Instanton-dyon ensembles reproduce deconfinement and chiral restoration phase transitions

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CPOD Stony Brook
Aug.7-11,2017

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Outline

- **Instanton-dyons and their ensembles in QCD-like theories**

- ***analytic (mean field) approach for dense ensemble (T<Tc) (1503.03058, 1503.09148 with Lui and Zahed)***

- **numerical studies at all densities: deconfinement (1504.03341 with Larsen) and chiral restoration (Nc=Nf=2)**

- **both transitions were found to be strongly influenced by (flavor dependent) quark periodicity phases (imaginary chem.potetials) that it nearly uniquely fixes the mechanism (1605.07474 with Larsen)**

1998

Instantons => Nc selfdual dyons (KvBLL, Pierre van Baal legacy)

<P> nonzero Polyakov line

=> <A_4>=v is non-zero

=> new solutions

Instanton liquid
4d+short range

→

Dyonic plasma
3+1d long range

instanton-dyons in SU(2)

name	E	M	mass
M	+	+	v
$\bar{M}$	+	-	v
L	-	-	$2\pi T - v$
$\bar{L}$	-	+	$2\pi T - v$

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

calorons=M+L are E and M neutral

the color circle
for any Nc

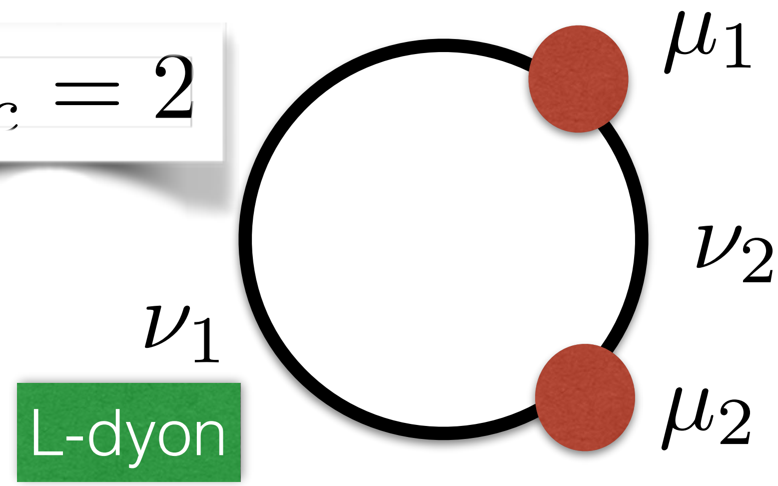
holonomy pameters

$$A_4(\infty) = 2\pi T \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_N),$$

$$\mu_1 \le \mu_2 \le \ldots \mu_N \le \mu_1 + 1, \quad \sum_{m=1}^{N} \mu_m = 0.$$

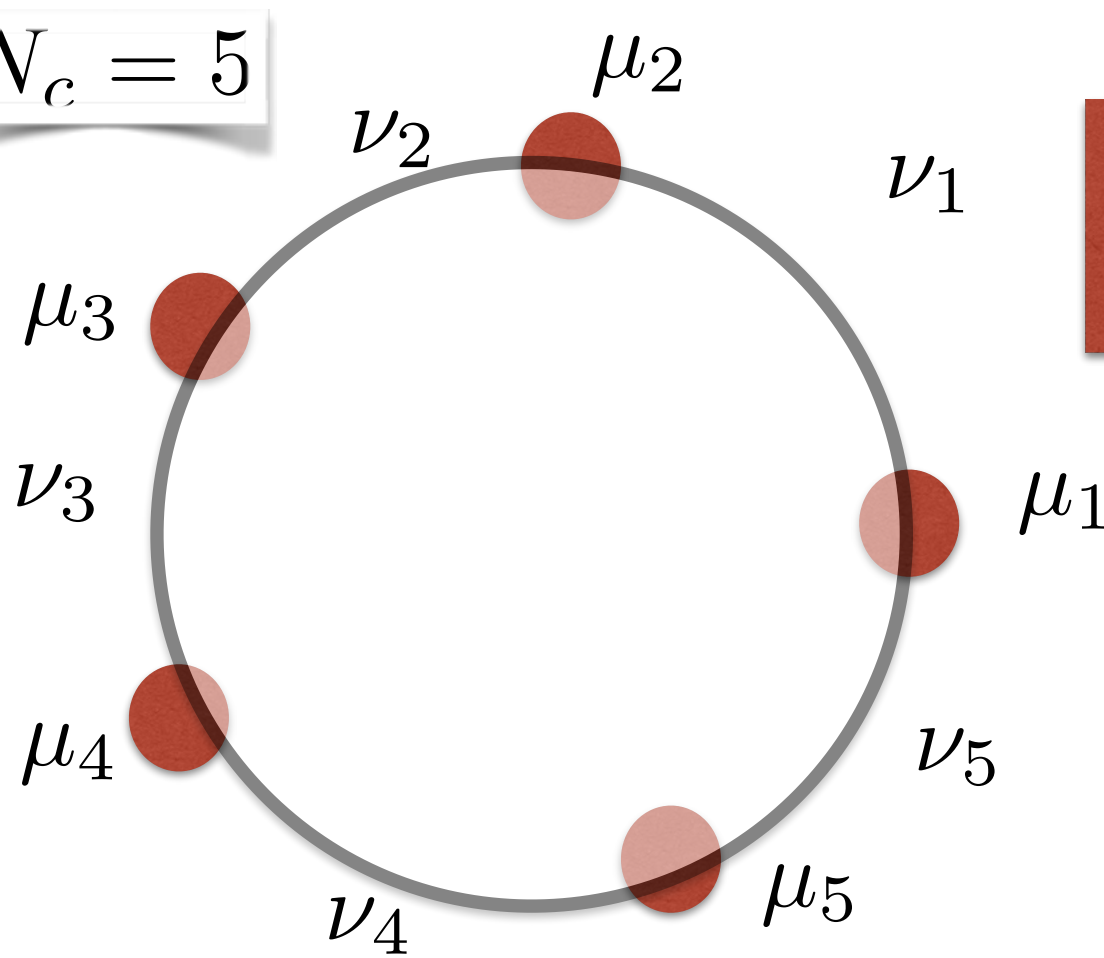
$$\nu_m \equiv \mu_{m+1} - \mu_m, \quad \sum_{m=1}^{N} \nu_m = 1.$$

$N_c = 2$



M dyon
at trivial field mu_i->0 gets massless

$N_c = 5$



all nu's fill the circle
sum of dyon masses
makes full instanton

Interacting Ensemble of the Instanton-dyons and Deconfinement Phase Transition in the SU(2) Gauge Theory

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Instanton-dyons, also known as instanton-monopoles or instanton-quarks, are topological constituents of the instantons at nonzero temperature and holonomy. We perform numerical simulations of the ensemble of interacting dyons for the SU(2) pure gauge theory. Unlike previous studies, we focus on back reaction on the holonomy and the issue of confinement. We calculate the free energy as a function of the holonomy and the dyon densities, using standard Metropolis Monte Carlo and integration over parameter methods. We observe that as the temperature decreases and the dyon density grows, its minimum indeed moves from small holonomy to the value corresponding to confinement. We then report various parameters of the self-consistent ensembles as a function of temperature, and investigate the role of inter-particle correlations.

Like in [12], instead of toroidal box with periodic boundary conditions in all coordinates, our simulations have been done on a S^3 sphere (in four dimensions), to simplify treatment of the long range Coulombic forces.

