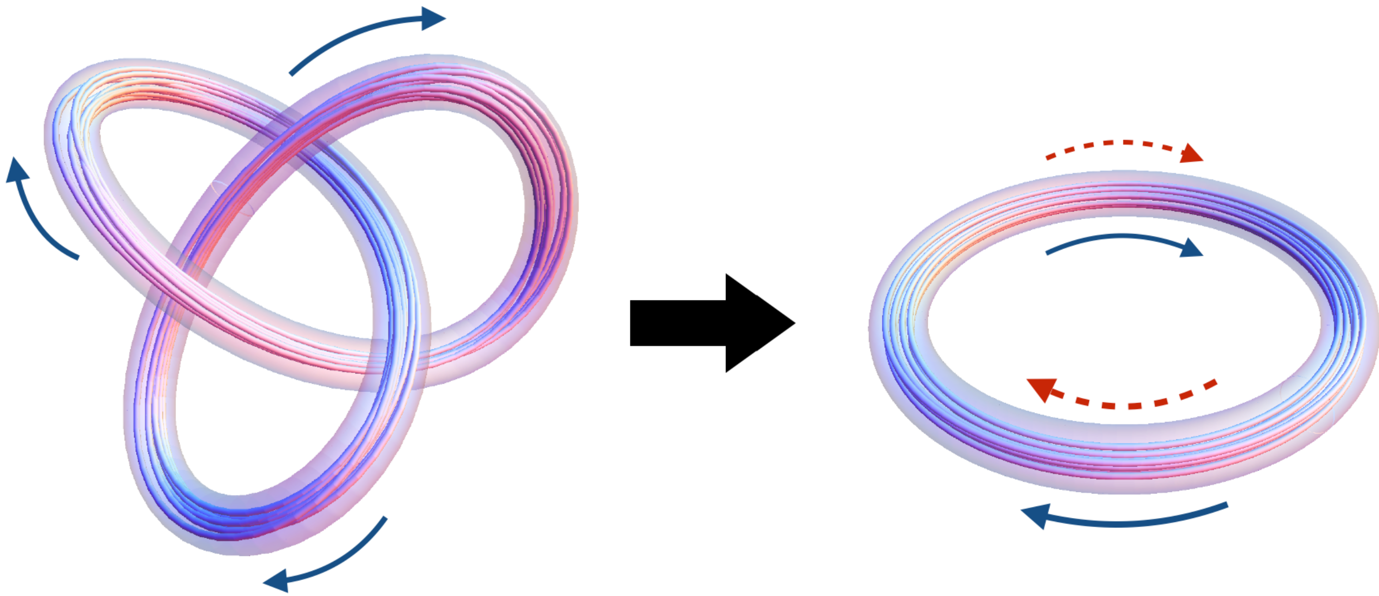

Properties of Chiral magnetohydrodynamics

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BEST COLLABORATION

Anomalous transport effects

$$\boldsymbol{j}_{\text{anom}} = \kappa_B \boldsymbol{B} + \kappa_\omega \boldsymbol{\omega}$$

$$\boldsymbol{j}_{5,\text{anom}} = \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega}$$

Chiral magnetic effect requires chirality imbalance

- Signature of **chiral symmetry restoration**

- Axial charge generation from color fields

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$

Theoretical frameworks

- **Anomalous hydrodynamics**

  [Son-Surowka 2009; …]

- **Chiral kinetic theory**

  [Son-Yamamoto; Stephanov-Yin; …]

Chiral Fluid

EM fields

Chiral MHD

MHD = Magnetohydrodynamics

Systems described by chiral MHD

- Heavy-ion collisions
  - For the CME search, reliable estimate of the life-time of **B** is important

- Early Universe

- Weyl/Dirac semimetals

CME currents from magnetic reconnections

[Hirono-Kharzeev-Yin, PRL'16]

Magnetic & fermionic helicities

$$\partial_\mu j_A^\mu = C_A \boldsymbol{E} \cdot \boldsymbol{B}$$

Magnetic & fermionic helicities

$$\partial_\mu j_A^\mu = C_A \boldsymbol{E} \cdot \boldsymbol{B}$$

$$\Rightarrow \quad \boxed{\frac{d}{dt}\left[\mathcal{H} + \mathcal{H}_F\right] = 0}$$

$$\mathcal{H} = \int d^3x \boldsymbol{A} \cdot \boldsymbol{B}$$

Magnetic helicity

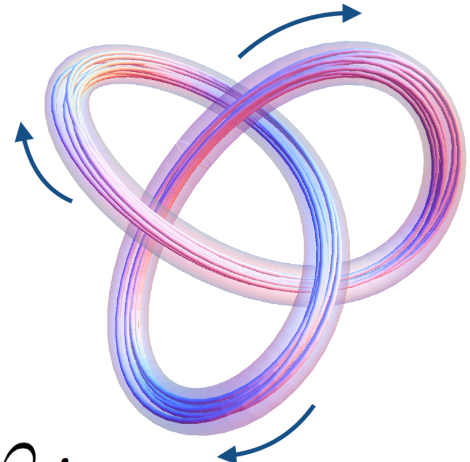
$$\mathcal{H}_F = \frac{2}{C_A} \int d^3x \; n_A$$

