Measurement of the cumulants of net-proton multiplicity distribution by STAR

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Plan of the talk

- Introduction
- Search for the QCD critical point with the first to fourth-order cumulants
- Measurement of the sixth-order cumulant
- Data-driven Monte-Carlo approach
- Summary

Fluctuation of Conserved Quantities

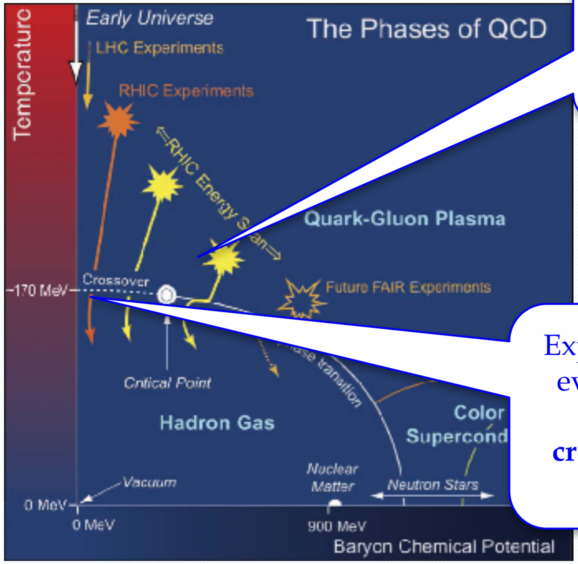
- Connection to the susceptibility of the system (χ)

$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial (\mu_q/T)^n} \qquad q = B, Q, S$$

$$\frac{\chi_q^{(4)}}{\chi_q^{(2)}} = \frac{C_{4,q}}{C_{2,q}} \qquad \frac{\chi_q^{(6)}}{\chi_q^{(2)}} = \frac{C_{6,q}}{C_{2,q}}$$

- Higher order cumulants are more sensitive to the signatures of QCD phase transition

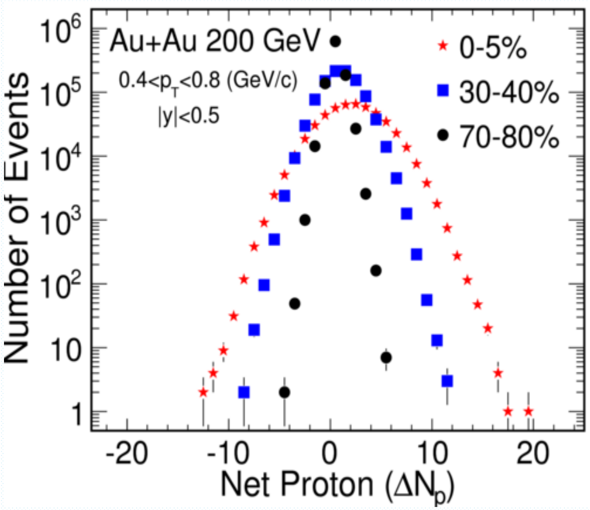
M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009).
M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).



