Lifshitz points in smectics, gravity, and the Beam Energy Scan

RDP, V. Skokov, & A. Tsvelik, work in progress

1. When is QCD quarkyonic? Chiral spirals ~ liquid crystal

3. Usual phase diagram & critical dimensions

4. Chiral Spirals in mean field theory: Lifshitz point

5. Effective reduction to 1-dimension, Brazovski 1st order transition

6. Pseudo-Lifshitz point and fluctuations

7. Phase diagram with fluctuations: *two* critical points in QCD?

8. *Fluctuations from a pseudo-Lifshitz point at the Beam Energy Scan?*

Quarkyonic & 1-D patches

Cold, quark matter as "Quarkyonic" matter: McLerran & RDP 0706.2191

Fermi surface ~ *confined*, deep in Fermi sea ~ perturbative

Valid at large N_c: $N_c = 3$? At $T \neq 0$, $\mu = 0$: $\Lambda_{ren} \sim 2 \pi T$

We suggest: $T = 0$, $\mu \neq 0$: quarkyonic for $\mu_{quark} < 1$ GeV, for *any* N_c, N_f

At $\mu \neq 0$, $T << \mu$ confining potential $\sim 1/(p^2)^2$ tends to form 1-dim *patches*

Kojo, Hidaka, McLerran & RDP 0912.3800; Kojo, Hidaka, Fukushima, McLerran, RDP 1107.2124 +...

Width of patch $\sim \Lambda_{QCD}$, so for large μ,

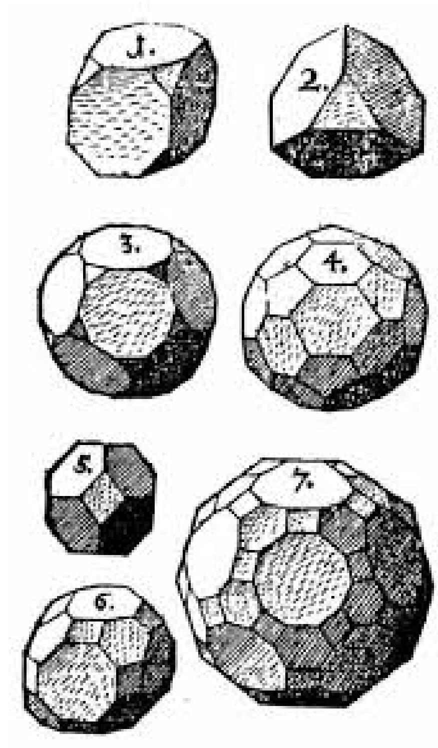
Fermi surface is covered with patches

At $T = 0$, phase transitions as number/geometry

of patches increases:

Series of (weak) first order transitions?

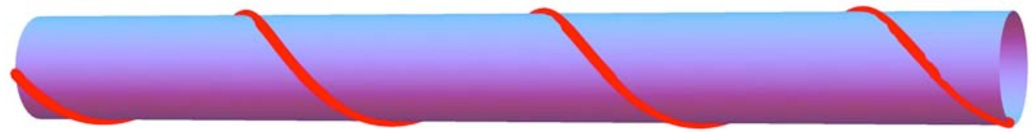
Analogous to Kepler's perfect solids...



Chiral Spirals in 1+1 dimensions

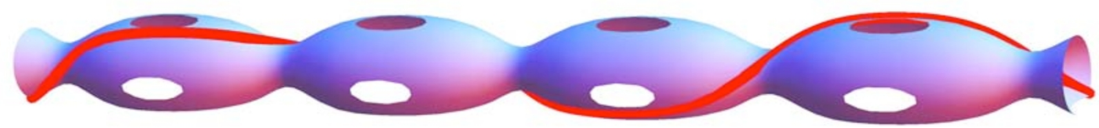
Chiral Spiral (CS) ~ Migdal's pion condensate:

