Chiral magnetic effect & anomalous transport in real-time

Soeren Schlichting

Based on:
N. Mueller, S. Schlichting and S. Sharma, PRL 117 (2016) no.14, 142301
M. Mace, N. Mueller, S. Schlichting and S. Sharma, PRD95 (2017) no.3, 036023
T. Lappi, S. Schlichting in preparation

“Critical Point and Onset of Deconfinement”, Stonybrook, 09 Aug 2017

UNIVERSITY *of* WASHINGTON

CME in Heavy-Ion Collisions

High-energy heavy-ion collisions provide an exciting environment to explore anomalous transport phenomena

Chiral Magnetic Effect: $\vec{j}_v \propto j_a^0 \vec{B}$

axial charge density

magnetic field

— *Strong magnetic field $eB \sim m_\pi{}^2$ present in off-central collisions*

— *Non-conservation of axial charge*

$$\partial_\mu j^\mu_{5,f} = 2m_f \bar{q}\gamma_5 q - \frac{g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

expected to lead to lead to significant fluctuations e.g. due to sphaleron transitions

Chiral magnetic effect presents exciting opportunity to further explore dynamics of QGP, e.g. topological properties

CME in Heavy-Ion Collisions

Challenges in theoretical description:

Since life time of magnetic field is presumably very short (~0.1-1 fm/c) system is out-of-equilibrium during the time scales relevant for CME & Co.

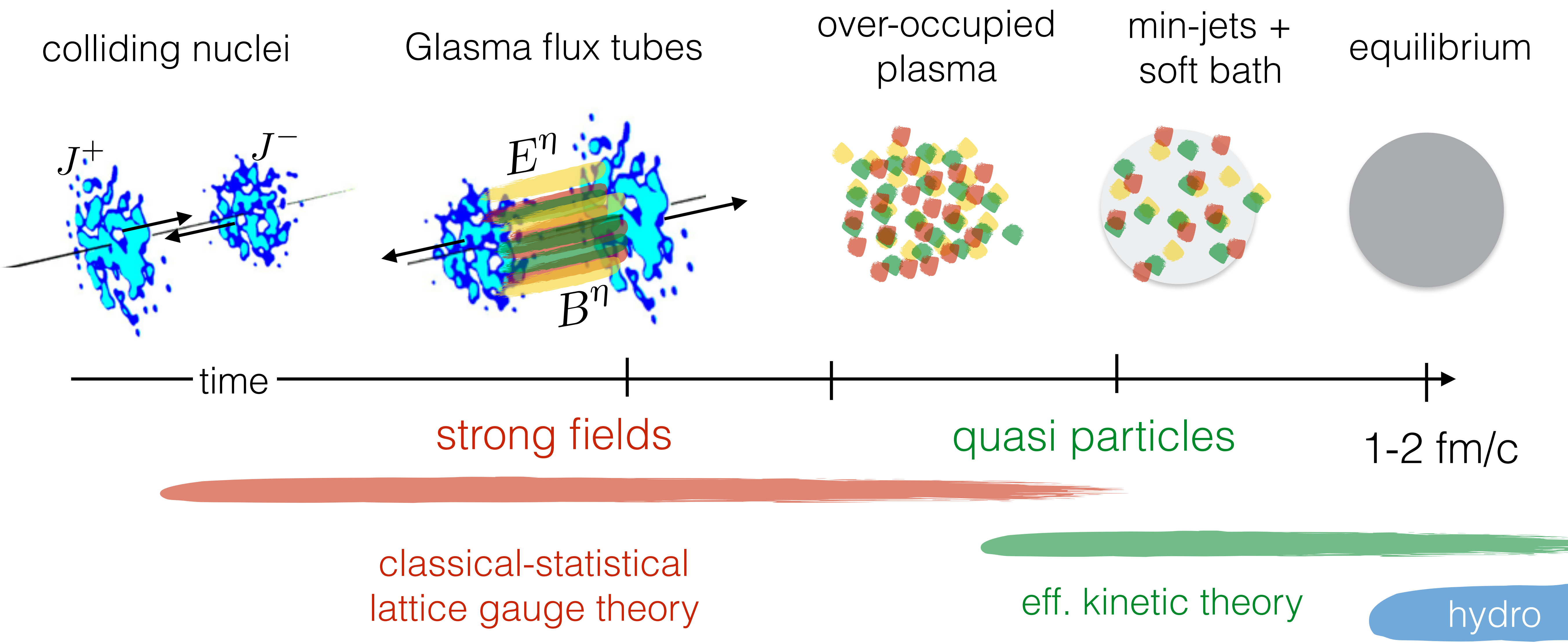
Need to understand non-equilibrium dynamics of axial and vector charges during the early-time pre-equilibrium phase

Existing theoretical approaches such as anomalous hydro or chiral kinetic theory effectively treat axial charge as a conserved quantity

In order to correctly describe generation of axial charge imbalance (e.g. due to sphalerons) field theoretical description is required

-> Develop field theoretical approach to describe early time dynamics and possibly devise improved macroscopic description of anomalous transport

Early-time dynamics of HIC



colliding nuclei

Glasma flux tubes

over-occupied plasma

min-jets + soft bath

equilibrium

time

strong fields

quasi particles

1-2 fm/c

classical-statistical lattice gauge theory

eff. kinetic theory

hydro

Early time dynamics described in terms of classical field dynamics amenable to non-perturbative real-time lattice simulations

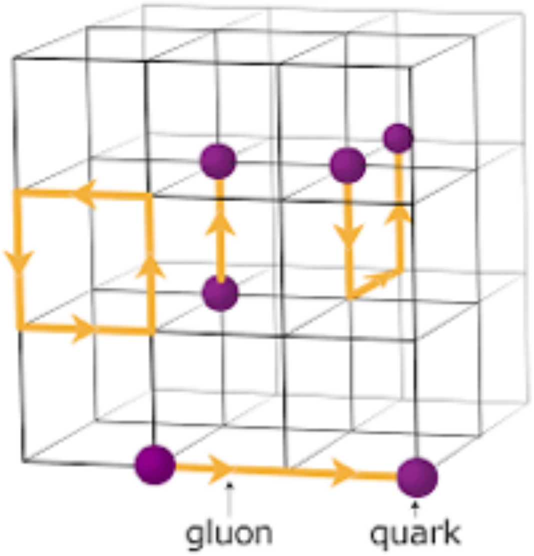
-> Include dynamical fermions to study anomalous transport

Simulation technique

Classical-statistical lattice simulation with dynamical fermions

(Aarts, Smit; Berges, Hebenstreit,Kasper, Mueller; Tranberg, Saffin; …)

- Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism



- Solve operator Dirac equation in the presence of SU(N) and U(1) gauge fields

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

- Compute expectation values of vector and axial currents to study anomalous transport processes

$$j^\mu_v(x) = \langle \hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\rangle \quad j^\mu_a(x) = \langle \hat{\bar{\psi}}(x)\gamma^\mu\gamma^5\hat{\psi}(x)\rangle$$

Dynamical fermions

Solving the operator Dirac equation can be achieved by expanding the fermion field in operator basis at initial time

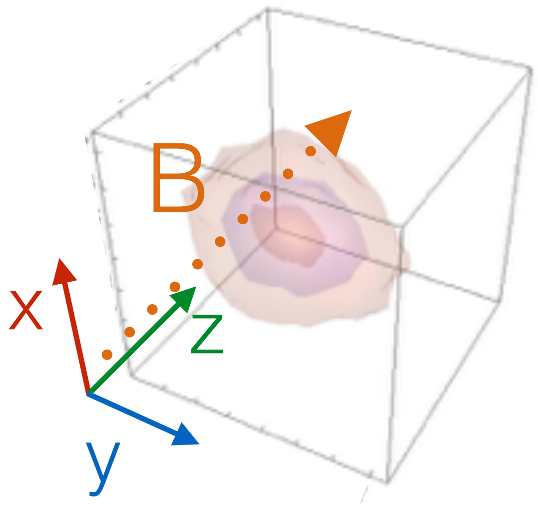
$$\hat{\psi}(x,t) = \sum_{p,\lambda} \hat{b}_{p,\lambda}(t=0)\phi_u^{p,\lambda}(x,t) + \hat{d}^{\dagger}_{p,\lambda}(t=0)\phi_v^{p,\lambda}(x,t)$$

and solving the Dirac equation for evolution of $4N_cN^3$ wave-functions

Computationally extremely demanding (~TB memory, ~M CPU hours)
Not clear to what extent stochastic estimators are useful to reduce problem size

