Hydrodynamics with critical slowing down ("Hydro+")

Hydro. Hydro.+

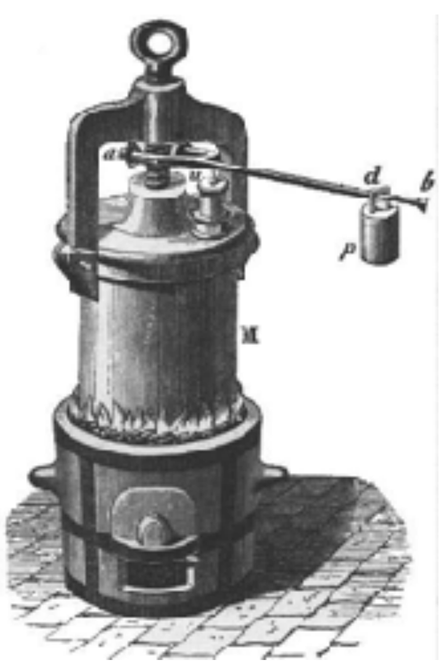
Yi Yin

(M. Stephanov and YY, in preparation)

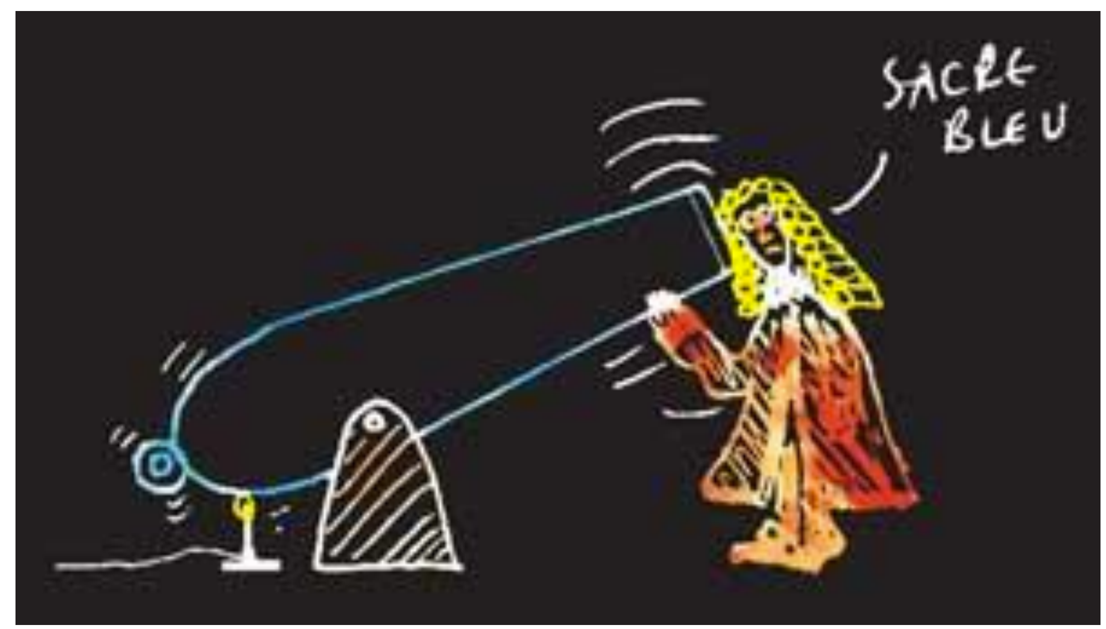
StonyBrook, Aug.8, 2017

Search for the critical point in 19th c.

- Heating and compressing liquid (matter under extreme conditions) in a digester.
- Listening to discontinuities in the splashing sound (critical signature).



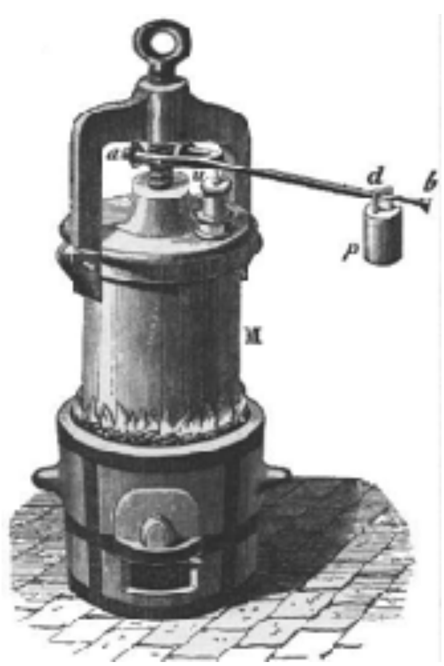
(Digester)



High Energy Frontier of 19th c. (digester made from canon barrel).

Search for the QCD critical point in 21 c.

- Creating QCD matter under extreme conditions .
- Identifying observables sensitive to criticality (fluctuations of hadron multiplicities etc).



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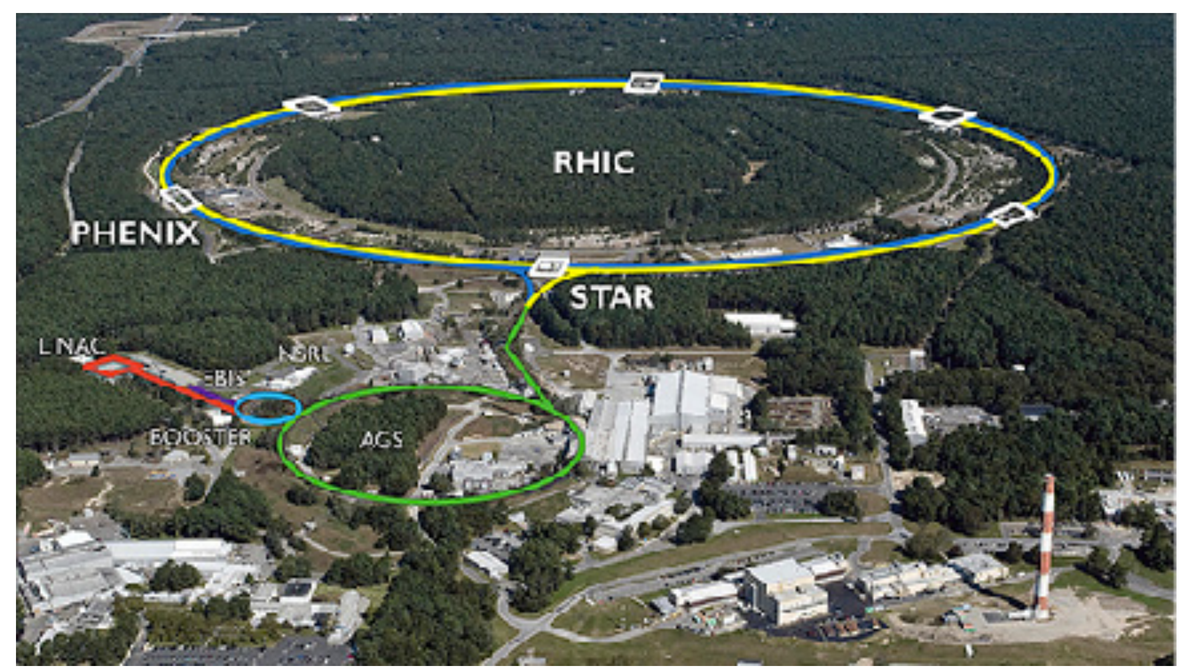
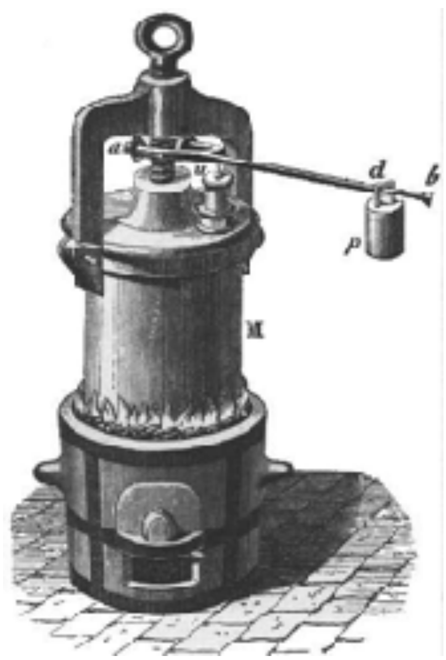


