## Rapidity Correlations

W.J. Llope for the STAR Collaboration

Wayne State University


STAR net-p multiplicity cumulant ratios



A wider acceptance increased the multiplicities and made the deviations from Poisson larger

In a small acceptance, you will see Poissonian cumulant ratios, CP or not....
e.g. V. Koch, RIKEN BNL Research Center Workshop on Fluctuations, Correlations and RHIC Low Energy Runs, October 3-5, 2011
http://quark.phy.bnl.gov/~htding/fcrworkshop/Koch.pdf
decreasing rapidity acceptance in the analysis also drives the $\mathrm{K}_{4} / \mathrm{K}_{2}$ values to Poisson:
see also D. Mahapatra et al., Int. J. Mod. Phys A 17, 675 (2002)


## Net-baryon Acceptance:



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Net-baryon Acceptance:

| $0 \%$ | Zero fluctuations <br> Poisson <br> fluctuations <br> No signal? |
| :---: | :---: |
| (baryon \# conservation) |  |

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Net-baryon Acceptance:

$\mathrm{R}_{2}\left(\mathrm{y}_{1}, \mathrm{y}_{2}\right)$ - developed at ISR \& FNAL in 1970s to describe two particle correlations in (pseudo)rapidity $\mathrm{R}_{2}>0$ correlations, $\mathrm{R}_{2}<0$ anticorrelations, $\mathrm{R}_{2}=0$ uncorrelated.

$$
R_{2}=\frac{C_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}=\frac{\rho_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}-1 \quad \text { same event } \quad \text { mixed events or tensor product of 1D }
$$

lead to "cluster" picture...

- clusters decay to FS particles
- clusters uncorrelated w/ each other
- isotropic decay of clusters in their rest frames
- Lorentz-invariant translation of clusters in pseudorapidity

Exposes short and long-range correlations: E \& p conservation minijets HBT
L. Foà, Phys. Lett. C22, 1 (1975)
H. Bøggild, Ann. Rev. Nucl. Sci. 24, 451 (1974)
M. Jacob, Phys. Rep. 315, 7 (1999)

(A)



Figure 3.5: $R_{2}^{c c}$ for $p+p$ collisions at FNAL (a-b)and CERN ISR (c-d): $\sqrt{s}=13.7$, $27,23,63 \mathrm{GeV}$.

Małgorzata Janik, X Workshop on Particle Correlations and Femtoscopy, Gyöngyös, Hungary, Aug 26, 2014 „Away-side" ( $\Delta \phi \sim \pi$ ) jet correlations:

Correlation of particles between back-to-back jets

Momentum conservation:
$\sim-\cos (\Delta \phi)$

"Near-side" ( $\Delta \phi \sim 0$ ) jet peak:
Resonances, string fragmentation Correlation of particles within a single jet

Fit 4-5 functions to the 2D correlators to extract strengths of near-side peak, momentum conservation, $\mathrm{v}_{\mathbf{1}}$ "dipole", $\mathrm{v}_{\mathbf{2}}$ "quadrupole", etc...

Recall how fourier decomposition of azimuthal angle distrubutions leads to all sorts of interesting information on elliptic flow, flow fluctuations, triangularity....

A similar approach can be applied to study the shape of the fireball in the longitudinal direction!
Long-range rapidity correlations as fluctuating rapidity density of the fireball:
A. Bialas, A. Bzdak, and K. Zalewski, Phys. Lett. B 710, 332 (2012).
A. Bialas and K. Zalewski, Acta Phys. Pol. B 43, 1357 (2012).
...possibly with a significant asymmetric component in fireball's rapidity shape:
B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 103, 172301 (2009).
...Generalize!
A. Bzdak and D. Teaney, Phys. Rev. C 87, 024906 (2013)

$$
C\left(y_{1}, y_{2}\right) \equiv \rho_{2}\left(y_{1}, y_{2}\right)-\rho\left(y_{1}\right) \rho\left(y_{2}\right)
$$

...decompose rapidity correlator onto Chebyshev polynomials...

$$
\frac{C_{2}\left(y_{1}, y_{2}\right)}{\left\langle\rho\left(y_{1}\right)\right\rangle\left\langle\rho\left(y_{2}\right)\right\rangle}=\sum_{i, k}\left\langle a_{i} a_{k}\right\rangle T_{i}\left(y_{1} / Y\right) T_{k}\left(y_{2} / Y\right)
$$



information on the number of sources, baryon stopping mechanisms, viscosity, ...