So far first results on small lattices 24 x 24 x 64 in a clean theoretical setup

SU(N): Single sphaleron transition
U(1): constant magnetic field



Axial anomaly in real-time

Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Naive fermion discretization: Cancellation of axial anomaly due to Fermion doublers

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) >$$

Exploit knowledge from Euclidean lattice simulations

Wilson fermions: Explicit symmetry breaking term added to the Hamiltonian to decouple doublers (c.f. Aarts,Smit)

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) > + r_W < W(x) > \xrightarrow{\text{cont. limit}} -\frac{g^2}{8\pi^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$$

Overlap fermions: Non-local derivative operator with chiral properties on the lattice

Axial anomaly in real-time

Non-trivial cross check of axial charge production (B=0)

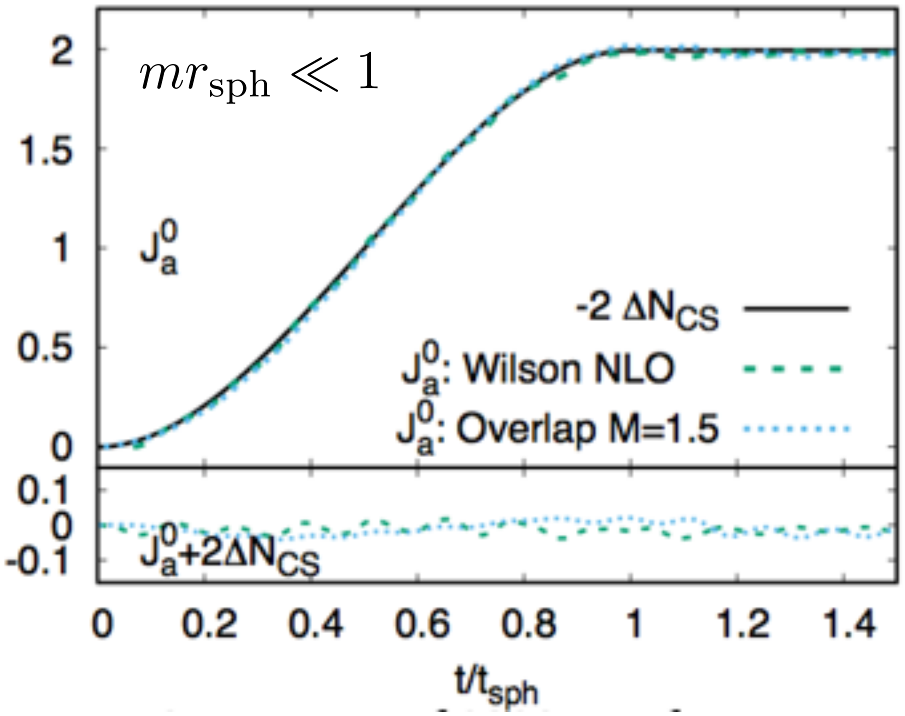
Over the coarse of the sphaleron transition Chern-Simons number

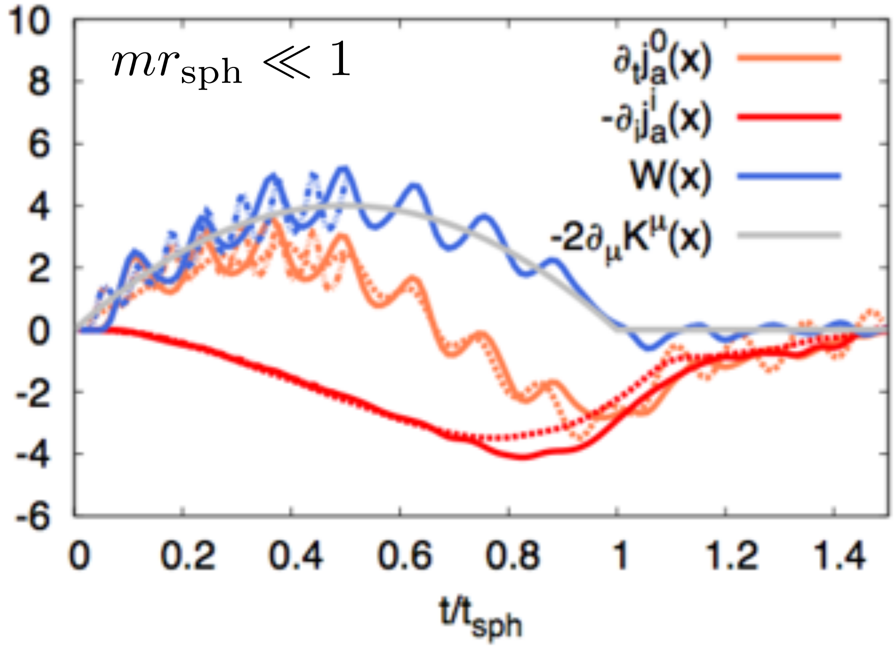
$$\Delta N_{CS} = \frac{g^2}{8\pi^2}\int d^4x\ \vec{E}_a\vec{B}_a$$

changes by an integer amount leading to an imbalance of axial charge

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f\int d^4x\langle\bar{\psi}i\gamma_5\psi\rangle$$

Excellent agreement for (almost) massless fermions from simulations with improved Wilson fermions and Overlap fermions



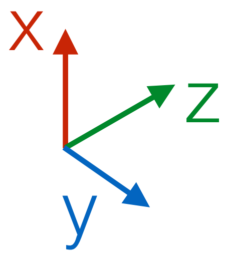


(Mace,Mueller,SS, Sharma, 1612.02477)

CME Dynamics

Axial charge j_5^0

Sphaleron transition induces local imbalance of axial charge density



Vector current j_V^z

Non-zero magnetic field B_z leads to vector current j_V^z in z-direction

Vector charge j_V^0

Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

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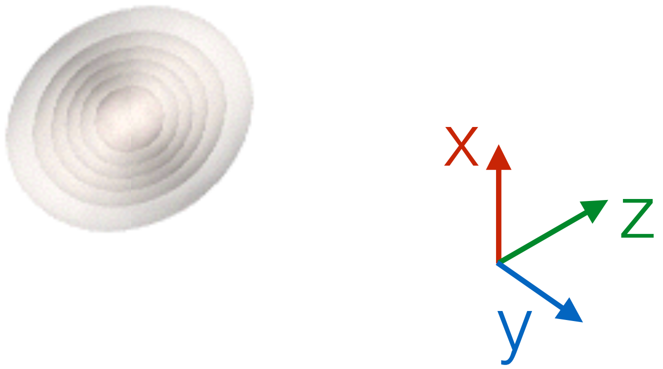

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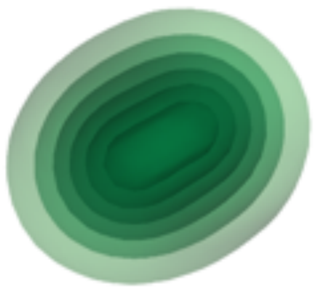

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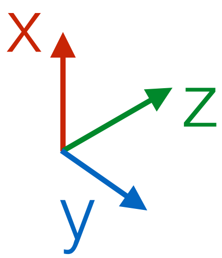
CMW Dynamics

Vector charge imbalance j_V^0 generates an axial current j_5^z so that axial charge also flows along the B-field direction

Axial charge j_5^0



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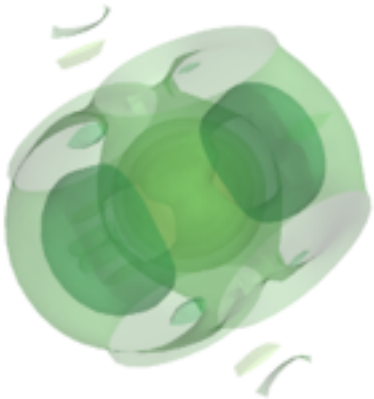

