Recent lattice QCD results at non-zero baryon densities

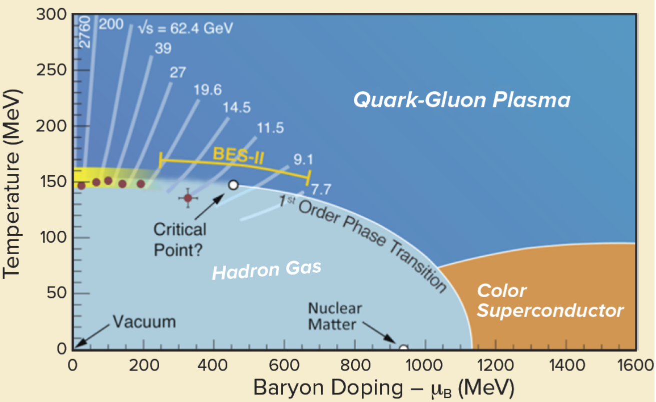
Jana Günther

August 7th 2017

SFB TR55

WB collaboration

The (T, μ_B)-phase diagram of QCD



Our observables:
T_c, Equation of state, Fluctuations

The sign problem

The QCD partition function:

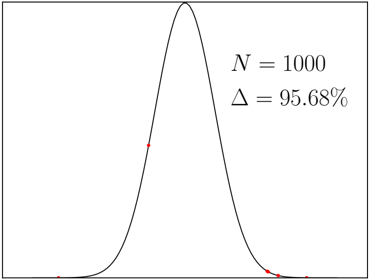
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \det M(U) e^{-\beta S_G(U)}$$

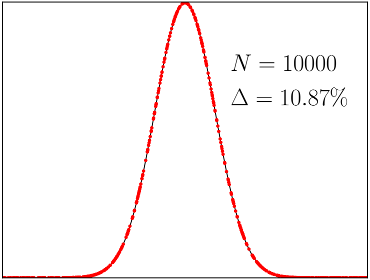
- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}}\cdot\frac{200}{N}$$

The x_i are drawn from a uniform distribution in the interval $[-100, 100]$

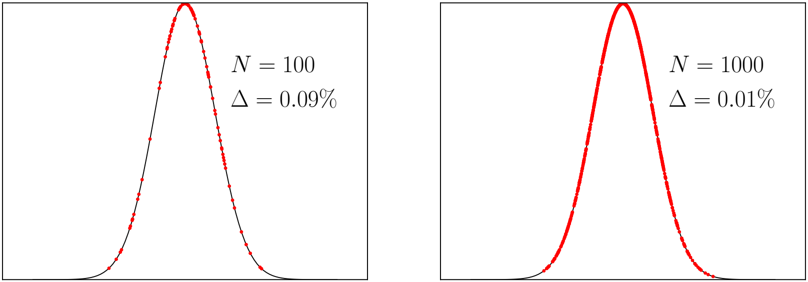




Importance sampling

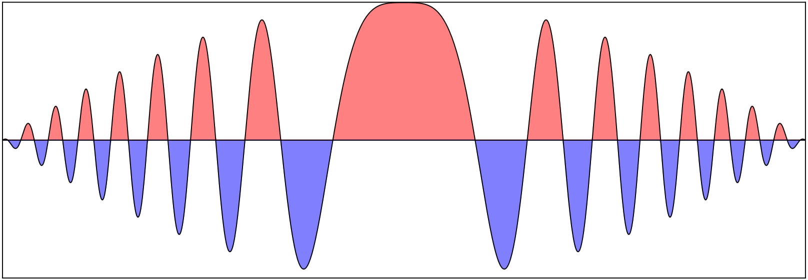
$$\int_{-\infty}^{\infty} (100 - x^2) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N} (100 - x_i^2) \cdot \frac{1}{N}$$

The x_i are drawn from a normal distribution



The sign problem

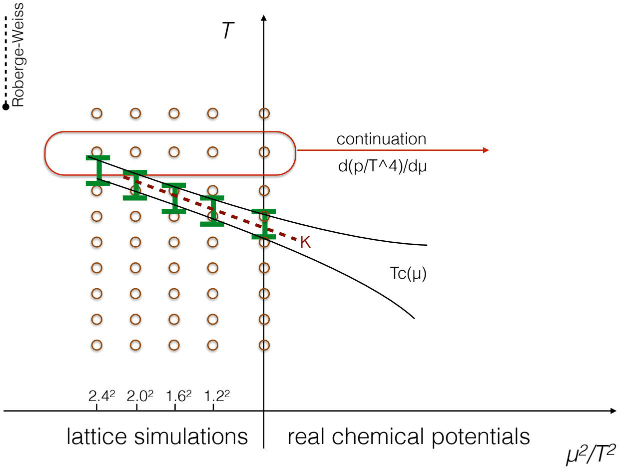
$$\int_{-\infty}^{\infty}(100 - x^2)\frac{e^{-\frac{\mathrm{i}}{2}x^2}}{\sqrt{2\pi}}$$