- The expanding fireball is not the liquid in a digest : dynamical effects are important !



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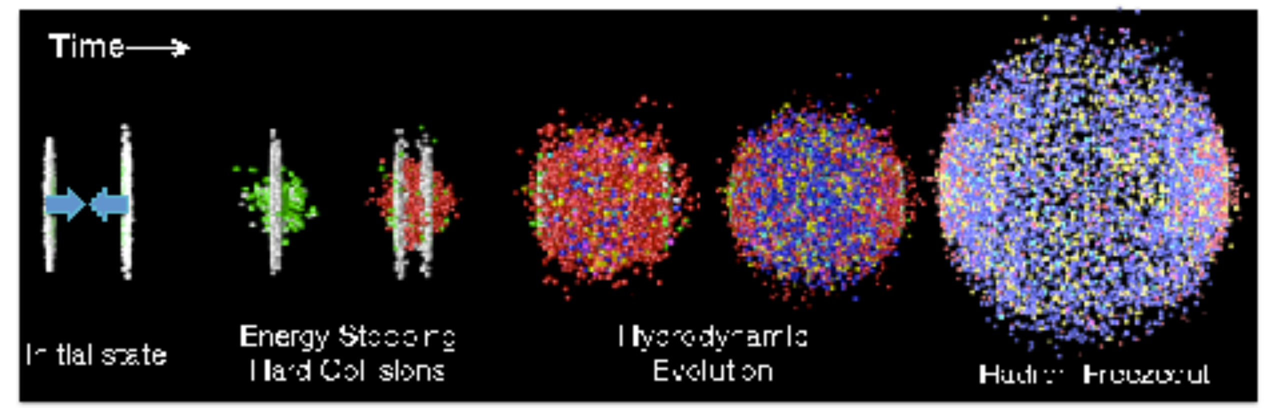


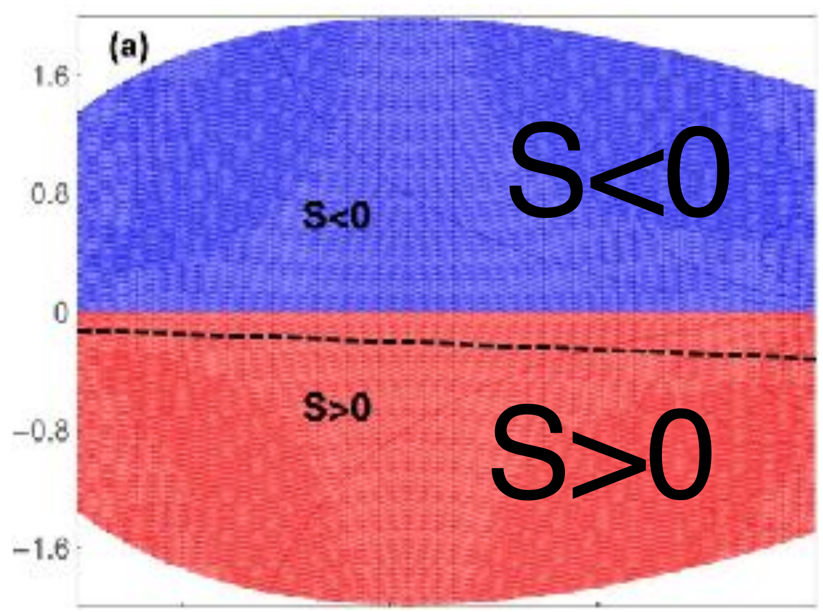
- The equilibration time of critical fluctuation grows (critical slowing down).

$$\text{Relaxation time} \sim \xi^{z} \gg \tau_{mic} \qquad (z \approx 3)$$

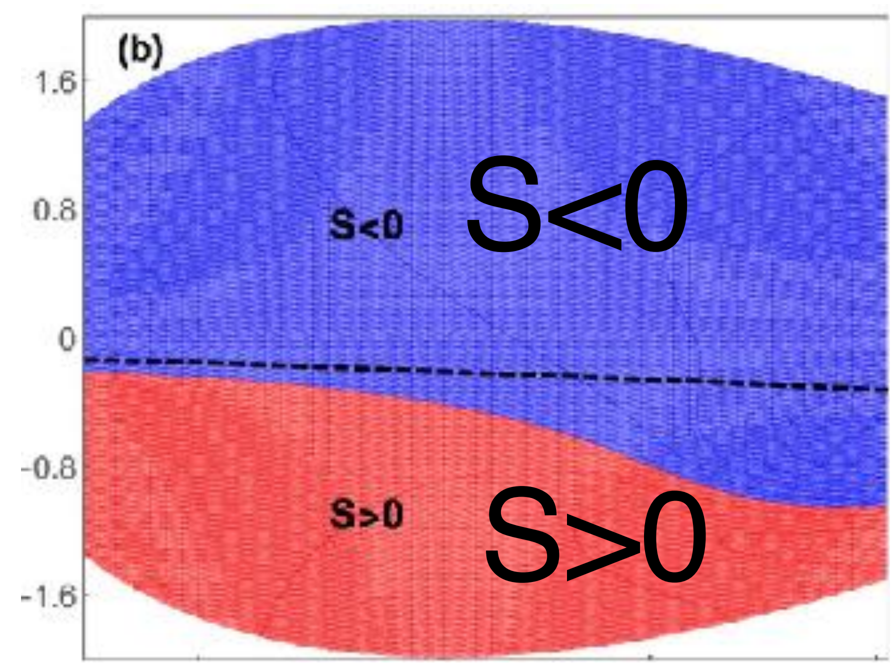
Or $\quad \omega_{Critical} \sim 1/(\xi)^3$

- Off-equilibrium effects limit the growth of correlation length. **(Berdinkov-Rajagopal, 99')**

- Critical cumulants can be different from the equilibrium expectation qualitatively !