The partition function we simulate depends on several parameters, changed from one simulation set to another. Those include (i) the number of the dyons N_M, N_L; (ii) the radius of the S^3 sphere r; (iii) the action parameter S; (iv) the value of the holonomy ν, (v) the value of the Debye mass M_D; (vi) the auxiliary factor λ, which is then integrated over as explained in section IV.

$$f = \frac{4\pi^2}{3}\nu^2\bar{\nu}^2 - 2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{\bar\nu} e}{n_L}\right] + \Delta f$$

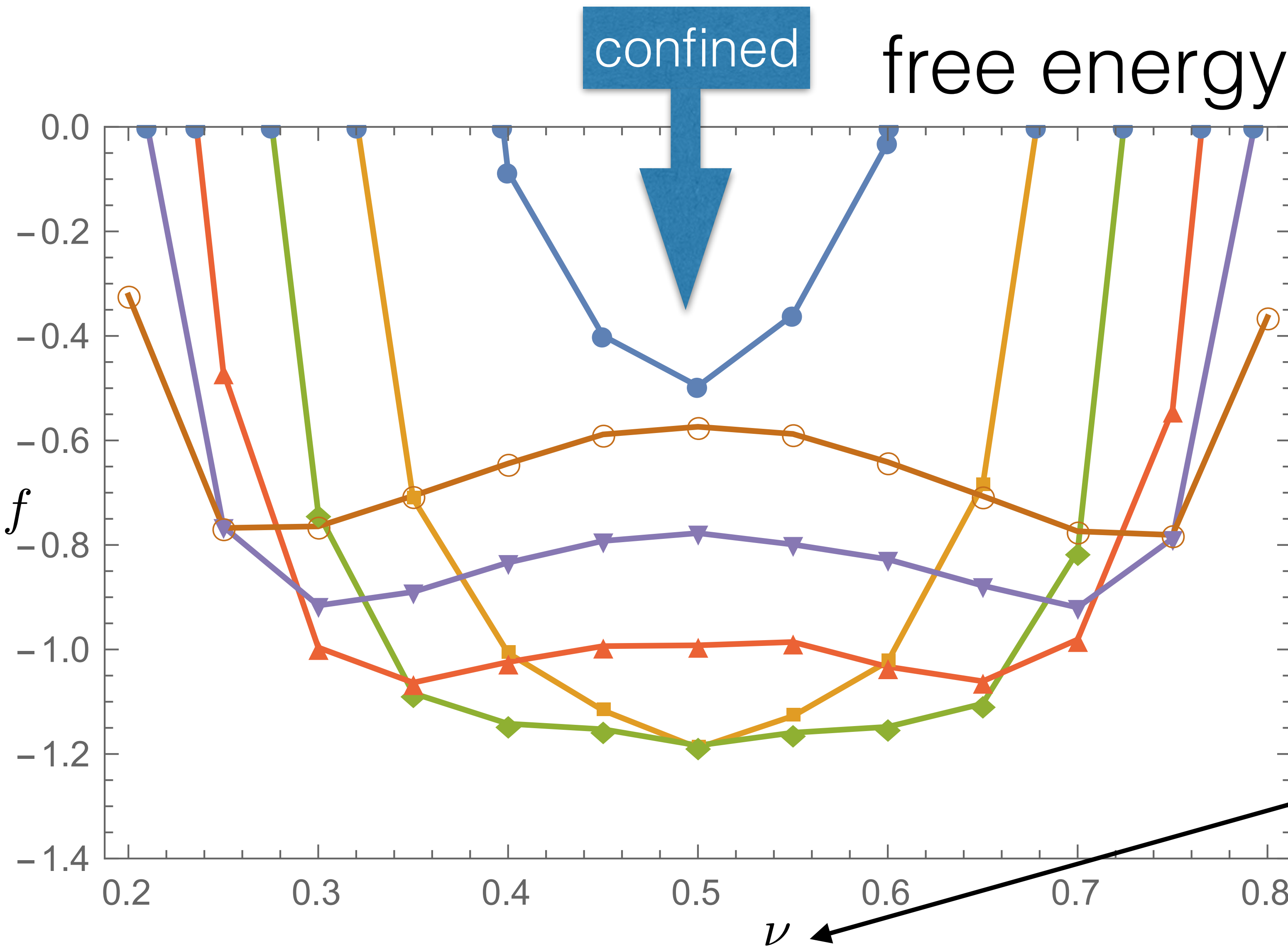
Gross-Pisarski-Yaffe perturbative term +free dyons+ interaction

$\nu = 0$ *is the trivial case*

$\nu = 1/2$ *confining*

Larsen's talk at parallel session tuesday afternoon

free energy vs holonomy



$$\langle A_4^3 \rangle = v\frac{\tau^3}{2} = 2\pi T \nu \frac{\tau^3}{2}$$

$$< P >= cos(\pi\nu) \to 0$$

$$if\ \nu = 1/2$$

holonomy

$$\nu = 0\ is\ the\ trivial\ case$$

$$\nu = 1/2\ confining$$

So, as a function of the dyon density the potential changes its shape and confinement takes place

show only the “selfconsistent” input set.

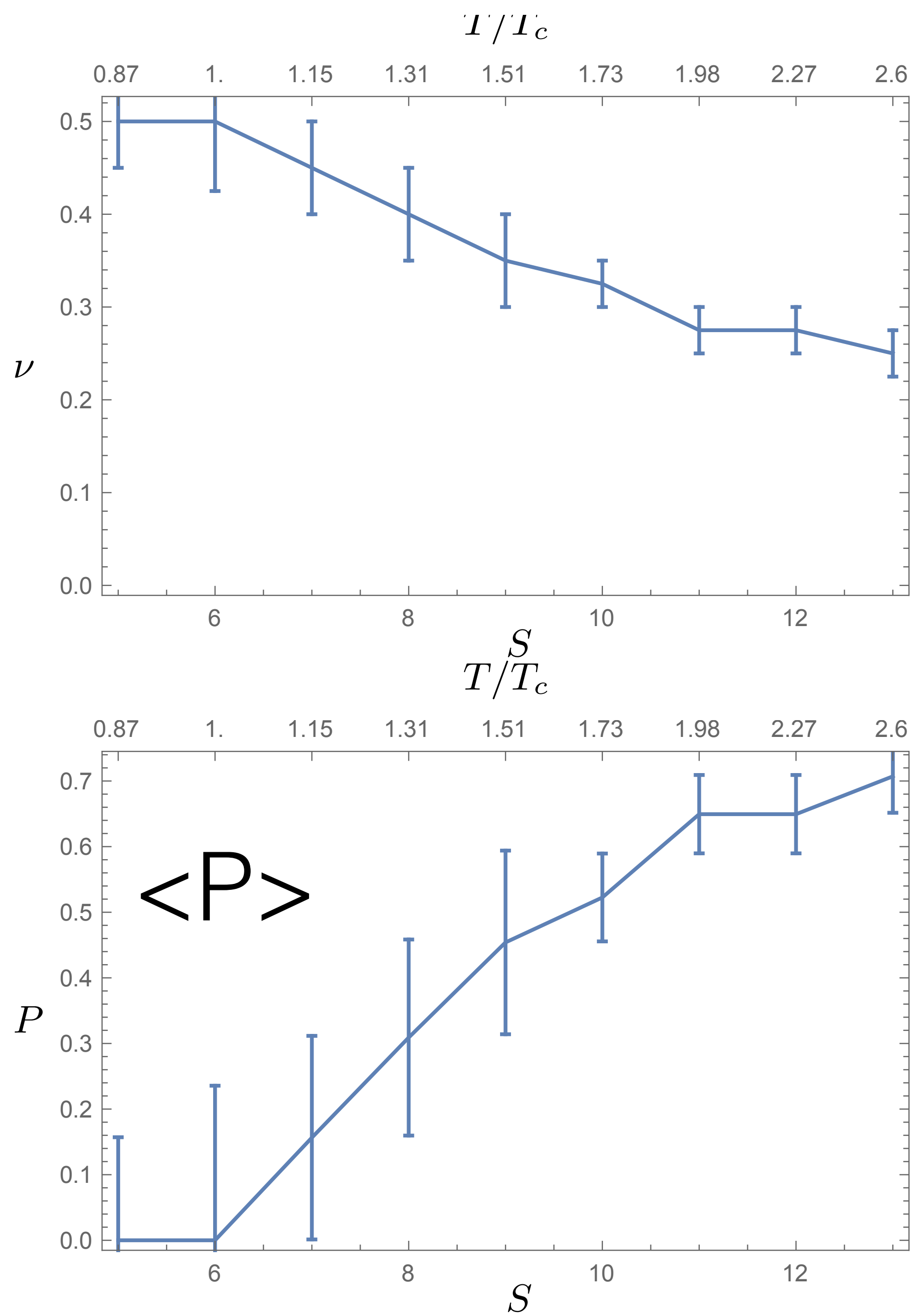


FIG. 6: Self-consistent value of the holonomy ν (upper plot) and Polyakov line (lower plot) as a function of action S (lower scales), which is related to T/T_c (upper scales). The error bars are estimates based on the fluctuations of the numerical data.