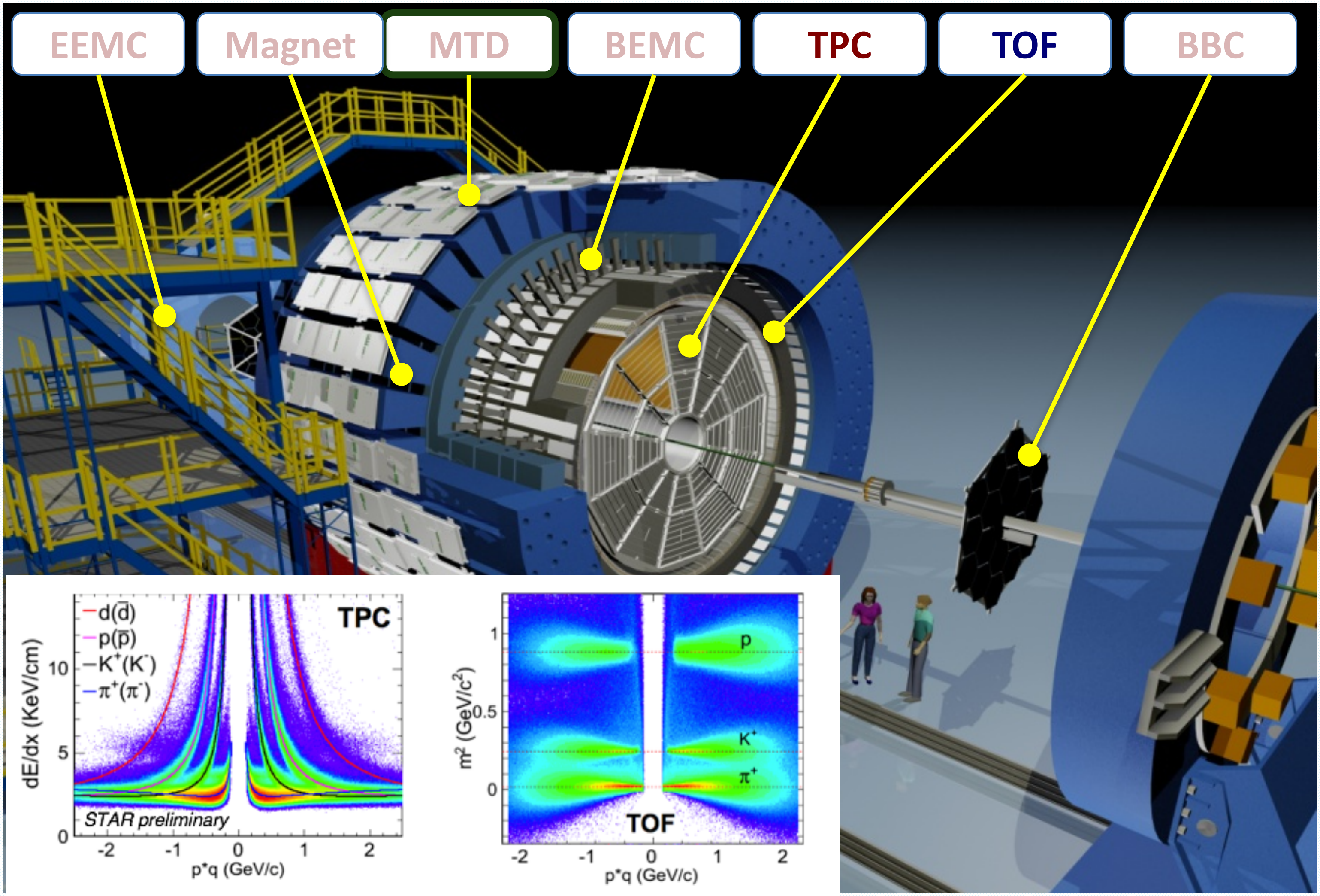
QCD **critical point** at large μ_B

Experimental evidence of **smooth crossover** at $\mu_B = 0$

STAR Collaboration, Phys.Rev.Lett. 105 (2010) 022302



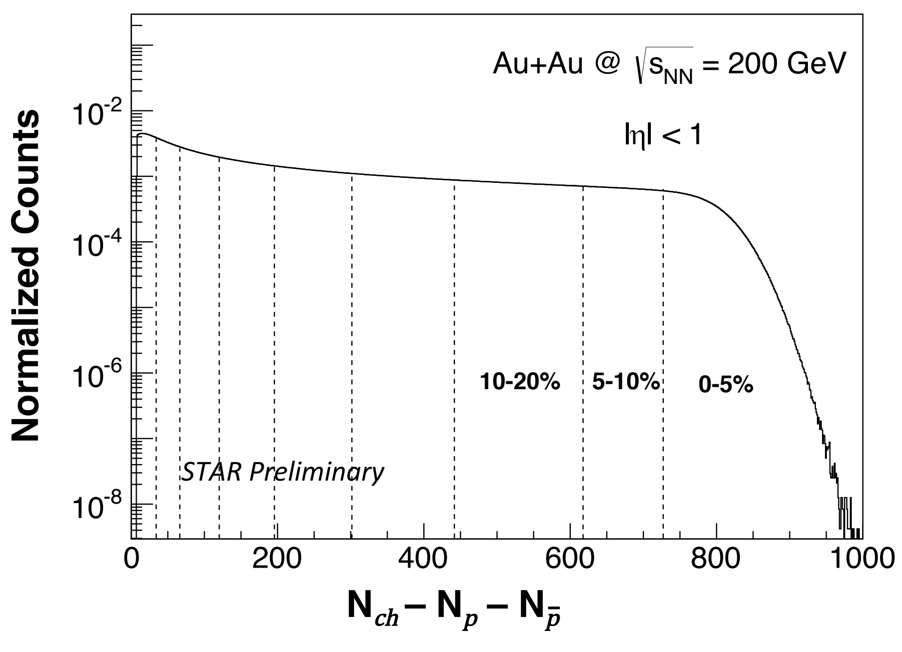
STAR Detector



Analysis Technique

- **Centrality determination**

  Use charged particles within $|\eta| < 1$, excluding protons and anti-protons, to avoid **auto-correlations**



- **Centrality bin width correction**

  Evaluate cumulants for each centrality bin to suppress **volume fluctuations**

X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017).
STAR Collaboration, Phys.Rev.Lett. 105 (2010) 022302.
STAR Collaboration, Phys.Rev.Lett. 113 (2014) 092301 .

Analysis Technique

➢ Error estimation

Statistical errors are based on **Bootstrap technique** or the **Delta Theorem**.

$$Error\ (C_r)\ \propto\ \frac{\sigma^r}{\sqrt{n}}$$

$$Error\ (C_r/C_2)\ \propto\ \frac{\sigma^{r-2}}{\sqrt{n}}$$

σ : width of the distribution
n : number of events

B. Efron et al. An Introduction to Bootstrap, Chapman & Hill (1993).
X. Luo, J. Xu, B. Mohanty, N. Xu, J. Phys. G 40, 105104 (2013) .

➢ Efficiency correction

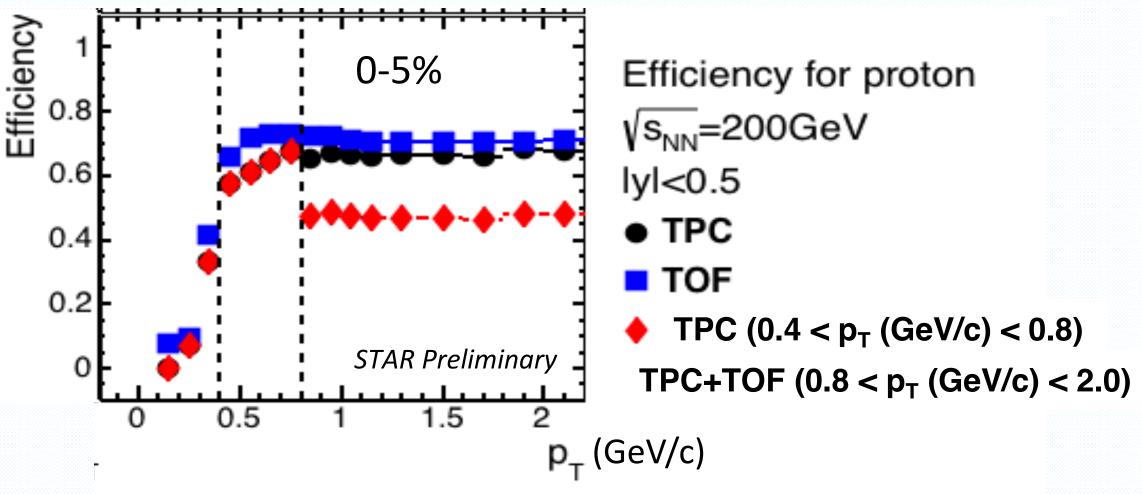
Express the cumulants in terms of the factorial moments or factorial cumulants, which can be easily efficiency corrected by assuming **binomial response function for efficiency**.

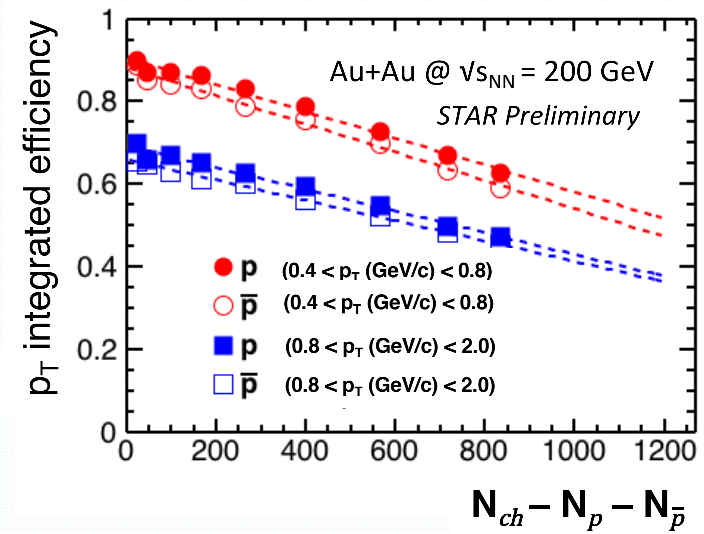
$$F_{ij}(N_p, N_{\bar{p}}) = \frac{f_{ij}(N_p, N_{\bar{p}})}{\varepsilon_p^i \varepsilon_{\bar{p}}^j}$$

$F_{ij}(N_p, N_{\bar{p}})$: corrected factorial moments
$f_{ij}(N_p, N_{\bar{p}})$: measured factorial moments
ε_p, $\varepsilon_{\bar{p}}$: efficiencies of proton and anti-proton

Based on factorial cumulants: T. Nonaka, M. Kitazawa and S. Esumi, Phys. Rev. C 95, 064912(2017).
Based on factorial moments: A. Bzdak and V. Koch, Phys. Rev. C 91, 027901 (2015). X. Luo, Phys. Rev. C 91, 034907(2015). X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)

Detector Efficiency





$$< \epsilon > = \frac{\int_{p_{T1}}^{p_{T2}} \epsilon(p_T)\, f(p_T)\, dpT}{\int_{p_{T1}}^{p_{T2}} f(p_T)\, dp_T}$$

- p_T – integrated efficiency is calculated as a function of multiplicity
- Efficiency correction is applied at each multiplicity bin

PART - I

Search for the QCD Critical Point

Cumulants and Correlation Functions

- Higher order cumulants are more sensitive to the correlation lengths

$$C_2 = \langle(\delta N)^2\rangle \sim \xi^2;\ \ C_3 = \langle(\delta N)^3\rangle \sim \xi^{4.5};\ \ C_4 = \langle(\delta N)^4\rangle \sim \xi^7 \quad \text{with} \quad \delta N = N - \langle N\rangle$$

M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).
M.Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009); Y. Hatta, M. Stephanov, Phys. Rev. Lett. 91, 102003 (2003).