Equilibrium



Off-Equilibrium

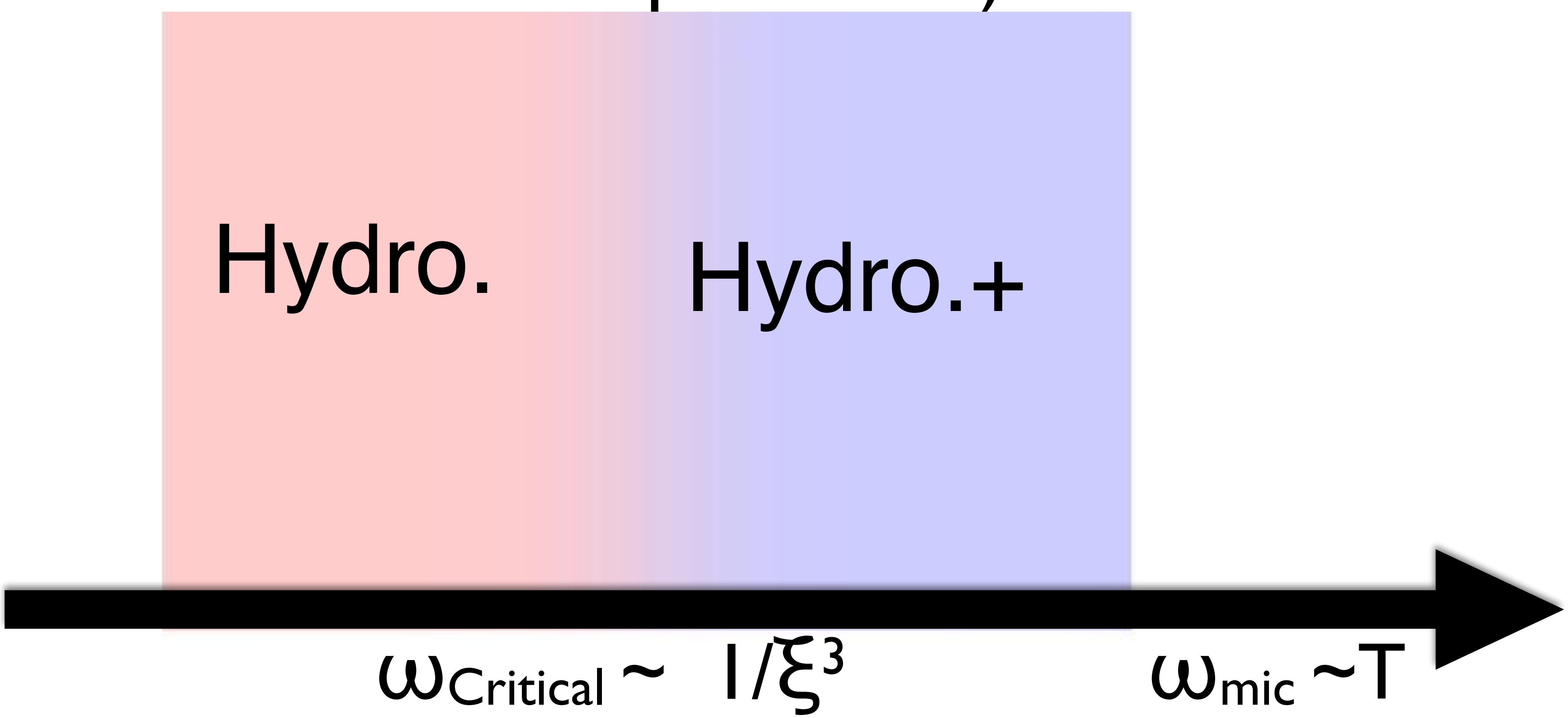
(S. Mukherjee, R. Venugopalan and YY, PRC, 2015)

Challenging for hydro. near a critical point

- The applicability of Hydro. near C.P. is limited by the critical slowing down ($\omega < \omega_{Critical} \sim 1/\xi^3$).
  - The critical E.o.S only applies to the situation that critical fluctuations are in equilibrium.
  - Another symptom: the growth of bulk viscosity $\zeta_{Kubo} \sim \xi^3$.

The goal of hydro+critical slowing (or "Hydro+")

- Formulating a hydro-like theory which is applicable at scale $\omega > \omega_{Critical} \sim 1/\xi^3$.
  - "+": adding critical slow modes (parametrically longer life than other microscopic modes).



Outline

- Construction of "Hydro+" critical slowing down.
- An example in the expanding background (briefly).
- Summary.

A warm-up exercise: adding one slow mode to hydro.

General strategy

- Step I: writing down a general local theory with additional slow mode(s) ϕ and relaxation rate Γ_ϕ .
- Step II: Fixing inputs of "hydro+" as much as possible from dynamical/static critical universality.

E.o.S and dynamical equation for ϕ

- Generalized entropy (E.o.S) : $s_{(+)}(\varepsilon,n,\phi)$.
  - Equilibrium entropy is the maximum of $s_{(+)}(\varepsilon,n,\phi)$:

$$s(\varepsilon,n) = s_{(+)}(\varepsilon,n,\phi)\,|_{\pi=0} \quad \text{where}$$

$$\pi(\varepsilon,n,\phi) = \partial s_{(+)}(\varepsilon,n,\phi)/\partial\phi$$

- E.o.M for ϕ (slow equilibration of ϕ) :

$$(u^\mu\,\partial_\mu)\phi = -\,\gamma_\phi\,\pi \;+ \text{gradient term} \qquad \gamma_\phi \propto \Gamma_\phi$$

(γ_ϕ *is functions of* ε,n,ϕ)

Hydro part of "hydro+"

- $T^{\mu\nu} = \ \varepsilon\, u^\mu\, u^\nu + p_{(+)}(\varepsilon,n,\phi)\; g^{\mu\nu} + \Delta T^{\mu\nu}$.

  $s_{(+)}(\varepsilon,n,\phi) \;\Rightarrow\; p_{(+)}(\varepsilon,n,\phi)$

- $\Delta T^{\mu\nu} = \ -\ \zeta_{(+)}\, \Delta^{\mu\nu}\, (\partial\ u)$ - shear viscous term (similar for ΔJ^μ)