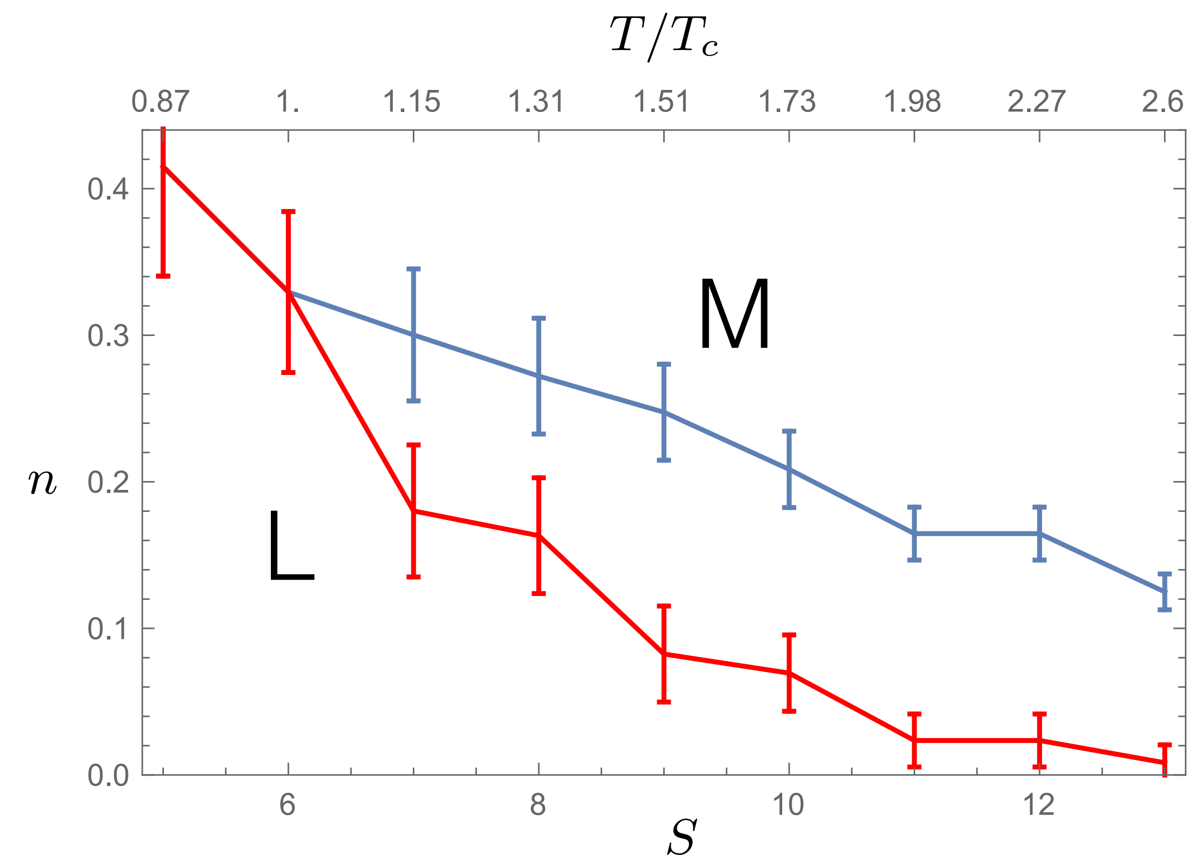


FIG. 8: (Color online). Density n (of an individual kind of dyons) as a function of action S (lower scale) which is related to T/T_c (upper scale) for M dyons(higher line) and L dyons (lower line). The error bars are estimates based on the density of points and the fluctuations of the numerical data.

confining phase is symmetric
$n_L = n_M$

$$S = \left(\frac{11N_c}{3} - \frac{2N_f}{3}\right) log\left(\frac{T}{\Lambda_T}\right).$$

Light Quarks in the Screened Dyon-Anti-Dyon Coulomb Liquid Model II

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(Dated: April 1, 2015)

We discuss an extension of the dyon-anti-dyon liquid model that includes light quarks in the dense center symmetric Coulomb phase. In this work, like in our previous one, we use the simplest color SU(2) group. We start with a single fermion flavor $N_f = 1$ and explicitly map the theory onto a 3-dimensional quantum effective theory with a fermion that is only $\mathrm{U}_V(1)$ symmetric. We use it to show that the dense center symmetric plasma develops, in the mean field approximation, a nonzero chiral condensate, although the ensuing Goldstone mode is massive due to the $\mathrm{U}_A(1)$ axial-anomaly. We estimate the chiral condensate and σ, η meson masses for $N_f = 1$. We then extend our analysis to several flavors $N_f > 1$ and colors $N_c > 2$ and show that center symmetry and spontaneous chiral symmetry breaking disappear simultaneously when $x = N_f/N_c \geq 2$ in the dense plasma phase. A reorganization of the dense plasma phase into a gas of dyon-antidyon molecules restores chiral symmetry, but may preserve center symmetry in the linearized approximation. We estimate the corresponding critical temperature.

The main issue discussed in this paper is the behavior (pairing or collectivization) of the fermionic zero modes into what is called in the literature the "Zero Mode Zone" (ZMZ). The approximations used in its description follows closely the construction, developed for instantons and described in detail in refs [14]. The fermionic determinant can be viewed as a sum of closed fermionic loops connecting all dyons and antidyons. Each link – or hopping – between L-dyons and $\bar{\mathrm{L}}$-anti-dyons is described by the elements of the "hopping chiral matrix" $\tilde{\mathbf{T}}$

$$\tilde{\mathbf{T}}(x,y) \equiv \begin{pmatrix} 0 & \mathbf{T}_{ij} \\ -\mathbf{T}_{ji} & 0 \end{pmatrix} \tag{9}$$

with dimensionality $(K_L + K_{\bar{L}})^2$. Each of the entries in $\mathbf{T}_{ij}$ is a "hopping amplitude" for a fermion between an L-dyon and an $\bar{\mathrm{L}}$-anti-dyon, defined via the zero mode φ_D of the dyon and the zero mode $\varphi_{\bar{D}}$ (of opposite chirality) of the anti-dyon

$$\mathbf{T}_{ij} \equiv \mathbf{T}(x_i - y_j) = \int d^4z\, \varphi^\dagger_{\bar{D}}(z - x_i) i(\gamma\cdot\partial)\varphi_D(z - y_j) \tag{10}$$

$$\int \frac{d^3p}{(2\pi)^3}\frac{M^2(p)}{p^2+M^2(p)} = \frac{n_D}{4}$$

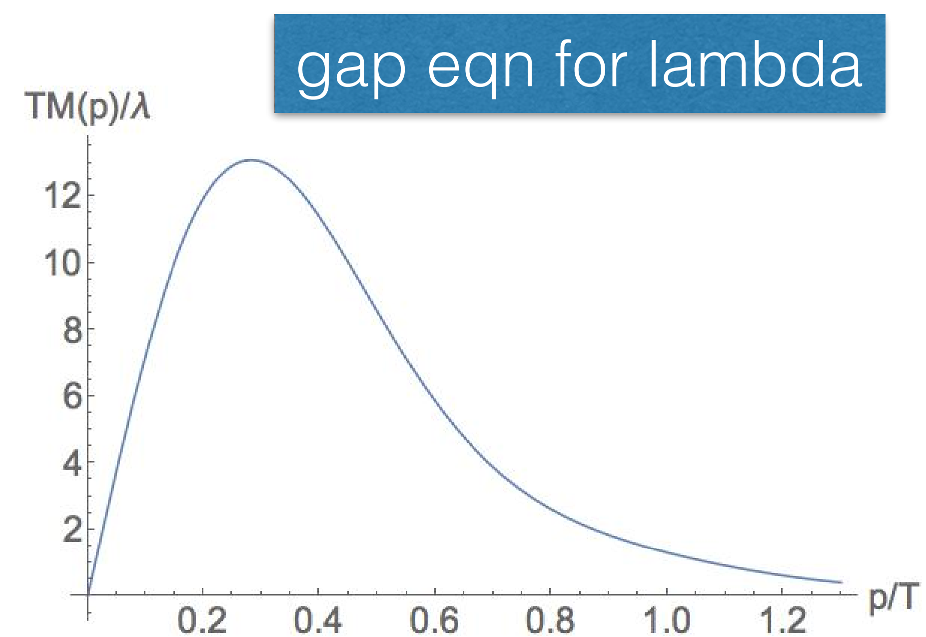


FIG. 1: The momentum dependent quark constituent mass $TM(p)/\lambda$ versus p/T.

chiral symmetry breaking for different Nf

Nc=2,Nf=1 solution is studied in detail

$$\frac{|\langle\bar{q}q\rangle|}{T^3} \approx 1.25\left(\frac{n_D}{T^3}\right)^{1.63}$$

For general $x = N_f/N_c$, the saddle point equation in Σ of (85) gives

$$\Sigma = \left(\frac{\tilde{\lambda}}{2x\alpha(N_c)}\right)^{\frac{1}{x-1}} \tag{88}$$

after the shift $-i\lambda \to \lambda$ and $\tilde{\lambda} = N_f\lambda$. With this in mind and inserting (88) into (85) yields

$$-\mathcal{V}/\mathbb{V}_3 = -2\alpha(N_c)\,(x-1)\left(\frac{\tilde{\lambda}}{2x\alpha(N_c)}\right)^{\frac{x}{x-1}} + xN_c\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\frac{\tilde{\lambda}^2}{N_f^2}\mathbf{T}^2(p)\right) \tag{89}$$

The effective potential (89) has different shapes depending on the ratio of the number of flavors to the number of colors x. Let us explain that in details for four cases:

(i) If $x < 1$ the first term in (89) has a positive coefficient and a negative power, so it is decreasing at small $\tilde{\lambda}$. At large value of $\tilde{\lambda}$ the second term is growing as $\ln\tilde{\lambda}$. Thus a minimum in between must exist. This minimum is the physical solution we are after.

