Color-flavor center symmetry of QCD and its order parameter

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Order parameter and QCD : The common lore

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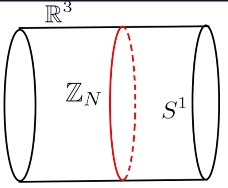

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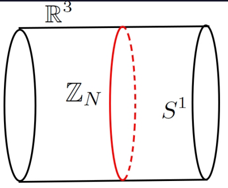

- Adding fundamental fermions spoils center symmetry $\longrightarrow$ Polyakov loop not an order parameter any more.

Order parameter and QCD : The common lore

Exploring phase transitions in QCD involves subtleties :-

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- Massive quarks break chiral symmetry explicitly $\longrightarrow$ $\langle \sum_a \bar{q}_a q_a \rangle$ no longer an order parameter.
- Hence QCD with massive dynamical fermions lacks non-trivial order parameter at zero baryon density.

Can we do better ?

Possible with degenerate quark flavors !

- On $\mathcal{R}_4$ there is $U(n_{\rm f})_V$ flavor symmetry as well as flavor permutation symmetry.
- Consider the theory on $\mathcal{R}_3 \times \mathcal{S}_1$.
- Use flavor-twisted boundary condition on quarks ($\mathbb{Z}_{n_{\rm f}}$ symmetric)

$$q_a(x_1+L) = \mathcal{U}^{ab}\, q_b(x_1)\,.$$

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, M. Yahiro, K. Kashiwa, T. Iritani, E. Itou, T. Misumi, T. Hirakida, J. Takahashi, T. Hirakida, J. Sugano

Can we do better ?

- Choose $\mathbb{Z}_{n_\mathrm{f}}$ symmetric BC.

$$\mathcal{U} = \mathrm{diag}(1, \nu, \cdots, \nu^{n_\mathrm{f}-1}), \qquad \nu \equiv e^{2\pi i/n_\mathrm{f}}\,, \tag{1a}$$

or

$$\mathcal{U} = \mathrm{diag}(\nu^{1/2}, \nu^{3/2}, \cdots, \nu^{n_\mathrm{f}-1/2})\,. \tag{1b}$$

- key observation : For

$$d \equiv \gcd(n_\mathrm{f}, N) > 1\,, \tag{2}$$

center transformation followed by a $\mathbb{Z}_d$ cyclic flavor permutation is a symmetry of the theory $\equiv$ CFC symmetry.

QCD phase diagram ?

keep in mind that

- Twisting thermal bc will tamper with thermal physics - so don't.
- Compactify one of the spatial directions (Length= L) and apply flavor twist along it.
- Ask about the Polyakov loop along the spatial circle.
- We need to make L large to comment on the phase diagram $L \gg \frac{1}{\mu}$, $L \gg \frac{1}{T}$.

$\mu \gg \Lambda$ and $g\mu \gg T$

one-loop effective potential for Ω : $V_{\text{eff}}(\Omega) = V_{\text{g}}(\Omega) + V_{\text{f}}(\Omega)$.

where gluon contribution :

$$V_{\text{g}}(\Omega) = -\frac{1}{L^4} \sum_{n=1}^{\infty} f_n \left(|\text{tr}\, \Omega^n|^2 - 1 \right),$$

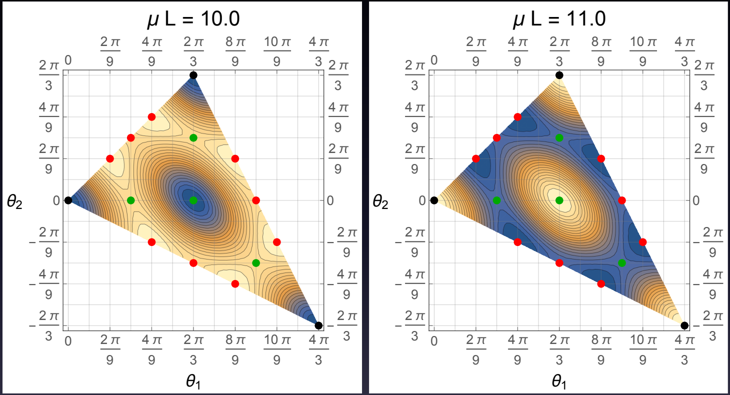
with $f_n \sim e^{-n m_{\text{g}} L}$, when $m_{\text{g}} L \gg 1$ and fermion contribution :

$$V_{\text{f}}(\Omega) = \frac{\pm 2T\, e^{-n_{\text{f}} \pi L T}}{n_{\text{f}} \pi L^2} \left[\mu \sin(n_{\text{f}} \mu L) + \pi T \cos(n_{\text{f}} \mu L) \right] \\ \times \left(\text{tr}\, \Omega^{n_{\text{f}}} + \text{h.c.} \right) + (\text{holonomy-independent}),$$

$V_{\text{eff}}(\Omega) \propto \mathrm{Re}\,\mathrm{tr}\,\Omega^{n_f} \times$ oscillating function of $n_f \mu L$. Four categories of extrema :