Dealing with the sign problem

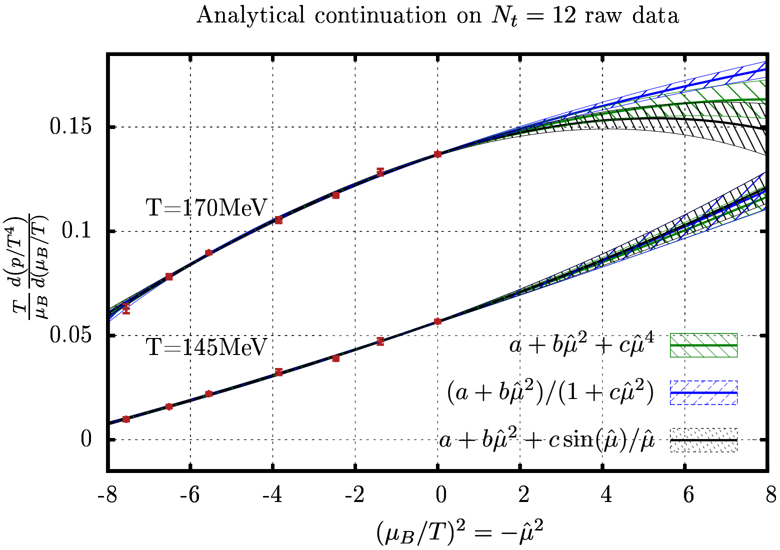
- Reweighting technics
- Canonical ensemble
- Complex Langevin
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Taylor expansion $\longrightarrow$ Talk by C. Schmidt this morning
- Imaginary μ
- ...

Analytic continuation



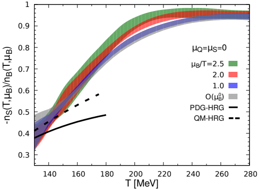
Common technique: [de Forcrand, Philipsen, deForcrand:2002hgr], [Bonati et al., Bonati:2015bha], [Cea et al., Cea:2015cya], [D'Elia et al., DElia:2016jqh] . . .

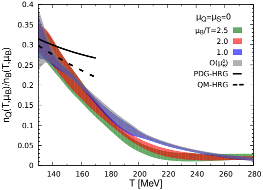
Different functions



Strangeness Neutrality

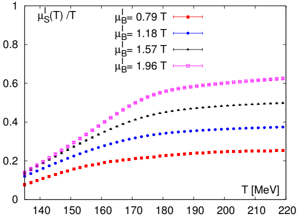
In heavy ion collisions: $\langle n_S \rangle = 0$ and $0.4\langle n_B \rangle \approx \langle n_Q \rangle$
Simulations with $\mu_S = \mu_Q = 0$ lead to:

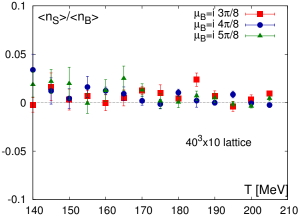




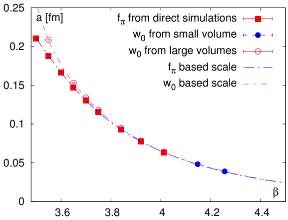
[Bazavov et al., Bazavov:2017dus]

Therefore tuning/extrapolating to $\langle n_S \rangle = 0$ and $0.4\langle n_B \rangle \approx \langle n_Q \rangle$





Simulation details

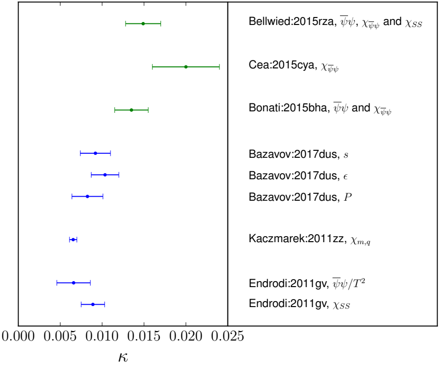


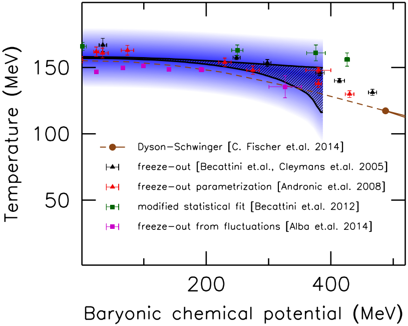
- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, in contrast to simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $(32^3 \times 8)$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5$ and 6
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