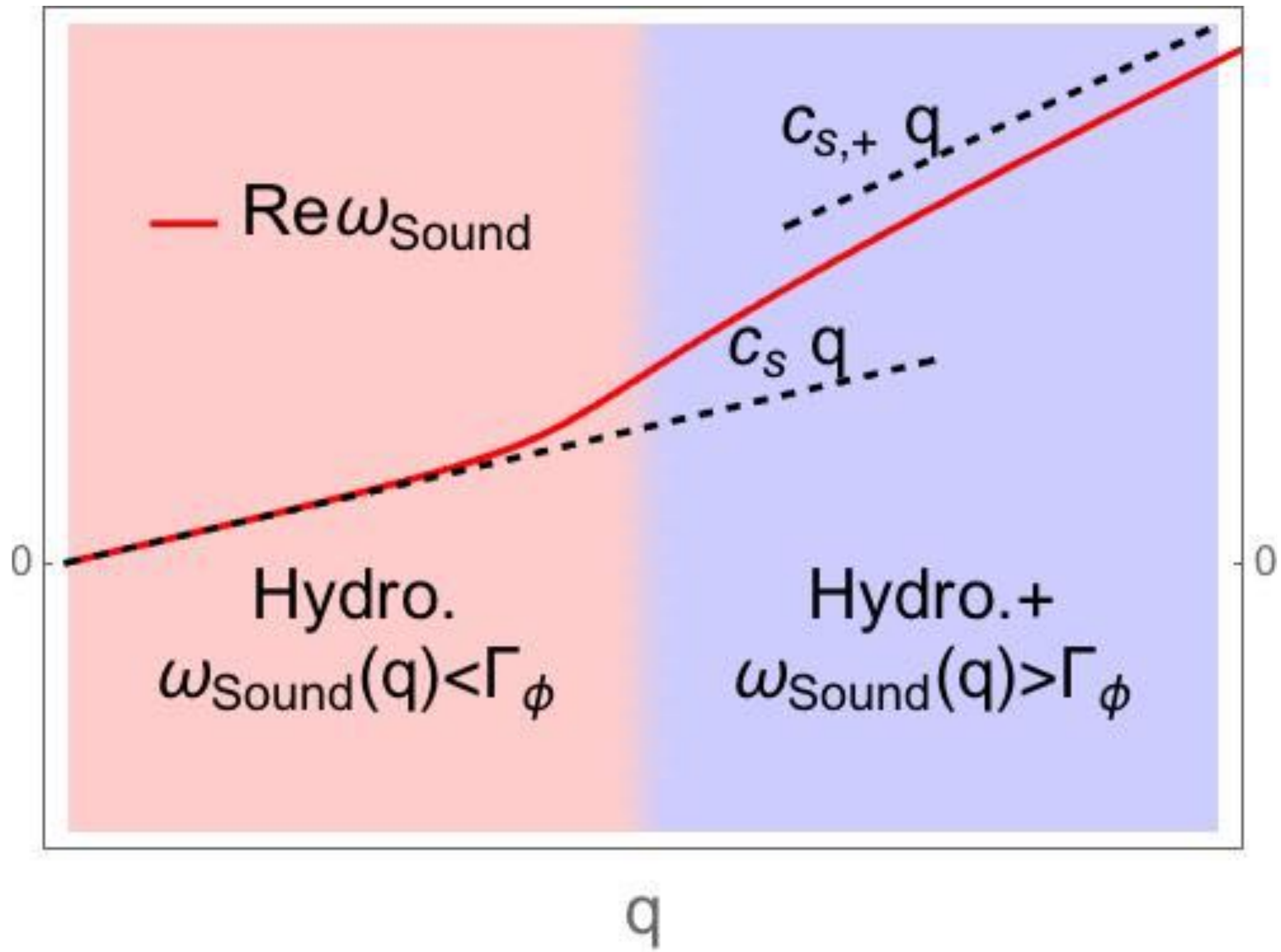
- Constraints:

  - 2nd law: $\Gamma_\phi,\ \zeta_{(+)}$, etc >0 .

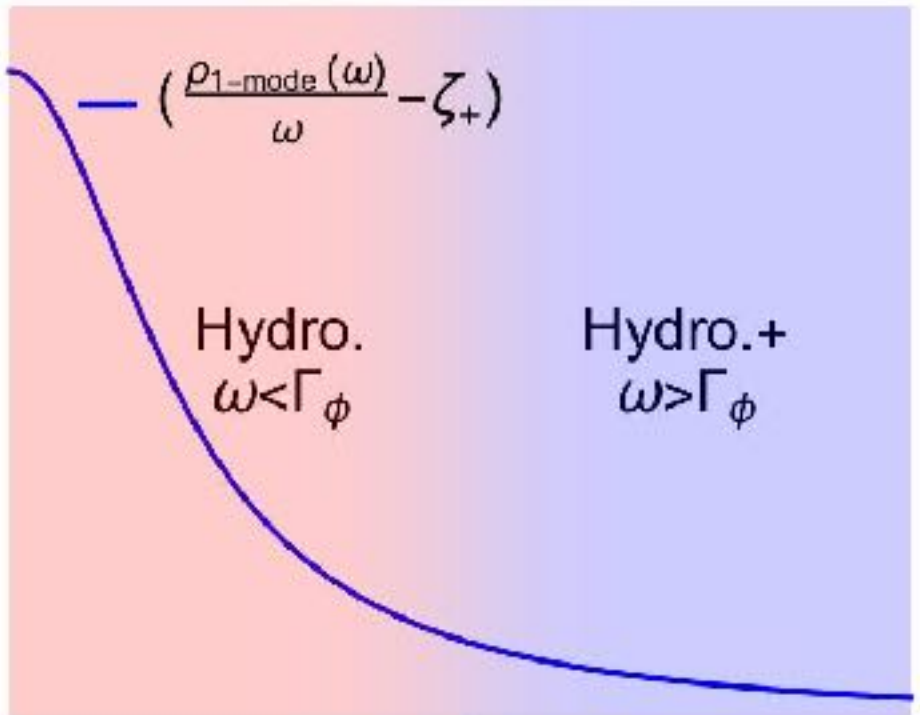
  - No double counting:

    $\zeta_{Kubo} = \zeta_{(+)}$ + contributions from ϕ ($\propto 1/\Gamma_\phi$)

Non-equilibrium effects modify E.o.S



$$c_s^2 = \left(\frac{\partial p(\epsilon, n)}{\partial \epsilon} \right)_{s/n} \qquad c_+^2 = \left(\frac{\partial p_{(+)}(\epsilon, n, \phi)}{\partial \epsilon} \right)_{s/n,\phi}$$



$(\rho_{Bulk}(\omega) \sim \mathrm{Im} \langle T^i{}_i T^i{}_i \rangle)$

- Transport coefficient is related to $1/\Gamma_\phi$.

$$\zeta_{Kubo} - \zeta_+ = [(c_{s,+})^2 - (c_s)^2]/\Gamma_\phi$$

  - Entropy production will be over-estimated without including the additional slow mode.

- Area: shift of sound velocity.

