Global spin polarization in heavy ion collisions

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Outline

- **Introduction**
- **Theoretical models on particle polarization: [Spin-orbital coupling, Statistical-hydro, Kinetic]**
- **Vorticity and Λ polarization in AMPT model**
- **Correlation in Λ polarization as probe to the most vortical fluid**
- **Summary**

Global OAM and Magnetic field in HIC

- **Huge global orbital angular momenta are produced**

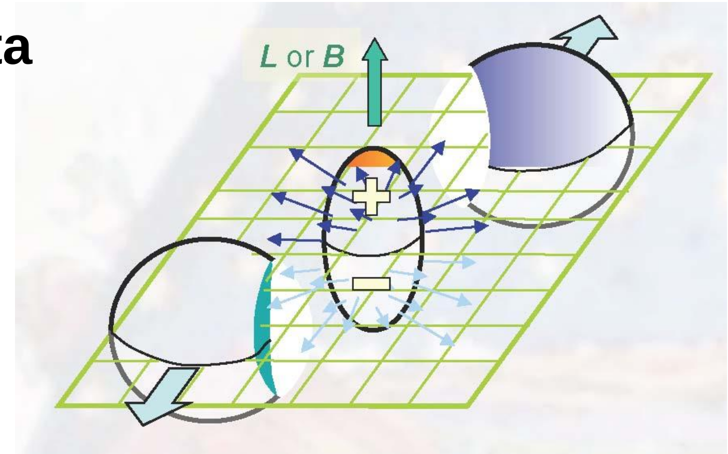
$$L \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$B \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **Can and how does orbital angular momentum be transferred to the matter created?**

- **Any way to measure angular momentum?**



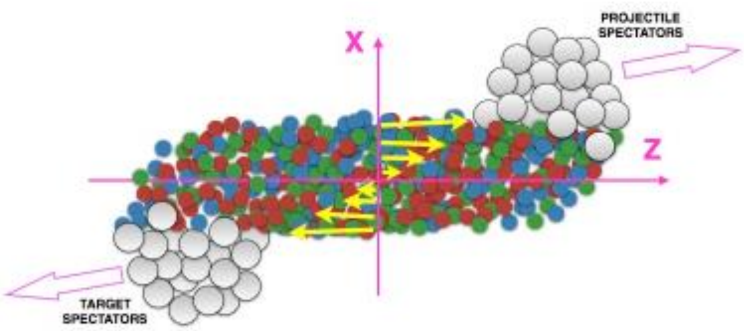
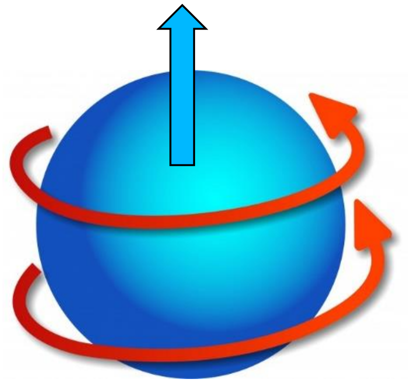


Figure taken from Becattini et al, 1610.02506

Rotation vs Polarization



- **Barnett effect: rotation to polarization**

  uncharged object in rotation

  → spontaneous magnetization

  → polarization (spin-orbital coupling)

  [Barnett, Rev.Mod.Phys.7,129(1935)]

- **Einstein-de Haas Effect: polarization to rotation**

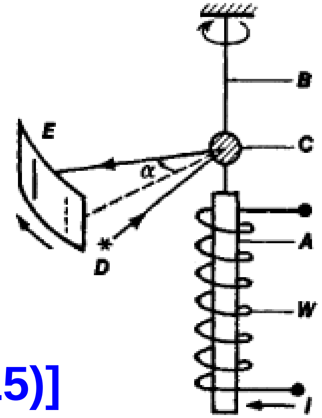
  magnetic field (impulse)

  → polarization of electrons

  → ΔL_electron

  → ΔL_mechanical = - ΔL_electron

[Einstein, de Haas, DPG Verhandlungen 17, 152(1915)]



Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

- **Polarizations of Λ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

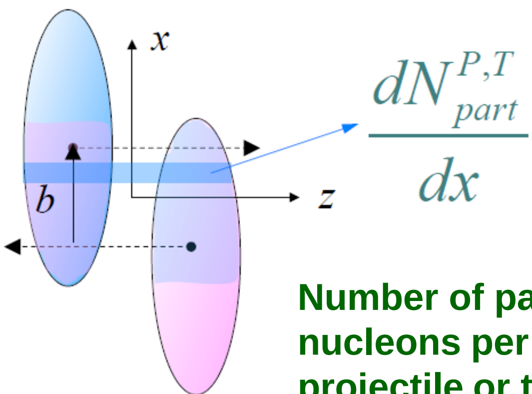
- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Angular momentum conservation in HIC**
- **-- Becattini, Piccinini, Rizzo, PRC 77, 024906 (2008) [0711.1253]**

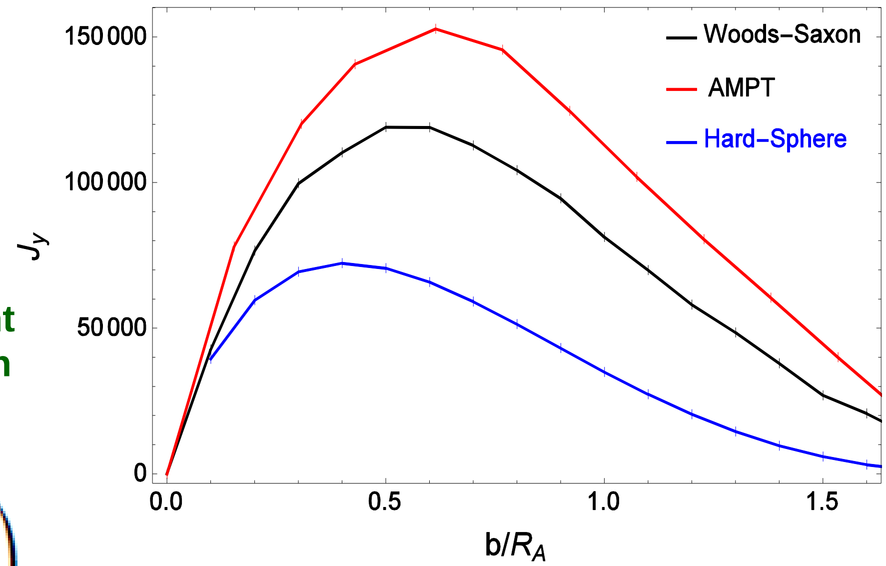
Spin-orbital coupling model

Global OAM in HIC

- **Non-central collisions produce global orbital angular momentum**



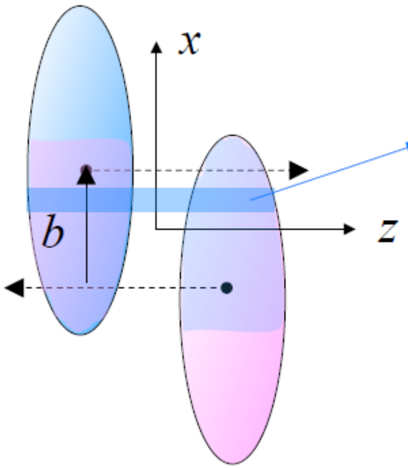
Number of participant nucleons per unit x in projectile or target



$$L_y = -p_{\rm in} \int x dx \left(\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx} \right)$$

Liang & Wang, PRL 94, 102301(2005); PLB 629, 20(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Huang, Huovinen, Wang,PRC 84,054910(2011); Jiang, Lin, Liao, PRC 94,044910(2016); Deng,Huang, PRC 93,064907(2016); many others ……