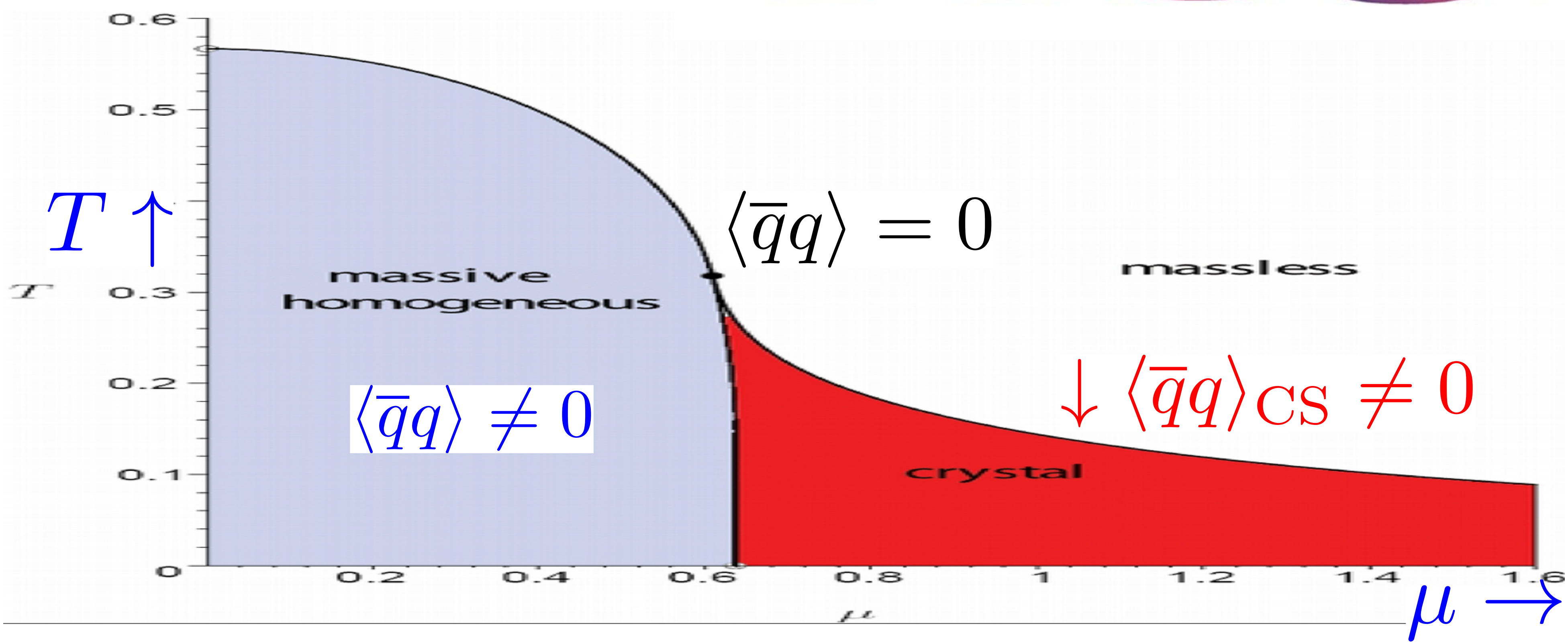
$$(\sigma, \pi^0) = f_\pi(\cos(k_0 z), \sin(k_0 z))$$



Ubiquitous in 1+1 dimensions: Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

Wealth of exact solutions, phase diagrams...