See also:
A. Bzdak, Phys. Rev. C 85, 051901(R) (2012)
T. Lappi \& L. McLerran, Nucl. Phys. A 832, 330 (2010)
A. Monnai, B. Schenke, PLB 752, 317 (2016)
A. Bzdak (QM2015) 29/9/2015 16:00-16:20
B. Schenke (QM2015) 30/9/2015 9:20-09:40

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$$


...decompose rapidity correlator onto Legendre polynomials

$$
\frac{C_{2}\left(y_{1}, y_{2}\right)}{\left\langle\rho\left(y_{1}\right)\right\rangle\left\langle\rho\left(y_{2}\right)\right\rangle}=\sum_{i, k}\left\langle a_{i} a_{k}\right\rangle T_{i}\left(y_{1} / Y\right) T_{k}\left(y_{2} / Y\right)
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$\mathrm{R}_{2}\left(\mathrm{y}_{1}, \mathrm{y}_{2}\right)$ - developed at ISR \& FNAL in 1970s to describe two particle correlations in (psuedo)rapidity $\mathrm{R}_{2}>0$ correlations, $\mathrm{R}_{2}<0$ anticorrelations, $\mathrm{R}_{2}=0$ no correlations.

Recently, this variable has reappeared with a new name: $C\left(y_{1}, y_{2}\right) \ldots \quad C\left(y_{1}, y_{2}\right)=R_{2}\left(y_{1}, y_{2}\right)+1$

$$
R_{2}=\frac{C_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}=\frac{\rho_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}-1
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C\left(y_{1}, y_{2}\right)=1+\frac{1}{2}<a_{0} a_{0}>+\frac{1}{\sqrt{2}} \sum_{n=1}^{\infty}<a_{0} a_{n}>\left(T_{n}\left(y_{1}\right)+T_{n}\left(y_{2}\right)\right)+\sum_{n, m=1}^{\infty}<a_{n} a_{m}>\frac{T_{n}\left(y_{1}\right) T_{m}\left(y_{2}\right)+T_{n}\left(y_{2}\right) T_{m}\left(y_{1}\right)}{2}
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$$

With a special normalization, the residual centrality dependence is largely eliminated.

$$
\begin{aligned}
& C_{N}\left(y_{1}, y_{2}\right)=\frac{C\left(y_{1}, y_{2}\right)}{C_{p}\left(y_{1}\right) C_{p}\left(y_{2}\right)} \\
& C_{p}\left(y_{1}\right)=\frac{\int_{-Y}^{Y} C\left(y_{1}, y_{2}\right) d y_{2}}{2 Y}, C_{p}\left(y_{2}\right)=\frac{\int_{-Y}^{Y} C\left(y_{1}, y_{2}\right) d y_{1}}{2 Y}
\end{aligned}
$$

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With a special normalization, the residual centrality dependence is largely eliminated.
$C_{N}\left(y_{1}, y_{2}\right)=\frac{C\left(y_{1}, y_{2}\right)}{C_{p}\left(y_{1}\right) C_{p}\left(y_{2}\right)}$
$C_{p}\left(y_{1}\right)=\frac{\int_{-Y}^{Y} C\left(y_{1}, y_{2}\right) d y_{2}}{2 Y}, C_{p}\left(y_{2}\right)=\frac{\int_{-Y}^{Y} C\left(y_{1}, y_{2}\right) d y_{1}}{2 Y}$
$C_{N}\left(y_{1}, y_{2}\right)=1+\sum_{n, m=1}^{\infty}<a_{n} a_{m}>\frac{T_{n}\left(y_{1}\right) T_{m}\left(y_{2}\right)+T_{n}\left(y_{2}\right) T_{m}\left(y_{1}\right)}{2}$