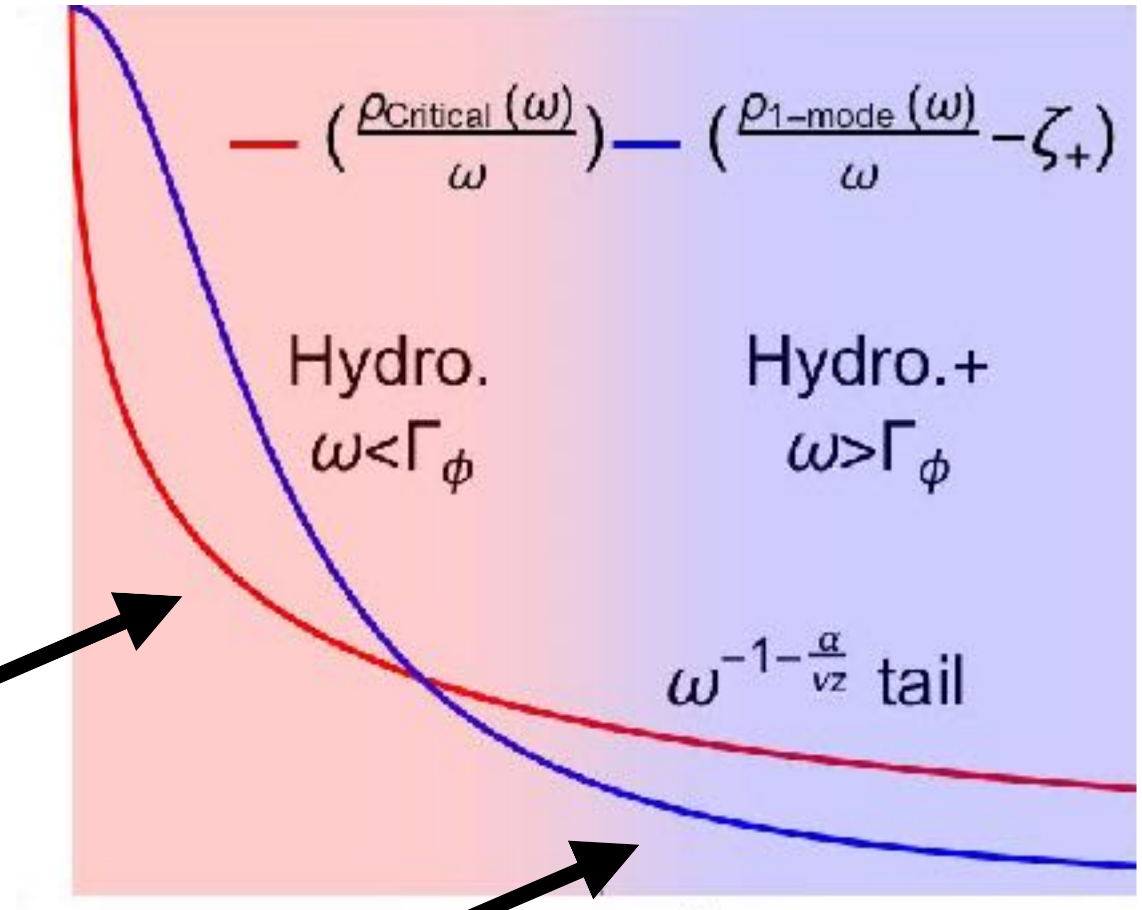
$$[(c_{s,+})^2 - (c_s)^2] \propto \int d\omega \, (\rho(\omega)/\omega - \zeta_{(+)})$$

Fixing inputs from matching to critical dynamics

- Peak: (critical) bulk viscosity

$$\zeta_{Kubo} \sim \xi^{3} \quad \rightarrow \quad \Gamma_{\phi} \sim \xi^{-3}$$

- A pleasing feature: the input of "hydro+" ζ_+ is finite.

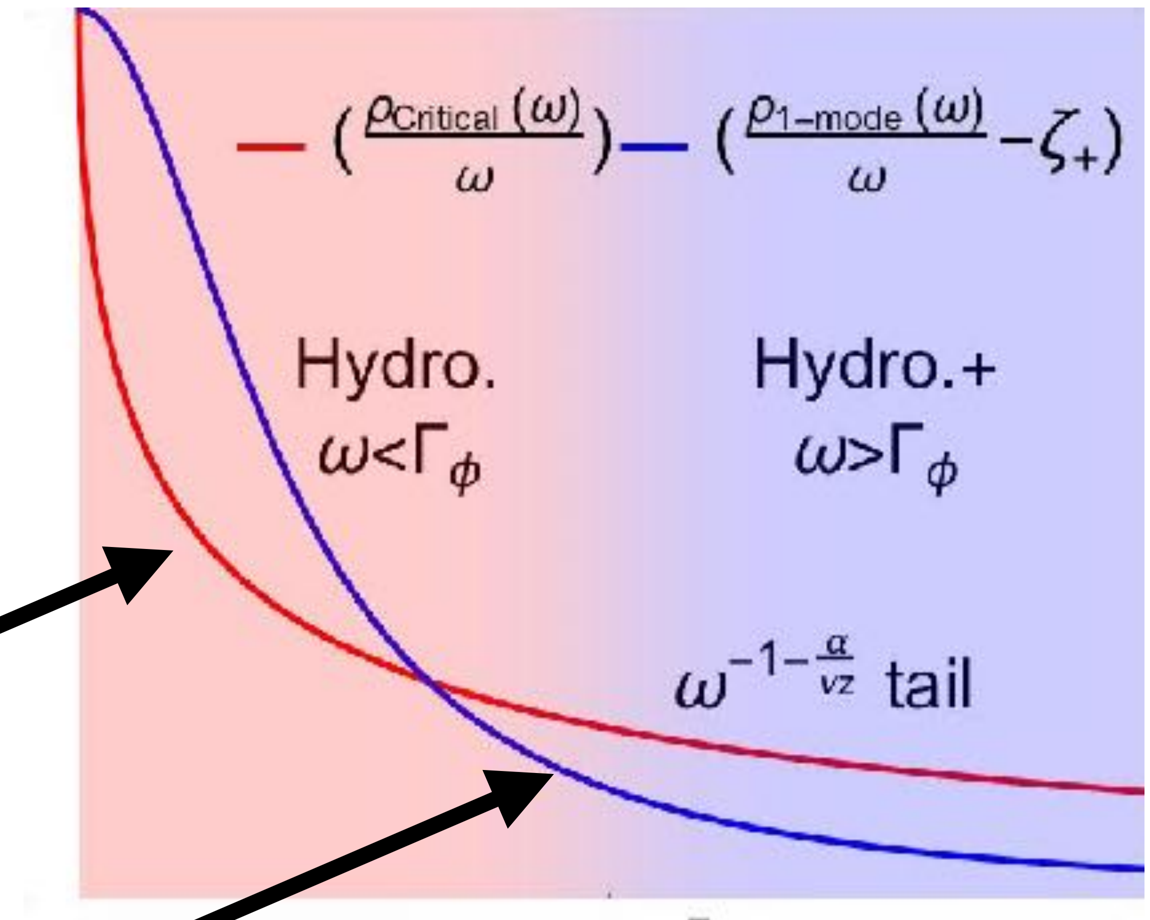


($\rho_{Bulk}(\omega)$ from mode H)

($\rho_{Bulk}(\omega)$ from hydro+one mode)

- Matching area fixes $(c_{s,+})^2$.

- "Hydro+" one mode qualitatively captures the transition from hydro regime $\omega < 1/\xi^3$ to "hydro+" regime $\omega > 1/\xi^3$.



($\rho_{\mathrm{Bulk}}(\omega)$ from mode H)

($\rho_{\mathrm{Bulk}}(\omega)$ from hydro+one mode)

- One mode is not enough to fully capture the critical dynamic behavior.

- Next step: Hydro+ a spectrum of slow modes.