Fermionic helicity

Magnetic helicity knows topology

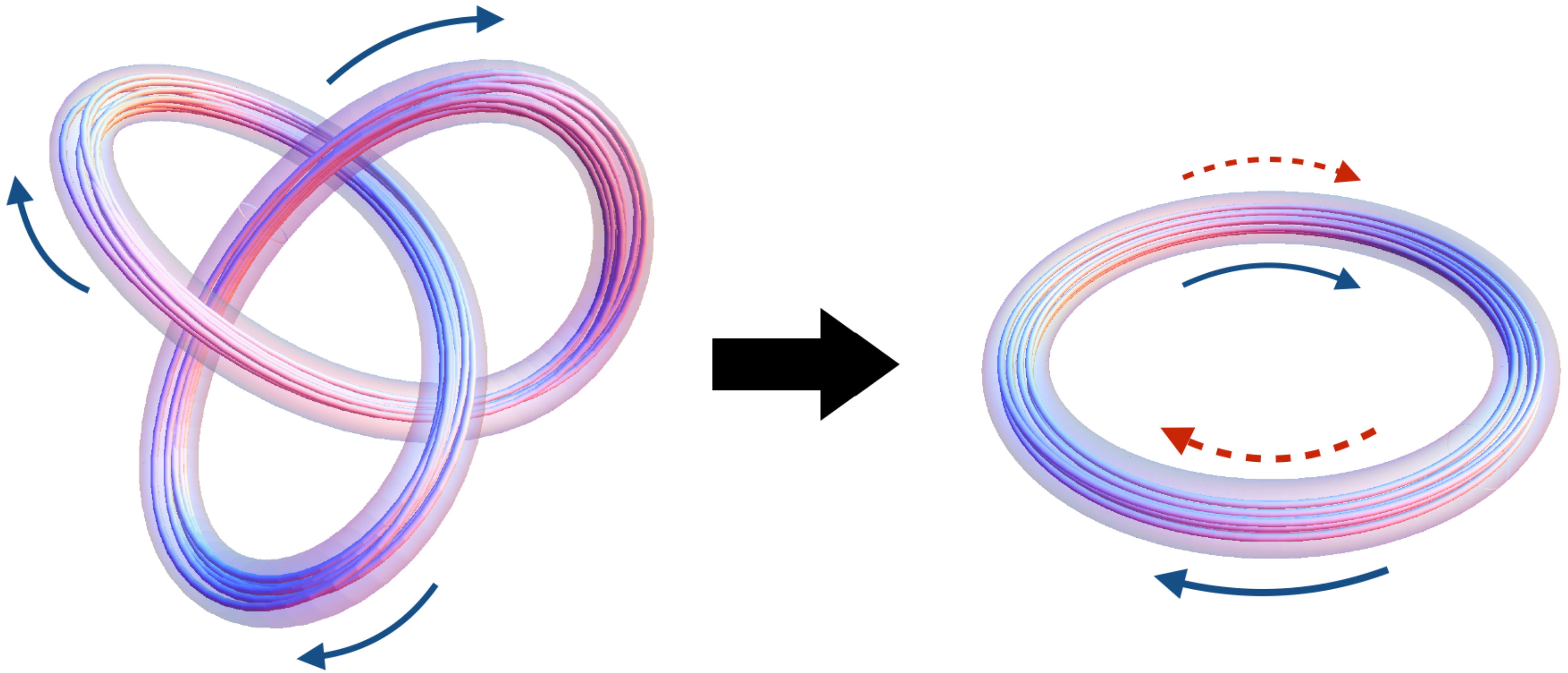
$$\mathcal{H} = \int d^3x \boldsymbol{A} \cdot \boldsymbol{B}$$

$$= \sum_i \mathcal{S}_i \varphi_i^2 + 2 \sum_{i,j} \mathcal{L}_{ij} \varphi_i \varphi_j$$