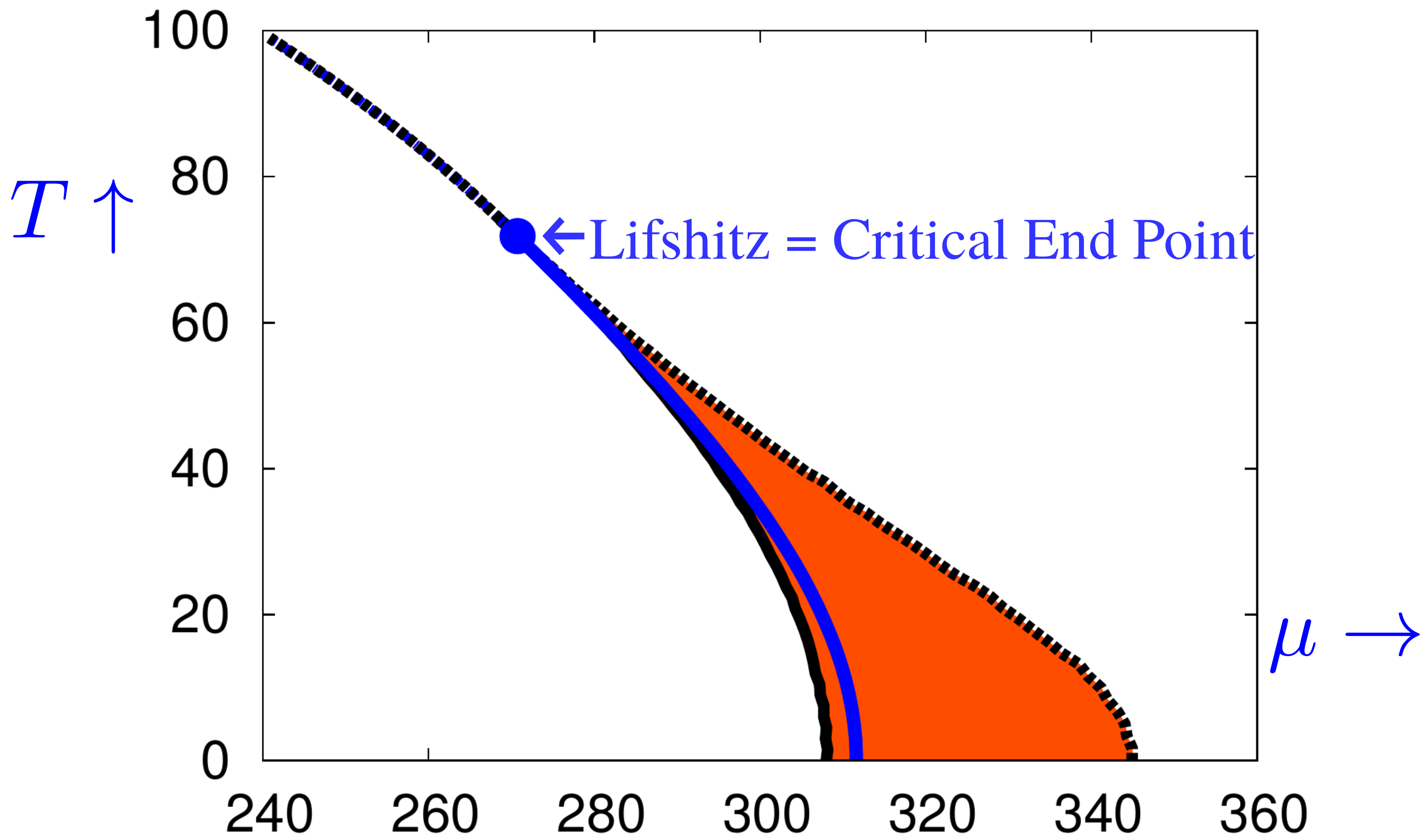




Chiral Spirals in 3+1 dimensions

In 3+1, *common* in NJL models: Nickel, 0902.1778 + ...Buballa & Carignano 1406.1367 + ...

Other CS's possible, between σ & $\bar{q}\gamma_0\gamma_z\gamma_5 q$



Fluctuations in Chiral Spirals

In Chiral Spiral, <φ>≠0 *locally* but <φ>=0 *globally*.

Spon. breaking of global symmetry => interactions of Goldstone Bosons ~ ∂^2

In CS, spon. bkg's of global *plus* rotational sym. implies interactions in transverse momenta ~ $\partial_\perp{}^2$ *cancel*. Interactions ~ $(\partial_\perp{}^2)^2$ ~ $\partial_\perp{}^4$. U = GB:

$$\mathcal{L}_{\rm CS} = f_\pi^2 |(\partial_z - i k_0) U|^2 + \kappa |\partial_\perp^2 U|^2 + \ldots$$

Hidaka, Kamikado, Kanazawa & Noumi 1505.00848; Nitta, Sasaki & Yokokura 1706.02938

Transverse fluctuations *dis*order: *large* fluctuations about $k_z \sim k_0$:

$$\int d^2 k_\perp \, dk_z \, \frac{1}{(k_z - k_0)^2 + (k_\perp^2)^2} \sim \int d^2 k_\perp \, \frac{1}{k_\perp^2} \sim \log \Lambda_{\rm IR}$$