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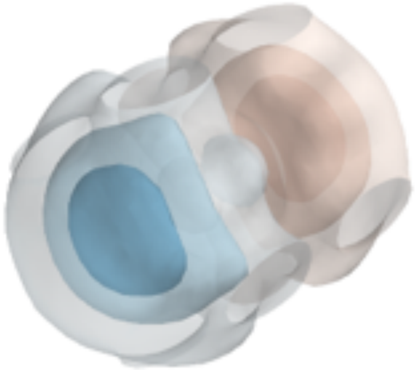

Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

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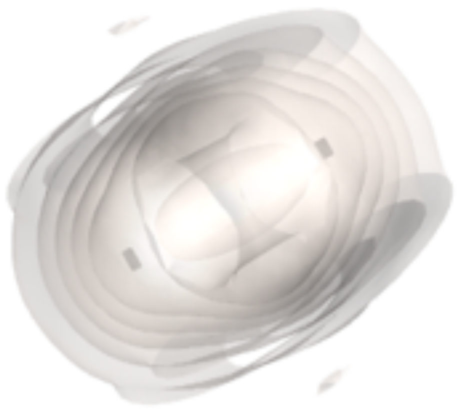

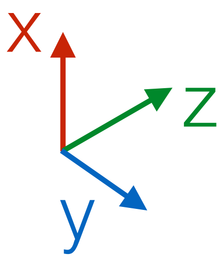

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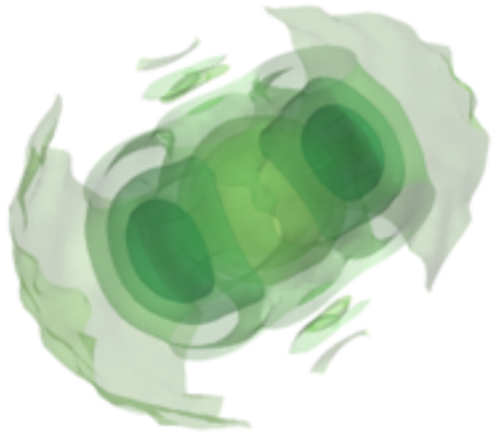

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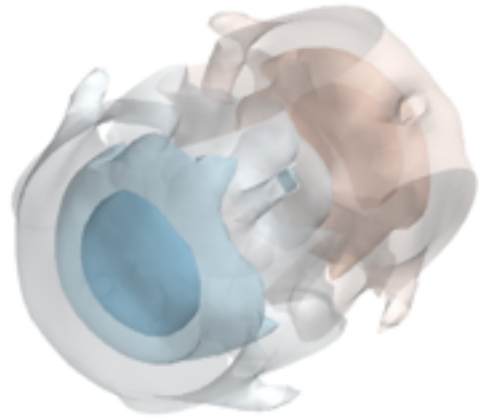

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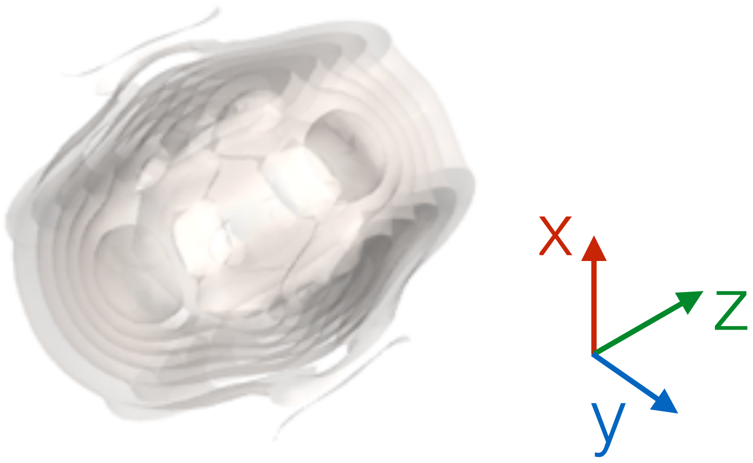

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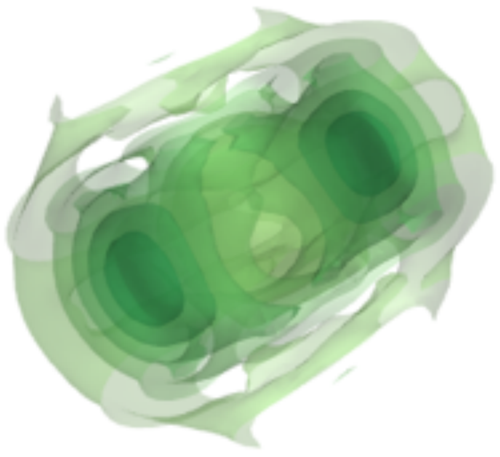

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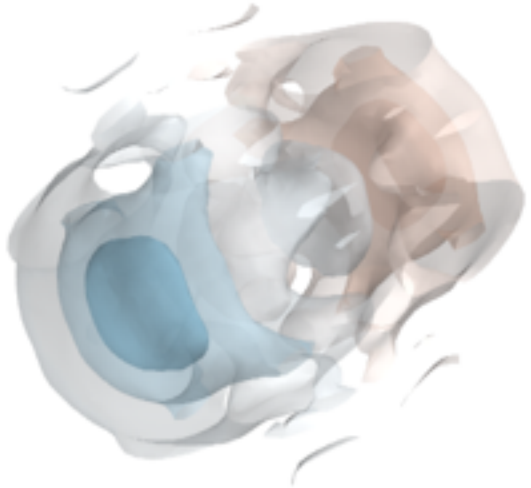

Emergence of a Chiral Magnetic Shock-wave of vector charge and axial charge propagating along B-field direction

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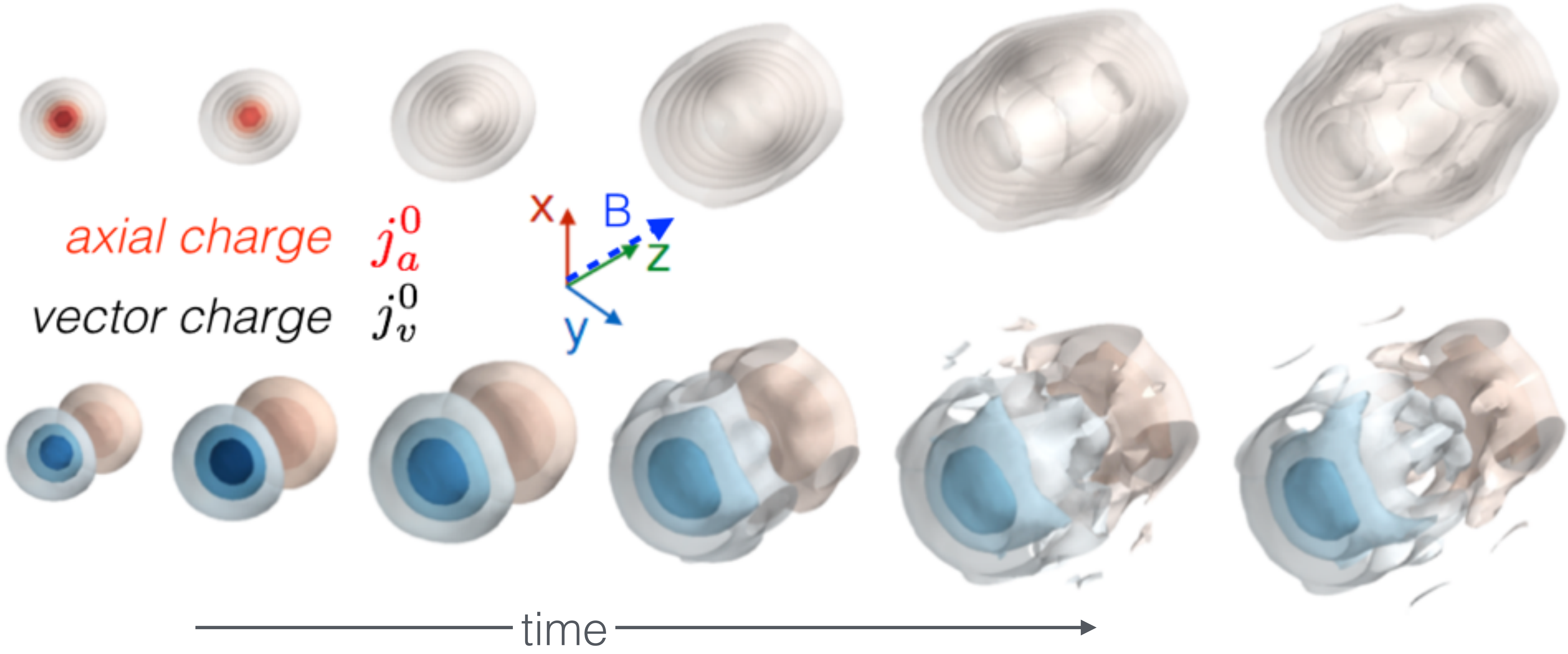
Non-equilibrium dynamics of vector and axial charges