- Relation between cumulants (C_n) and correlation functions ($\hat{\kappa}_n$)

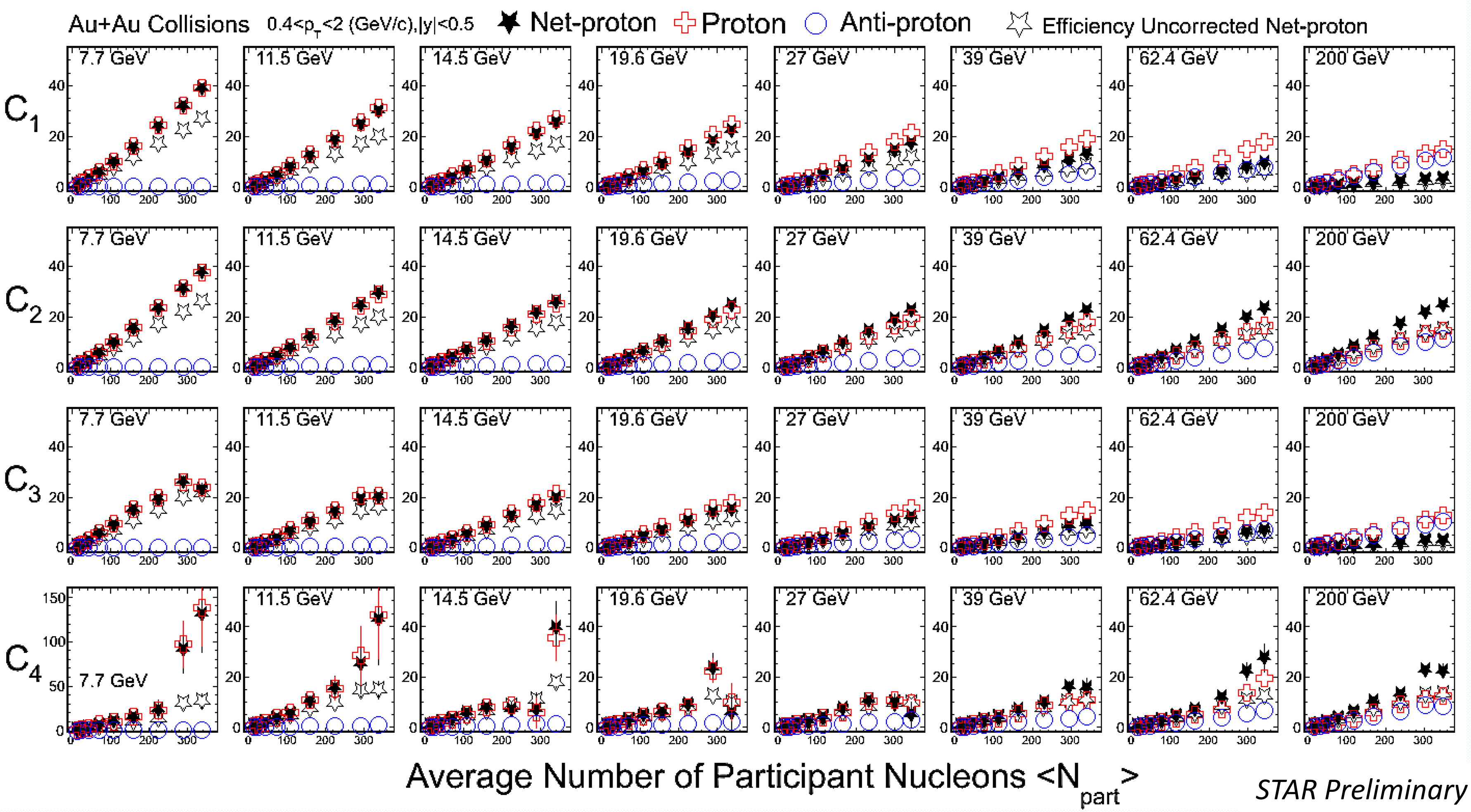
$$\hat{\kappa}_1 = C_1$$
$$\hat{\kappa}_2 = C_2 - C_1$$
$$\hat{\kappa}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{\kappa}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

$$C_1 =< N >$$
$$C_2 =< N > + \hat{\kappa}_2$$
$$C_3 =< N > + 3\hat{\kappa}_2 + \hat{\kappa}_3$$
$$C_4 =< N > + 7\hat{\kappa}_2 + 6\hat{\kappa}_3 + \hat{\kappa}_4$$

$$\hat{\kappa}_2 \propto \xi^2, \hat{\kappa}_3 \propto \xi^{4.5}, \hat{\kappa}_4 \propto \xi^7$$

B. Ling, M. Stephanov, Phys. Rev. C 93, 034915 (2016); A. Bzdak, V. Koch, N. Strodthoff, Phys. Rev. C 95, 054906 (2017).
A. Bzdak, V. Koch, V. Skokov, Eur. Phys. J. C 77, 288 (2017).