Dynamical shape fluctuations (and correlations) can be quantified by decomposing the measured distributions onto a basis set of Legendre polynomials, with "strength" coefficients $<\mathrm{a}_{\text {mn }}>$

Rapidity analog of decomposition of azimuthal anistropies onto $\cos (\mathrm{n} \varphi \ldots)$ bases with strengths $\mathrm{v}_{\mathrm{n}}$
Note: $<\mathrm{a}_{(\mathbf{n}, \mathbf{m})}>,<\mathrm{a}_{\mathbf{n}} \mathrm{a}_{\mathbf{m}}>$, and $<\mathrm{a}_{\mathbf{m} \mathbf{n}}>$ are all the same thing... (different people use different nomenclatures)

$$
r_{2}=\frac{\int d y_{1} d y_{2}\left[\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)\right] R_{2}\left(y_{1}, y_{2}\right)}{\int d y_{1} d y_{2}\left[\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)\right]} \quad \text { and } \quad K_{2}=\langle N\rangle+\langle N\rangle^{2} r_{2}
$$

$$
\begin{gathered}
R_{2}=\frac{\rho_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}-1 \rightarrow r_{2}=\frac{\int d y_{1} d y_{2} \rho_{2}\left(y_{1}, y_{2}\right)-\int d y_{1} d y_{2} \rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}{\int d y_{1} d y_{2} \rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)} \\
\int d y \rho_{1}(y)=\langle N\rangle \\
\int d y_{1} d y_{2} \rho_{2}\left(y_{1}, y_{2}\right)=\langle N(N-1)\rangle \\
r_{2}=\frac{\int d y_{1} d y_{2} \rho_{2}\left(y_{1}, y_{2}\right)-\int d y_{1} \rho_{1}\left(y_{1}\right) \int d y_{2} \rho_{1}\left(y_{2}\right)}{\int d y_{1} \rho_{1}\left(y_{1}\right) \int d y_{2} \rho_{1}\left(y_{2}\right)} \\
K_{2}=\langle N\rangle+\langle N\rangle^{2} \frac{\langle N(N-1)\rangle-\langle N\rangle^{2}}{\langle N\rangle^{2}}
\end{gathered}
$$

$$
K_{2}=\langle N\rangle+\left[\langle N(N-1)\rangle-\langle N\rangle^{2}\right] \quad \begin{aligned}
& \text { see also: } \\
& \text { E.L. Berger, NPB 85, } 61 \text { (1975) } \\
& \text { p Carruthers } \rho+1
\end{aligned}
$$

$$
\text { P. Carruthers et al., PRL 63, } 1562 \text { (1989) }
$$

$$
K_{2}=\langle N\rangle+\left[\left\langle N^{2}\right\rangle-\langle N\rangle-\langle N\rangle^{2}\right]
$$

$$
\text { P. Carruthers, PRA 43, } 2632 \text { (1991) }
$$

$$
\text { A. Bzdak et al., PRC 95, } 054906 \text { (2017) }
$$

$$
K_{2}=\left\langle N^{2}\right\rangle-\langle N\rangle^{2} \quad(\text { variance })
$$

integrals of $\mathrm{R}_{\mathrm{k}}$ give multiplicity cumulants $\mathrm{K}_{\mathrm{k}} \ldots \quad \mathrm{K}_{3} / \mathrm{K}_{2}=\mathrm{S} \mathrm{\sigma}, \mathrm{~K}_{4} / \mathrm{K}_{2}=\kappa \sigma^{2}$
"mixing" $\quad R_{2}=\frac{\rho_{2}\left(y_{1}, y_{2}\right)}{\rho_{2}^{\text {mix }}\left(y_{1}, y_{2}\right)}-1$
shown at QM2017 (S. Jowzaee) offsets in low multiplicity events
"convolution" $R_{2}=\frac{\rho_{2}\left(y_{1}, y_{2}\right)}{\rho_{1}\left(y_{1}\right) \rho_{1}\left(y_{2}\right)}-1$
new in this talk multiplicity baseline correction:

$$
R_{2}^{\text {baseline }}=\frac{\langle N(N-1)\rangle}{\langle N\rangle^{2}}-1
$$