Clear separation of electric charge j_V^0 along the B-field direction

Non-equilibrium dynamics of vector and axial charges

Comparison with anomalous hydro
(light quarks $m r_{\mathrm{sph}} \ll 1$)

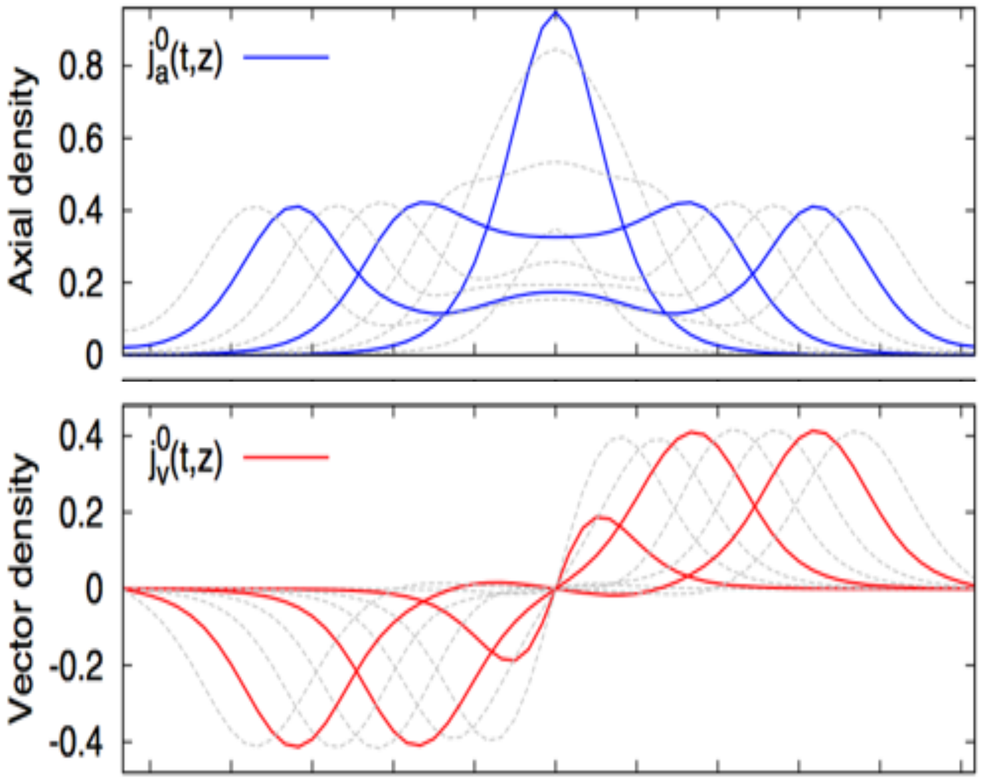
$$\partial_\mu j_a^\mu = S(x)\ , \qquad \partial_\mu j_v^\mu = 0$$

$$j^\mu_{v/a} = n_{v/a} u^\mu + \sigma^B_{v/a} B^\mu$$

Strong field limit ($B \gg r_{\mathrm{sph}}^{-2}, m^2$)

$$\partial_t \begin{pmatrix} j_v^0(t,z) \\ j_a^0(t,z) \end{pmatrix} = -\partial_z \begin{pmatrix} j_a^0(t,z) \\ j_v^0(t,z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t,z) \end{pmatrix}$$

Simulation results for light quarks

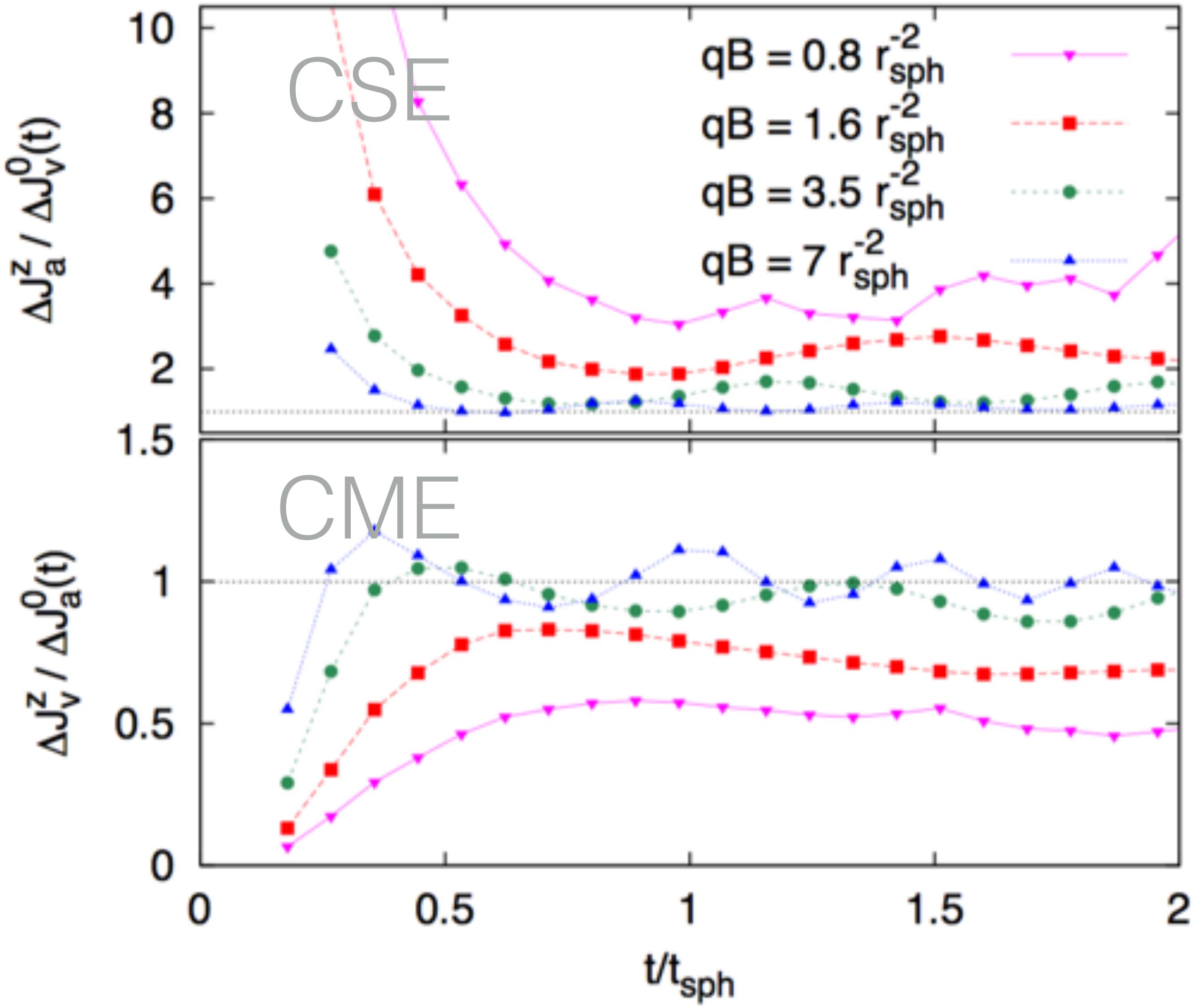


Chiral magnetic shock-wave

$$j^0_{v/a}(t > t_{\mathrm{sph}}, z) = \frac{1}{2}\int_0^{t_{\mathrm{sph}}} dt' \left[S(t', z - c(t-t')) \mp S(t', z + c(t-t')) \right]$$