Self-linking number

Linking number

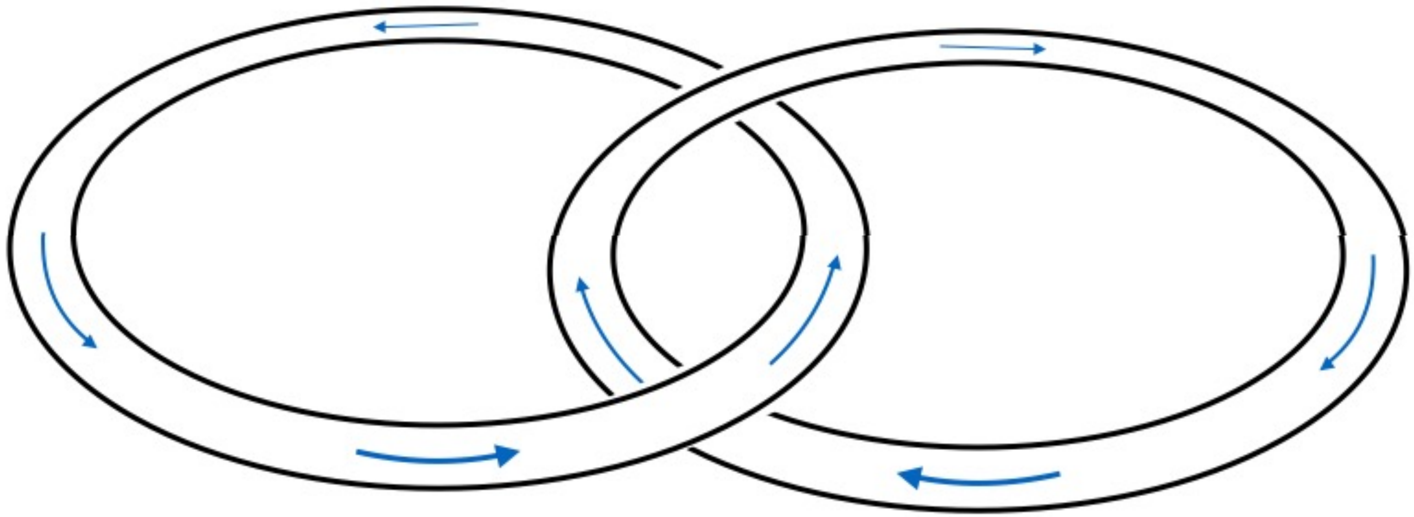


CME currents from reconnections of **B**

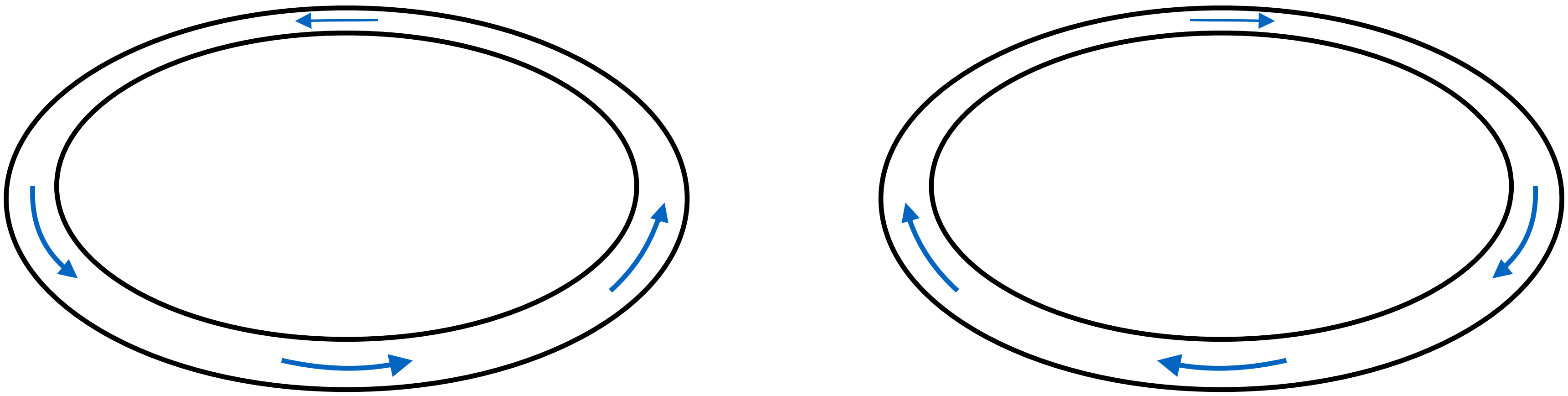
[Hirono-Kharzeev-Yin PRL'16]

$$\sum_i \oint_{C_i} \Delta \boldsymbol{J} \cdot d\boldsymbol{x} = -\frac{e^3}{2\pi^2} \Delta \mathcal{H}$$

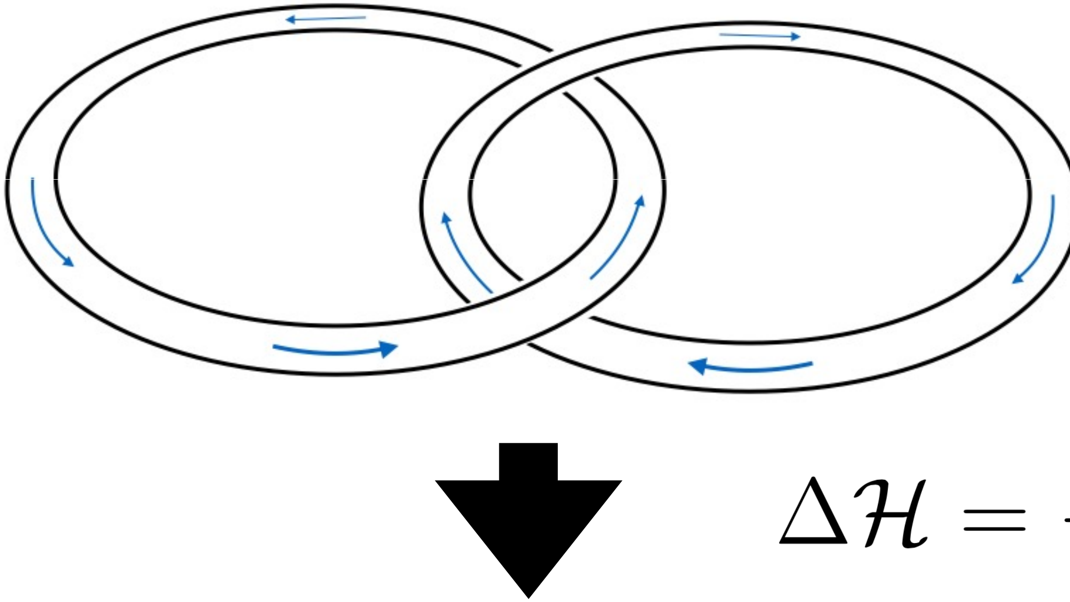
Change of topology induces CME currents!