Global OAM in HIC

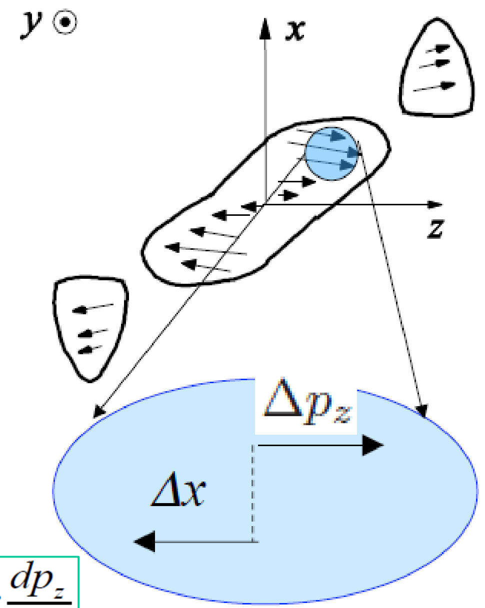


$$\frac{dN^{P,T}_{part}}{dx}$$

Number of participant nucleons per unit x in projectile or target

Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2c(s)} \frac{\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx}}{\frac{dN^P_{\rm part}}{dx} + \frac{dN^T_{\rm part}}{dx}}$$



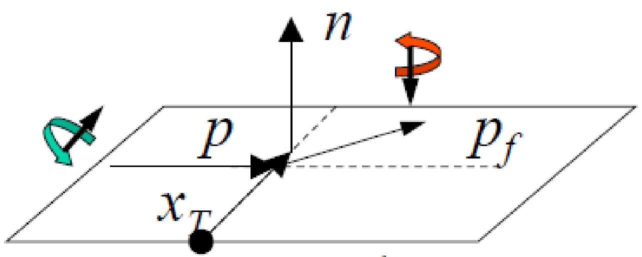
$$\boxed{\Delta p_z = \Delta x \frac{dp_z}{dx}}$$

$$\begin{aligned} L_y &= -\Delta x \Delta p_z \\ &= -(\Delta x)^2 \frac{dp_z}{dx} \sim \beta\omega_y \end{aligned}$$

Liang & Wang (2005); Gao, et al. (2008); Betz, Gyulassy, Torrieri (2007); Becattini, Piccinini, Rizzo (2008); Jiang, Lin, Liao (2016); Deng, Huang (2016); many others ……

Quark scatterings in potential

- **Quark scatterings at small angle in static potential with screening mass**
- **Unpolarized and polarized cross sections**



$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$$\boxed{\mu \sim T\sqrt{\alpha_S}}$$

$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \boxed{\vec{n}\cdot(\vec{x}_T \times \vec{p})}$$

Polarization vector **OAM** **Spin-Orbital coupling**

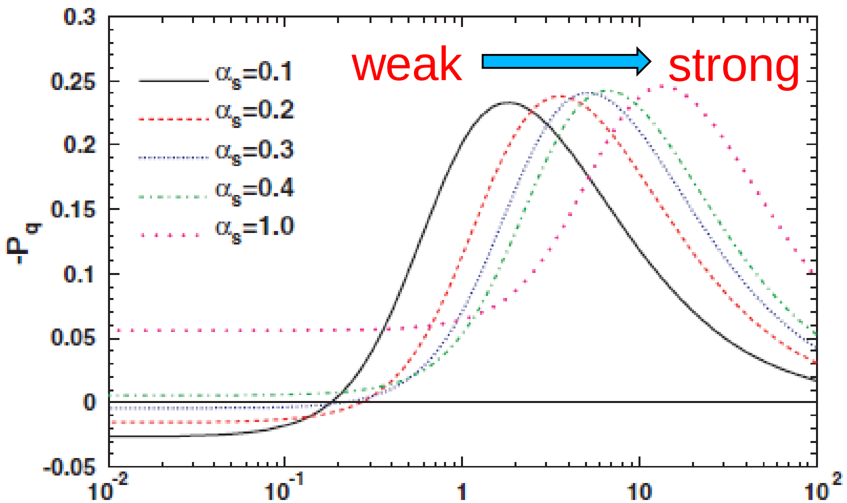
- **Polarization for small angle scattering and** $m_q \gg p, \mu$

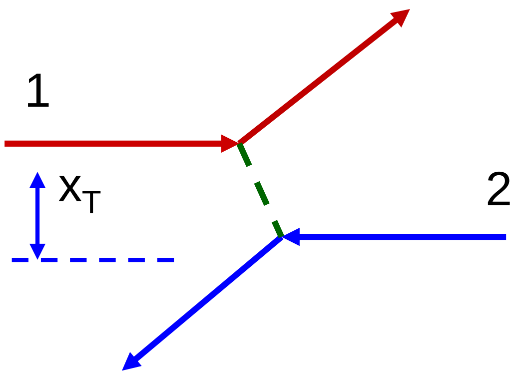
$$P_q \approx -\pi\frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

Quark-quark scattering

- **Beyond small angle approximation with HTL gluon progagator**





$\sqrt{\hat{s}}/T$ → **Local OAM or vorticity** → $\boxed{L \sim \langle x \rangle p \sim \frac{p}{\mu} \sim \beta\omega}$

Quark polarization as functions of the square root of parton-parton scattering energy over T [≈ local OAM or vorticity] which increases with α_S

Liang, Wang, PRL 94, 102301(2005); PLB 629, 20(2005);
Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008)

Statistical-hydro model

Covariant form of quantum statistical physics (local equilibrium)

- **To obtain covariant form in local equilibrium, one can use principle of maximal entropy with conservation of total energy-momentum and particle number,**

$$\boxed{d\Sigma_\mu = d\Sigma n_\mu}$$

space-like hyper-surface
n^μ is time-like vector

$$\hat{\rho}_{\mathrm{LE}} = \frac{1}{Z}\exp\left[\int_\Sigma d\Sigma_\mu \left(-\hat{T}^{\mu\nu}\beta_\nu + \zeta\hat{j}^\mu\right)\right]$$

Zubarev (1979);
Weert (1982);
Becattini et al. (2012-2015);
Hayat, et al. (2015);
Floerchinger (2016)

- **Given n^μ, one can determine β^μ and ζ by**

$$\underline{n_\mu \left\langle \hat{T}^{\mu\nu}(x)\right\rangle_{\mathrm{LE}}(\beta^\alpha,\zeta) = n_\mu T^{\mu\nu}(x)}, \quad \underline{n_\mu \left\langle \hat{j}^{\mu}(x)\right\rangle_{\mathrm{LE}}(\beta^\alpha,\zeta) = n_\mu j^{\mu}(x)}$$

Energy condition **Particle number condition**

- **where statistical average is defined by** $\boxed{\left\langle \hat{O}(x)\right\rangle_{\mathrm{TE}} = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{TE}}\hat{O}(x)\right]}$

Global equilibrium and stationary conditions

- **Stationary conditions**

Becattini (2012); Becattini, Bucciantini, Grossi, Tinti (2015) Becattini, Grossi (2015)

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \qquad \partial_\mu \zeta = 0$$

Killing equation

Thermal vorticity tensor

Killing vector solution ⇒

$$\beta^\mu = \underline{b^\mu} + \underline{\varpi^{\mu\nu}} x_\nu$$

$$\boxed{\varpi^{\mu\nu} = -\frac{1}{2}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu)}$$

$$\boxed{b^\mu = \frac{1}{T} u^\mu}$$

Total particle number

Density operator at global equilibrium

$$\boxed{\hat{\rho}_{\mathrm{GE}} = \frac{1}{Z} \exp\left[-\beta u_\nu \underline{\hat{P}^\nu} + \frac{1}{2} \underline{\hat{J}^{\nu\rho}} \varpi_{\nu\rho} + \zeta \underline{\hat{Q}} \right]}$$

4-momentum vector operator

Total angular momentum tensor (OAM+spin)