- center-symmetric extremum at $\Omega = \mathrm{diag}\,(1, e^{2\pi i/3}, e^{4\pi i/3})$
- three center-broken extrema with $\Omega = \mathrm{diag}\,(e^{(2k-1)i\pi/3}, e^{2ki\pi/3}, e^{(2k+1)i\pi/3})$, $k = 0, 1, 2$;
- nine center-broken "$SU(2) \times U(1)$" extrema $\Omega = \mathrm{diag}\,(e^{ki\pi/9}, e^{ki\pi/9}, e^{-2ki\pi/9})$ with $k \bmod 6 = 2$, 3 or 4;
- three center-broken "$SU(3)$" extrema, $\Omega = \mathrm{diag}\,(e^{2ki\pi/3}, e^{2ki\pi/3}, e^{2ki\pi/3})$, $k = 0, 1, 2$.

Contour plot for V_f



(d) V_f

Darker colors indicate lower values of $L^4 V_{\text{eff}}$. The center-symmetric point $(\theta_1, \theta_2, \theta_3) = (0, 2\pi/3, 4\pi/3)$ lies at the center while the corners are the coinciding eigenvalue points $(0, 0, 0)$ and $\pm(2\pi/3, 2\pi/3, 2\pi/3)$. Dots denote critical points of $\widehat{V}_{\text{f}}$.

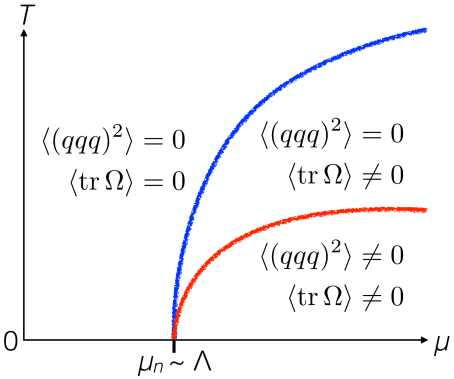
Quantum oscillation and multiphase point

- Quantum oscillations as a function of μL.
- Two groups of degenerate extrema :
  - center-symmetric extremum and three $SU(3)$ extrema
  - six $SU(2) \times U(1)$ extrema
- $L \to \infty$ is an accumulation point or multiphase point.
- The small residual gluon contribution to V_{eff} favors $SU(3)$ extrema. Hence multi-phase region $\longrightarrow$ broken CFC symmetry, with $\langle \text{tr}\,\Omega \rangle \neq 0$

Rest of the phase diagram

- Small T, small μ regime : lattice studies imply that $\langle\Omega\rangle = 0$.
- High T regime with $T \gg \max(\Lambda, \mu)$: the dynamics on spatial scales large compared to $(g^2 T)^{-1}$ are described by pure 3D YM theory which confines, so $\langle \mathrm{tr}\, \Omega\rangle = 0$.
- We expect the high-temperature region to be smoothly connected to the region near $T = \mu = 0$.

The full picture : Phase diagram



Sketch of a possible phase diagram of circle-compactified $SU(3)_V$ symmetric QCD at $m_q > 0$, as a function of T and μ, in the large L limit.

Conformal window

Let $x \equiv n_{\rm f}/N$, and $m_q = T = 0$.

- If $x > \frac{11}{2}$, QCD becomes an infrared-free theory.
- For x below some $x_\chi < \frac{11}{2}$, chiral symmetry is believed to be spontaneously broken.
- In the range $x \in (x_\chi, \frac{11}{2}) \longrightarrow$ non-trivial infrared (IR) fixed point, NO chiral symmetry breaking $\equiv$ "Conformal Window".

Conformal window at large N

- Choose N, n_{f} such that $d = \gcd(n_{\mathrm{f}}, N)$ is fixed and greater than 1, while the ratio $x = n_{\mathrm{f}}/N$ approaches a non-zero limit.
- If $\epsilon \equiv \frac{11}{2} - x \to 0^+$, $\exists$ an IR fixed point with a parametrically small coupling $\lambda_{\mathrm{IR}} = \frac{64}{75}\pi^2\epsilon \ll 1$.[1, 2]
- Compute the quantum effective potential $V_{\mathrm{eff}}(\Omega)$ for λ at all scales when $\epsilon \ll 1$: analysis valid for all L.
- Classically, $V_{\mathrm{eff}}(\Omega)$ is zero.