(ii) If $1 < x < 2$ the coefficient of the first term is negative but its power is now positive. So again there is a decrease at small $\tilde{\lambda}$ and thus a minimum.

(iii) If $x > 2$ the leading behavior at small $\tilde{\lambda}$ is now dominated by the second term which goes as $\tilde{\lambda}^2$ with positive coefficient. One may check that the potential is monotonously increasing for any $\tilde{\lambda}$ with no extremum. There is no gap equation, which means chiral symmetry cannot be broken in the mean-field approximation.

(iv) If $x = 2$ there are two different contributions of opposite sign to order $\tilde{\lambda}^2$ at small $\tilde{\lambda}$. An extremum forms only if the following condition is met

$$\int\frac{d^3p}{(2\pi)^3}\mathbf{T}^2(p) < \frac{N_c}{4\alpha(N_c)} = \mathcal{O}\left(\frac{1}{N_c}\right) \tag{90}$$

Using the exact form (13) and the solution to the gap equation at $T = T_0$, we have

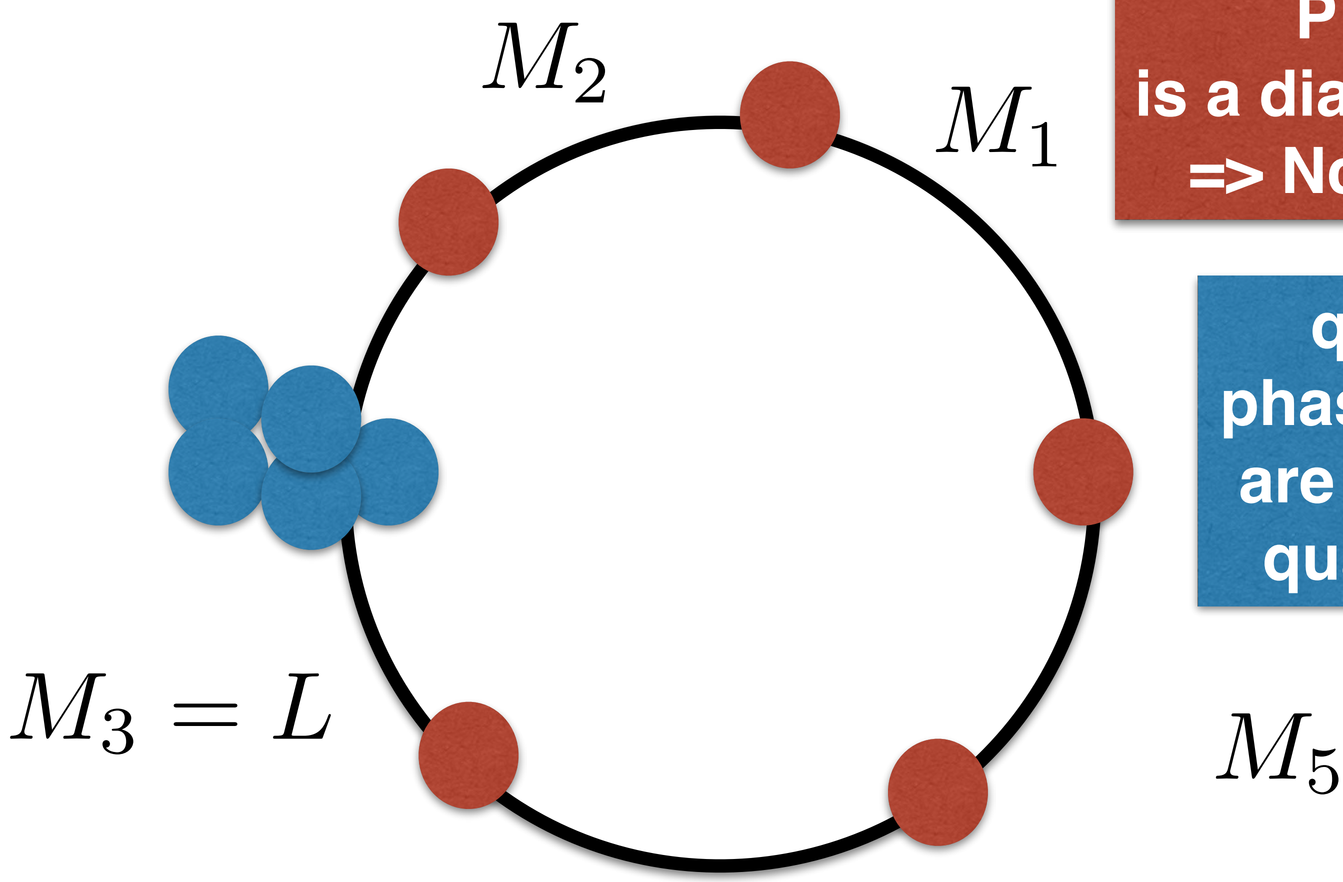
$$\int\frac{d^3p}{(2\pi)^3}\mathbf{T}^2(p) = \frac{10.37}{T_0} \tag{91}$$

which shows that (90) is in general upset, and this case does *not* possess a minimum.

Critical Nf/Nc=2 for mean field treatment

lattice: Nc=3,Nf=4 broken, NF=8 probably not

Ordinary Nc=Nf=5 QCD



P without a trace
is a diagonal unitary matrix
=> Nc phases (red dots)

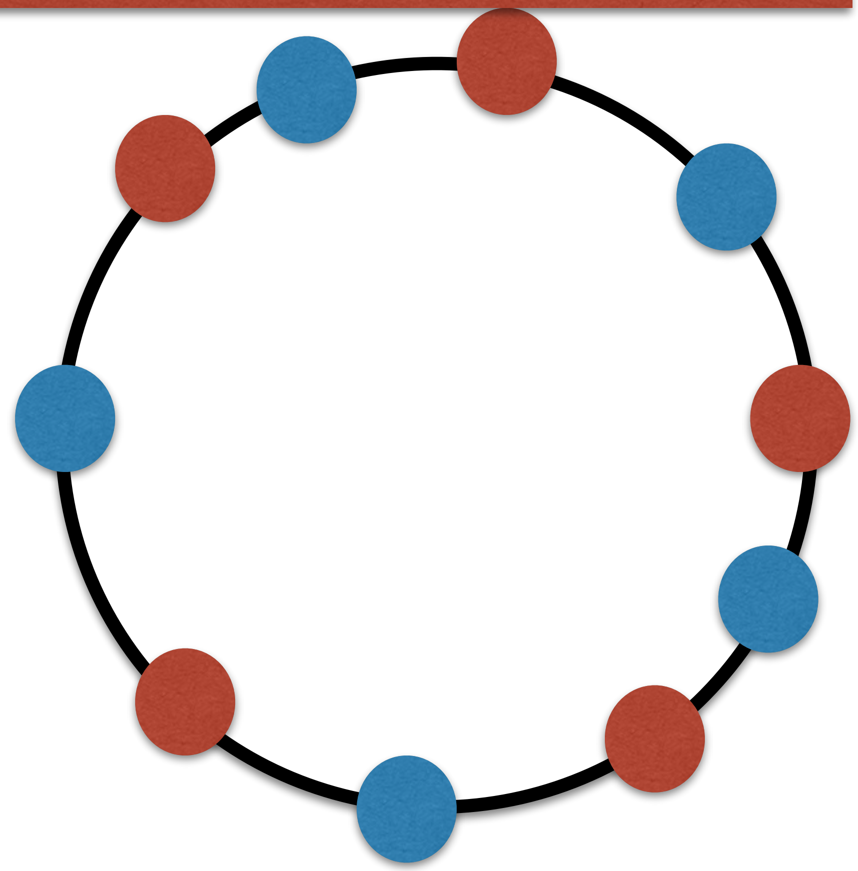
quark periodicity
phases => Nf blue dots
are in this case all =pi
quarks are fermions

as a consequence,
out of 5 types of instanton-dyons
only L has zero modes

still Nc=Nf=5 but with
“most democratic” arrangement
ZN-symmetric QCD

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki and M. Yahiro, J. Phys. G 39, 085010 (2012).

the idea: quarks can be
not fermions but “anyons”



quark periodicity
phases => Nf blue dots
are in this case
flavor-dependent
(but no connection to
instanton-dyons in
this work, but PNJL)

In this case each dyon type has
one zero mode
with one quark flavor
=>N independent topological ZMZ’s!