Spin and polarization

- **Spin (Pauli-Lubanski) pseudo-vector**

$$\begin{aligned}\hat{S}^{\mu} &= -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\hat{J}^{\mathrm{S}}_{\nu\rho}\hat{P}_{\sigma}\\ S^{\mu} &= \mathrm{Tr}(\hat{\rho}_{\mathrm{GE}}\hat{S}^{\mu})\\ \Pi^{\mu} &= \frac{1}{S}S^{\mu}\end{aligned}$$

$$\boxed{\begin{array}{l}[\hat{S}^{\mu},\hat{P}^{\nu}]=0, \quad \hat{S}^{\mu}\hat{P}_{\mu}=0\\ \hat{S}^{\mu}\hat{S}_{\mu}=-S(S+1)\end{array}}$$

properties of spin vector

phase space spin density for spin ½-fermions

$$\boxed{S^{\mu}(x,p)=-\frac{1}{8m}[1-n_F(x,p)]\epsilon^{\mu\rho\sigma\tau}p_{\tau}\varpi_{\rho\sigma}}$$

particle number at freezeout

$$\boxed{N=\int\frac{d^3p}{E_p}\int d\Sigma_{\lambda}p^{\lambda}n_F(x,p)}$$

spin at freezeout hypersurface

$$\boxed{S^{\mu}=\frac{1}{N}\int\frac{d^3p}{E_p}\int d\Sigma_{\lambda}p^{\lambda}n_F(x,p)S^{\mu}(x,p)}$$

Becattini, et al., 1610.02506;
Karpenko, Becattini, 1610.04717;
Fang, Pang, QW, Wang,
PRC 94,024904(2016);

Kinetic approach with Wigner function

- **To describe polarization for massive or massless spin ½ fermions, we have to explicitly know their momentum p, therefore we need to know information in phase space (t,x,p), that's why we use kinetic approach**
- **Classical kinetic approach: f(t,x,p)**
- **Quantum kinetic approach: W(t,x,p)**

Chiral (massless fermions) Kinetic Theory:
Son, Yamamoto, PRL 109 (2012) 181602;
Stephanov, Yin, PRL 109 (2012) 162001;
Chen, Pu, QW, Wang, PRL 110 (2013) 262301;
Mueller, Venugopalan, PRD 96 (2017) 016023.

Wigner functions for fermions in background EM field

- The Wigner function for spin 1/2 fermions in constant EM field satisfies EOM, which can be solved perturbatively in $(F_{\mu\nu})^i$ and $(\partial_x)^i$.
- Wigner function can be decomposed in 16 generators of Clifford algebra

$$W=\frac{1}{4}\left[\mathscr{F}+i\gamma^5\mathscr{P}+\gamma^\mu\mathscr{V}_\mu+\gamma^5\gamma^\mu\mathscr{A}_\mu+\frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

4x4 matrix scalar p-scalar vector axial-vector tensor

$$j^\mu=\int d^4p\mathscr{V}^\mu,\qquad j_5^\mu=\int d^4p\mathscr{A}^\mu,\qquad T^{\mu\nu}=\int d^4pp^\mu\mathscr{V}^\nu$$

Heinz, Phys.Rev.Lett. 51, 351 (1983);
Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986).

Spin tensor component

- **Spin tensor component of Wigner function**

Fang, Pang, QW, Wang, PRC 94,024904(2016);

QW, 1704.04022

$$\begin{aligned}\mathscr{M}^{\alpha\beta}(x,p) &\equiv \frac{1}{2}\mathrm{Tr}\left[\gamma_0\sigma^{\alpha\beta}W(x,p)\right]\\ &= \frac{1}{2}\left[-\epsilon^{0\alpha\beta\rho}\mathscr{A}_\rho + ig^{\alpha0}\mathscr{V}^\beta - ig^{\beta0}\mathscr{V}^\alpha\right]\end{aligned}$$

- **For αβ=ij (space indices)**

$$\mathscr{M}^{ij}(x,p) = \frac{1}{2}\epsilon^{ijk}\mathscr{A}^k(x,p) \Longrightarrow \begin{aligned}A^i(x) &= \psi^\dagger(x)\Sigma_i\psi(x)\\ &= \int d^4p\mathscr{A}^i(x,p)\end{aligned}$$

Pauli matrices

- **We can regard axial vector as spin vector (up to ½)**

$$\begin{aligned}\Pi^\mu(x) &\sim \frac{1}{2}\int d^4p\underline{\mathscr{A}^\mu(x,p)} && \textbf{Non-relativistic limit}\\ &\sim \frac{1}{2}\int d^4p\underline{\frac{|p_0|}{m}}\mathscr{A}^\mu(x,p) && \textbf{To match Pauli-Lubanski pseudo-vector}\end{aligned}$$

Axial vector component of Wigner function for massive fermions

- **Axial vector component: zero (i=0) and first (i=1) order in $(F_{\mu\nu})^i$ and $(\partial_x)^i$:**

$$\mathscr{A}^\mu = \mathrm{Tr}[\gamma^\mu\gamma^5 W]$$

$$\mathscr{A}^\mu_{(0)}(x,p) = m\left[\theta(p_0)\underline{n^\mu(\mathbf{p},\mathbf{n})} - \theta(-p_0)\underline{n^\mu(-\mathbf{p},-\mathbf{n})}\right]\delta(p^2-m^2)\underline{A}$$

$$\mathscr{A}^\alpha_{(1)}(x,p) = -\frac{1}{2}\hbar\beta\tilde{\Omega}^{\alpha\sigma}p_\sigma\frac{d\underline{V}}{d(\beta p_0)}\delta(p^2-m^2) - Q\hbar\tilde{F}^{\alpha\lambda}p_\lambda\underline{V}\frac{\delta(p^2-m^2)}{p^2-m^2}$$

where A and V are related to distribution functions

$$\tilde{F}^{\alpha\lambda} = \frac{1}{2}\epsilon^{\alpha\lambda\rho\sigma}F_{\rho\sigma}$$

$$\tilde{\Omega}^{\alpha\lambda} = \frac{1}{2}\epsilon^{\alpha\lambda\rho\sigma}\Omega_{\rho\sigma}$$

$$\Omega_{\rho\sigma} = \frac{1}{2}(\partial_\rho u_\sigma - \partial_\sigma u_\rho)$$

- **Spin (pseudo-)vector in Lab frame**

$$\underline{n^\mu(\mathbf{p},\mathbf{n})} = \underline{\Lambda^\mu_{\ \nu}(-\mathbf{v}_p)}\,\underline{n^\nu(0,\mathbf{n})} = \left(\frac{\mathbf{n}\cdot\mathbf{p}}{m}, \mathbf{n} + \frac{(\mathbf{n}\cdot\mathbf{p})\mathbf{p}}{m(m+E_p)}\right)$$

Spin in Lab frame

Lorentz boost from cms to Lab frame

Spin in cms frame

Fang, Pang, QW, Wang, PRC 94,024904(2016);
Fang, Pang, QW, Wang, PRD 95, 014032(2017)

Polarization (spin) vector

- **Polarization at zeroth order is vanishing if we assume that the chemical potential for spin-up and spin-down fermions are equal.**
- **Polarization vector at the first order**