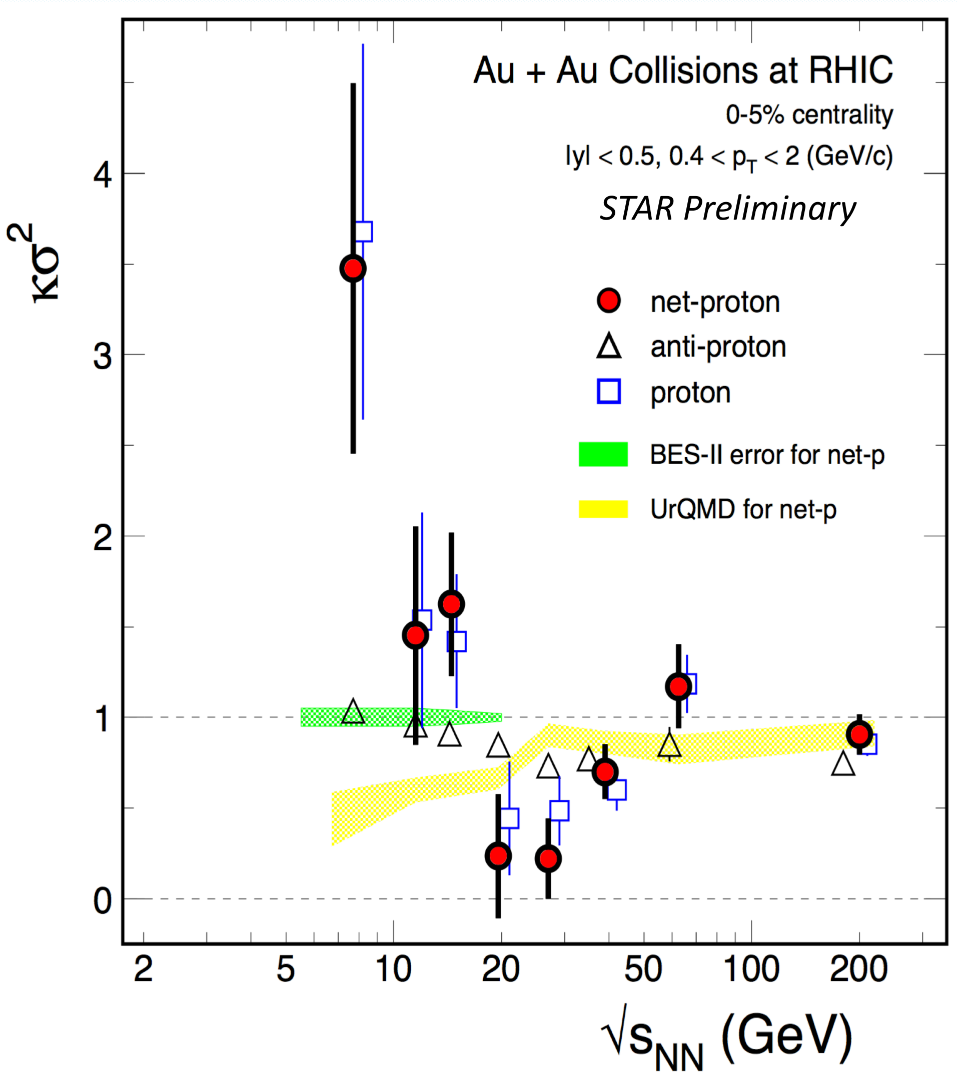
Cumulants vs centrality



STAR Preliminary

- In general, cumulants are linearly increasing with collision centrality.
- At low energies, the proton cumulants are close to the net-proton cumulants.

Net-Proton Fourth-Order Fluctuation

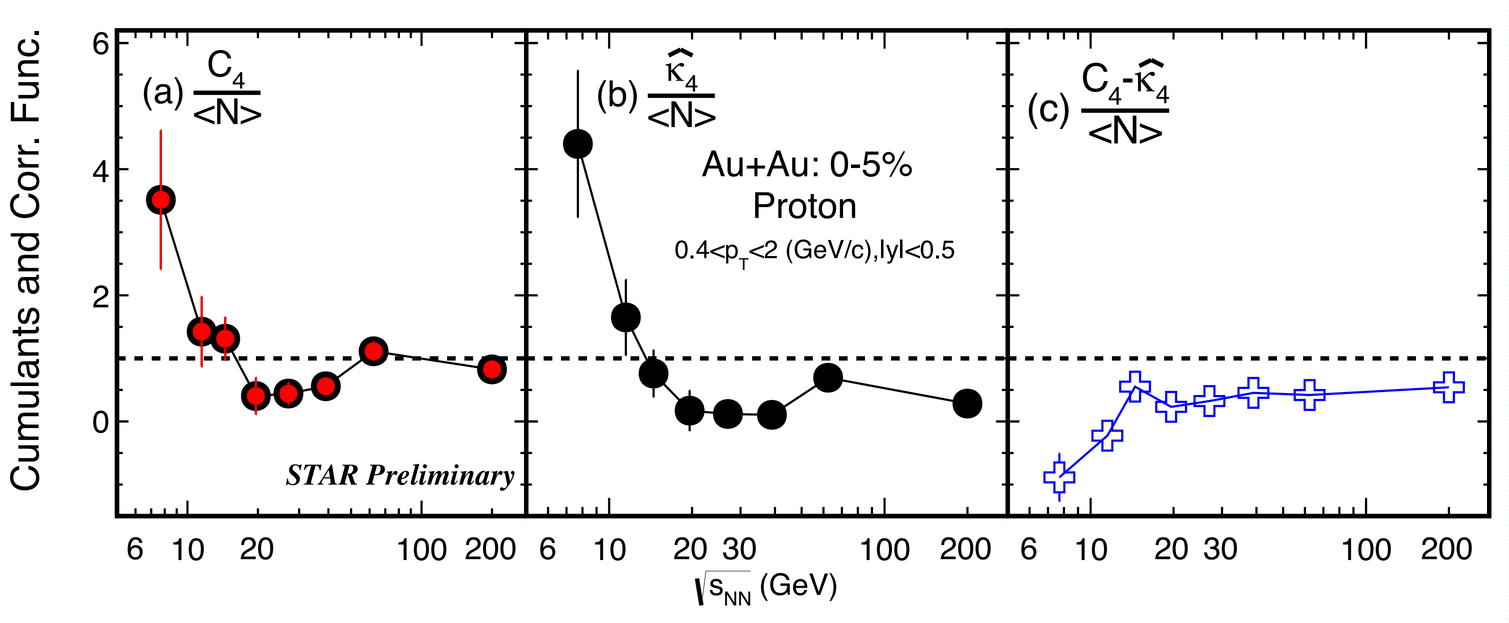


- Non-monotonic energy dependence is observed for 4th order net-proton, proton fluctuations in most central Au+Au collisions.

$$\kappa\sigma^2 = \frac{C_4}{C_2}$$

- UrQMD results show monotonic decrease with decreasing collision energy.

Contributions from Four-Particle Correlations



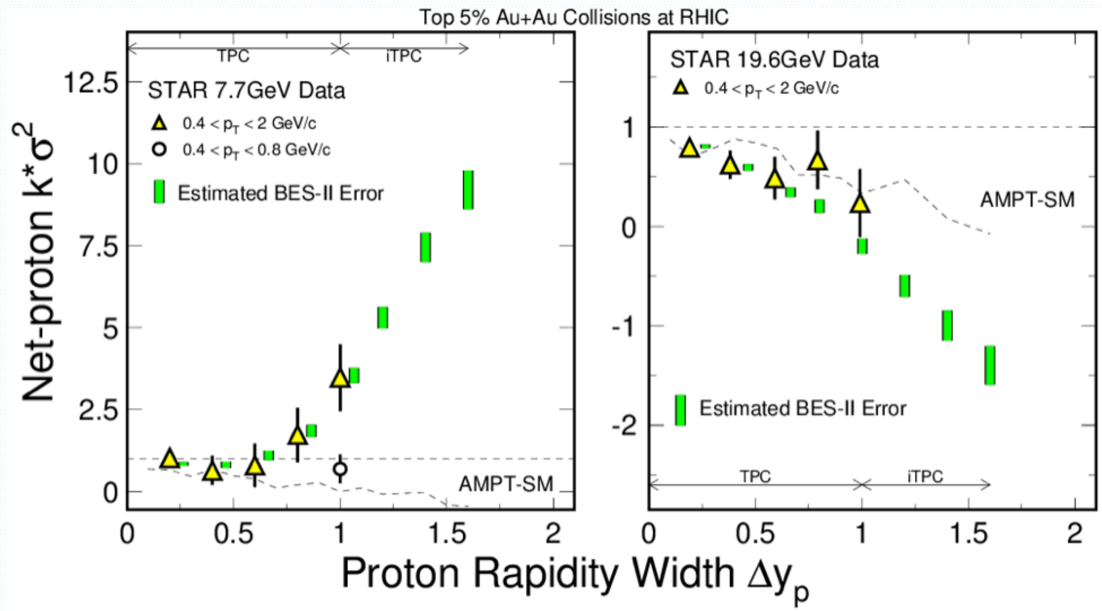
- Four-particle correlations contribute dominantly to the observed non-monotonicity.

Summary - I

- Non-monotonic energy dependences of net-proton and proton C_4/C_2 are observed for 0–5% central Au+Au collisions.

- Four-particle correlations contribute dominantly to the observed non-monotonicity.

- More data will be collected in BES-II at $\sqrt{s_{NN}}$ = 7.7 – 19.6 GeV in 2019–2020 with detector upgrades.