Hydro+a spectrum of slow modes

Construction of hydro+$\phi(t,x;Q)$

$s_+(\varepsilon,n,\phi)$ $\longrightarrow$ $s_+(\varepsilon,n,\phi(Q))$ (thus $p_+(\varepsilon,n,\phi(Q))$)

$\pi = \partial s_+(\varepsilon,n,\phi)/\partial\phi$ $\longrightarrow$ $\pi\,(Q) = \delta s_+(\varepsilon,n,\phi(Q))/\delta\phi(Q)$

$(u^\mu\,\partial_\mu)\phi = -\,\gamma_\phi\,\pi - \ldots.$ $\longrightarrow$ $(u^\mu\,\partial_\mu)\phi(Q) = -\,\gamma_\phi(Q)\,\pi(Q) - \ldots$

To proceed, a more "microscopic" understanding of critical slow modes is needed

Slow modes near a critical point

- A general critical point: slow modes include order parameter (M), and <δMδM> (and potentially higher cumulants…).

- QCD critical point: hydro + <δMδM>.
  - M is a linear combination of ϵ, n and chiral condensate σ. σ equilibrates at microscopic time scale and the evolution of σ simply traces the evolution of ϵ, n ⇒ ~~Eq. for M~~.

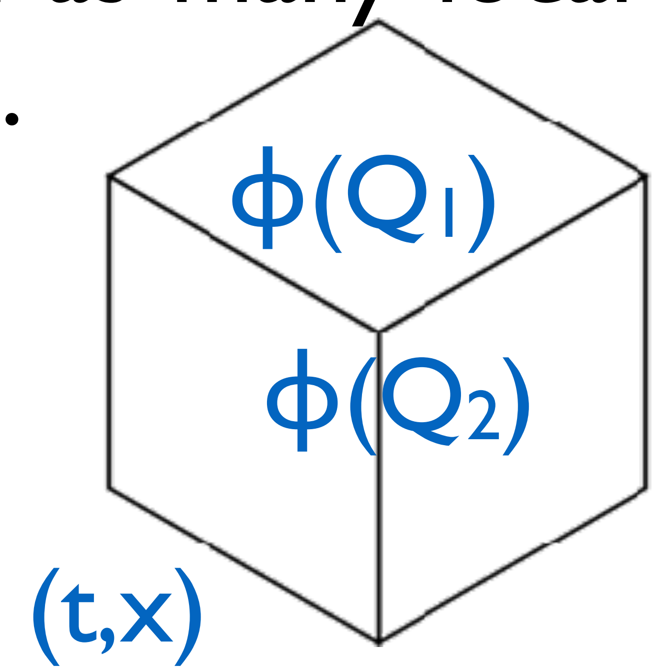
(Son-Stephanov, 04')

Relation between <δMδM> and $\phi(t, x\,; Q)$

- The Wigner transform of $\langle \delta M \delta M \rangle \Rightarrow \phi(t, x\,; Q)$

$$\phi(t, x\,; Q) = \int d^3\Delta x \, \langle \delta M(t, x+\Delta x)\; \delta M(t, x-\Delta x) \rangle \, e^{-i\, Q\, \Delta x}$$

$\phi(t,x\,;Q)$ may be viewed as many local slow modes with label Q at a fluid cell (t,x).



- In equilibrium: $\phi_{eq}(Q) = 1/[(\chi_M)^{-1}+Q^2]$ ($\phi_{eq}(Q=0)=\chi_M \sim \kappa_2$).

Generalized Entropy $s_+(\epsilon,n,\phi(Q))$

- The generalized entropy $s_+(\epsilon,n,\phi(Q))$ can be *derived* following the formalism of 2PI effective action in QFT.

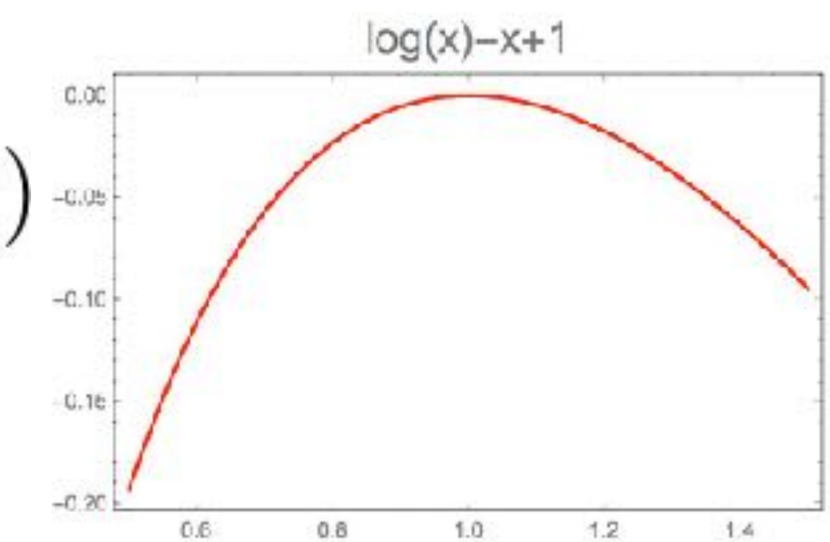
(J. M. Cornwall, R. Jackiw, E. Tomboulis, 1974')

  - NB: 2PI effective action is a useful tool to study non-equilibrium effects. (e.g. J. Berges et al, hep-ph/0409123)

- A simple form at the leading order in "loop expansion":

$$s_+(\epsilon,n,\phi(Q)) = s_{\rm eq}(\epsilon,n) + \frac{1}{2}\int_Q \left\{ \log\left(\frac{\phi(Q)}{\phi_{eq}(Q)}\right) - \frac{\phi(Q)}{\phi_{eq}(Q)} + 1 \right\};$$

$$\pi(Q) = \frac{\delta s_+}{\delta\phi(Q)} = \phi_{\rm eq}^{-1}(Q) - \phi^{-1}(Q)$$

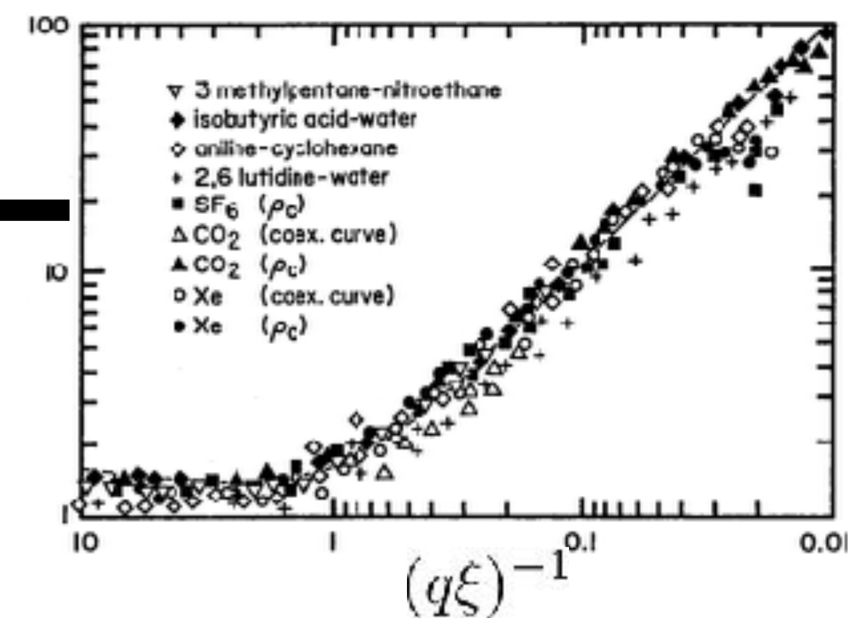


E.o.M for $\phi(Q)$

- A Q-dependent (phenomenological) relaxation equation for ϕ:

$$(u^\mu \partial_\mu)\phi = - \gamma_\phi \pi \quad \longrightarrow \quad (u^\mu \partial_\mu)\, \phi(Q) = -2\,\gamma(Q)\pi(Q)$$