Fang, Pang, QW, Wang, PRC(2016);
Aristova, Frenklakh, Gorsky, Kharzeev, JHEP (2016);
QW, 1704.04022

$$\Pi^{\alpha}_{(1)} \approx \frac{1}{2m}\hbar\beta\int\frac{d^3p}{(2\pi)^3}\left\{[E_p\omega^{\alpha}+QB^{\alpha}]\boxed{\frac{e^{\beta(E_p-\mu)}}{[e^{\beta(E_p-\mu)}+1]^2}} + [E_p\omega^{\alpha}-QB^{\alpha}]\boxed{\frac{e^{\beta(E_p+\mu)}}{[e^{\beta(E_p+\mu)}+1]^2}}\right\}$$

susceptiblility

+/- $\rightarrow$ particle/antiparticle

- **Polarization at freezeout**

$$E_p\frac{d\Pi^{\alpha}(p)}{d^3p} \approx \frac{\hbar}{2m}\beta\frac{1}{(2\pi)^3}\int d\Sigma_{\lambda}p^{\lambda}\left(E_p\omega^{\alpha}\pm QB^{\alpha}\right)\underline{f^{\pm}_{\mathrm{FD}}(x,p)\left[1-f^{\pm}_{\mathrm{FD}}(x,p)\right]}$$

vorticity

magnetic field

susceptiblility

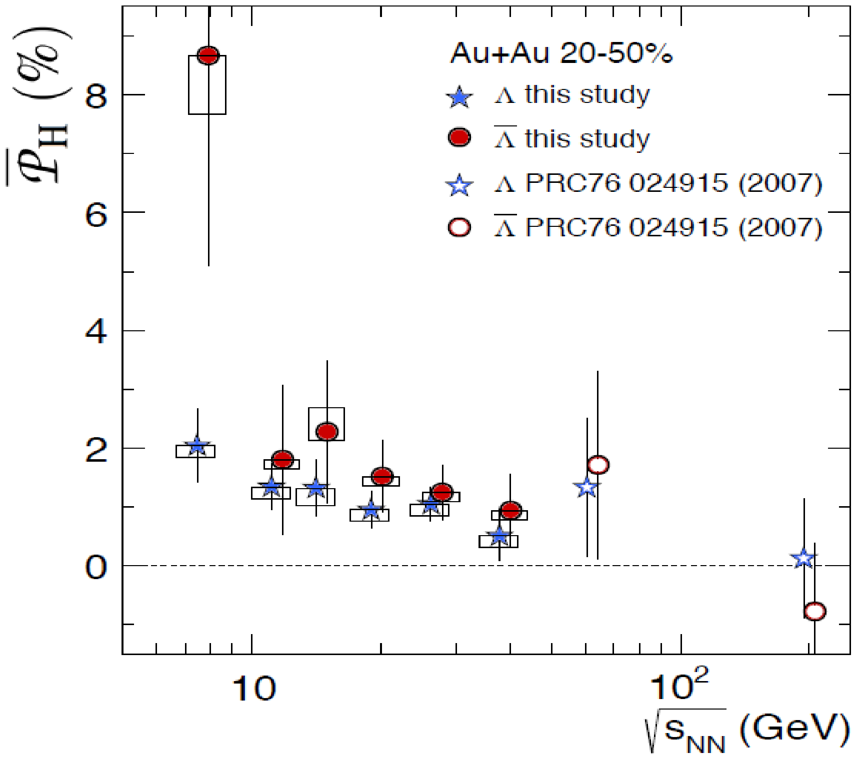
STAR data for Λ polarization

- At each energy, a positive polarization for Λ and Λ at 1.1-3.6σ level. On average over all data,

$$\mathscr{P}_{\Lambda} = (1.08 \pm 0.15)\%$$

$$\mathscr{P}_{\bar{\Lambda}} = (1.38 \pm 0.30)\%$$

- The polarization decreases with collisional energy



STAR collab., Nature, 548, 62(2017); arXiv: 1701.06657.

Global Λ polarization from AMPT model

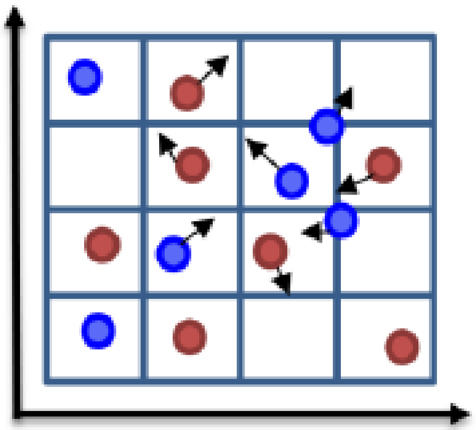
H. Li, L. Pang, QW, X.Xia, 1704.01507

Fluid velocity and vorticity from AMPT

- **Velocity:**

(a) average in cell

$$\mathbf{v}(t,\mathbf{x}) = \frac{\sum_i \mathbf{p}_i}{\sum_i E_i}$$



(b) Other method: Gaussian smearing

$$\mathbf{v}(t,\mathbf{x}) = \frac{\sum_i \mathbf{p}_i G(\mathbf{x}_i - \mathbf{x})}{\sum_i E_i G(\mathbf{x}_i - \mathbf{x})}$$

Other methods:
Oliinychenko, Petersen 2016
Deng, Huang 2016

(c) Sum is over particles and events

(d) 10^5 events at each energy in BES range

- **Vorticity: finite-difference method**

Vorticity fields from AMPT

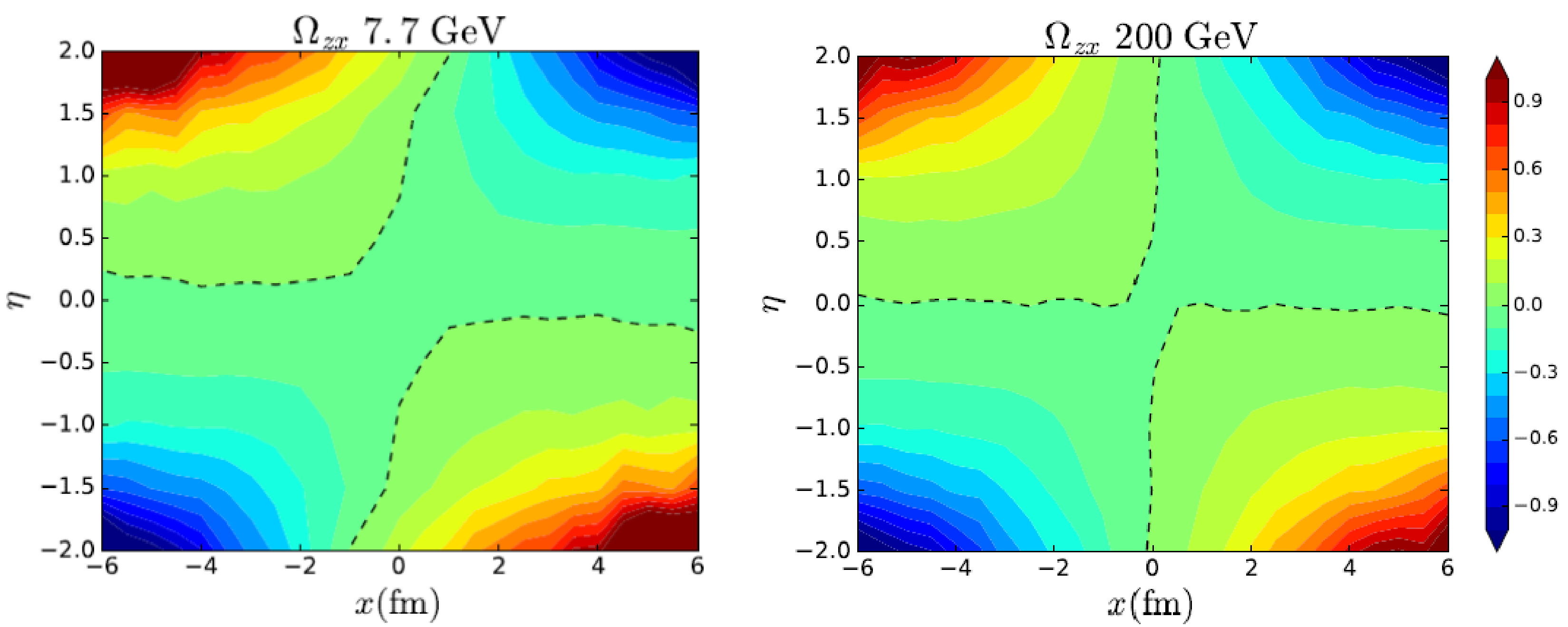
Vorticity field on reaction plane:

(a) Nearly odd function of x and η;