1. W. E. Caswell, Phys. Rev. Lett. 33, 244 (1974).
2. T. Banks and A. Zaks, Nucl. Phys. B196, 189 (1982).

Conformal window

At one loop $V_{\text{eff}}(\Omega) = V_{\text{g}}(\Omega) + V_{\text{f}}(\Omega)$ with gluon and fermion contributions given by

$$V_{\text{g}}(\Omega) = -\frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left(|\text{tr}\, \Omega^n|^2 - 1 \right),$$

and

$$V_{\text{f}}(\Omega) = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left(\text{tr}\, \mathcal{U}^{-n}\, \text{tr}\, \Omega^n + \text{tr}\, \mathcal{U}^n\, \text{tr}\, \Omega^{-n} \right)$$

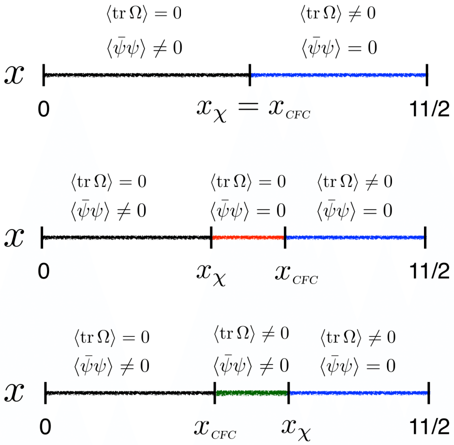
$$= \frac{2}{\pi^2 L^4 {n_{\text{f}}}^3} \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^4} \left(\text{tr}\, \Omega^{n_{\text{f}} n} + \text{h.c.} \right).$$

$V_{\text{g}} = O(N^2)$ and $V_{\text{f}} = O(N^{-2}) \longrightarrow V_{\text{g}}$ dominates setting $\Omega \propto 1$ $\longrightarrow$ breaks CFC.

Conformal Window

- When $\epsilon = \frac{11}{2} - x \ll 1$ the CFC symmetry is spontaneously broken at any L.
- At the pure Yang-Mill point, $x = 0$, center symmetry is certainly expected to be unbroken at large L.
- There must be at least one transition at some $x = x_{\rm CFC}$ where the realization of the CFC symmetry changes.
- This point may or may not coincide with the point x_χ where the chiral symmetry realization changes.

The full picture : Conformal window



Possible phase structures of massless QCD as a function of $x = n_\mathrm{f}/N$. The chiral and CFC symmetry realizations change at some $x = x_\chi$ and $x = x_\mathrm{CFC}$, respectively.

CFC and Local Order parameter

- CFC has local order parameters as well :

  $\mathcal{O}_{\Gamma}^{(p)} \equiv \sum_{a=1}^{n_{\rm f}} \nu^{-ap}\, \bar{q}_a \Gamma q_a$, where Γ is an arbitrary Dirac matrix, $p \bmod d \neq 0$ and $\nu \equiv e^{2\pi i/n_{\rm f}}$.

The action of the $\mathbb{Z}_d$ CFC symmetry is

$$\mathrm{tr}\, \Omega^p \;\to\; \omega^{Np/d}\, \mathrm{tr}\, \Omega^p\,, \quad \mathcal{O}_{\Gamma}^{(p)} \;\to\; \nu^{n_{\rm f} p/d}\, \mathcal{O}_{\Gamma}^{(p)}\,.$$

Center and confinement

- Consider

$$\langle \mathrm{tr}\, \Omega(\vec{x})\, \mathrm{tr}\, \Omega^\dagger(0) \rangle_{\mathrm{conn}} \equiv e^{-F(r)}\,, \qquad r = |\vec{x}|\,.$$

- Suppose $\exists$ non-zero lower bound E on the energy of states that contribute to the correlator, so $F(r) \sim E\, r$ as $r \to \infty$.
- For $n_f = 0$ $\mathbb{Z}_N$ symmetric ground state :
  - No intermediate local operator contributes to the correlator.
  - All contributions must involve flux tubes which wrap the compactified dimension, with $E = L\sigma$ with σ the string tension. $\longrightarrow$ "Confinement of static quarks by flux tubes".

Contrast between CFC and center

- Of course with $n_f \neq 0$ center is broken explicitly $\longrightarrow$ local operators ccontribute to the correlator $\longrightarrow$ string breaking : "deconfinement".

- Local order parameters transforming under CFC exist even when CFC is not broken explicitly or spontaneously.

- For example, states created by $\mathcal{A} \equiv \sum_{a=1}^{n_\mathrm{f}} \nu^{-ap}\, \bar{q}_a \gamma_1 D_1 q_a$ and $\mathcal{B} \equiv \sum_{a=1}^{n_\mathrm{f}} \nu^{-ap}\, \bar{q}_a \gamma_1 q_a$ can contribute to correlators of $\mathrm{Re}\,\mathrm{tr}\,\Omega^p$ and $\mathrm{Im}\,\mathrm{tr}\,\Omega^p$, respectively.

- In summary, for $n_\mathrm{f} > 0$ No relation between the presence of a non-zero string tension and the existence, or realization, of CFC.

Summary

- Well-defined and non-trivial order parameters for quantum and thermal phase transitions exists in QCD compactified on a circle with degenerate massive flavors of quarks.
- A calculation of the one-loop gluon contribution to $V_{\text{eff}}(\Omega)$ in the hard dense loop approximation would give a better estimate for the CFC symmetry restoration temperature $T_{\text{CFC}}(\mu)$.
- The role of explicit $SU(3)_V$ symmetry breaking should be explored.
- It would be interesting to study local order parameters for CFC symmetry.

references

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