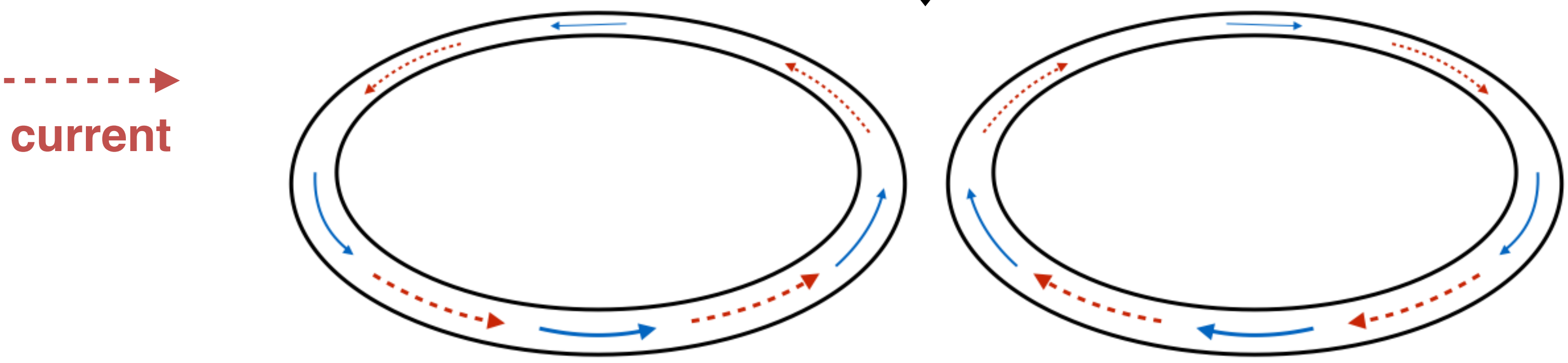
$$\mathcal{H} = 2\varphi^2$$

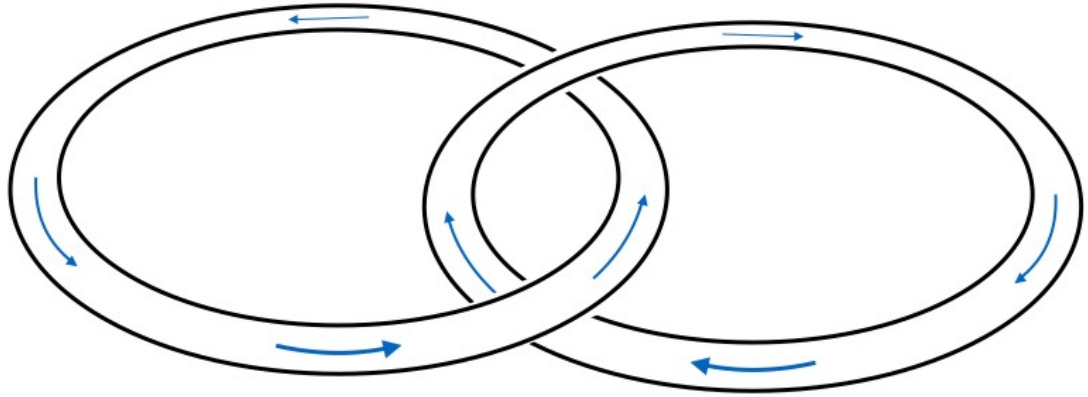


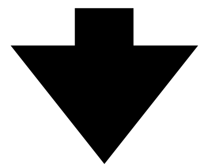
$$\mathcal{H} = 0$$



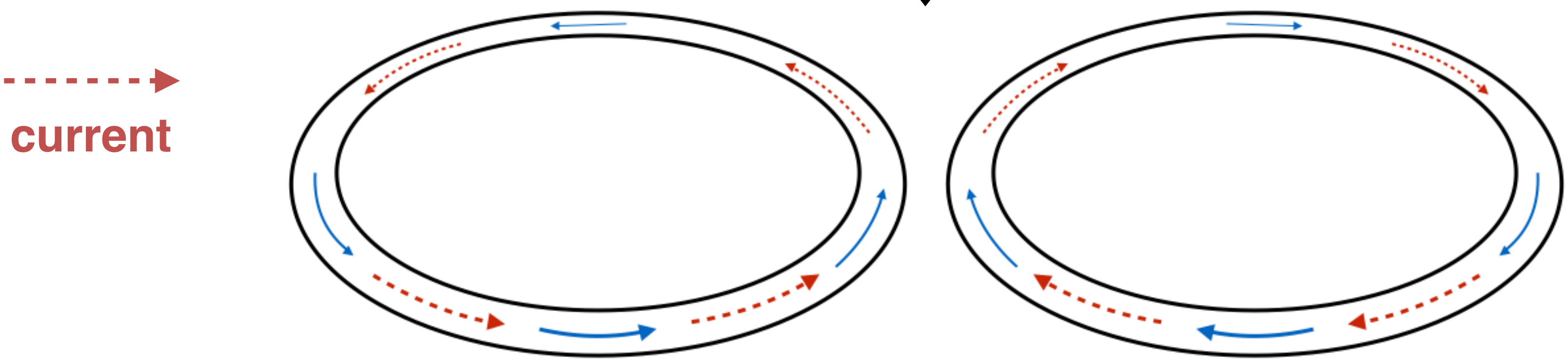
$$\Delta\mathcal{H} = -2\varphi^2$$







$$\Delta\mathcal{H} = -2\varphi^2$$



$$\sum_i \oint_{C_i} \Delta\boldsymbol{J}\cdot d\boldsymbol{x} = \frac{e^3\varphi^2}{\pi^2}$$

Formulation & waves of chiral MHD

[Hattori-Hirono-Yee-Yin, in preparation]

Chiral MHD

- MHD & chiral MHD can be understood as a low-energy effective theory basing on derivative expansion

- A new anomaly-induced instability

EOM of MHD

$$\partial_\mu T^{\mu\nu}_{\text{fluid}} = F^{\nu\rho} j_\rho$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

EOM of MHD

$$\partial_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_{\rm fluid} + T^{\mu\nu}_{\rm EM}$$

$$T^{\mu\nu}_{\rm EM} = -F^{\mu}{}_{\alpha} F^{\nu\alpha} + \frac{1}{4}\eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

EOM of MHD

$$E^\mu \equiv F^{\mu\nu} u_\nu, \quad B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$$

