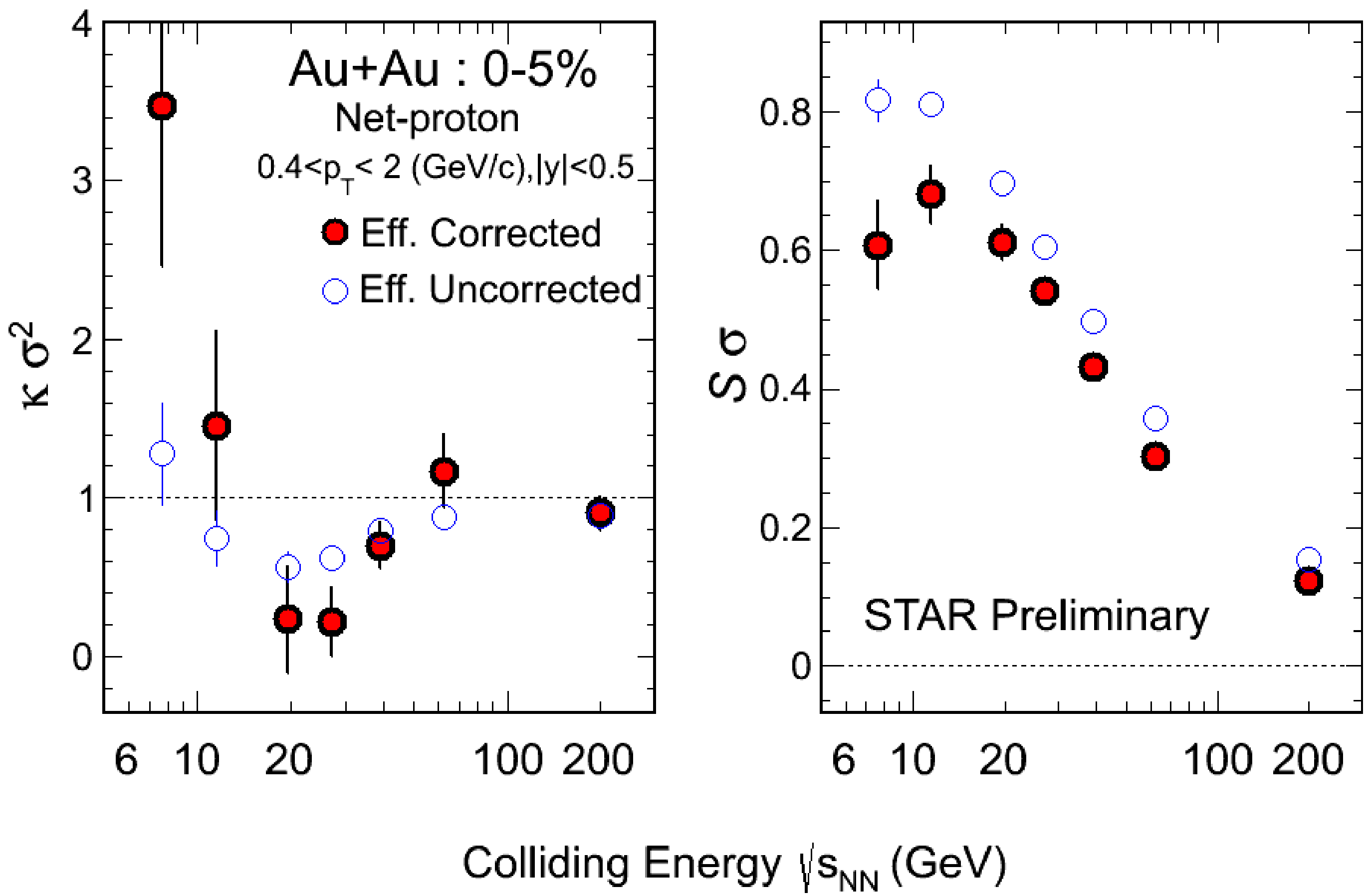




At 7.7 GeV we see $1/N^2$ for small N_{part} then c_3 **changes sign** and stays roughly constant…

Similar story for c_4

Preliminary STAR data



K_4/K_2 my notation K_3/K_2

Genuine three-proton correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + \boldsymbol{c_2}(y_1, y_2\ ) + \cdots + \boldsymbol{c_3}(y_1, y_2, y_3)]$$

$$F_3 = \langle N(N-1)(N-2)\rangle = \langle N\rangle^3 + 3\langle N\rangle^3 \boldsymbol{c_2} + \langle N\rangle^3 \boldsymbol{c_3}$$

$$\boldsymbol{c_3} = \frac{\int \rho(y_1)\rho(y_2)\rho(y_3)\boldsymbol{c_3}(y_1, y_2, y_3)dy_1 dy_2 dy_3}{\int \rho(y_1)\rho(y_2)\rho(y_3)dy_1 dy_2 dy_3}$$

and the third order cumulant

$$K_3 = \langle N\rangle + \underbrace{3\langle N\rangle^2 \boldsymbol{c_2}}_{3\boldsymbol{C_2}} + \underbrace{\langle N\rangle^3 \boldsymbol{c_3}}_{\boldsymbol{C_3}}$$