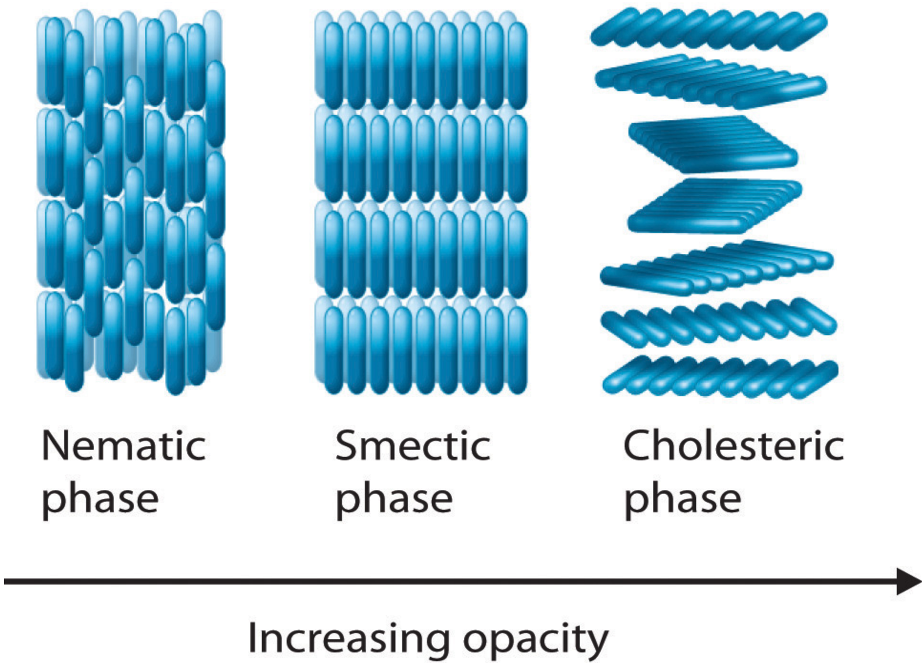
No true long range order (Landau-Peierls) ~ *smectic liquid crystal*

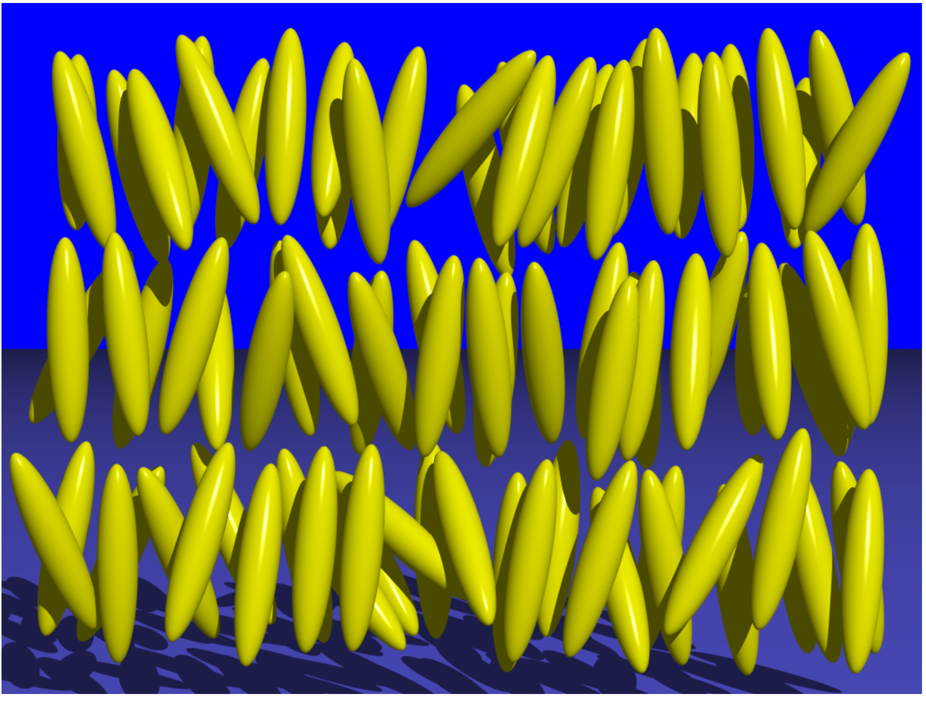
Varieties of liquid crystals

Nematics: rotational ordering (vector with
no direction)