T_c

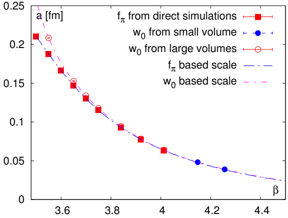
Curvature function:
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$
For error analysis we also fit:

$$C_1(x) = 1 + ax + bx^2$$
$$C_2(x) = \frac{1+ax}{1+bx}$$
$$C_3(x) = \frac{1}{1+ax+bx^2}$$





Simulation details



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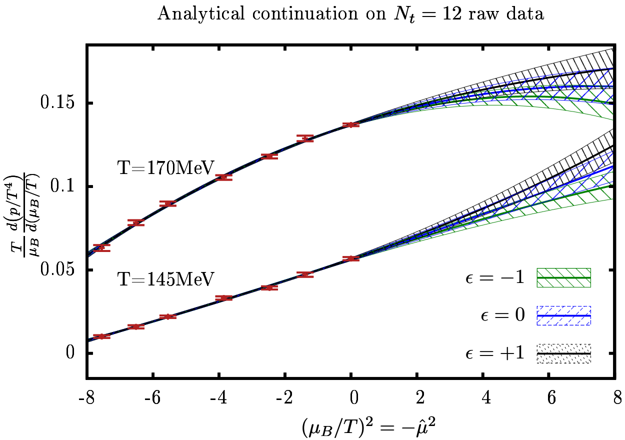
- Lattice sizes: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5, 6,$ 6.5 and 7
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

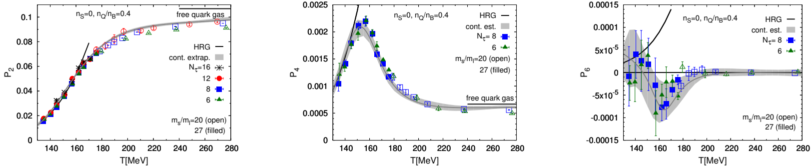
Different functions

Condition: $\chi_8 \lesssim \chi_4 \longrightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$

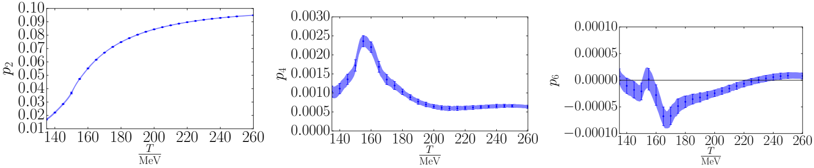


Taylor coefficients of the pressure

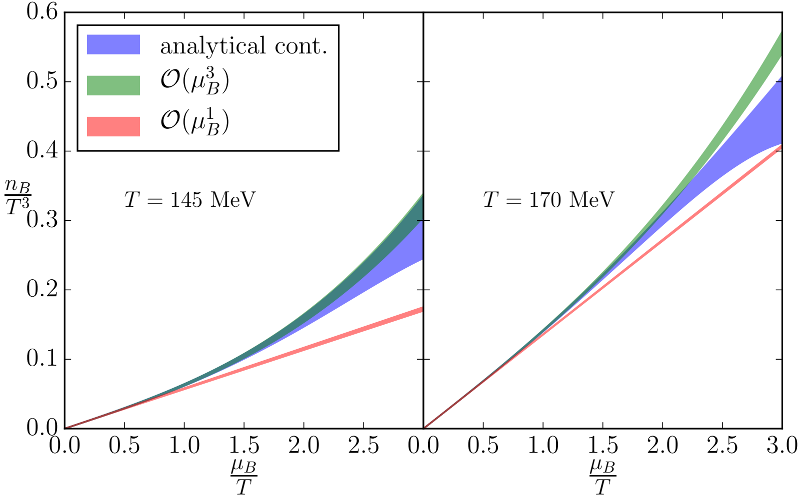
$$\frac{P}{T^4} = p_0 + p_2 \frac{\mu_B^2}{T^2} + p_4 \frac{\mu_B^4}{T^4} + p_6 \frac{\mu_B^6}{T^6} + \ldots$$