-> Evolution for light quarks and strong magnetic fields well described by anomalous hydrodynamics at late times

Validity of constitutive relations



(Mace,Mueller,SS, Sharma, 1612.02477)

Verify ratios vector/axial currents and axial/vector charge

$$C_{\mathrm{CME}}(t) = \frac{\Delta J_v^z(t)}{\Delta J_a^0(t)}\,, \qquad C_{\mathrm{CSE}}(t) = \frac{\Delta J_a^z(t)}{\Delta J_v^0(t)}\,.$$

In the strong field limit related to thermodynamic constitutive relations

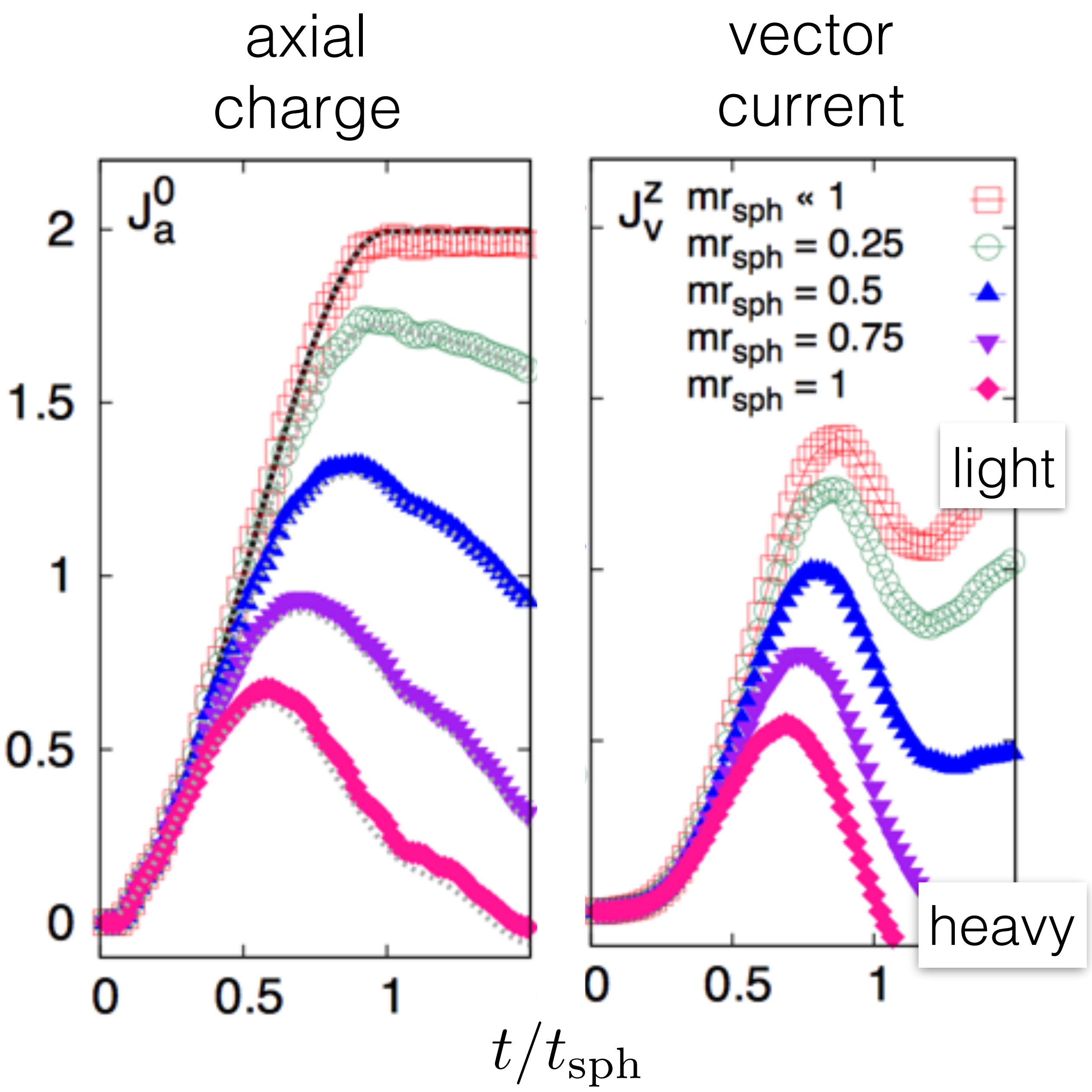
$$C_{CME} = 1\,, \qquad C_{CSE} = 1\,.$$

equal to time independent constants.

Simulation results indicate approach towards constant value with a finite relaxation time

Since lifetime of magnetic field in HIC is short this effect should also be incorporated in phenomenological approaches

Quark mass dependence



Explicit violation of axial charge conservation for finite quark mass

$$\partial_\mu j_a^\mu(x) = 2m\langle \hat{\bar{\psi}}(x) i\gamma_5 \hat{\psi}(x)\rangle + S(x)$$

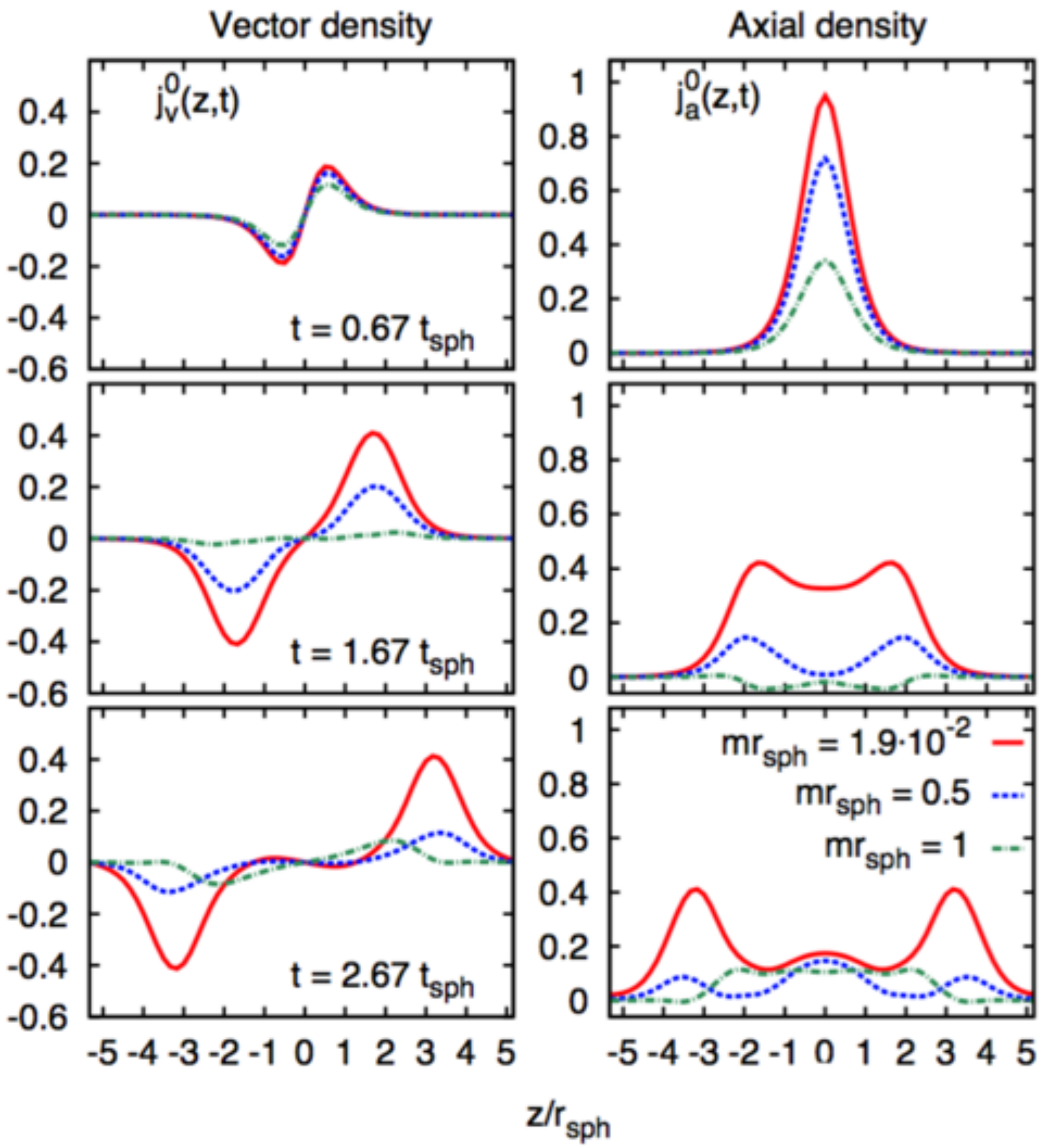
leads to damping of axial charge

Since chiral magnetic effect current is proportional to axial charge density it will also be reduced

$$\vec{j}_v \propto j_a^0 \vec{B}$$

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

Quark mass dependence



Light quarks ($\mathbf{mt_{sph} \ll 1}$)