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu - \epsilon^{\mu\nu\rho} B_\rho$$

$$\tilde{F}^{\mu\nu} = B^\mu u^\nu - B^\nu u^\mu + \epsilon^{\mu\nu\rho} E_\rho$$

$$\epsilon^{\mu\nu\alpha} \equiv \epsilon^{\mu\nu\alpha\beta} u_\beta$$

Hydrodynamic variables

- Parameters characterizing local thermal equilibrium

- Neutral fluid with a conserved charge $\{T(\boldsymbol{x}),\ u^\mu(\boldsymbol{x}),\ \mu(\boldsymbol{x})\}$

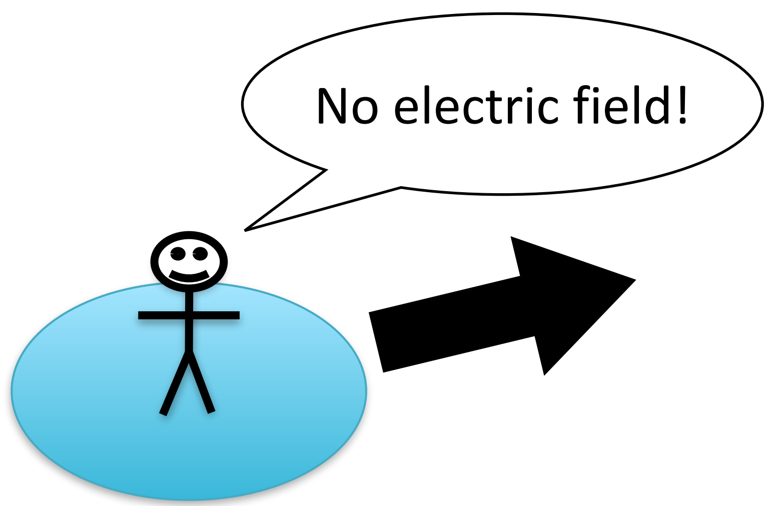
- MHD $\{T(\boldsymbol{x}),\ u^\mu(\boldsymbol{x}),\ B^\mu(\boldsymbol{x})\}$

# of hydro variables = # of equations = 7

No electric field in the fluid frame in ideal MHD

$$E^{\mu}_{(0)} = 0$$

Correspond to large conductivity limit



Constitutive relations for ideal MHD

$$T^{\mu\nu}_{\mathrm{tot}(0)} = (e+p)u^\mu u^\nu - p\eta^{\mu\nu} + \boldsymbol{B}^2\left[u^\mu u^\nu - b^\mu b^\nu - \frac{1}{2}\eta^{\mu\nu}\right]$$

$$B^\mu = |\boldsymbol{B}|b^\mu \quad b_\mu b^\mu = -1$$

$$F^{\mu\nu}_{(0)} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

CME doesn't play any role in ideal MHD!