Smectic: rotational ordering and in planes
disordered in the planes ("liquid")

Cholesteric: chiral ordering (with twist)





Smectic something like patches in QCD

Smectic – nematic transition has analogy,
to follow (1st order from reduction to 1-dim)

Standard phase diagram

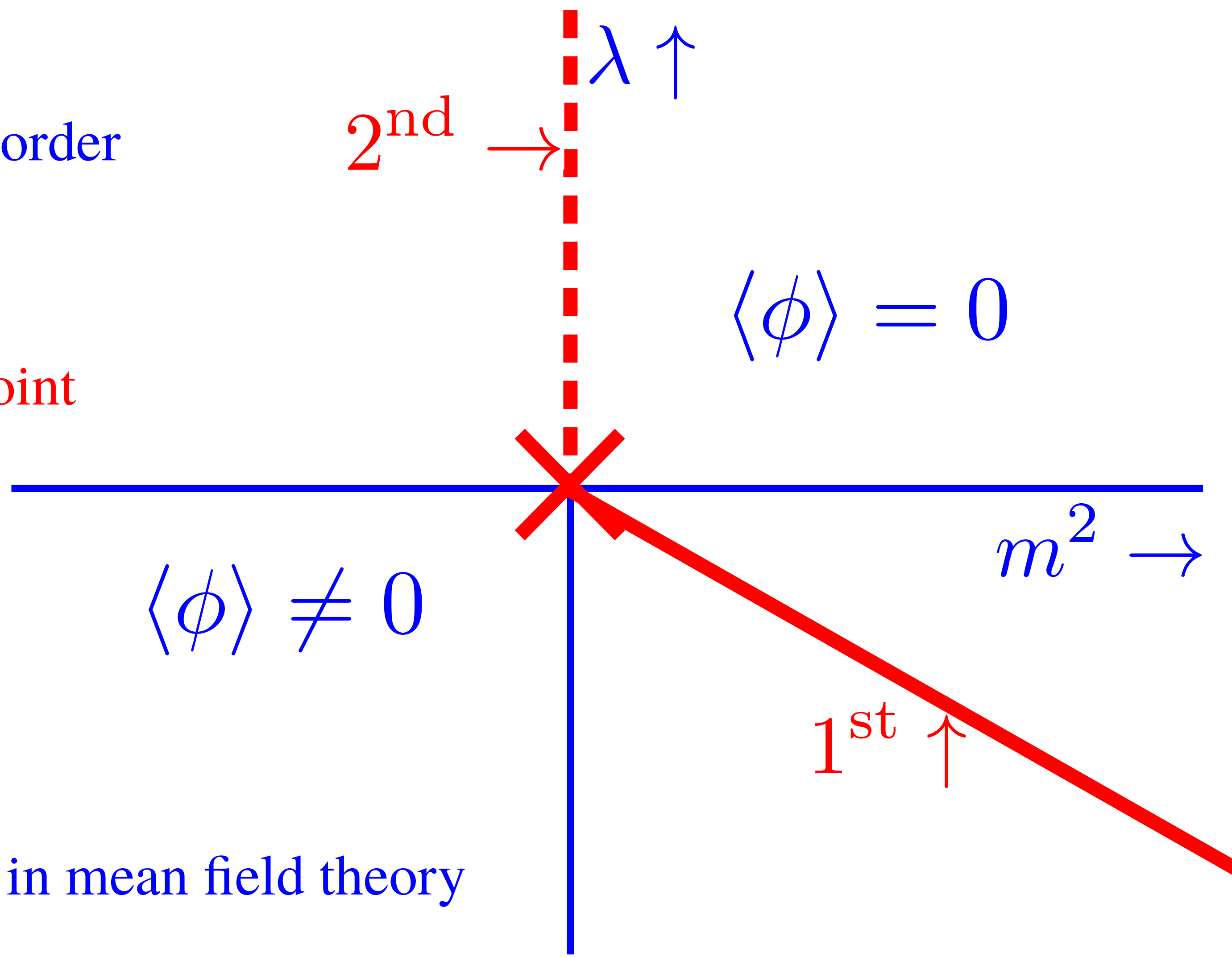
Trade T & μ for m^2 & λ.