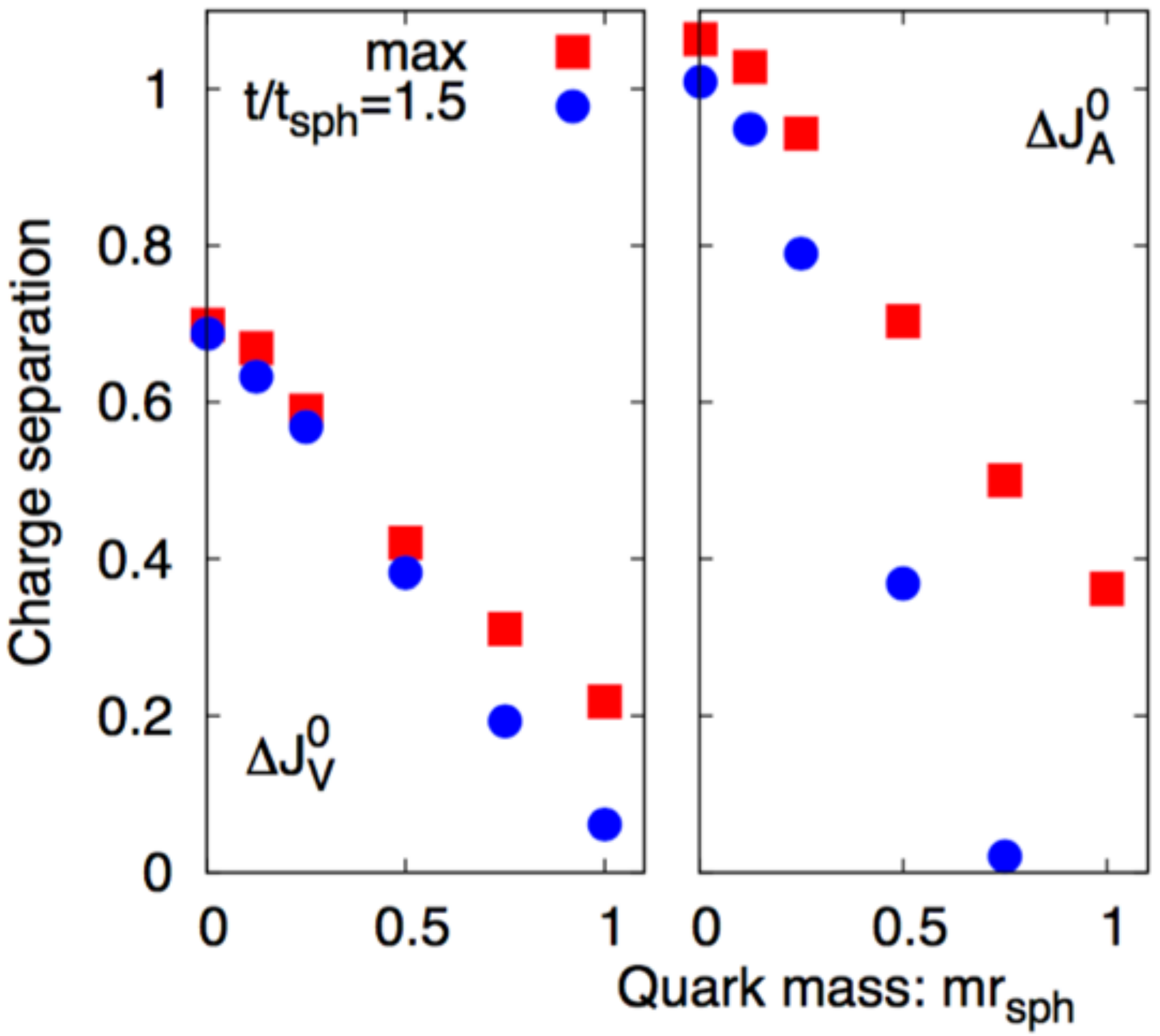
Chiral magnetic wave leads to non-dissipative transport of axial and vector charges

Heavy quarks ($\mathbf{mt_{sph} \sim 1}$)

Dissipation of axial charge leads to significant reduction of charge separation

(Mace,Mueller,SS, Sharma, PRD95 (2017) no.3, 036023)

Quark mass dependence



Significant reduction of the charge separation signal by factor ~5 already for moderate quark masses

Phenomenological consequences

Unlikely that strange quarks participate in CME

Desirable to include dissipative effects in macroscopic description

(Mace,Mueller,SS, Sharma, PRD95 (2017) no.3, 036023)

Outlook

Next step is to extend simulations towards a realistic heavy-ion environment

- Quark production & electro-magnetic response
- Chiral magnetic effect & anomalous transport in HIC

-> provide initial conditions for anon. hydro/ chiral kin. theory

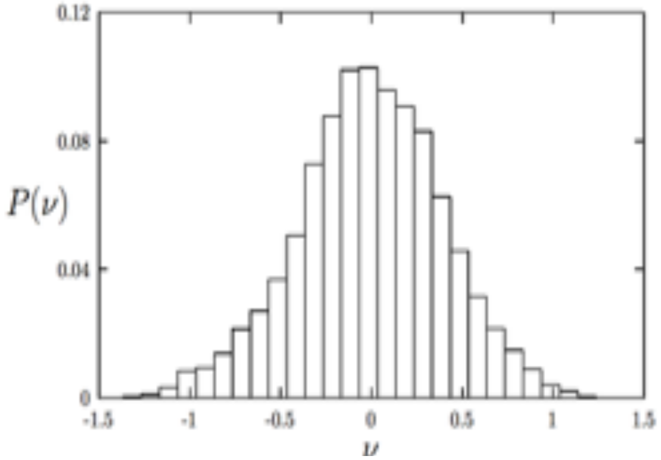
Challenge: Discretization effects for rough gauge field configurations
-> Currently performing detailed investigation in QED

Can already provide input for phenomenological modeling, by estimating axial charge dynamics from gauge field sector

Early time dynamics ($\tau \sim 1/Q_s$)

Early time dynamics of axial charge production can be addressed from considering gauge field dynamics

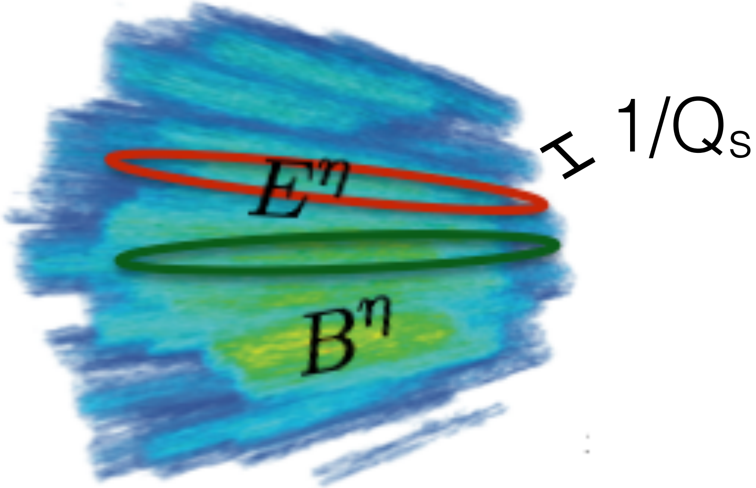
$$\left[\partial_\tau + \frac{1}{\tau}\right] j^\tau_{(5)}(\mathbf{x}) + \partial_i j^i_{(5)}(\mathbf{x}) + \partial_\eta j^\eta_{(5)}(\mathbf{x}) = -\frac{g^2 N_f}{8\pi^2} \mathrm{tr} F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{g^2 N_f}{2\pi^2}\dot{\nu}(\mathbf{x}),$$