Why?

$$j^\mu = \sigma E^\mu + \sigma_B B^\mu$$

$$\Rightarrow \quad E^\mu = \frac{1}{\sigma} j^\mu - \frac{\sigma_B}{\sigma} B^\mu$$

In the limit $\sigma \to \infty$

$$F^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

Chiral magnetic conductivity never appears in EOM

$$\partial_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

Conservation of topology of B

- Flux is "frozen in" to the fluid
- Magnetic helicity is conserved in ideal MHD
  - **No reconnection**

$$h^\mu_{\mathrm{B}} = \tilde{F}^{\mu\nu} A_\nu$$: helicity current

$$\partial_\mu h^\mu_{\mathrm{B}} = 2\tilde{F}^{\mu\nu} F_{\mu\nu} = 8 E^\mu B_\mu = 0$$

$$\mathcal{H} = \int d^3x\, h^0_{\mathrm{B}}$$ is conserved

First order in derivative expansion

Using the second law,

$$T^{\mu\nu}_{\mathrm{tot}(1)} = \zeta\Delta^{\mu\nu}\partial\cdot u + 2\eta\nabla^{<\mu}u^{\nu>}$$

E^{μ} : C-odd, P-odd

$$E^{\mu}_{(1)} = \frac{1}{\sigma\beta}\epsilon^{\mu\nu\alpha\beta}u_{\nu}\partial_{\alpha}(\beta B_{\beta}) - \epsilon_B B^{\mu}$$

CME

σ : electric conductivity

$\epsilon_B = \dfrac{\sigma_B}{\sigma}$ σ_B : chiral magnetic conductivity

First order in derivative expansion

$$\tilde{F}^{\mu\nu}_{(1)} = \epsilon^{\mu\nu\rho\sigma} E_{(1)\rho} u_\sigma$$

$$\partial_\mu \left[T^{\mu\nu}_{\text{tot}(0)} + T^{\mu\nu}_{\text{tot}(1)} \right] = 0$$

$$\partial_\mu \left[\tilde{F}^{\mu\nu}_{(0)} + \tilde{F}^{\mu\nu}_{(1)} \right] = 0$$