"weighting" $\quad R_{2}=\frac{\rho_{2}^{w}\left(y_{1}, y_{2}\right)}{\rho_{1}^{w}\left(y_{1}\right) \rho_{1}^{w}\left(y_{2}\right)}-1 \quad$ e.g. ALICE arXiv:1612.08975
$\rho_{2}^{w}\left(y_{1}, y_{2}\right)$ filled with weight $1 /[n(n-1)]$
$\rho_{1}^{w}(y)$ filled with weight $1 / n \quad n=$ multiplicity in each event
"Weighting" approach works fine for dealing with multiplicity effects
but destroys the mathematics of multiplicity cumulants from $R_{k}$ integrals
Will concentrate here on existing results from mixing, and new ones from convolution
Note, low multiplicity offsets do not affect $<\mathrm{a}_{\mathbf{m} \mathbf{n}}>$ values!

## Turning now to the $\lambda$ data...

Track crossing effects are a pain, standard techniques are applied... ( $\mathrm{P}_{\mathrm{T}}$ ordering, reflection)
Denominator from mixing (sampling, i.e. QM results) and now convolution
Not yet scaling $\mathrm{R}_{\mathbf{2}}$ by $\mathrm{N}_{\text {part }}$
Systematic uncertainities for convolution
results not yet determined.
Short-range correlations not subtracted...


LHC plots generally smoother - event sample sizes are similar, but the LHC has many more pairs/event.

Datasets: All 8 BES energies 200 GeV data from Run-10

POI:
$\mathrm{h}^{ \pm}, \pi^{ \pm}, \mathrm{K}^{ \pm}, \& \mathrm{p}^{ \pm}$
$2 \sigma$ on $\mathrm{dE} / \mathrm{dx}$, then require good TOF $\mathrm{m}^{2}$ reject electrons

Cuts:
$\mid$ Zvtx $\mid<30 \mathrm{~cm}$ at all $\sqrt{s}{ }_{\mathbf{N N}}$
Nhitsfit $>15$
$\mathrm{gDCA}^{2}<2 \mathrm{~cm}$
$\mathrm{p}_{\mathbf{T}^{\text {min }}:} \quad 0.2$ for $\mathrm{h} \pm \& \mathrm{~K} \pm, 0.4$ for $\mathrm{p} \pm$
$\mathrm{p}_{\mathbf{T}}{ }^{\text {max }}: \quad 2.0$
$\mathrm{p}^{\text {max }}: \quad 1.6$ for $\mathrm{h} \pm \& \mathrm{~K} \pm, 3.0$ for $\mathrm{p} \pm$

Centrality: $\quad \mathrm{N}_{\text {tracks }}$ with $0.5<|\eta|<1$ for $\mathrm{h} \pm \& \mathrm{~K} \pm$ $\mathrm{N}_{\pi, \mathrm{K}}$ with $-1<\eta<1$ for $\mathrm{p} \pm$

Cuts \& centrality intentionally very close to those used in recent $\star$ multiplicity cumulant analyses.

Detailed "bad run" and "bad event in good run" QA

EPreliminary








Better control of finite multiplicity effects from convolution Significant beam energy dependence


Better control of finite multiplicity effects from convolution LS proton anticorrelation for $\Delta \mathrm{y} \sim 0$. Weak beam energy dependence.









STAR म
W.J. Llope for STAR, CPOD2017, Aug. 8-11, 2017, Stony Brook, NY

open: mixing (presented at QM2017), solid: convolution
(SRC not subtracted)

proton anticorrelation for $\Delta y \sim 0$, beam energy dependence in pion correlations

A first comparison to model calculations from B. Schenke \& C. Shen
"net-protons" formed by convoluting pbar/p vs. y into a 2D histogram, then

$$
C_{2}^{n e t-p}=\frac{C_{2}^{p p}-r_{\bar{p} / p} C_{2}^{\bar{p} \bar{p}}}{1-r_{\bar{p} / p}} \quad \begin{aligned}
& r_{\bar{p} / p \text { taken from: }} \\
& \text { STAR, PRL 112, } 162301 \text { (2014) }
\end{aligned}
$$



Just starting these comparisons. We would love to collaborate with others too! Most interested in particles alone (not net-particles), $0-5 \%$ central...