Classical Yang-Mills simulations:

Global imbalance of axial charge is small at $\tau \sim 1/Q_s$

Kharzeev, Venugopalan, Krasnitz, PLB545 (2002) 298-306



Early time analytics:

Significant *local* imbalance of axial charge density created at $\tau \sim 1/Q_s$ on microscopic scales $\sim Q_s$

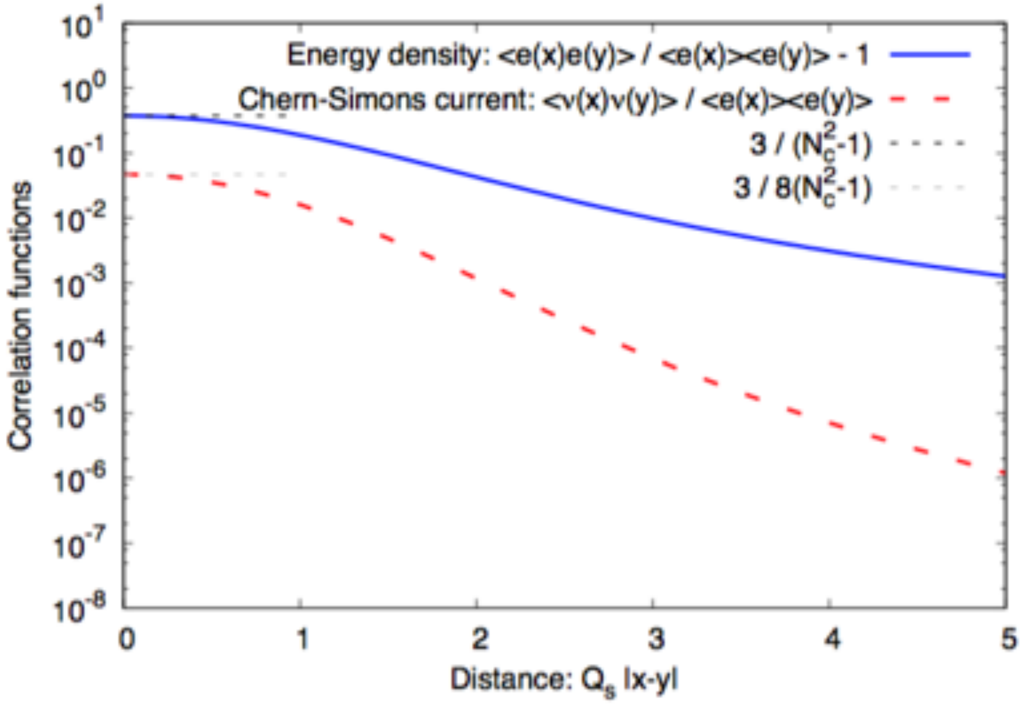
Lappi, SS in preparation

Early time dynamics ($\tau \sim 1/Q_s$)

Based on analytic solutions of gauge field at $\tau=0^+$ axial charge production at very early times can be estimated

$$\left.\frac{\mathrm{d}N_5}{\mathrm{d}^2\mathbf{x}\mathrm{d}\eta}\right|_{\tau\lesssim 1/Q_s} \approx \frac{\tau^2}{2}\frac{g^2N_f}{2\pi^2}\dot{\nu}(\mathbf{x},\tau=0^+)$$

Calculating relevant correlators of gluon fields diagrammatically in Glasma graph approximation



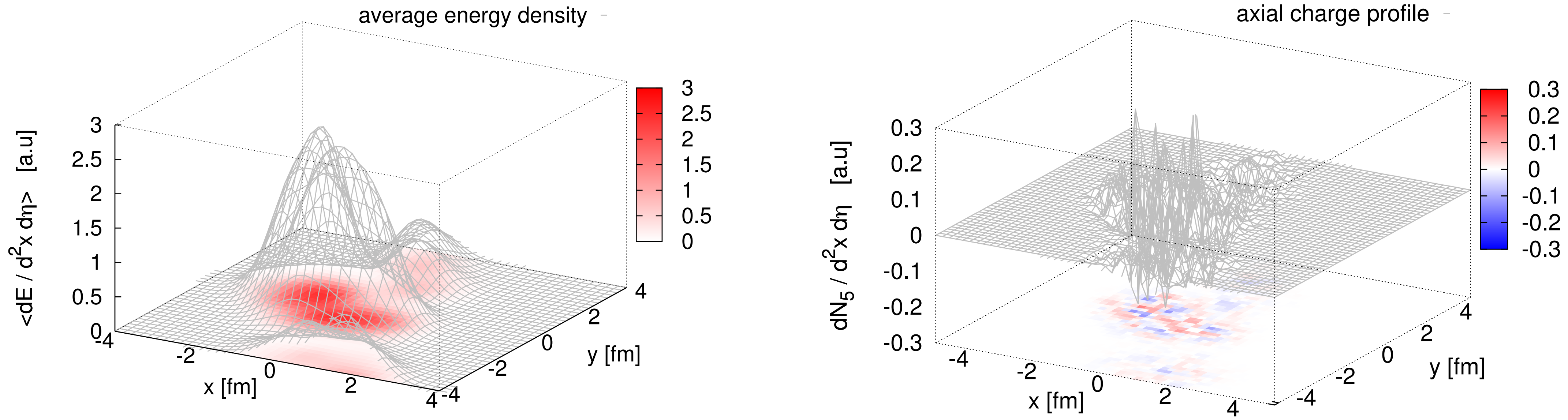
$$\langle\dot{\nu}(\mathbf{x})\dot{\nu}(\mathbf{y})\rangle = \frac{3g^4N_c^2(N_c^2-1)}{32}\left[\left(G^{(1)}_{(U)}(\mathbf{x},\mathbf{y})\right)^2\left(G^{(1)}_{(V)}(\mathbf{x},\mathbf{y})\right)^2-\left(h^{(1)}_{\perp(U)}(\mathbf{x},\mathbf{y})\right)^2\left(h^{(1)}_{\perp(V)}(\mathbf{x},\mathbf{y})\right)^2\right],$$

Event-by-event modeling

Based on estimate of axial charge correlation function in terms of energy density

$$\left\langle \frac{\mathrm{d}N_5}{\mathrm{d}^2\mathbf{x}\mathrm{d}\eta}\frac{\mathrm{d}N_5}{\mathrm{d}^2\mathbf{y}\mathrm{d}\eta} \right\rangle \approx \frac{3\alpha_s^2 N_f^2}{8\pi^2(N_c^2-1)}\,\langle\varepsilon(\mathbf{x})\rangle\langle\varepsilon(\mathbf{y})\rangle\,\tau^4\left(\frac{1-e^{-\frac{N_c}{4C_F}Q_s^2|\mathbf{x}-\mathbf{y}|^2}}{\frac{N_c}{4C_F}Q_s^2|\mathbf{x}-\mathbf{y}|^2}\right)^4$$

can sample event-by-event configurations this distribution which can be used as initial conditions for subsequent evolution