Waves in chiral MHD

- Linear fluctuations - 6 modes in total

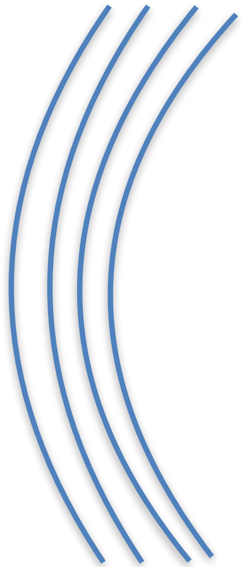
$$e \to e + \delta e,$$

$$B \to B + \delta B,$$

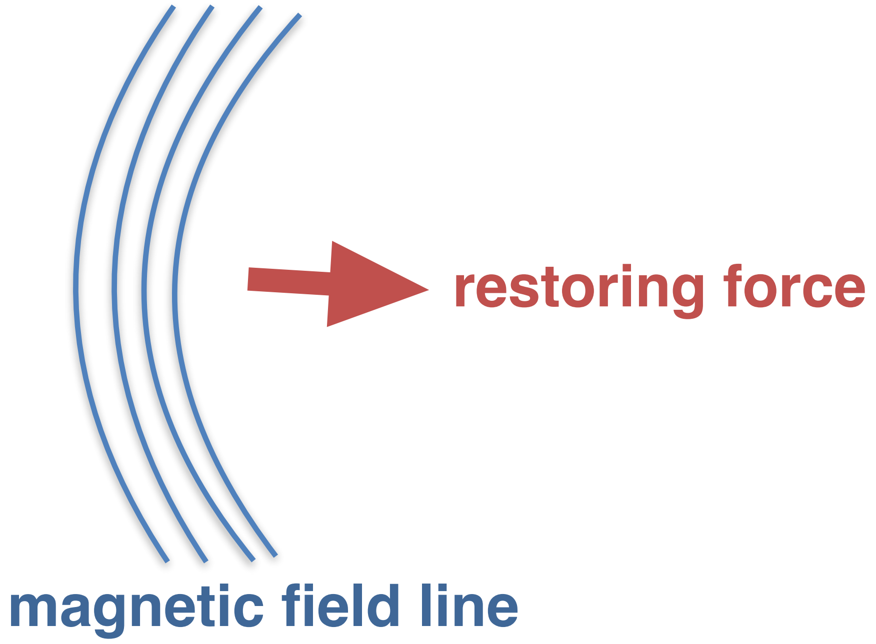
$$u^\mu \to u^\mu + \delta u^\mu,$$

$$b^\mu \to b^\mu + \delta b^\mu.$$

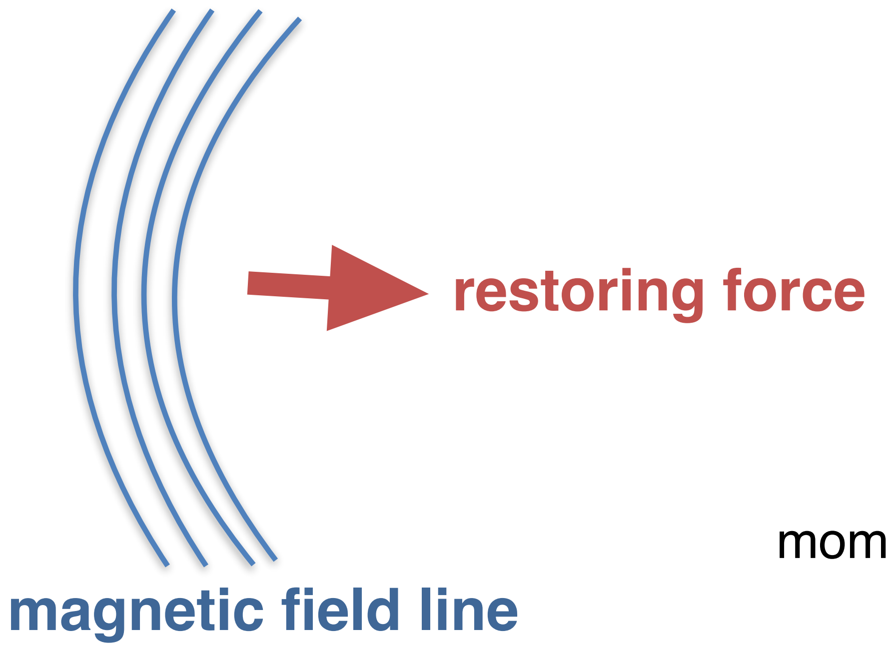
Alfven wave



Alfven wave



Alfven wave



momentum along the background **B**

Dispersion relation $\omega = \pm v_A k_{||}$

$$v_A^2 = \frac{\boldsymbol{B}^2}{e + p + \boldsymbol{B}^2}$$ **Alfven velocity**

Alfven wave in dissipative MHD

Dispersion relation

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2$$

damping

$$\lambda = \frac{1}{\sigma}$$

σ : electric conductivity

$$\bar{\eta} \equiv \frac{\eta}{e + p + \boldsymbol{B}^2}$$

η : shear viscosity

Alfven wave in chiral MHD

Including CME (when $\boldsymbol{k} \propto \boldsymbol{B}$)

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[(\bar{\eta}+\lambda)\,k_{||}^2 - s\epsilon_B k_{||}\right]$$

CME

$s = \pm 1$ indicates the helicity of the mode

$$i\boldsymbol{k} \times \boldsymbol{e}^{(s)} = s k \boldsymbol{e}^{(s)}$$

helicity eigenstate

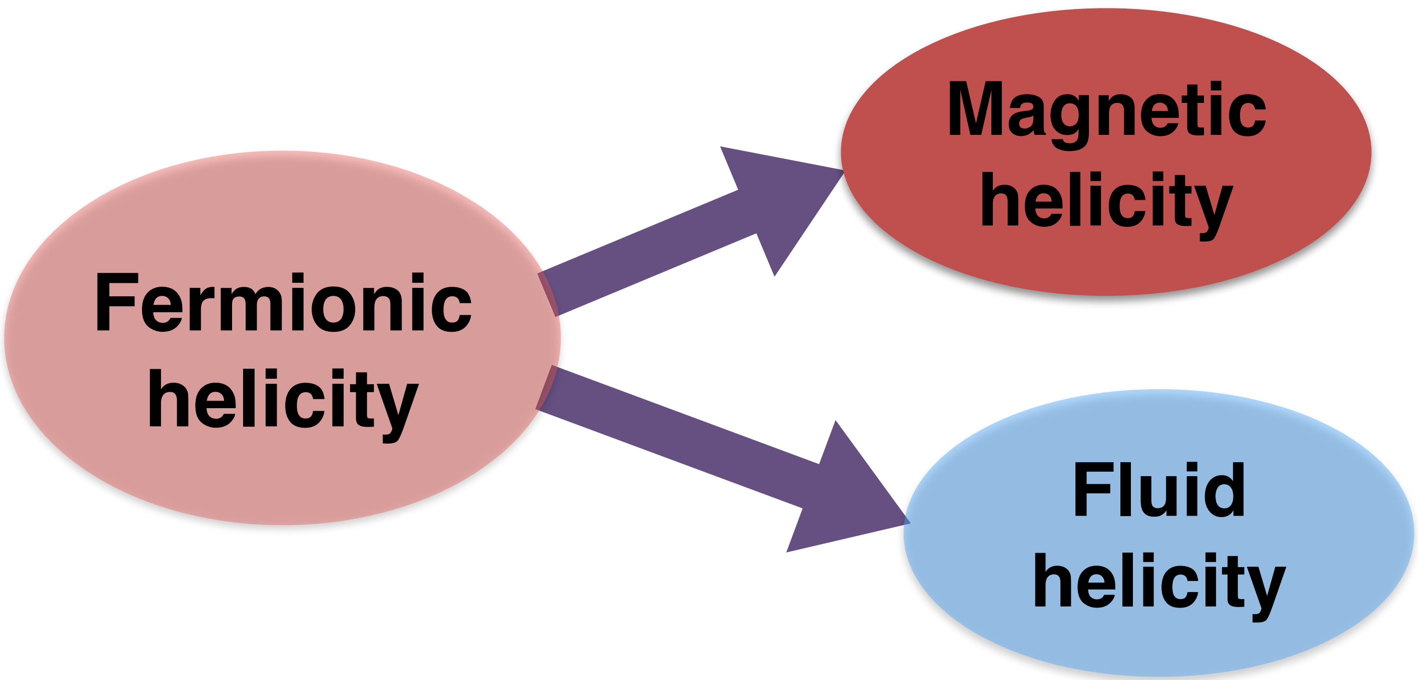
Instability in one of the helicity modes