Recall: effect is beam energy localized, charge independent, \& pions only Appears when TOF PID is required. $\mathrm{R}_{2}$ much larger and has no $\Delta \varphi$-ridge for $\mathrm{dE} / \mathrm{dx}$ PID TOF PID cleaner, and guarantees tracks are from the triggered crossing.

Not arising from specific Zvtx range, nor in some chronological section of the data
Seen in three completely independent analyses
Electrons? No, very few per event. Rejecting them makes no difference.
Bug in track crossing correction? No.
skip crossing correction, \& compare three tracking codes: Sti, StiCA, StiHR-



Seen at 19.6 GeV , less so at 27 GeV , both runs taken in same RHIC run (2011)... No

Was using Run- 10 data at 200 GeV . Check 200 GeV data from Run-11:



We are still investigating it - Still too early to ascribe "physics" to this $\Delta \varphi$-ridge.

Rapidity correlation variables $\mathrm{R}_{2}$ and $\mathrm{C}_{\mathrm{N}}$ studied for LS and US pions and protons as function of $\sqrt{s}^{S_{N \mathbf{N}}}$
$\mathrm{C}_{\mathrm{N}}$ decomposed using basis set of Legendre polynomials to quantify the importance of different shaped (anti)correlations.

This approach is the analog in the rapidity direction of quantifying azimuthal anistropies with $\mathrm{v}_{\mathbf{n}}$ observables.

Consistent results from two separate approaches (mixing, convolution) from two completely independent codes.

Two proton anticorrelations at $\Delta \mathrm{y} \sim 0\left(\mathrm{a}_{11}<0\right)$. Beam energy independent.
Significant beam energy dependence of two-pion correlations.
Appears as a ridge at small $\Delta y$ and extended in $\Delta \varphi \ldots$ ( $19.6-27 \mathrm{GeV}, \pi$ only, charge independent) Still investigating if this is experimental or physical.

First comparison of $<\mathrm{a}_{\mathrm{mn}}>$ in STAR BES data to viscous hydrodynamics.
Basic trends of $<\mathrm{a}_{\mathbf{m n}}>$ values vs. ( $\mathrm{m}, \mathrm{n}$ ) in data and theory are similar

## BACKUP SLIDES

open: mixing (presented at QM2017), solid: convolution
(SRC not subtracted)

proton anticorrelation for $\Delta y \sim 0$, beam energy dependence in pion correlations

$T_{4}\left(y_{1}\right) T_{4}\left(y_{2}\right)$


$\left[T_{1}\left(y_{1}\right) T_{3}\left(y_{2}\right)+T_{3}\left(y_{1}\right) T_{1}\left(y_{2}\right)\right] / 2$

$T_{3}\left(y_{1}\right) T_{3}\left(y_{2}\right)$

$\left[T_{2}\left(y_{1}\right) T_{4}\left(y_{2}\right)+T_{4}\left(y_{1}\right) T_{2}\left(y_{2}\right)\right] / 2$


## Structure of correlation functions

Department of Physics, University of Arizona, Tucson, Arizona 85721
$C_{2}\left(x_{1}, x_{2}\right)=\rho_{2}\left(x_{1}, x_{2}\right)-\rho_{1}\left(x_{1}\right) \rho_{1}\left(x_{2}\right)$,
$C_{3}\left(x_{1}, x_{2}, x_{3}\right)=\rho_{3}\left(x_{1}, x_{2}, x_{3}\right)-\sum_{(3)} \rho_{2}\left(x_{1}, x_{2}\right) \rho_{1}\left(x_{3}\right)+2 \rho_{1}\left(x_{1}\right) \rho_{1}\left(x_{2}\right) \rho_{1}\left(x_{3}\right)$,
$C_{4}\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=\rho_{4}\left(x_{1}, x_{2}, x_{3}, x_{4}\right)-\sum_{(4)} \rho_{3}\left(x_{1}, x_{2}, x_{3}\right) \rho_{1}\left(x_{4}\right)-\sum_{(3)} \rho_{2}\left(x_{1}, x_{2}\right) \rho_{2}\left(x_{3}, x_{4}\right)$

$$
+2 \sum_{(6)} \rho_{2}\left(x_{1}, x_{2}\right) \rho_{1}\left(x_{3}\right) \rho_{1}\left(x_{4}\right)-6 \rho_{1}\left(x_{1}\right) \rho_{1}\left(x_{2}\right) \rho_{1}\left(x_{3}\right) \rho_{1}\left(x_{4}\right) .
$$