Instanton-dyon Ensemble with two Dynamical Quarks: the Chiral Symmetry Breaking

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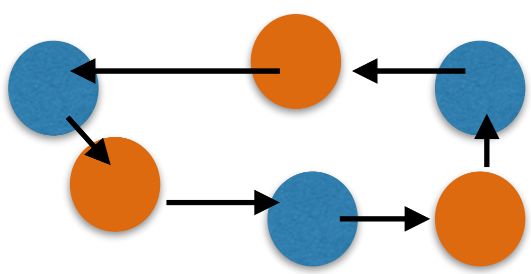
This is the second paper of the series aimed at understanding of the ensemble of the instanton-dyons, now with two flavors of light dynamical quarks. The partition function is appended by the fermionic factor, $(detT)^{N_f}$ and Dirac eigenvalue spectra at small values are derived from the numerical simulation of 64 dyons. Those spectra show clear chiral symmetry breaking pattern at high dyon density. Within current accuracy, the confinement and chiral transitions occur at very similar densities.

$$|<\bar{\psi}\psi>| \;=\; \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty}$$



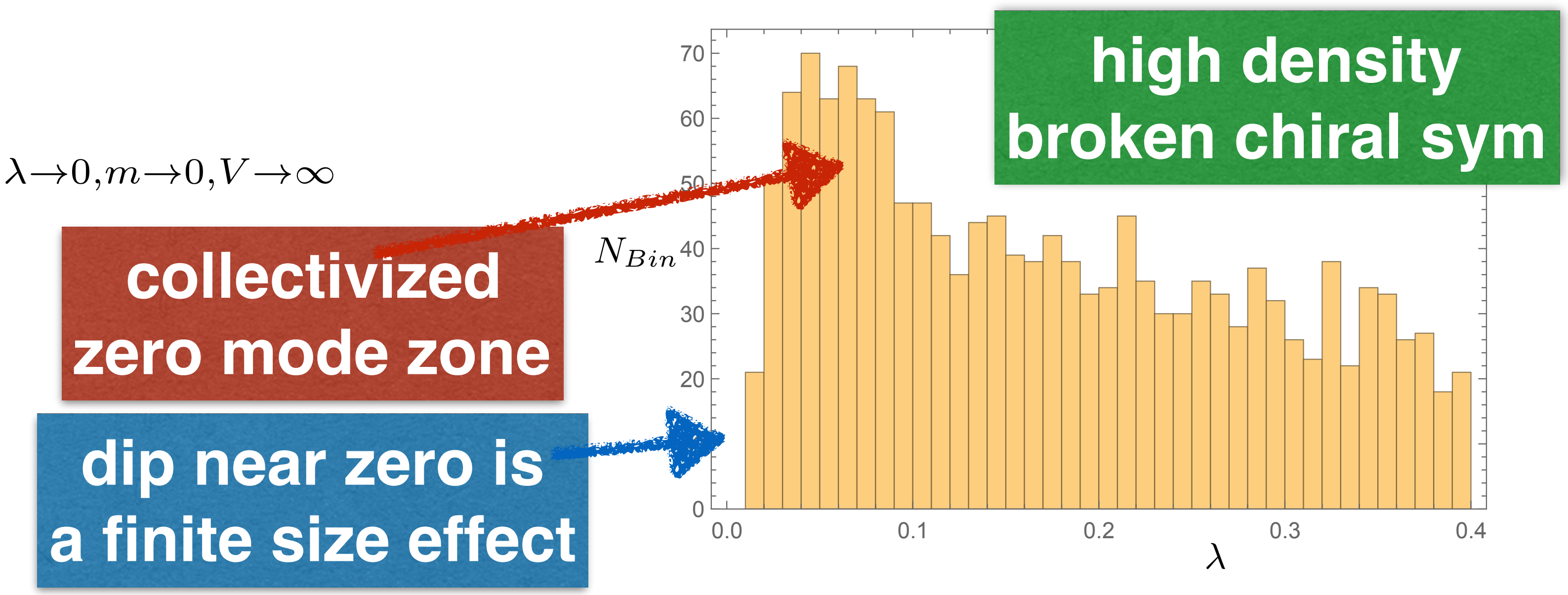


FIG. 1: Eigenvalue distribution for $n_M = n_L = 0.47$, $N_F = 2$ massless fermions.

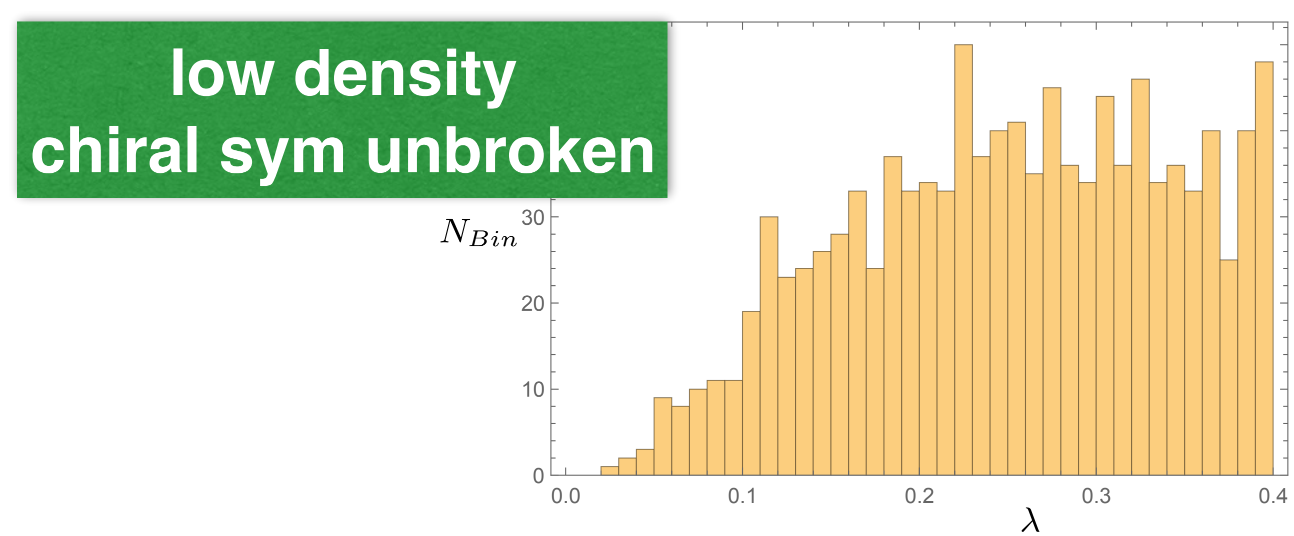
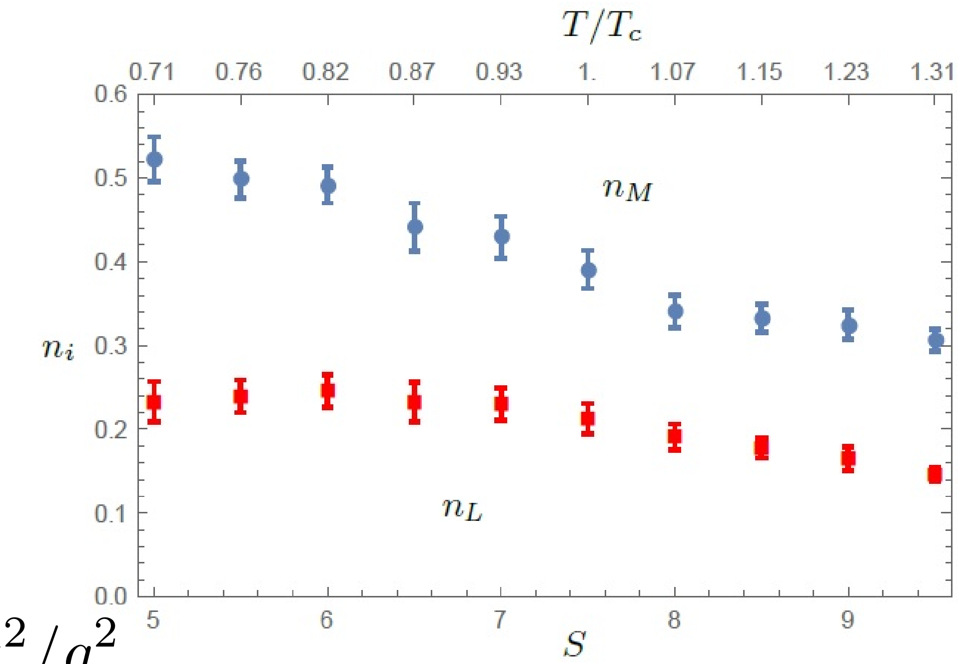


FIG. 2: Eigenvalue distribution for $n_M = n_L = 0.08$, $N_F = 2$ massless fermions.