Two phases, symmetric & broken

$$\mathcal{L} = (\partial_\mu \phi)^2 + m^2\phi^2 + \lambda\phi^4 + \kappa\phi^6$$

$m^2 = 0$, $\lambda > 0$: usual 2nd order

$m^2 = \lambda = 0$: tricritical point

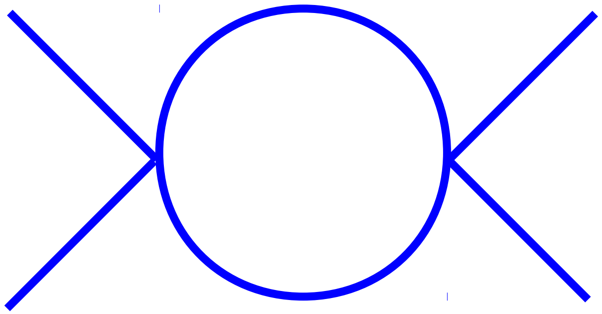


$m^2 > 0$, $\lambda < 0$: 1st order in mean field theory

Usual critical dimensions

φ^4: $d_{upper} = 4$: expand in $d = 4 - \varepsilon$ dimensions

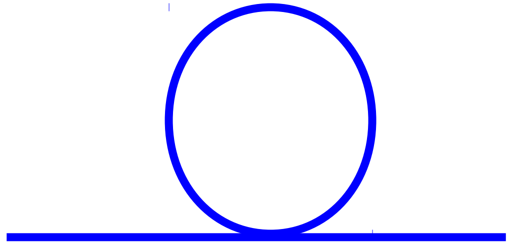
$$\int d^4k \, \frac{1}{(k^2)^2} \sim \log \Lambda_{\mathrm{UV}}$$



φ^4: $d_{lower} = 2$: expand in $2 + \varepsilon$ dimensions

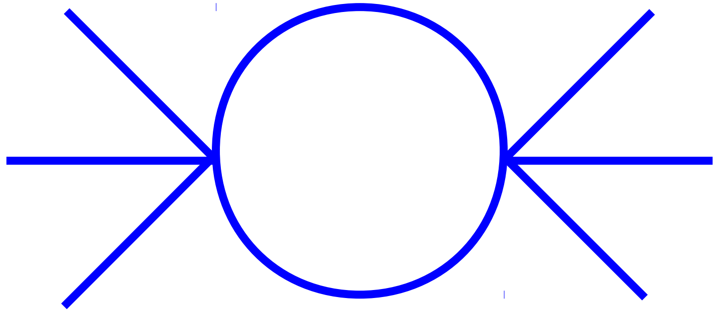
always disordered when $d < 2$

$$\int d^2k \, \frac{1}{k^2} \sim \log \Lambda_{\mathrm{IR}}$$



φ^6: $d_{critical} = 3$: at tricritical point, log corrections

$$\int d^3k_1 \int d^3k_2 \, \frac{1}{(k_1)^2(k_2)^2(k_1+k_2)^2} \sim \log \Lambda_{\mathrm{UV}}$$



Lifshitz points

To get a Chiral Spiral (CS):

$$\mathcal{L}_{CS} = (\partial_0 \phi)^2 + Z(\partial_i \phi)^2 + \frac{1}{\Lambda^2}(\partial_i^2 \phi)^2 + m^2\phi^2 + \lambda\phi^4$$

Need higher (spatial) derivatives for stability. Then CS occurs when $Z < 0$.

Can*not* have higher derivatives in time or theory is acausal.

In gravity, models with higher derivatives are renormalizable:

$$\mathcal{L}_{\mathrm{ren.gravity}} = \frac{1}{16\pi G} R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2$$

but acausal. Hořava-Lifshitz gravity: add higher derivatives only in space

Hořava 0901.3775 + ...

$$\mathcal{L}_{\mathrm{Horava-Lifshitz}} = \frac{1}{16\pi G} R + \beta_1 R_{ij}^2 + \ldots$$

Only two time derivatives, so causal. Flows into Einstein gravity in the infrared.