(b) Less odd-symmetric at lower energy

Li, Pang, QW, Xia, 1704.01507

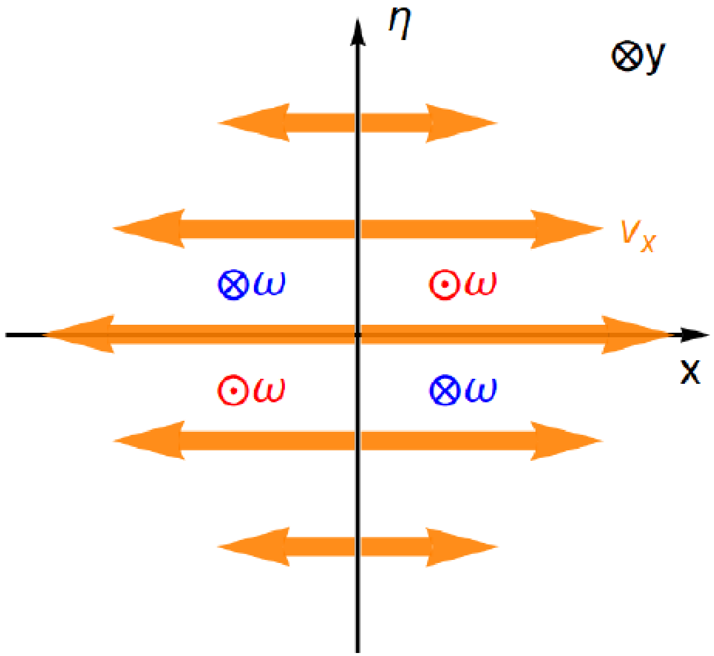
$$\langle \mathbf{S}^* \rangle \sim \int d^4x f_\Lambda(x) \Omega_{zx}(x)$$

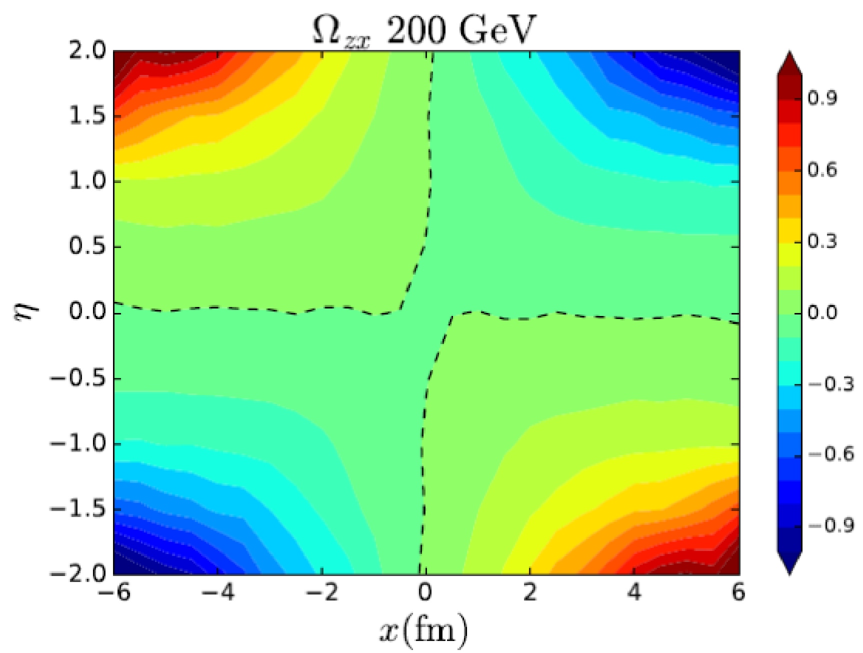


Vorticity fields in reaction plane

The odd-symmetry can be understood by the radial flow.

Li, Pang, QW, Xia, 1704.01507

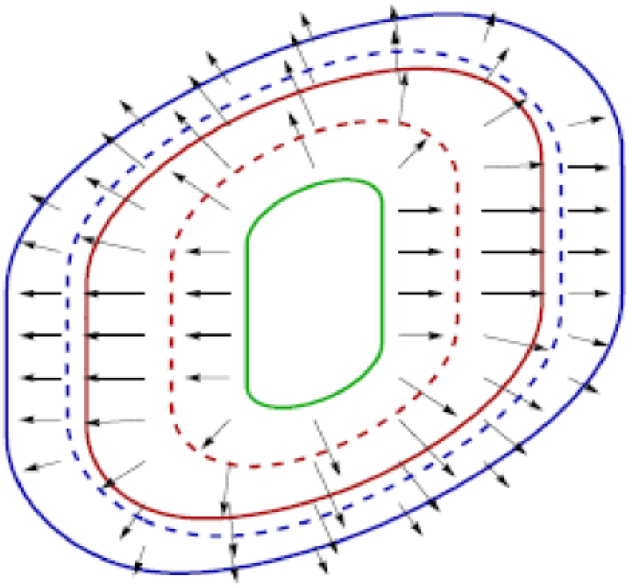




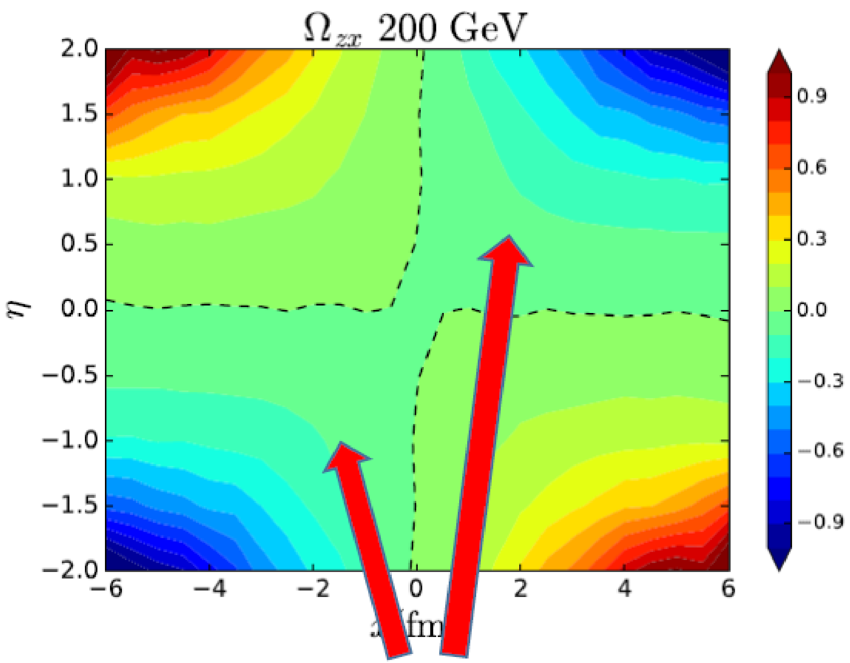
$$\Omega_{zx} = \partial_z u^x - \partial_x u^z$$