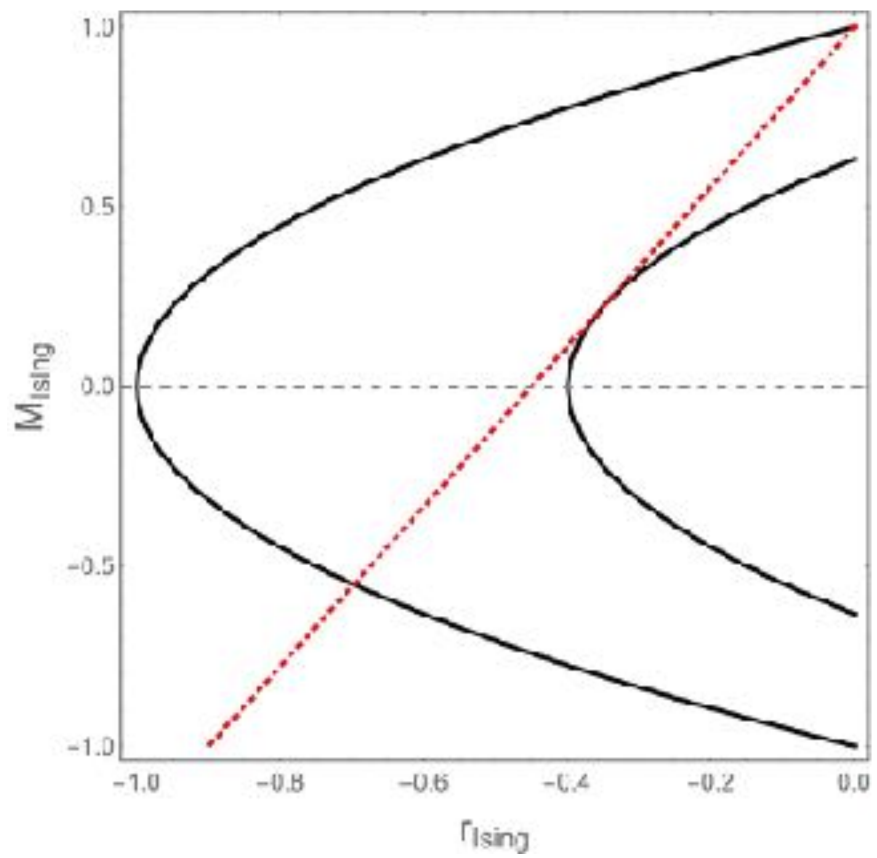
- $\Gamma(Q)=\gamma(Q)/(\phi_{eq}(Q))^2$ is known ⟵ from model H.



- $s_{(+)}(\epsilon, n, \phi(Q))$ together with $\Gamma(Q)$ successfully reproduces critical behavior of $\rho_{Bulk}(\omega) \sim \mathrm{Im} \langle T^i{}_i T^i{}_i \rangle$.

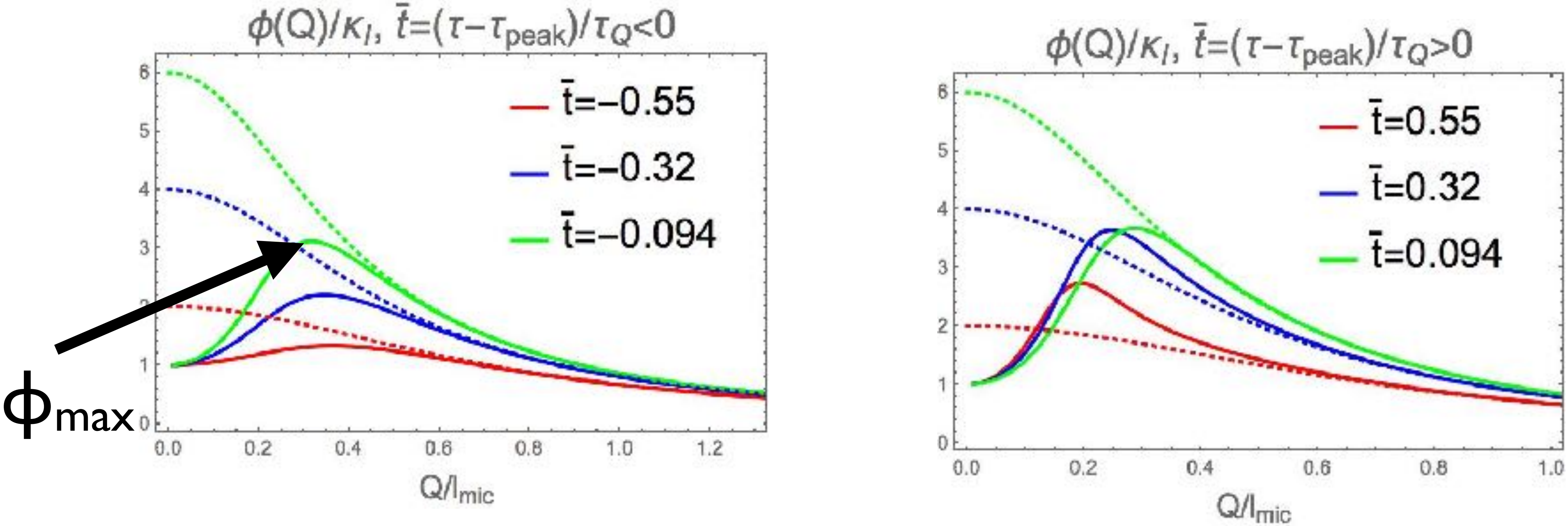
An example of hydro+ in an expanding QGP