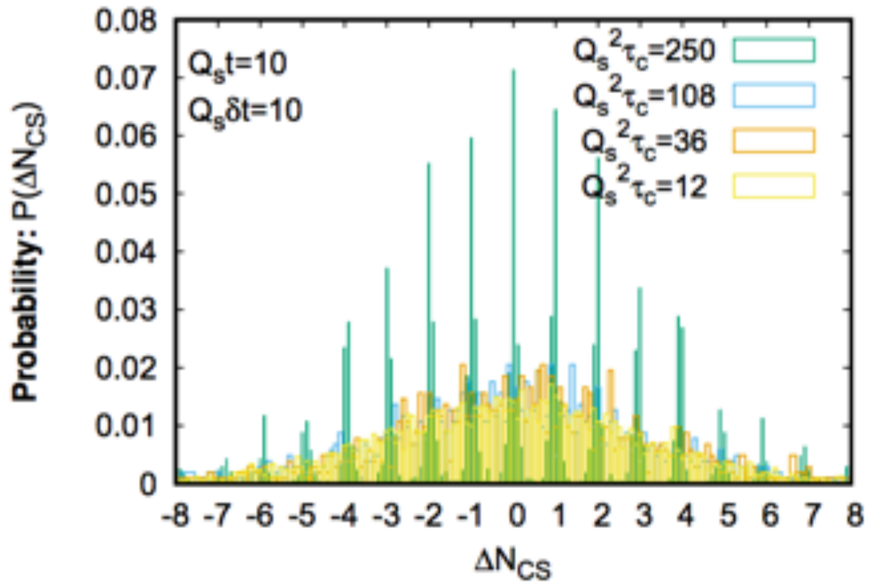
Pb+Pb Event (TRENTO) | b=11.4 fm | N_{part}=56

Evolution of axial charge imbalance

Beyond very early times sphaleron transitions in and out-of equilibrium dominate axial charge production on long time/distance scales

Strong color-fields at early times can lead to an enhancement of sphaleron transition rate during the pre-equilibrium stage

Mace, SS, Venugopalan PRD93 (2016) no.7, 074036



Sphaleron transitions persist throughout the evolution of the fire-ball and create/destroy axial charge in the plasma

Moore, Tassler, JHEP 1102 (2011) 105, …

Since axial charge is *not* conserved, knowledge of “initial condition” for axial charge is in general *not sufficient* to describe subsequent space-time evolution

Still a major source of uncertainty in theoretical description of CME

Conclusions

Development of first-principle techniques to study dynamics of vector and axial charges out-of-equilibrium

Successful microscopic description of CME & CMW

Observed importance of finite relaxation time and dissipative effects

Should be included in macroscopic descriptions

Development of simulations is in progress — stay tuned

Dynamics of axial charge production can also be addressed from Yang-Mills sector

Developed initial state model for phenomenological applications

Beware that chirality changing processes (sphalerons,…) take place throughout space-time evolution of the fireball

Backup

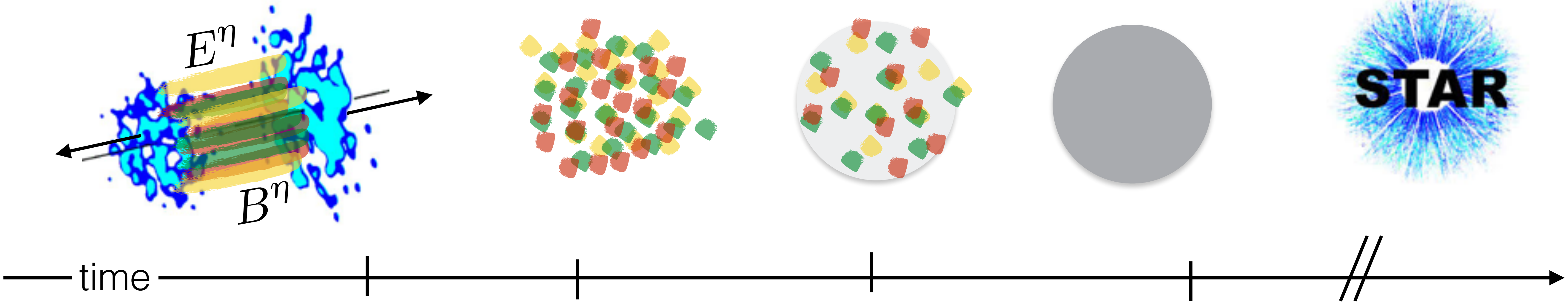
Chiral magnetic effect out-of-equilibrium

Glasma flux tubes

over-occupied plasma

min-jets + soft bath

equilibrium



semi-analytic treatment

non-equilibrium lattice simulations

chiral kinetic theory

anomalous hydrodynamics

color flux tubes

Kharzeev, Venugopalan, Krasnitz, PLB545 (2002) 298-306

SS, Lappi in preparation

out-of-equilibrium sphaleron transitions

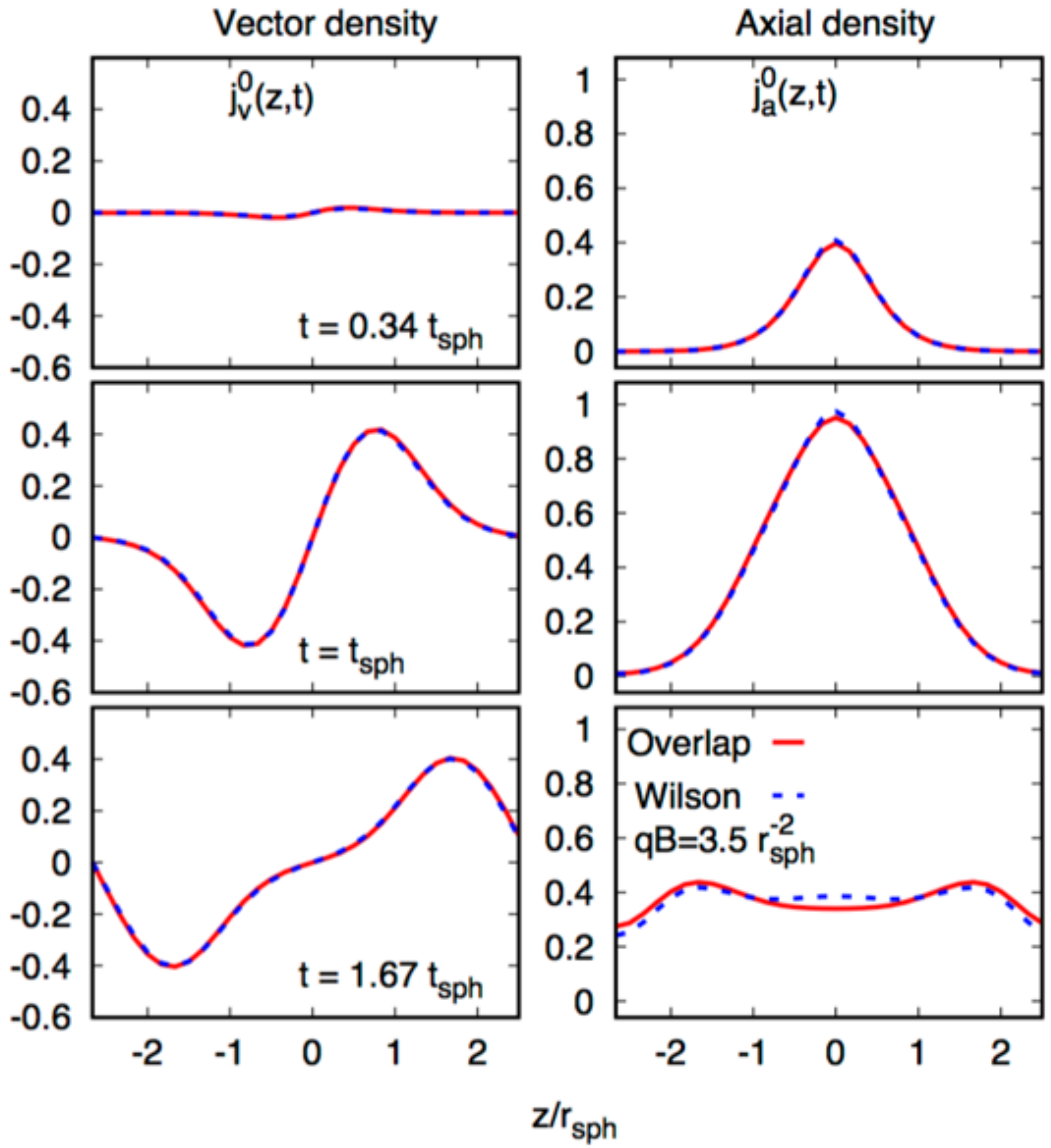
Mace, SS, Venguopalan PRD93 (2016) no.7, 074036

sphaleron transitions

Moore, Tassler, JHEP 1102 (2011) 105

…

Comparison of Wilson & Overlap



Magnetic field dependence