Vorticity fields and matter distribution

Due to global OAM, fireball or matter distribution is tilted



Bozek, Wyskiel 2010

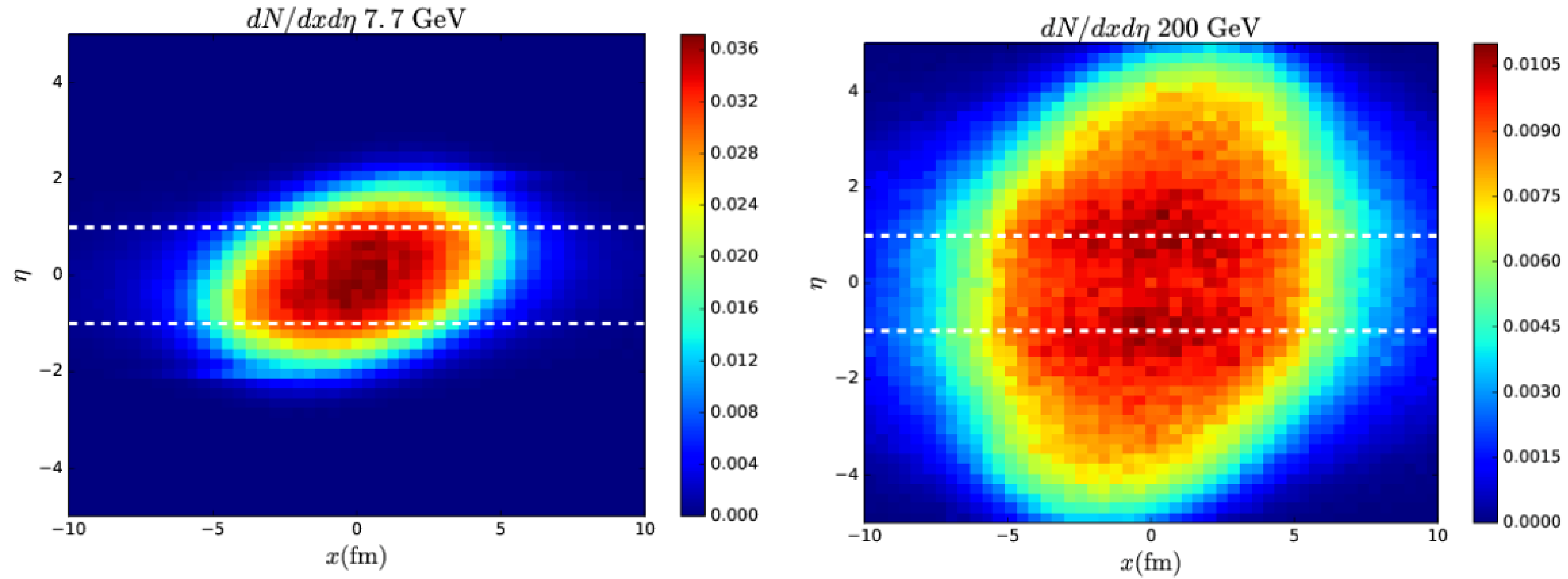


Li, Pang, QW, Xia, 1704.01507

Number distribution of Λ

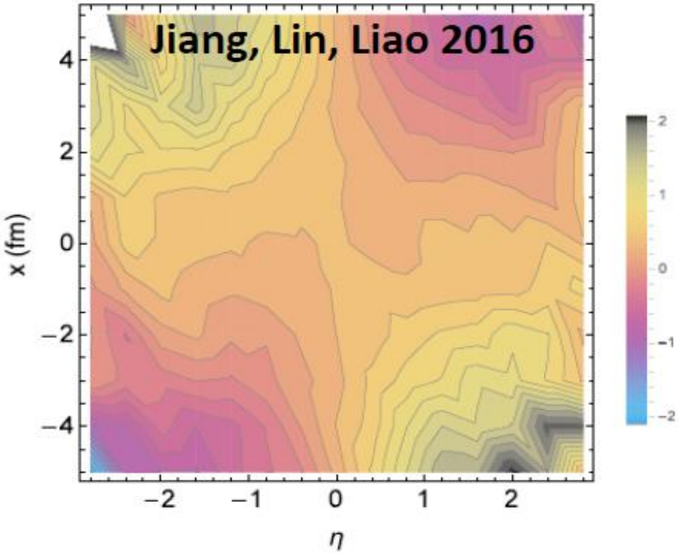
More tilted at 7.7 GeV

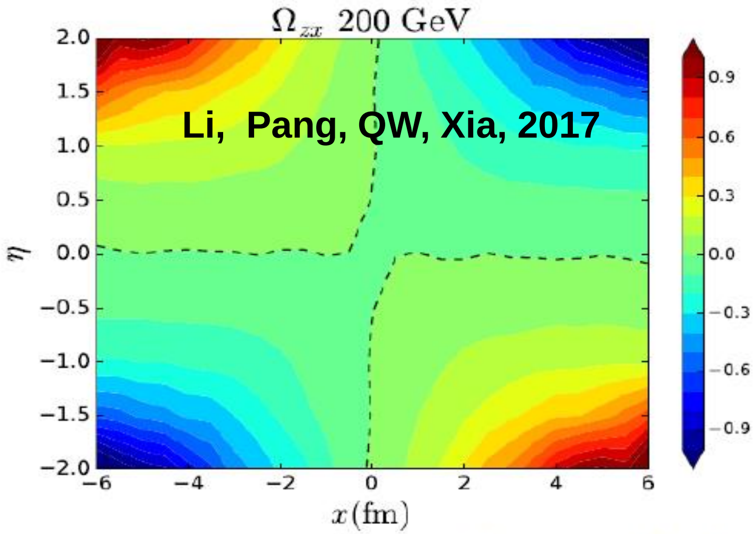
More symmetric at 200 GeV due to rapid expansion in beam direction