We find that the required condition for both the chiral symmetry breaking and confinement is basically sufficiently high density of the dyons.



$$S = 8\pi^2/g^2$$

FIG. 9: (Color online) Parameterization A: The density of the M (blue circles) and L (red squares) dyons as a function of action $S = 8\pi^2/g^2$ or temperature T/T_c.

Furthermore,
unlike in the case of pure gauge theory without quarks,
the holonomy dependence on the density is smoother.
We don’t observe holonomy vanishing,
and also the densities of the M and L type dyons does not become equal,
even at the lowest T we studied.

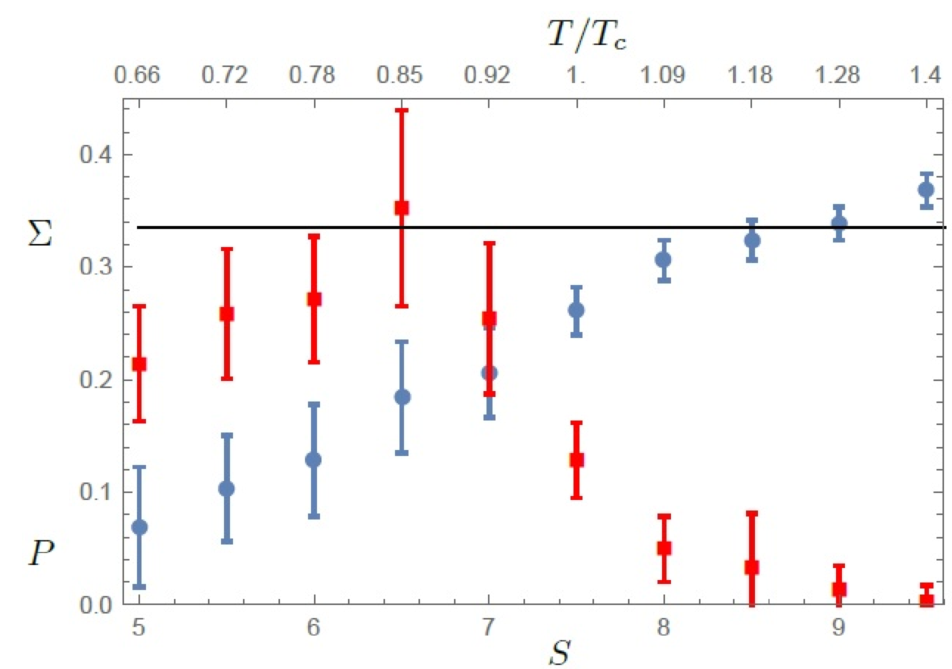
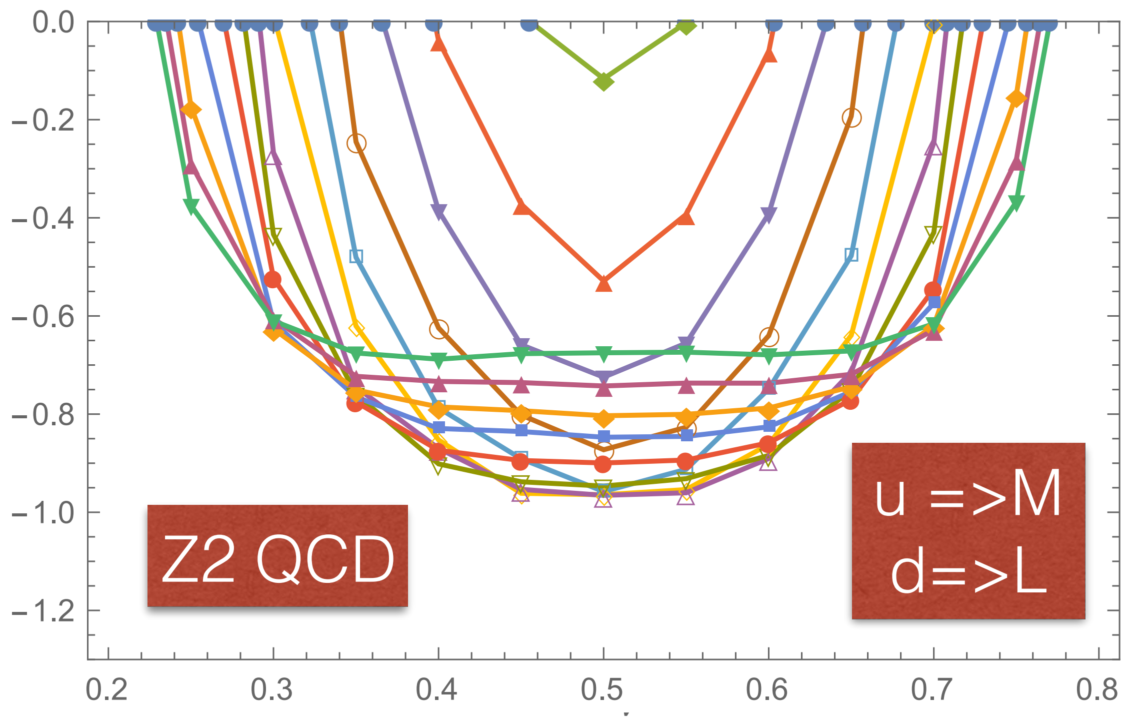


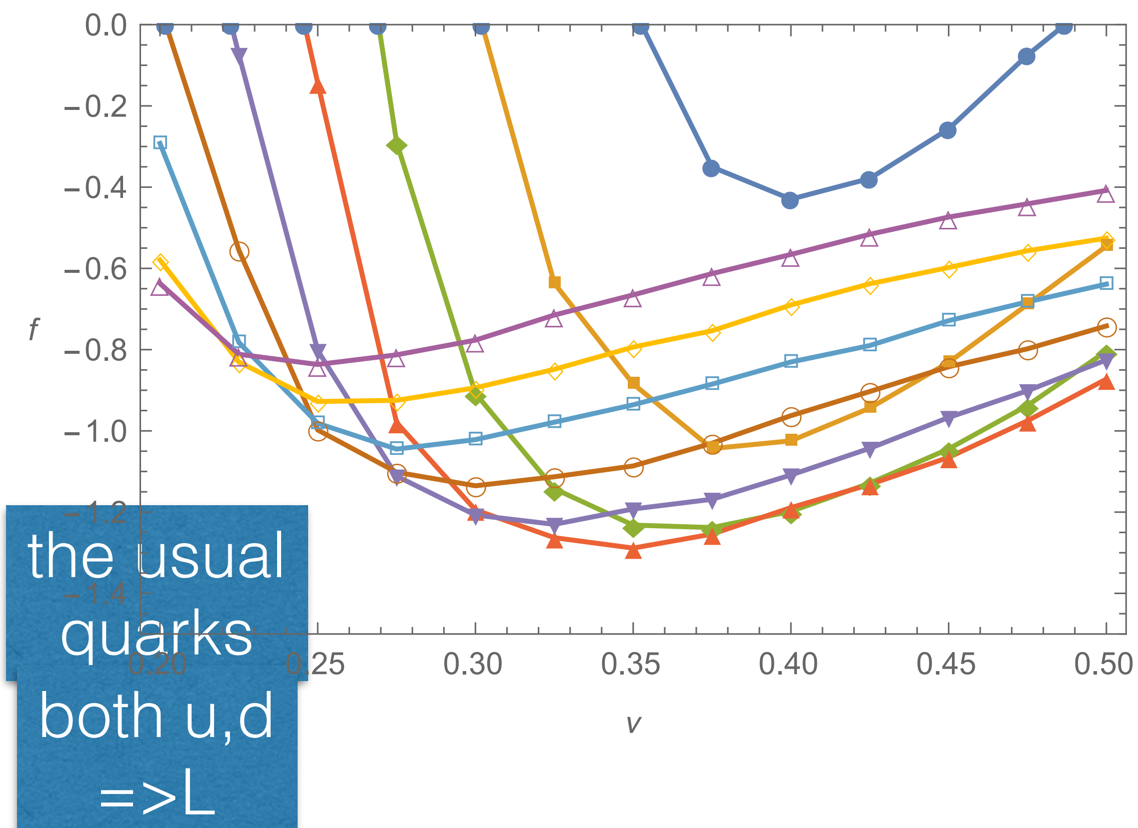
FIG. 10: (Color online) Parameterization A: The Polyakov loop P (blue circles) and the chiral condensate Σ (red squares) as a function of action $S = 8\pi^2/g^2$ or temperature T/T_c. A clear rise is seen around $S = 7.5$ for the chiral condensate. Σ is scaled by 0.2. The black constant line corresponds to the upper limit of Σ under the assumption that the entire eigenvalue distribution belong to the almost-zero-mode zone, i.e. the maximum of Σ_2.