STAR Collaboration, https://drupal.star.bnl.gov/STAR/starnotes/public/sn0619

PART – II

Measurement of the net-proton sixth-order cumulant at small μ_B and its comparison to Lattice QCD

Connections with Lattice QCD

- LQCD predicts a "crossover" for $\mu_B = 0$

  Y. Aoki, Nature 443, 675(2006)

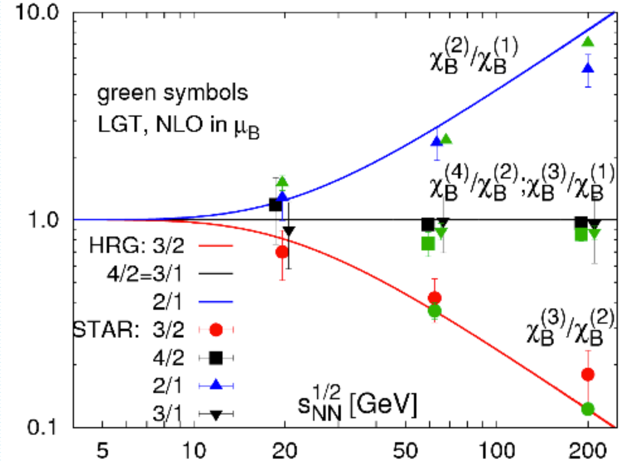
- No established approach to do QCD calculations at finite μ_B

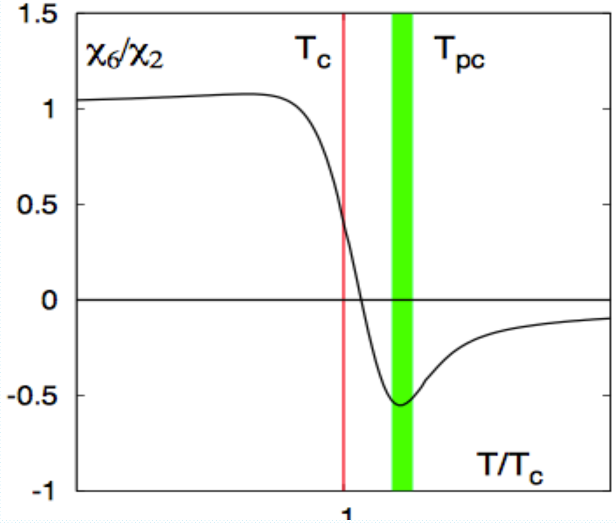
- By putting $\mu_Q = \mu_S = 0$ and using Taylor expansion, the equation of state for finite μ_B:

$$\frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{1}{2}\chi_2^B(T)\left(\frac{\mu_B}{T}\right)^2 \times \left[1+\frac{1}{4}\frac{\chi_4^B(T)}{\chi_2^B(T)}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\left(\frac{\mu_B}{T}\right)^4\right] + O(\mu_B^8)$$

- The sixth-order cumulant of baryon number is expected to be negative at chiral transition temperature

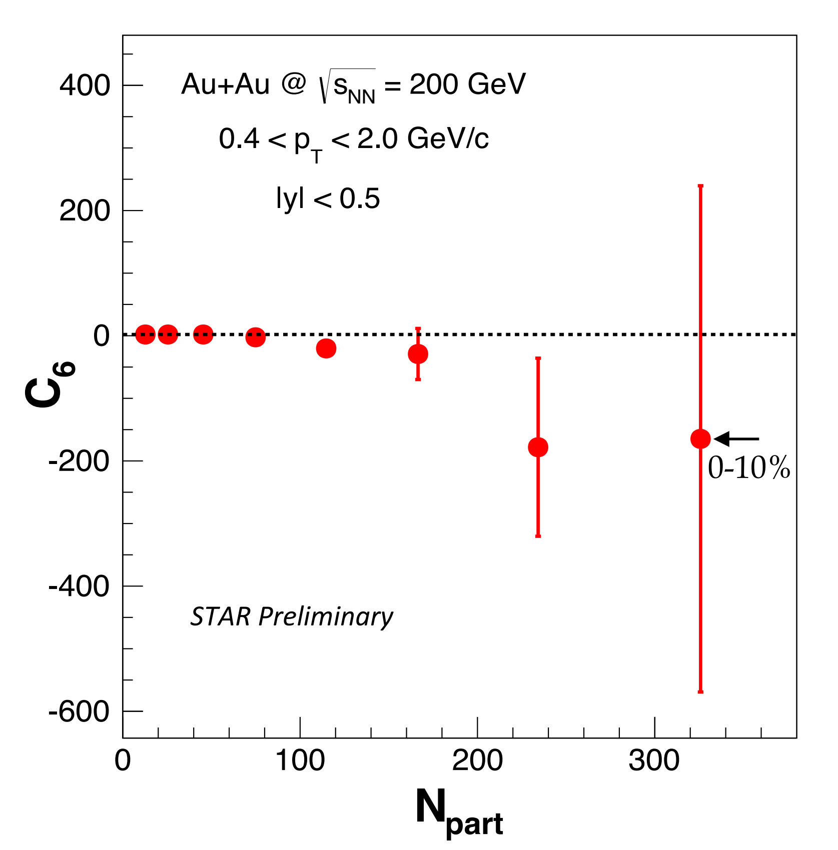
F. Karsch and K. Redlich, Phys. Lett. B 695, 136 (2011)
STAR Collaboration, Phys.Rev.Lett. 112 (2014) 032302.