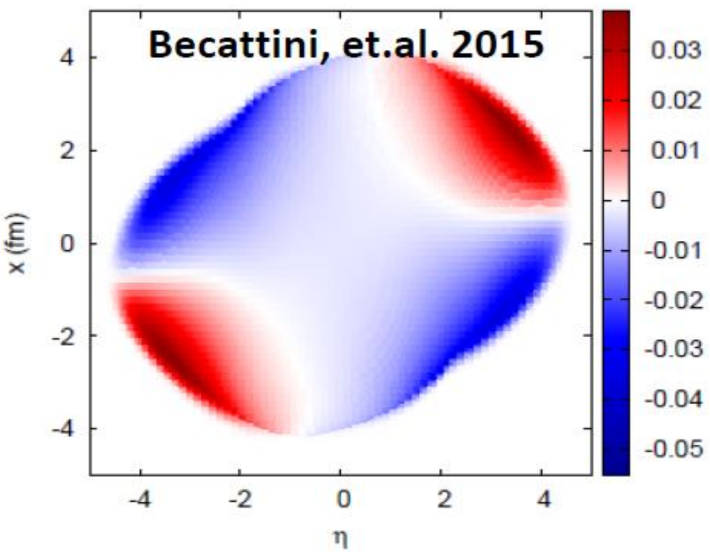


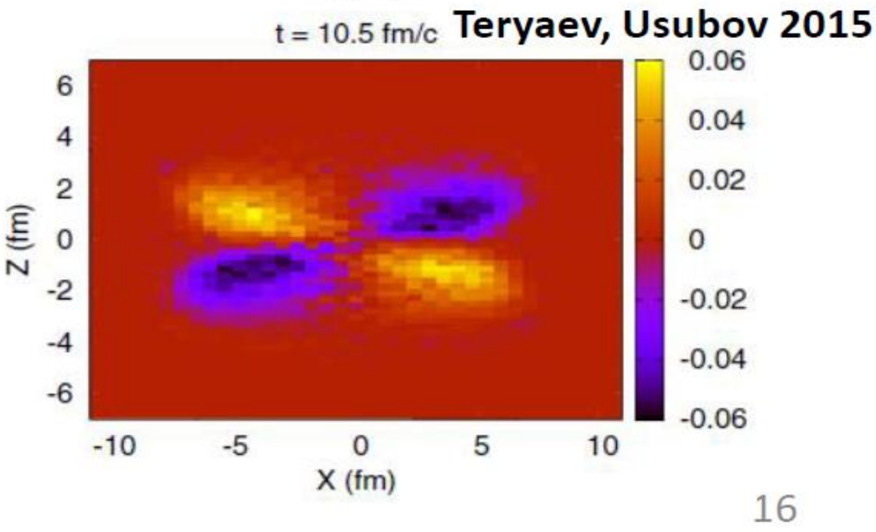
Li, Pang, QW, Xia, 1704.01507

Vorticity fields from other methods







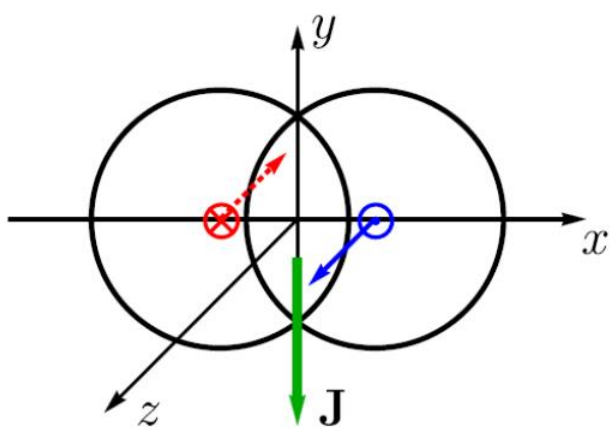


Global polarization of Λ from AMPT

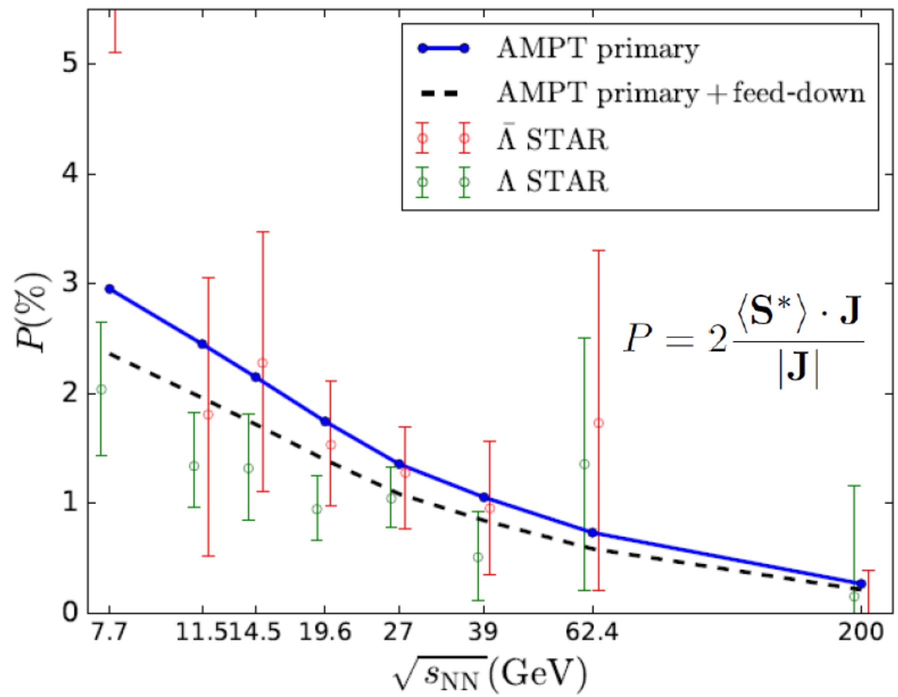
- **Polarization of Λ: average over events with |η|<1**

$$\langle \mathbf{S}^* \rangle = \frac{1}{N}\sum_{i=1}^{N} \mathbf{S}^*(x,p)$$

$$P = 2\frac{\langle \mathbf{S}^* \rangle \cdot \mathbf{J}}{|\mathbf{J}|}$$



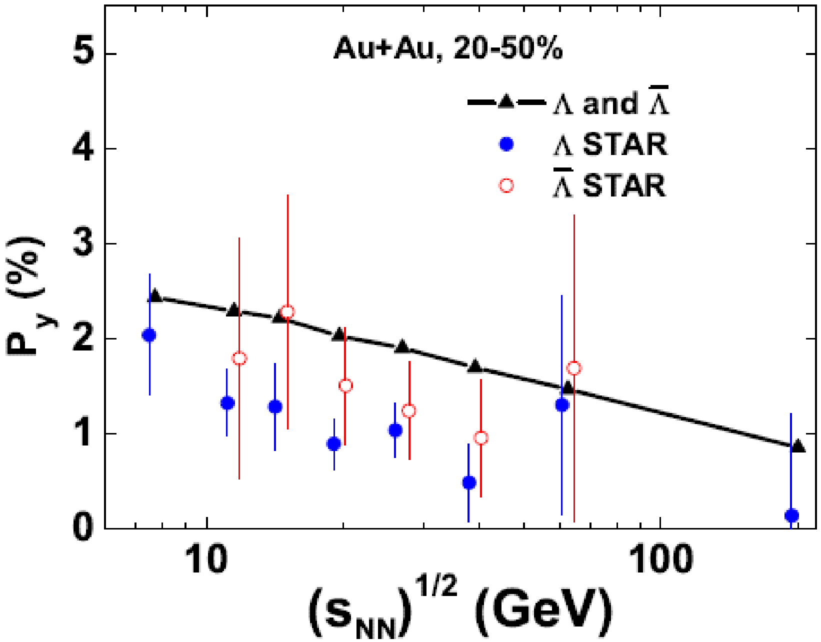
Au+Au, 20%-50%, with feed-down



Li, Pang, QW, Xia, 1704.01507

Global polarization of Λ from Chiral Kinetic approach

- Chiral kinetic approach+ AMPT model
- Spin polarizations of quarks and antiquarks
- Quarks and antiquarks are converted to hadrons via the coalescence Model

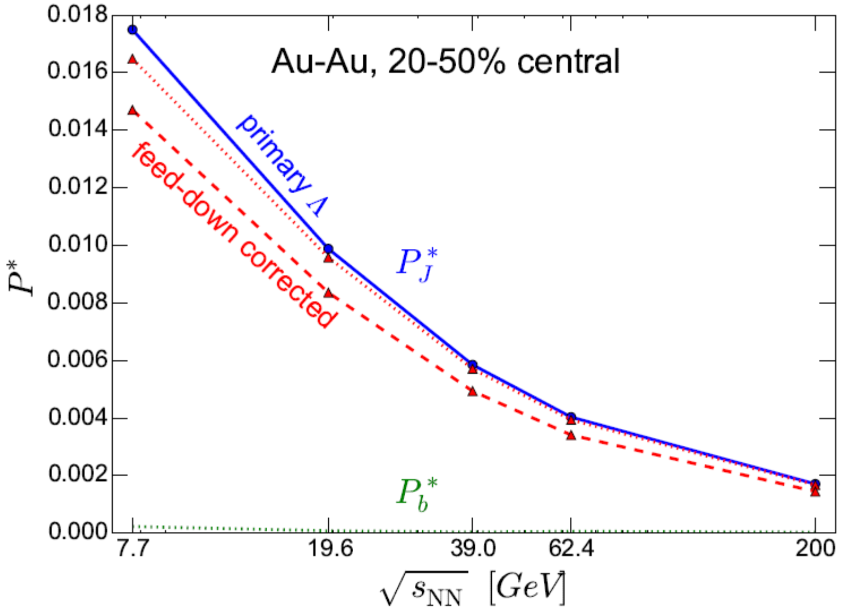


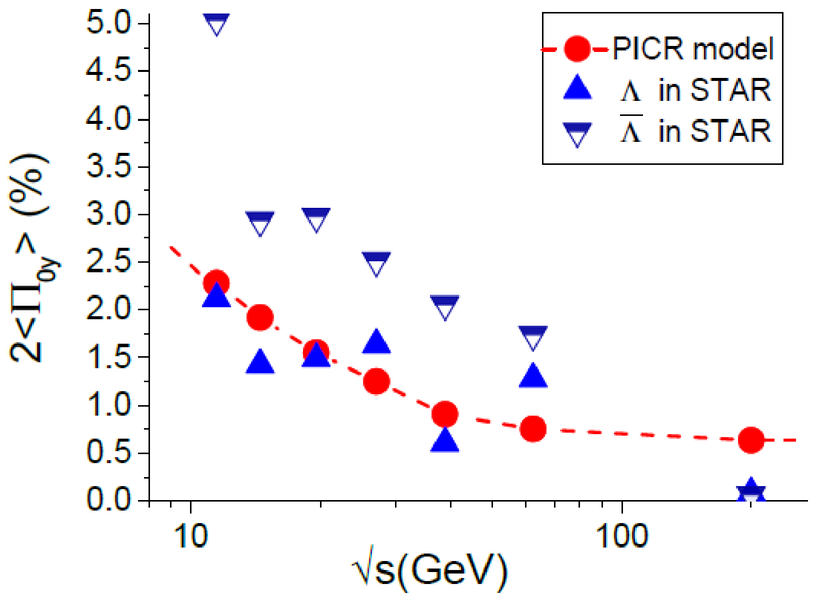
Chiral kinetic approach:

Son, Yamamoto, PRL 109 (2012) 181602;
Stephanov, Yin, PRL 109 (2012) 162001;
Chen, Pu, QW, Wang, PRL 110 (2013) 262301;
Mueller, Venugopalan, PRD 96 (2017) 016023.