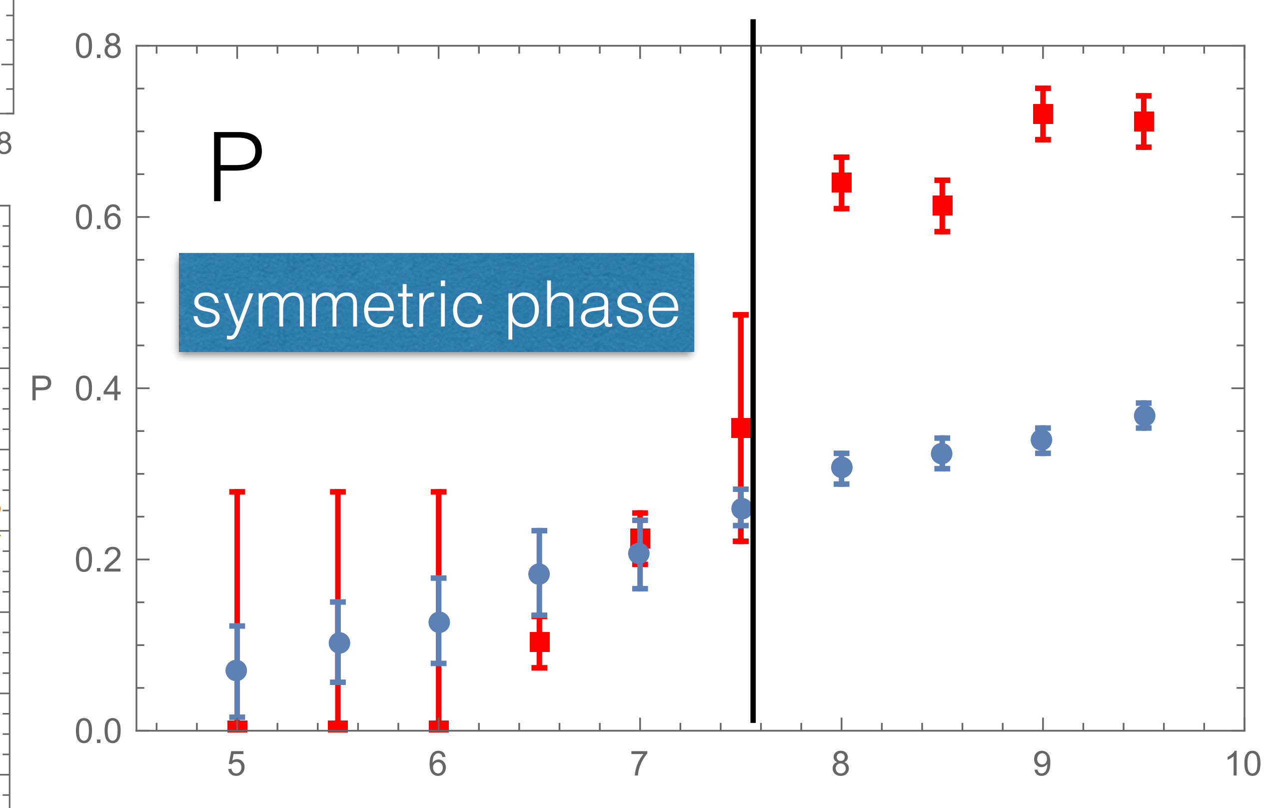
Instanton-dyon Ensembles III: Exotic Quark Flavors

Rasmus Larsen and Edward Shuryak



confining phase
gets much more
robust:
transition strong first order
mixed phase (flat F)
is observed at medium densities





$$S = \left(\frac{11N_c}{3} - \frac{2N_f}{3}\right)log\left(\frac{T}{\Lambda_T}\right).$$

chiral symmetry breaking is dramatically different

symmetric phase

u=>M

d=>L

$$< \bar{u}u > \neq < \bar{d}d >$$

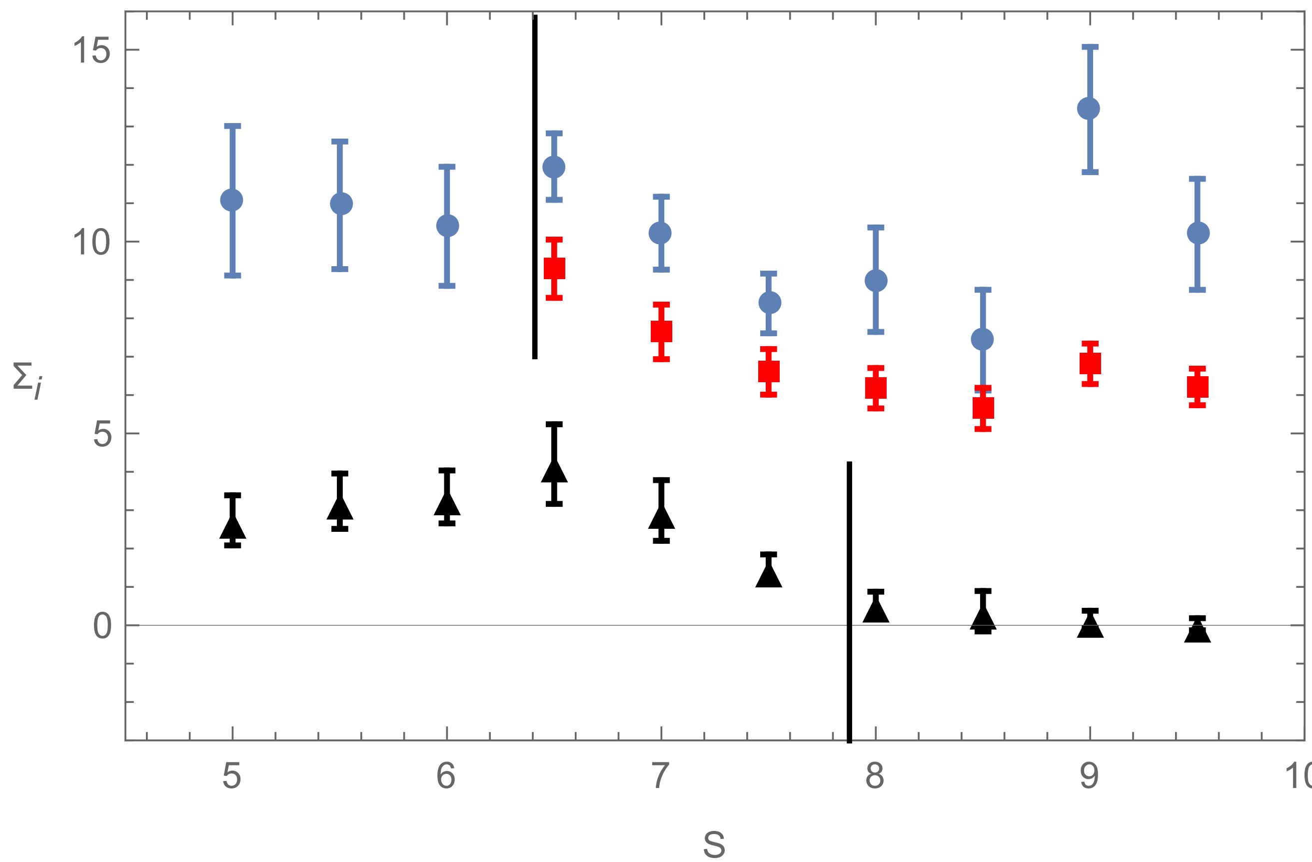


FIG. 6: Chiral condensate generated by u quarks and L dyons (red squares) and d quarks interacting with M dyons (blue circles) as a function of action S, for the Z_2-symmetric model. For comparison we also show the results from II for the usual QCD-like model with $N_c = N_f = 2$ by black triangles.