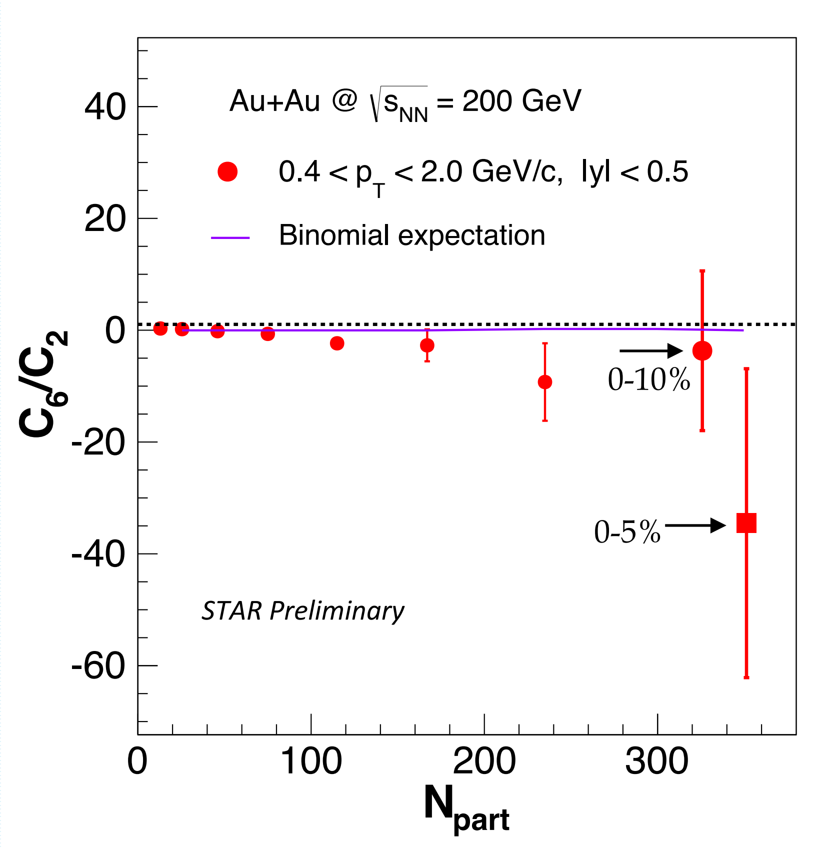




B. Friman, F. Karsch, K. Redlich, V. Skokov Eur. Phys. J. C 71, 1694 (2011)

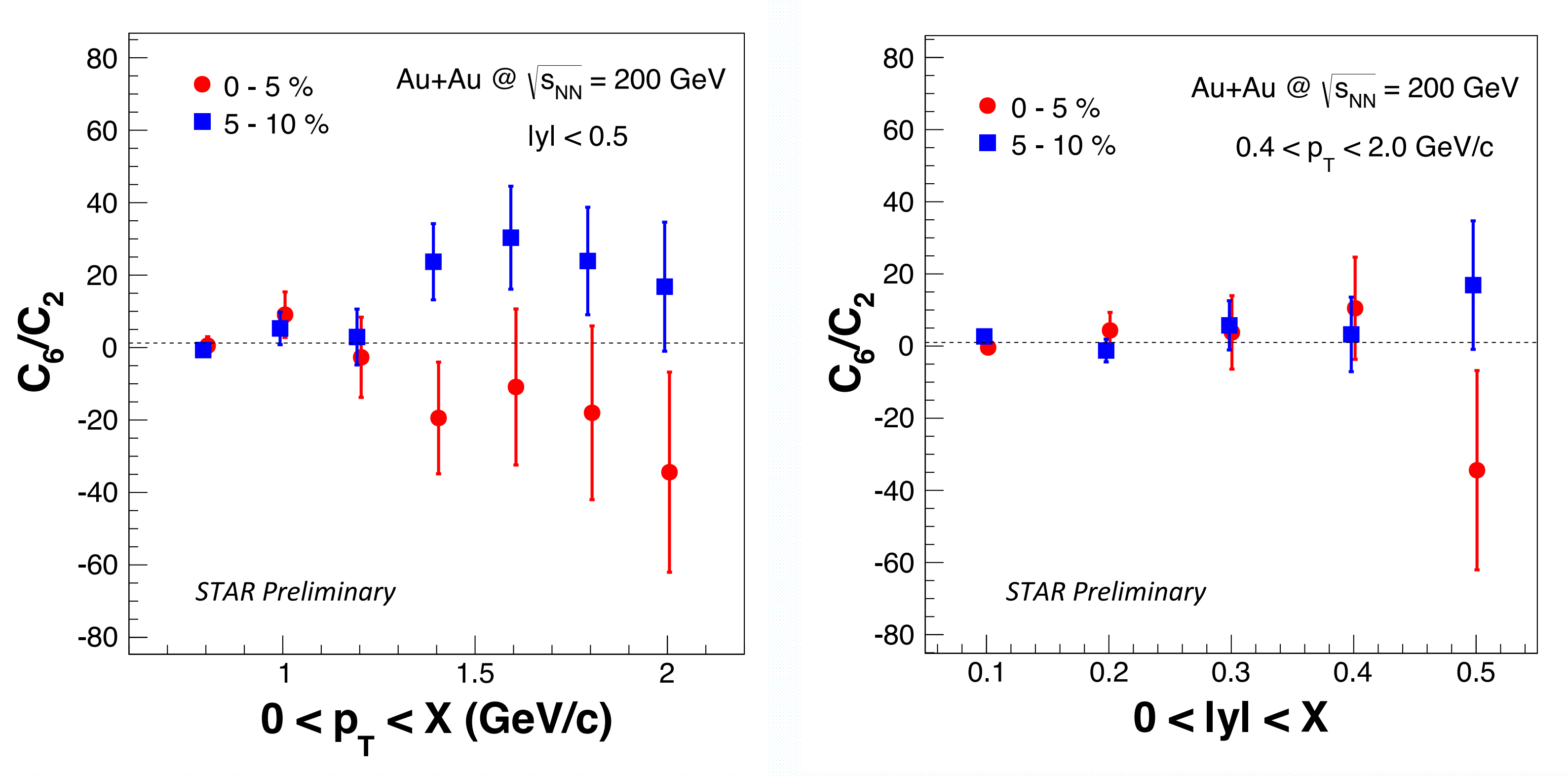
Net-Proton Sixth-Order Cumulant





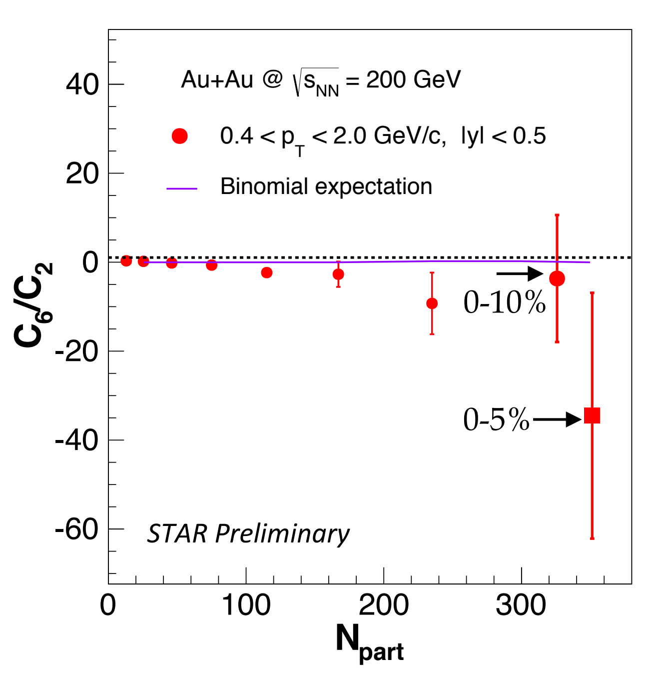
- Combined data of Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV from year 2010 and 2011: 200 M (0 – 10 %) and 650 M (10 – 80 %) events
- The C_6 and C_6/C_2 are negative for central collisions with large statistical uncertainties

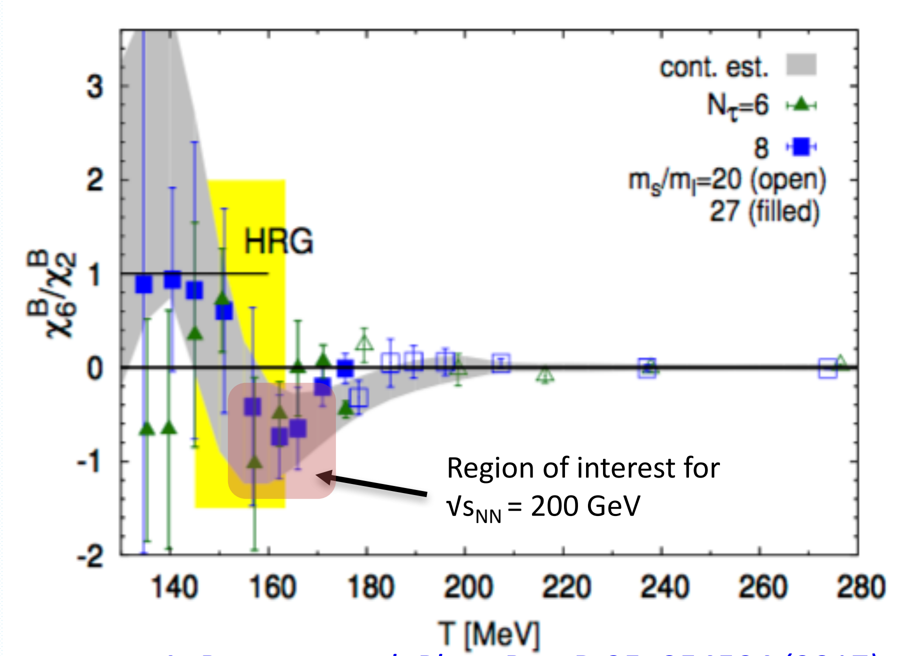
Acceptance Dependence



- Around 160 M events are analyzed for 0-10% central Au + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV with central trigger from year 2010