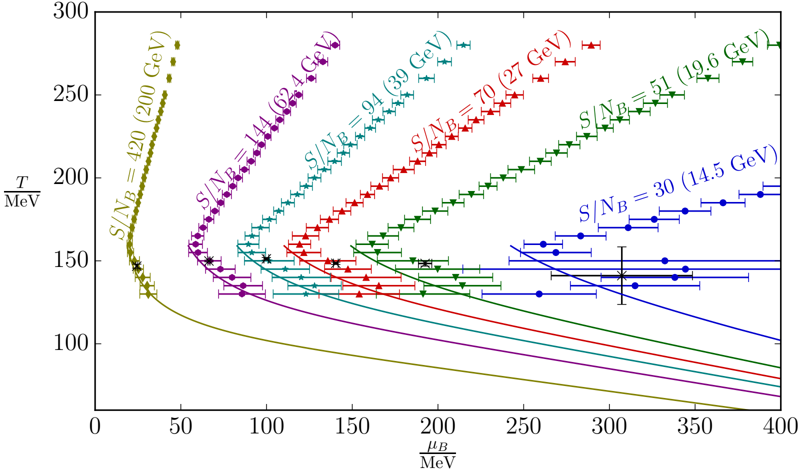
[Bazavov et al., Bazavov:2017dus]



Extrapolation

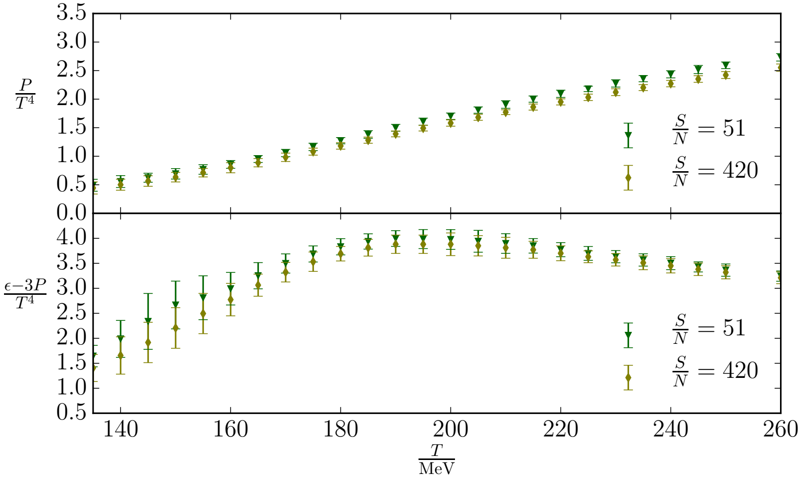


$\frac{S}{N_B}$ Trajectories

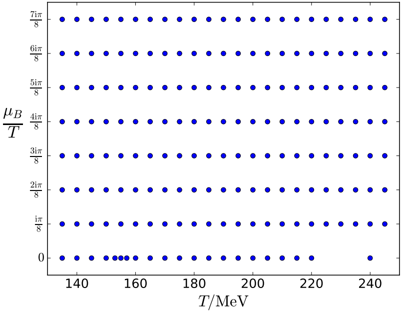


Black points from [Alba et al., Alba:2014eba]

Equation of State



Simulation details



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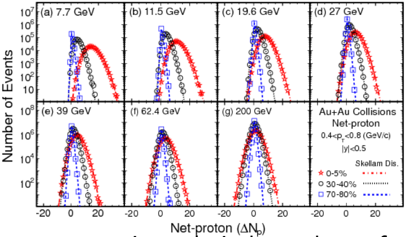
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\mu_S = \mu_Q = 0$
- Lattice size: $48^3 \times 12$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 1, 2, 3, 4, 5$, 6 and 7

Calculating observables

We have derivatives with respect to $\hat{\mu}_B$, $\hat{\mu}_Q$ and $\hat{\mu}_S$ of the pressure at $\mu_S = \mu_Q = 0$. Notation:

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k},$$



with $\hat{\mu}_i = \mu/T$.