Z2 QCD

has symmetric and asymmetric phases yet apparently no chiral symmetry restoration at any T

the usual QCD has chiral restoration

why can the quark condensate be much larger for Z2?

the first lattice study of Z3 QCD

Lattice study on QCD-like theory with exact center symmetry

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arXiv:1508.07132v3 [hep-lat] 5 Nov 2015

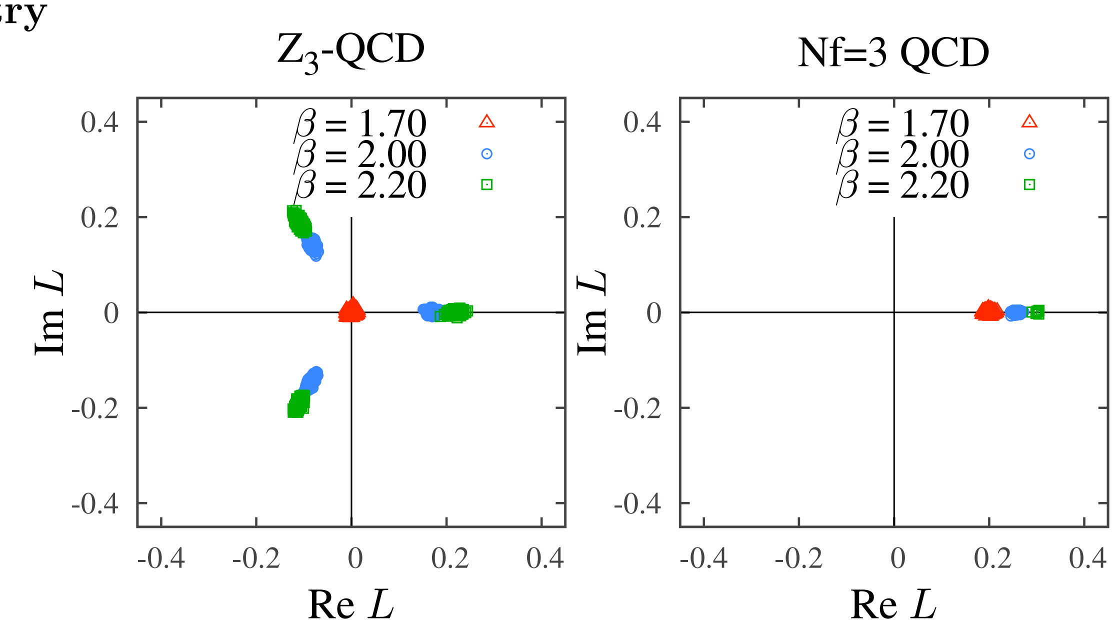
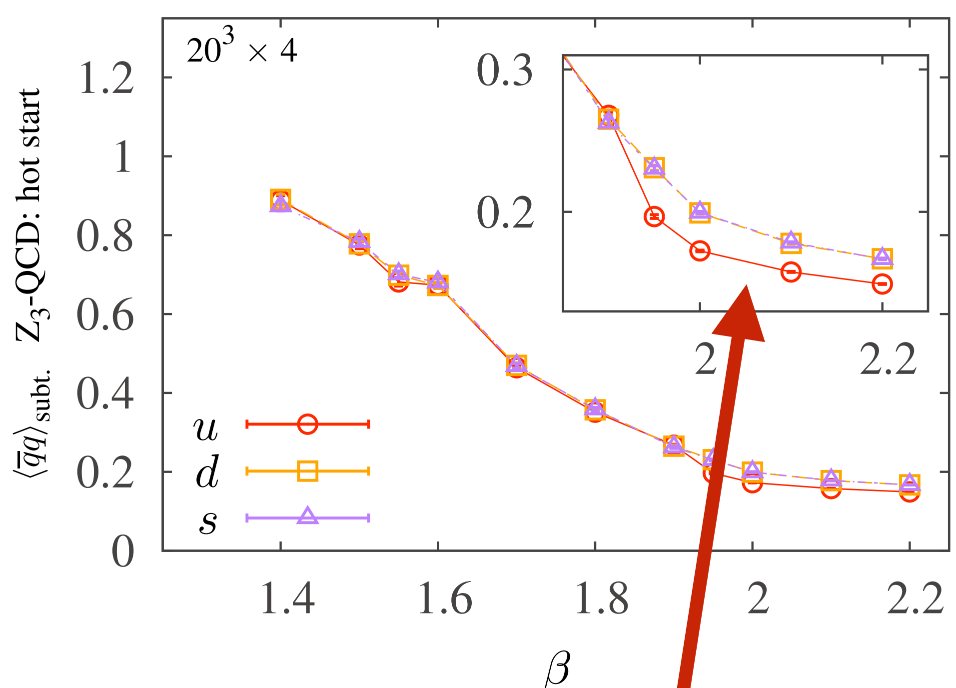
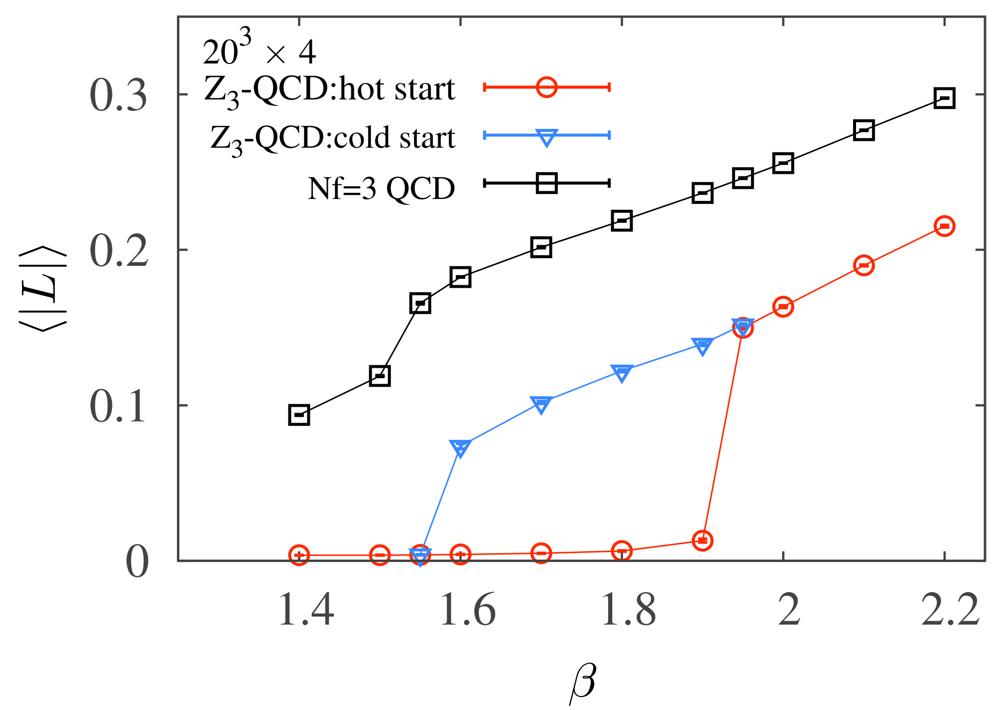


FIG. 1: Polyakov loop distribution plot in Z_3-QCD (left) and the standard three-flavor QCD (right). Based on $16^3 \times 4$ lattice for $\beta = 1.70, 2.00, 2.20$ with the same values of κ in both panels.





explanation: three flavors of quarks interact with three different ``liquids'' of M1,M2,L instanton-dyons!

Summary

Instanton-dyon ensembles:
in QCD-like theories the deconfinement
and chiral transitions
are driven just by sufficiently large dyon density
=> quasicritical Tdec and Tchir are about the same

But this changes in theories with
unusual fermions.
Nontrivial flavor holonomies
(phases in boundary conditions)
dramatically change both deconfinement
and chiral transitions:
interesting dependences seen.
It is an excellent tool to fix the microscopic mechanism

Yet direct identification
of the instanton-dyons
on the lattice,
study of their density etc are
still badly needed