Alfven wave in chiral MHD

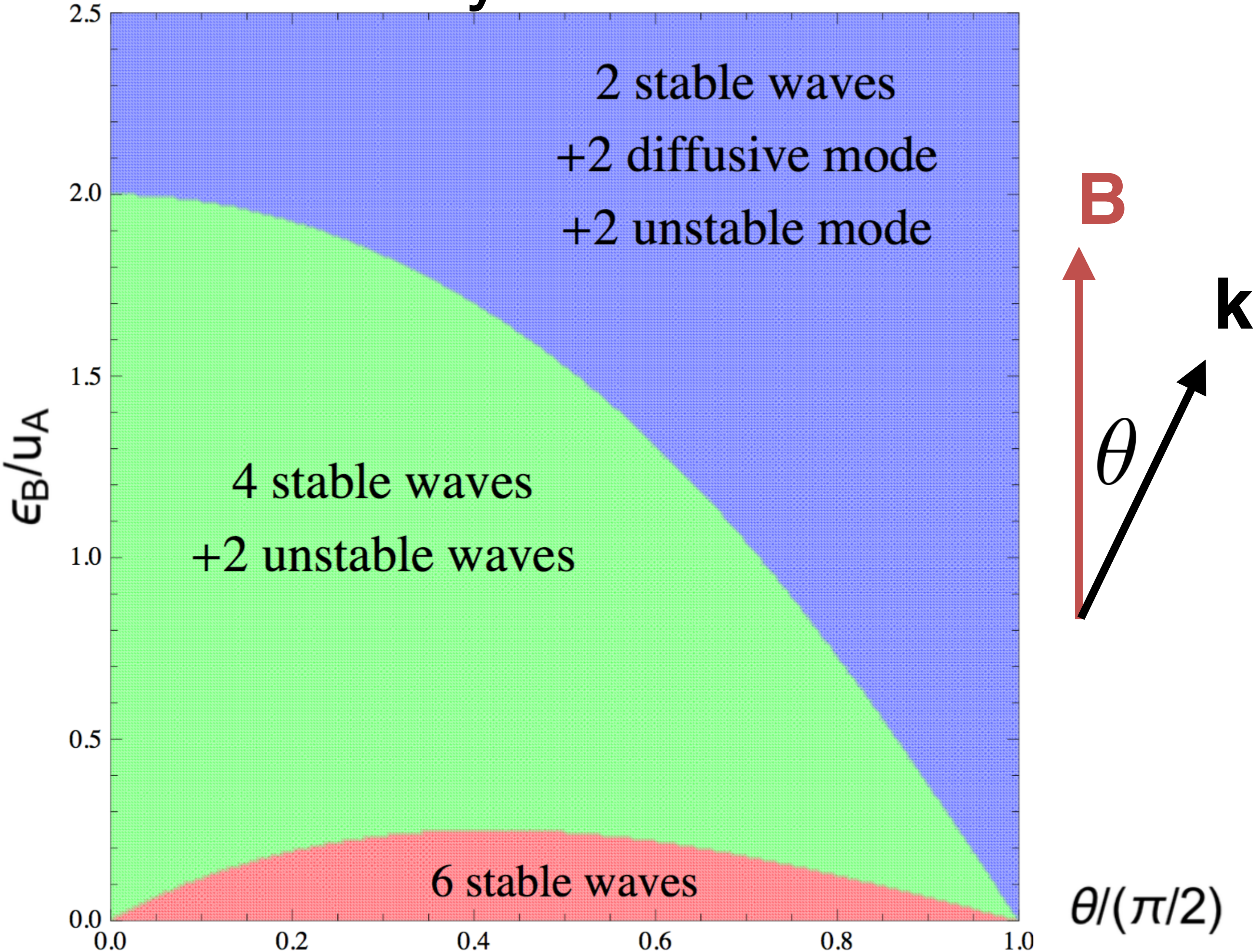
Including CME (when $\boldsymbol{k} \propto \boldsymbol{B}$)

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[(\bar{\eta} + \lambda)\, k_{||}^2 - \underline{s\epsilon_B k_{||}}\right]$$

CME



Stability of the waves



Summary

1. **B-field topology & CME currents**

   - Reconnections generate CME currents

2. **Chiral MHD**

   - Consistent theory of chiral fluids and EM fields
   - CME appears in the first order P-odd correction
   - Helicity dependent instability
   - generation of helical B fields and flows
   - Numerical implementation is under way

with Mace, Kharzeev, Inghirami et.al., in progress

Backup slides

Toward numerical impl. of chiral MHD

with Mace, Kharzeev, Inghirami et.al.

- Collaborating with ECHO-QGP
- Implement:
  - Finite resistivity + anomalous effects
  - Pre-equilibrium CME currents

B-field evolution