Solving equation for ϕ(Q) along a trajectory



$M \sim T - T_c$, $r_{Ising} \sim \mu - \mu_c$

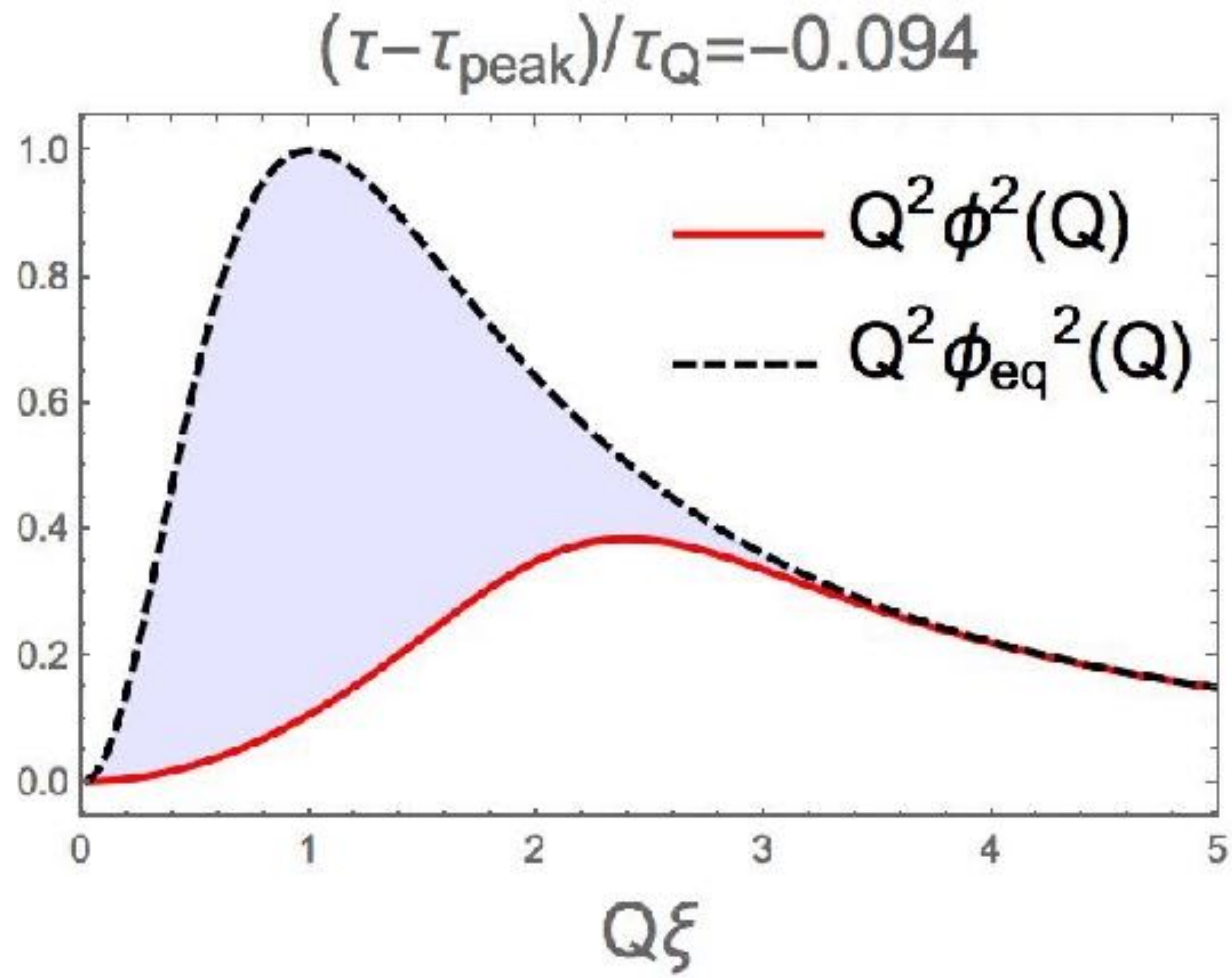
"Hydro+" describes the slow relaxation of critical fluctuations



$\tau<\tau_{peak}$, fall out of equilibrium. $\tau>\tau_{peak}$, memory.

- NB: $\phi(Q)$ can be related to the baryon number balance function (if supplemented with mapping and freeze-out prescription).

Hydro+ describes off-equilibrium modification of E.o.S

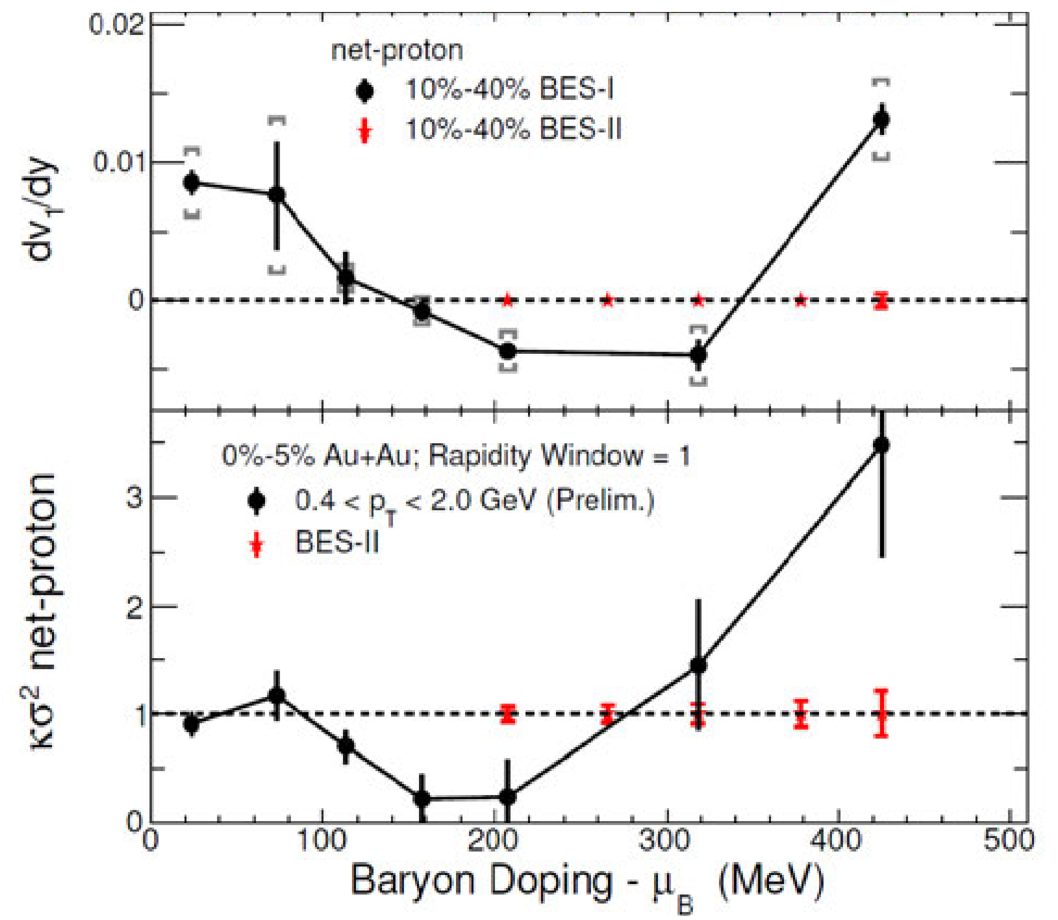


$(c_s)^{-2} - (c_{s,\,\text{off-equilibrium}})^{-2} \propto$ shaded area

Summary

- “Hydro+”: describing bulk evolution and fluctuations near the critical point in one and the same framework.

- Future: quantitative description for both flow observables and fluctuation measure.



Coupled evolution ⇒<flow cumulants>$_{data}$

Back-up