See also:
L. Foà, Phys. Lett. C22, 1 (1975)
H. Bøggild, Ann. Rev. Nucl. Sci. 24, 451 (1974)
M. Jacob, Phys. Rep. 315, 7 (1999)

Lower-order correlations explicitly removed. $\mathrm{R}_{\mathrm{k}}$ is just these rapidity cumulants $\mathrm{C}_{\mathrm{k}}$ scaled by the number of pairs, triplets, quadruplets, $\ldots$ $\mathrm{R}_{\mathrm{k}}$ thus manifestly independent of experimental inefficiencies by definition...

$$
\mathrm{R}_{2} \text { baseline: } R_{3}=\frac{\langle n(n-1)\rangle}{\langle n\rangle^{2}}-1 \quad \mathrm{R}_{3} \text { baseline: } R_{3}=\frac{\langle n(n-1)(n-2)\rangle}{\langle n\rangle^{3}}-3 \frac{\langle n(n-1)\rangle}{\langle n\rangle^{3}}\langle n\rangle+2
$$

Robust indicator of N -fold (anti)correlations, explicitly as a function of $\Delta \mathrm{y}$ and $<\mathrm{y}>\ldots$ By construction, independent of single-particle inefficiencies..

Pseudocorrelations

$$
<\mathrm{R}_{\mathbf{2}}>\operatorname{vs} \Delta \mathrm{y}
$$

low $\Delta y$ enhancement... not seen in UrQMD evts...




Color: $\begin{gathered}\text { data }\end{gathered}$
Gray: UrQMD 14.5 GeV

Caused by rapidity dependence of experimental efficiency coupled with Zvtx smearing...
See L. Tarini, Ph.D. Thesis, and his talk at the STAR Analysis Meeting, MIT, 7/10/2009


Analyze in 2cm-wide Zvtx bins then weight-average the results...


Very strong trench in $\mathrm{R}_{\mathbf{2}}$ when particle multiplicities/event of POI get large:
$h \pm$ for all centralities and $V_{s_{N N}}$, and only most central for $\mathrm{K} \pm$
Numerator and denominator of $\mathrm{R}_{2} \& \mathrm{C}_{\mathrm{N}}$ uses only measured tracks... but there is a slight 2-particle efficiency loss when two tracks are nearby $(\Delta y \sim 0)$

The STAR track-finder "sti" does not share spacepoints!
a new one does "stiCA" (10\%)


## like-sign


$p_{t, 1}<p_{t, 2}$
$\Delta \varphi_{12}>0$


Merging/crossing Hit losses, Pair Loss
$p_{t, 1}<p_{t, 2}$
$\Delta \varphi_{12}<0$


No merging/crossing
No losses


$$
\Delta \varphi_{12}<0
$$

Image from P. Pujahari
LS \& US: reflect clean area in $\Delta \varphi$ to replace problem area
US: nothing special in fill method
LS: pT order the tracks, fill numerator for upper triangle only, then symmetrize

The cut used is $|\Delta \mathrm{y}|<0.04$ and $-5 \pi / 12 \leq \Delta \varphi<0$

$$
T_{4}\left(y_{1}\right) T_{4}\left(y_{2}\right)
$$

Given this cut, I cannot bin the (y1,y2) parts of the TH3D too finely! (or there will never be any counts in the $\Delta y=0$ bins)
Rapidity bin width must be near or larger than $2^{*} 0.04 \ldots$
But this can cause non-physical artifacts in the $<\mathrm{a}_{\mathbf{m n}}>$ values!


Fixed perfectly by integrating $\mathrm{T}_{\mathrm{n}}(\mathrm{y})$ over the bin