Y.Sun, C.M. Ko, 1706.09467

Global polarization of Λ from other methods



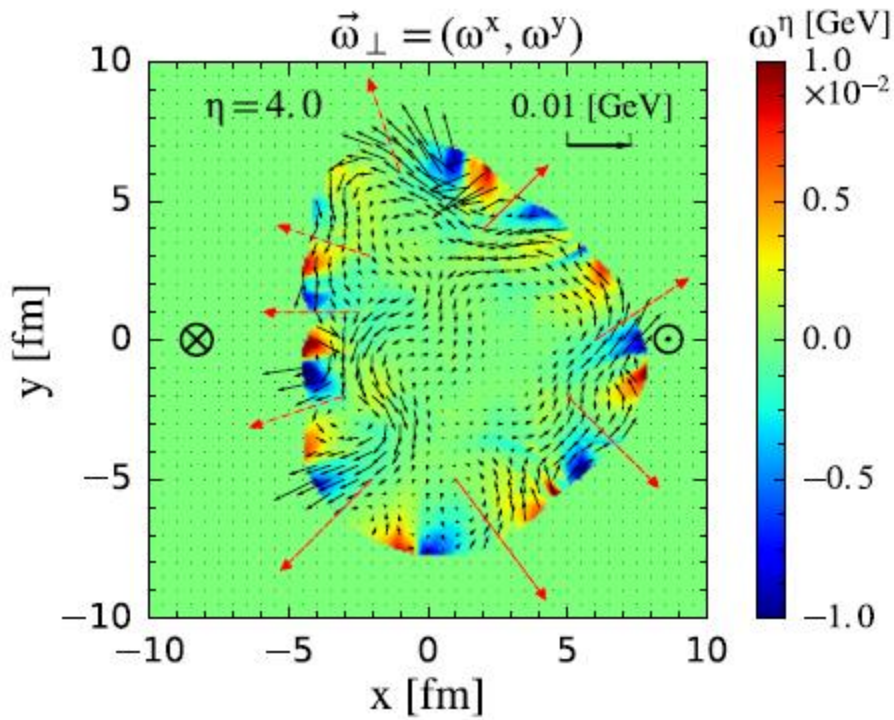


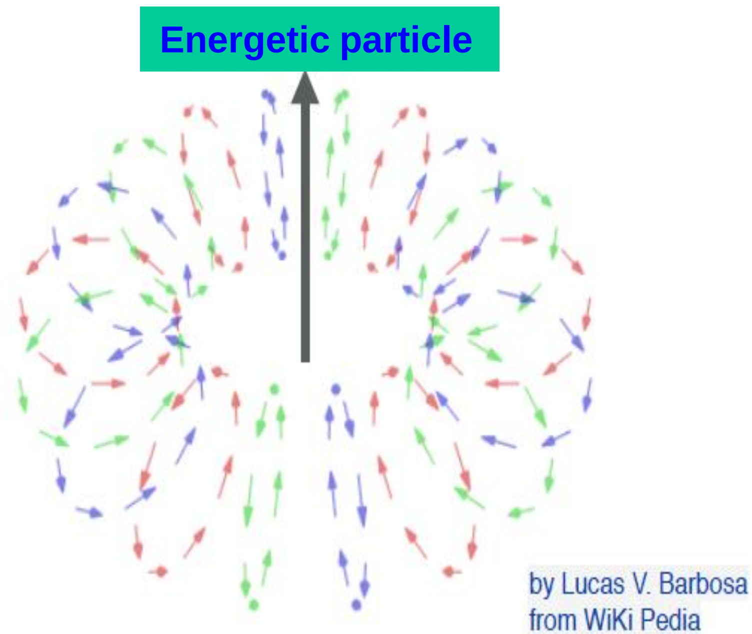
Karpenko, Becattini, EPJC 77,213(2017)
UrQMD + vHLLE hydro

Xie,Wang,Csernai,PRC 95,031901(2017)
PICR hydro

Other approach:
Aristova, Frenklakh, Gorsky, Kharzeev, JHEP 1610, 029 (2016)

Turbulence and vortices in high energy HIC



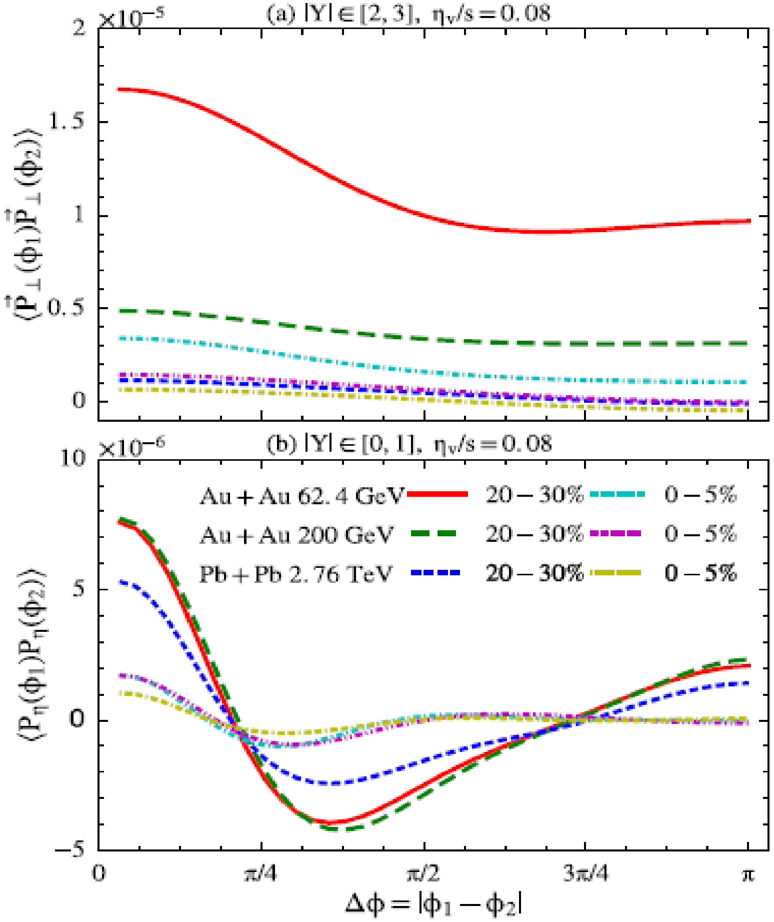


Spin-spin correlation of Λ can probe the vortical structure of sQGP

Pang, Petersen, QW, Wang, PRL 117, 192301 (2016)

Correlation in Λ Polarization as probe to vortical fluid

- **(a) The offset of transverse spin correlation indicates global polarization, which is stronger at lower collisional energies.**
- **(b) $\cos(\Delta\phi)$ -type azimuthal distribution in transverse spin correlation is due to circular structure of ω along beam direction.**
- **(c) Longitudinal spin correlation (pair structure) is due to transverse energetic particles. The beam energy dependence for longitudinal spin is weak.**



Pang, Petersen, QW, Wang, PRL 117, 192301 (2016)

Summary

- **Λ polarization provides a measurement of global angular momentum in HIC**
- **STAR data in Beam Energy Scan program show a clear non-vanishing global polarization for Λ**
- **A few theoretical models for hadron polarization: microscopic spin-orbital coupling model, statistical-hydro models, (Wigner function) kinetic approach etc.**
- **"Discovery of global Λ polarization opens new directions in the study of the hottest, least viscous – and now, most vortical – fluid ever produced in the laboratory." --- from STAR Collab., Nature, 548, 62-65 (2017)**

It is just the beginning, stay tuned !