Comparison with LQCD





A. Bazavov et al, Phys. Rev. D 95, 054504 (2017).

- C_6/C_2 for most central collisions is negative with large uncertainties in the STAR data
- Some differences between STAR measurements and LQCD calculations:
  1. Net-proton is not equivalent to net-baryon
  2. Limited phase space
  3. $\mu_B \neq 0$ ($\mu_B \sim 20$ MeV at $\sqrt{s_{NN}} = 200$ GeV)

Summary - II

- We report the efficiency-corrected sixth-order cumulant of the net-proton multiplicity distribution for Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV.

- Centrality, transverse momentum and rapidity dependences of the ratio C_6/C_2 are presented.

- C_6 and C_6/C_2 are negative for central collisions with large statistical uncertainties.

- Assessment of systematic uncertainties is underway.

- Combining the data taken in year 2014 and 2016, with more than 2 billion events, we can get a better control on the statistical uncertainties for C_6/C_2.

PART – III

Data-driven approach for efficiency corrections

(Simulation studies)

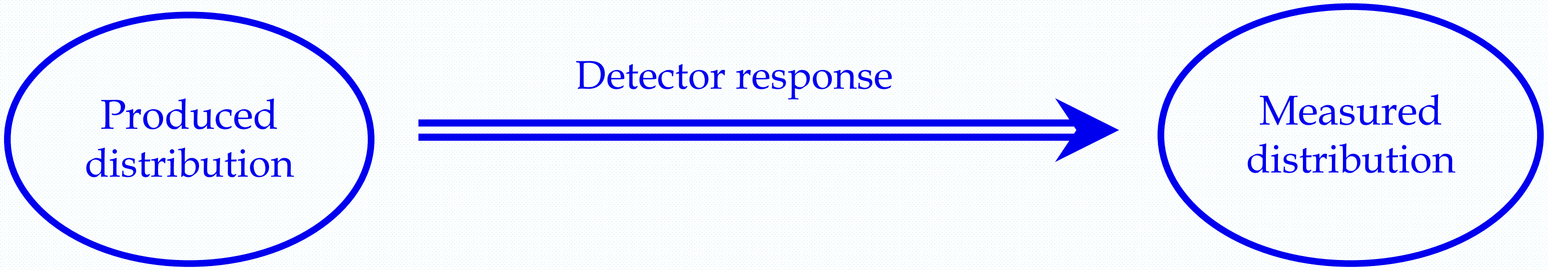
Motivation

- Efficiency correction is an important ingredient in order to reliably calculate the higher-order cumulants
- According to A. Bzdak *et al*, there could be noticeable consequences of the multiplicity-dependent behavior in detection efficiency on measured higher-order cumulants

A. Bzdak, R. Holzmann and V. Koch, Phys.Rev. C 94, 064907 (2016)

- We need to develop an approach to explore these issues adequately, which we have not done previously in our data analyses.

Methodology



Number of produced particles	Number of particles detected
10	8
11	7
8	7
9	7
10	9
11	8
8	7
9	7

Get the distribution of the number of produced particles for a given number of measured particles using embedding

Algorithm

Ingredients

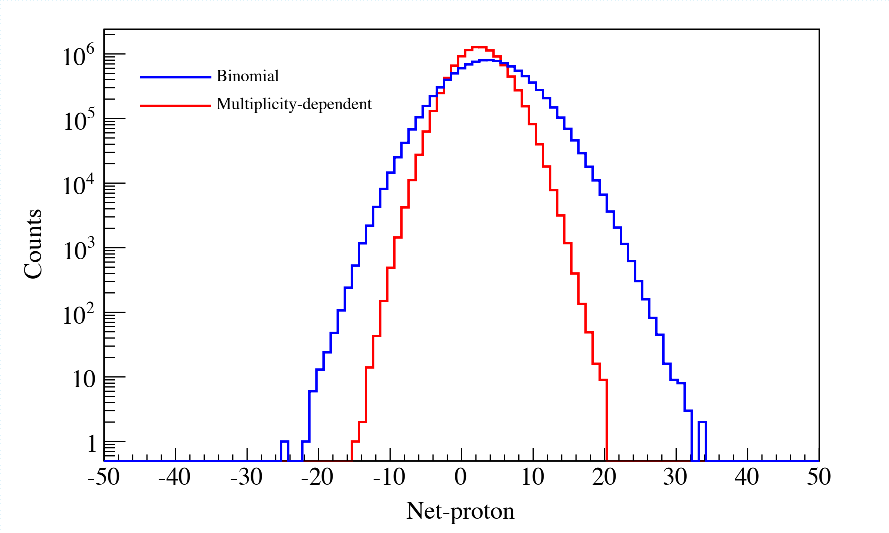
- Correlation histogram
  - Contains the number correlation between measured protons and anti-protons
- Response histograms
  - Contains the distribution of produced particles for every detected number of particles
  - Obtained using information from embedding

Steps

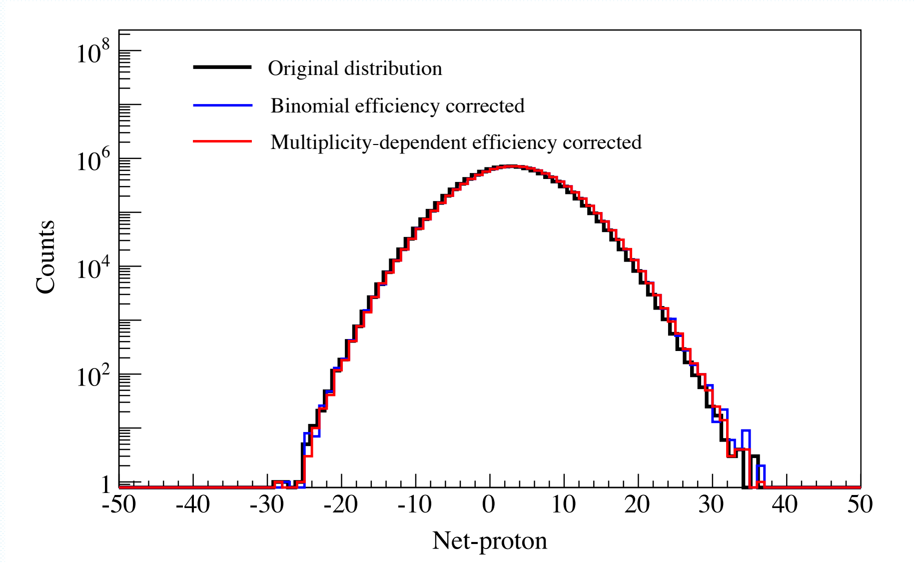
- Sample an event from the correlation histogram and correct the number of protons and antiprotons using the response histograms
- Repeat the above process M times with the same number of events as in the true distribution (correlation histogram).
- Evaluate the cumulants for each of these M copies.
  - The mean will give us the values of cumulants and the width will be the respective error