We want ratios of the cumulants that are approximately independent of the volume at $\mu_B > 0$, $\langle n_S\rangle = 0$ and $\langle n_Q\rangle = 0.4\langle n_B\rangle$:

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \ldots$$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \ldots$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \ldots$$

Calculating observables II

The μ_B dependence can be written in terms of the Taylor expansion:

$$\chi^{BQS}_{i,j,k}(\hat{\mu}_B) = \chi^{BQS}_{i,j,k}(0) + \hat{\mu}_B\left[\chi^{BQS}_{i+1,j,k}(0) + q_1\chi^{BQS}_{i,j+1,k}(0) + s_1\chi^{BQS}_{i,j,k+1}(0)\right]$$

$$+\frac{1}{2}\hat{\mu}_B^2\left[\chi^{BQS}_{i+2,j,k}(0) + s_1^2\chi^{BQS}_{i,j+2,k}(0) + q_1^2\chi^{BQS}_{i,j,k+2}(0)\right.$$

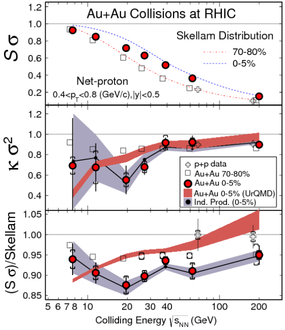
$$\left.+2q_1s_1\chi^{BQS}_{i,j+1,k+1}(0) + 2s_1\chi^{BQS}_{i+1,j+1,k}(0) + 2q_1\chi^{BQS}_{i+1,j,k+1}(0)\right] + \dots$$

with $q_j = \frac{1}{j!}\frac{d^j\hat{\mu}_Q}{(d\hat{\mu}_B)^j}(0) \qquad s_j = \frac{1}{j!}\frac{d^j\hat{\mu}_S}{(d\hat{\mu}_B)^j}(0)$

From $\langle n_S\rangle = 0$ and $\langle n_Q\rangle = 0.4\langle n_B\rangle$ we get the conditions

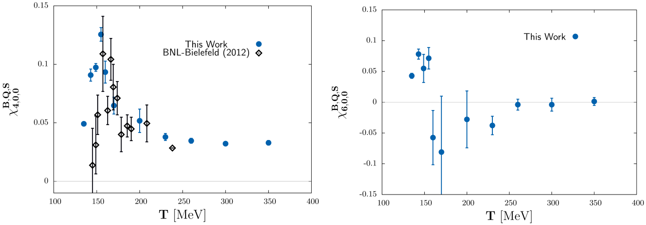
$$\chi^Q_1 = 0.4\chi^B_1\,, \qquad \chi^S_1 = 0$$

After some calculations we arrive at formulas for $\frac{M_B}{\sigma_B^2}$, $\frac{S_B\sigma_B^3}{M_B}$ and $\kappa_B\sigma_B^2$.

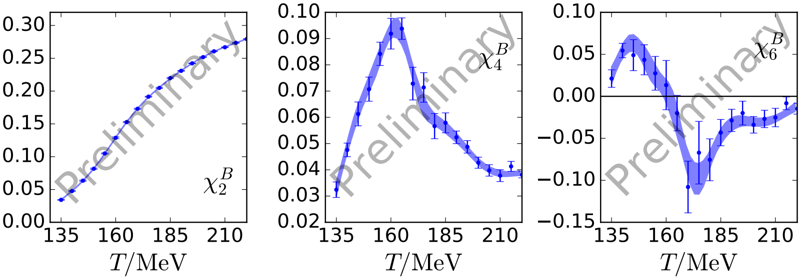


[STAR, Adamczyk:2013dal]

χ_2^B, χ_4^B and χ_6^B

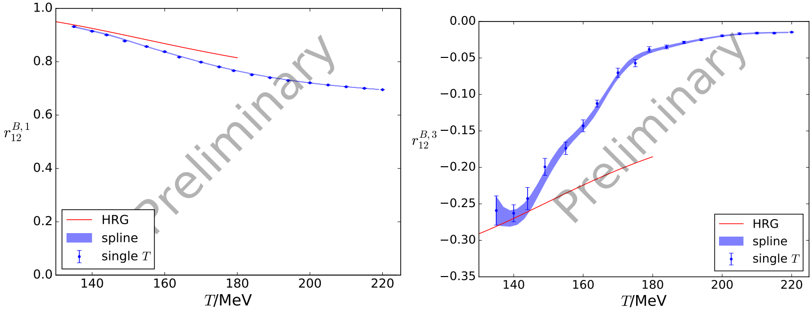


[D'Elia et al., DElia:2016jqh]