Lifshitz phase diagram in mean field theory

Phase diagram in Z & m^2: *three* phases, symmetric, broken, *and* Chiral Spiral

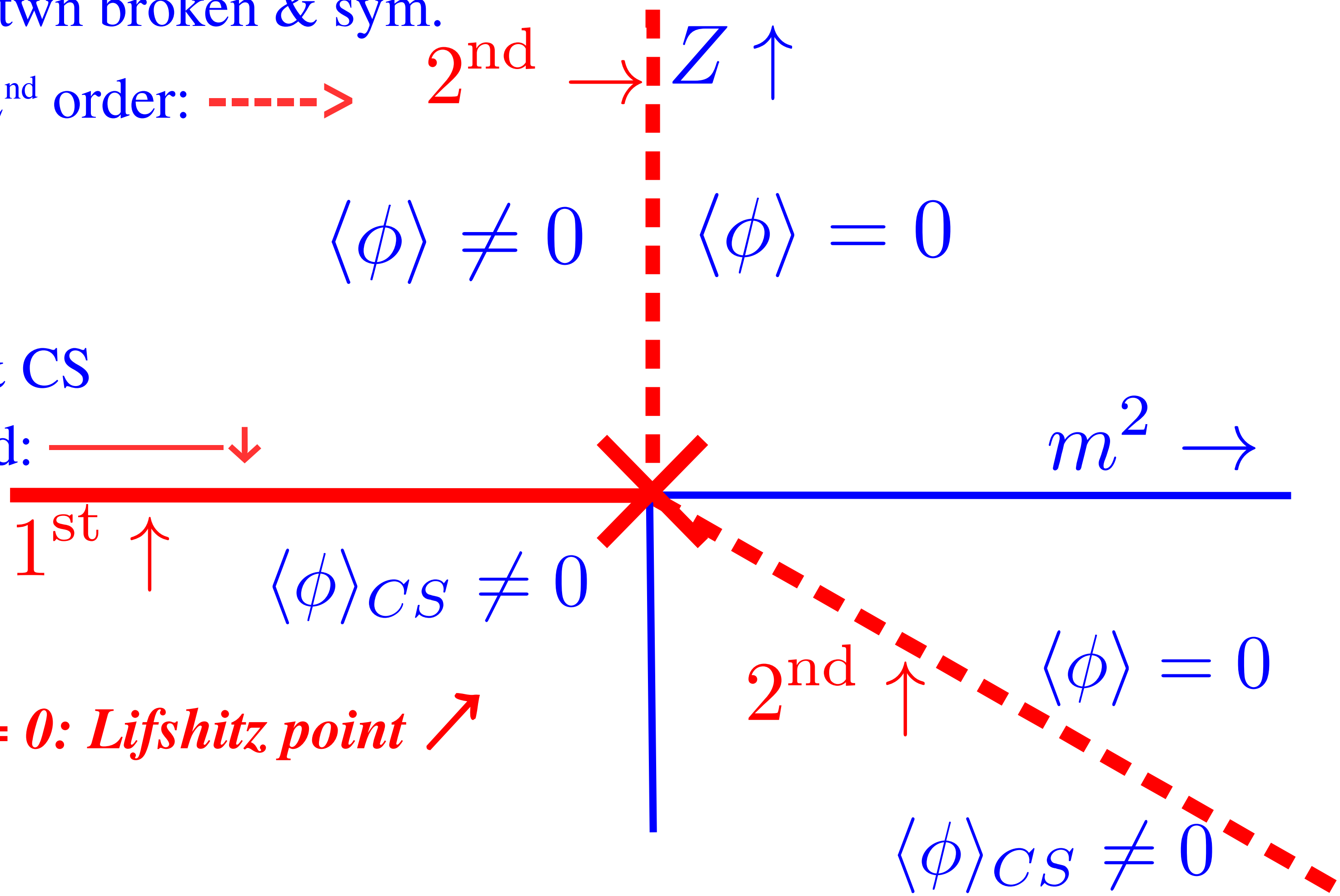
Hornreich, Luban, Shtrikman, PRL '75, Hornreich J. Magn. Matter '80...Diehl, cond_mat/0205284 + ...