Results

Measured distribution



Produced distribution

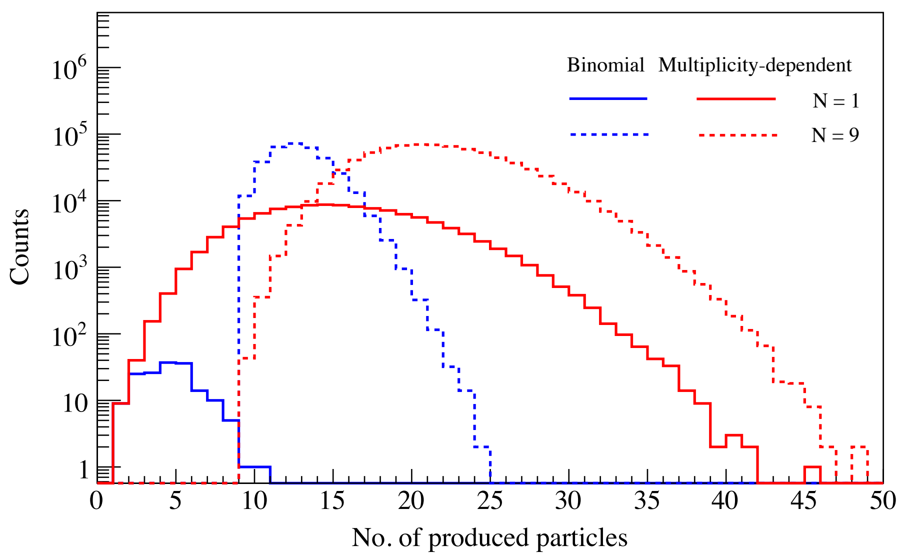


An example of measured and efficiency-corrected distributions using AMPT with Binomial and multiplicity-dependent efficiencies for 0-5% central Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV

For the purpose of simulation, we have 10 M events and 1000 copies.

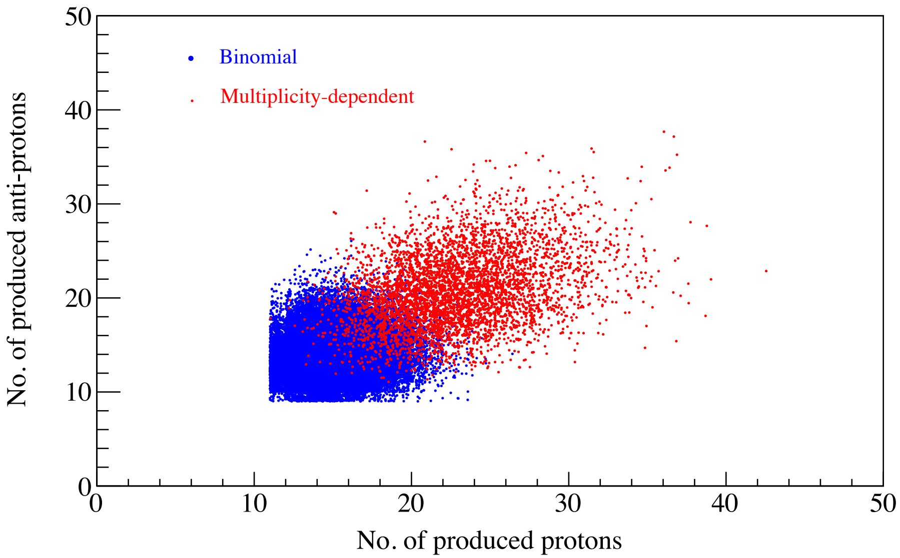
Results

Response histogram 1D



An example of the distribution of produced particles for a given number of detected particles (N)

Response histogram 2D



An example of the distribution of produced particles for a given number of detected particles (N = 11 for proton and N = 9 for anti-proton)

Results

- *Poisson distribution for protons and antiprotons with **Binomial efficiency***

Poisson mean for proton = 10
Poisson mean for antiproton = 9

Efficiency for proton = 0.8
Efficiency for antiproton = 0.7

Cumulant for net-proton distribution	**Skellam (Analytically)**	**Efficiency corrected (this method)**	**Efficiency corrected (factorial moment method)**
C_1	1	0.9996 $\pm$ 0.0005	1.001 $\pm$ 0.0006
C_2	19	18.990 $\pm$ 0.003	18.990 $\pm$ 0.004
C_3	1	1.035 $\pm$ 0.023	1.045 $\pm$ 0.031
C_4	19	19.29 $\pm$ 0.39	18.696 $\pm$ 0.28

Results

- *Poisson distribution for protons and antiprotons with **multiplicity-dependent efficiency***

Poisson mean for proton = 10
Poisson mean for antiproton = 9

Efficiency for proton = 0.8 - 0.0003*N_{proton}
Efficiency for antiproton = 0.7 - 0.0003*$N_{antiproton}$

Cumulant for net-proton distribution	Skellam (Analytically)	Efficiency corrected (this method)	Efficiency corrected (factorial moment method)
C_1	1	1.000 $\pm$ 0.0004	0.998 $\pm$ 0.0006
C_2	19	19.00 $\pm$ 0.0038	18.78 $\pm$ 0.0040
C_3	1	1.021 $\pm$ 0.024	1.096 $\pm$ 0.031
C_4	19	19.11 $\pm$ 0.34	16.99 $\pm$ 0.28

The coefficient (0.0003) is the expected order of magnitude of the multiplicity dependence of efficiency

- *Even a seemingly small non-binomial effect could have a noticeable consequence on higher-order cumulants -- Pointed out by* A. Bzdak *et al.*

A. Bzdak, R. Holzmann and V. Koch, Phys.Rev. C 94, 064907 (2016)

Results

- *AMPT model with **multiplicity-dependent efficiency** for 0-5% central Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV*

  Efficiency for proton = 0.8 - 0.0003*(N_{charge} - N_{proton} - $N_{antiproton}$)
  Efficiency for antiproton = 0.7 - 0.0003*(N_{charge} - N_{proton} - $N_{antiproton}$)

Cumulant for net-proton distribution	True distribution	Efficiency corrected (2D response matrix)	Efficiency corrected (1D response matrix)	Efficiency corrected (factorial moment method)
C_1	2.7990 ± 0.0017	2.7994 ± 0.0019	2.8001 ± 0.0020	2.5502 ± 0.0011
C_2	31.436 ± 0.015	31.435 ± 0.014	49.777 ± 0.019	12.632 ± 0.012
C_3	8.43 ± 0.15	8.45 ± 0.14	9.33 ± 0.24	2.58 ± 0.04
C_4	91.33 ± 1.57	90.95 ± 1.98	88.89 ± 3.49	12.49 ± 0.28

2D response matrix : Protons and anti-protons are corrected simultaneously
1D response matrix : Protons and anti-protons are corrected separately
Factorial moment method assumes binomial efficiency correction. CBWC is applied.

- The coefficient (0.0003) is the expected order of magnitude of the multiplicity dependence of efficiency
- *Even a seemingly small non-binomial effect could have a noticeable consequence on higher-order cumulants -- Pointed out by* A. Bzdak *et al.*

A. Bzdak, R. Holzmann and V. Koch, Phys.Rev. C 94, 064907 (2016)

Summary - III

- Through Monte-Carlo simulations, we demonstrated a data-driven efficiency correction method to take into account the possible experimental effects like multiplicity-dependent detection efficiency.

- Implementation of this method in future STAR data analyses is under study.

Thank you !