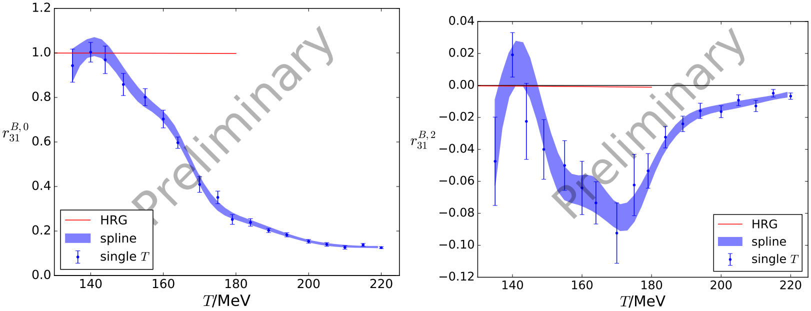
M_B/σ_B^2

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \dots$$

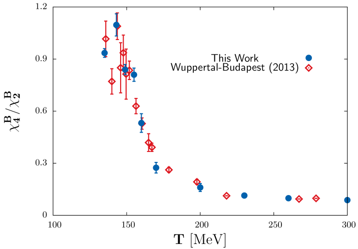


$S_B \sigma_B^3$

$$\frac{S_B \sigma_B^3}{M_B} = \frac{\chi_3^B(T, \hat{\mu}_B)}{\chi_1^B(T, \hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \ldots$$

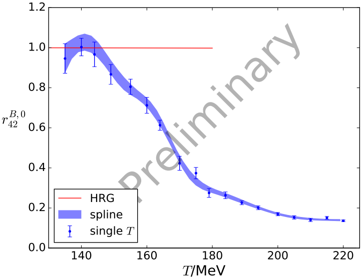


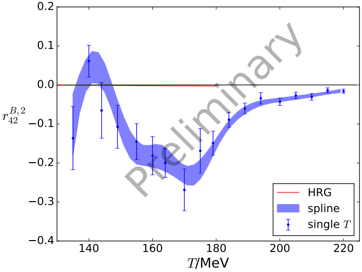
$\kappa_B \sigma_B^2$



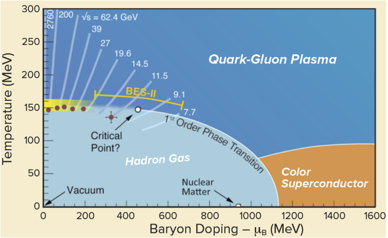
$$\kappa_B \sigma_B^2 = \frac{\chi_4^B(T, \hat{\mu}_B)}{\chi_2^B(T, \hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \ldots$$

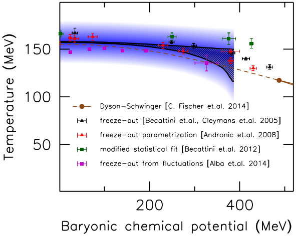
[D'Elia et al., DElia:2016jqh]

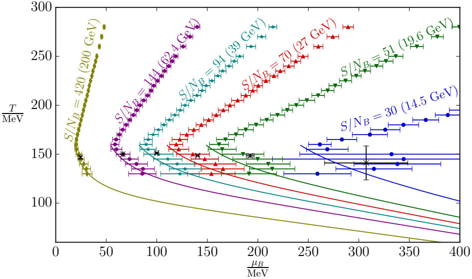


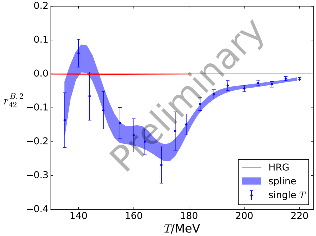


Summary


Temperature (MeV)
Baryon Doping – μB (MeV)
√s = 62.4 GeV
2760
200
39
27
19.6
14.5
11.5
9.1
7.7
BES-II
Quark-Gluon Plasma
Critical Point?
1st Order Phase Transition
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter


Temperature (MeV)
Baryonic chemical potential (MeV)
Dyson–Schwinger [C. Fischer et.al. 2014]
freeze–out [Becattini et.al., Cleymans et.al. 2005]
freeze–out parametrization [Andronic et.al. 2008]
modified statistical fit [Becattini et.al. 2012]
freeze–out from fluctuations [Alba et.al. 2014]


T/MeV
μB/MeV
S/NB = 420 (200 GeV)
S/NB = 144 (62.4 GeV)
S/NB = 94 (39 GeV)
S/NB = 70 (27 GeV)
S/NB = 51 (19.6 GeV)
S/NB = 30 (14.5 GeV)


r42^B,2
T/MeV
Preliminary
HRG
spline
single T