$m^2 = 0$, $Z > 0$: btwn broken & sym.

usual 2nd order: ----->

$m^2 < 0$, $Z = 0$: broken & CS

1st order in mean field: ——↓



X at origin, $m^2 = Z = 0$: Lifshitz point ↗

$m^2 > 0$, $Z < 0$: btwn CS & broken ----- ↗

2nd order in mean field, but *fluctuations?*

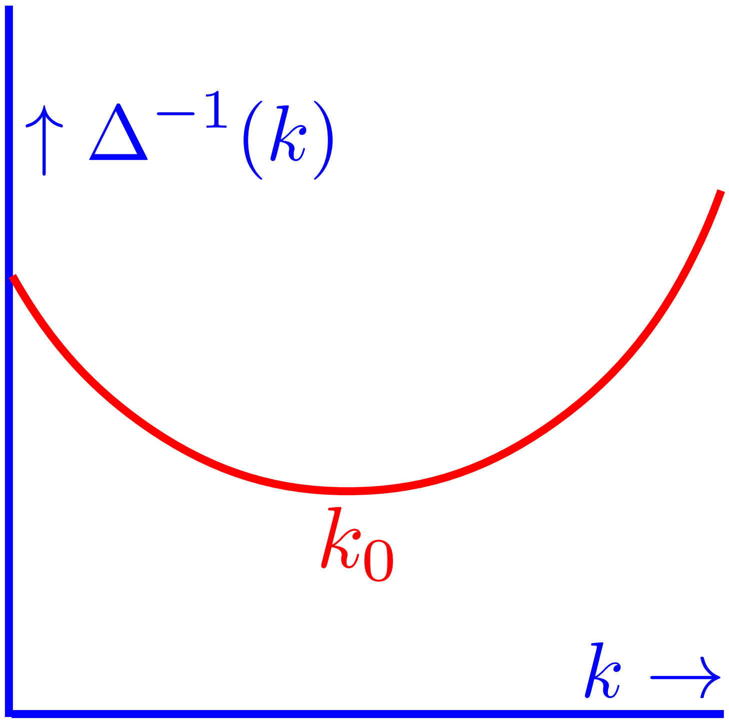
Symmetric to CS: 1D (Brazovski) fluctuations

Consider $m^2 > 0$, $Z < 0$: minimum in propagator at *non*zero momentum

Brazovski '75; Hohenberg & Swift '95 + ... ;

Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Yoshiike, Lee & Tatsumi 1702.01511

$$\Delta^{-1} = m^2 + Zk^2 + k^4/\Lambda^2$$
$$\sim m^2_{\text{eff}} + (k - k_0)^2 + \ldots$$



Reduction from 3 to 1 dimension,
along the *radial* direction:

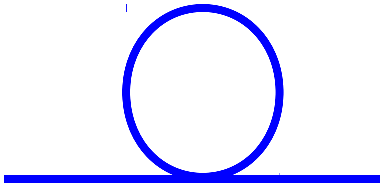
$$\int d^3k \, \frac{1}{(k-k_0)^2 + m^2_{\text{eff}}} \sim k_0^2 \int \frac{d(k-k_0)}{(k-k_0)^2 + m^2_{\text{eff}}} \sim \frac{1}{m_{\text{eff}}}$$

Effective reduction to 1-d for any spatial dimension d, any global symmetry

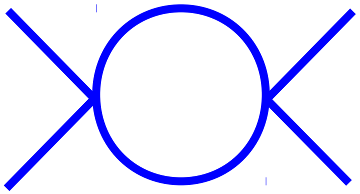
1^{st} order transition in 1-dim.

Strong infrared fluctuations in 1-dim., both in the mass:

$$\Delta m^2 \sim \lambda \int d^3k \, \frac{1}{(k-k_0)^2 + m_{\rm eff}^2} \sim \frac{\lambda}{m_{\rm eff}}$$



and for the coupling constant:



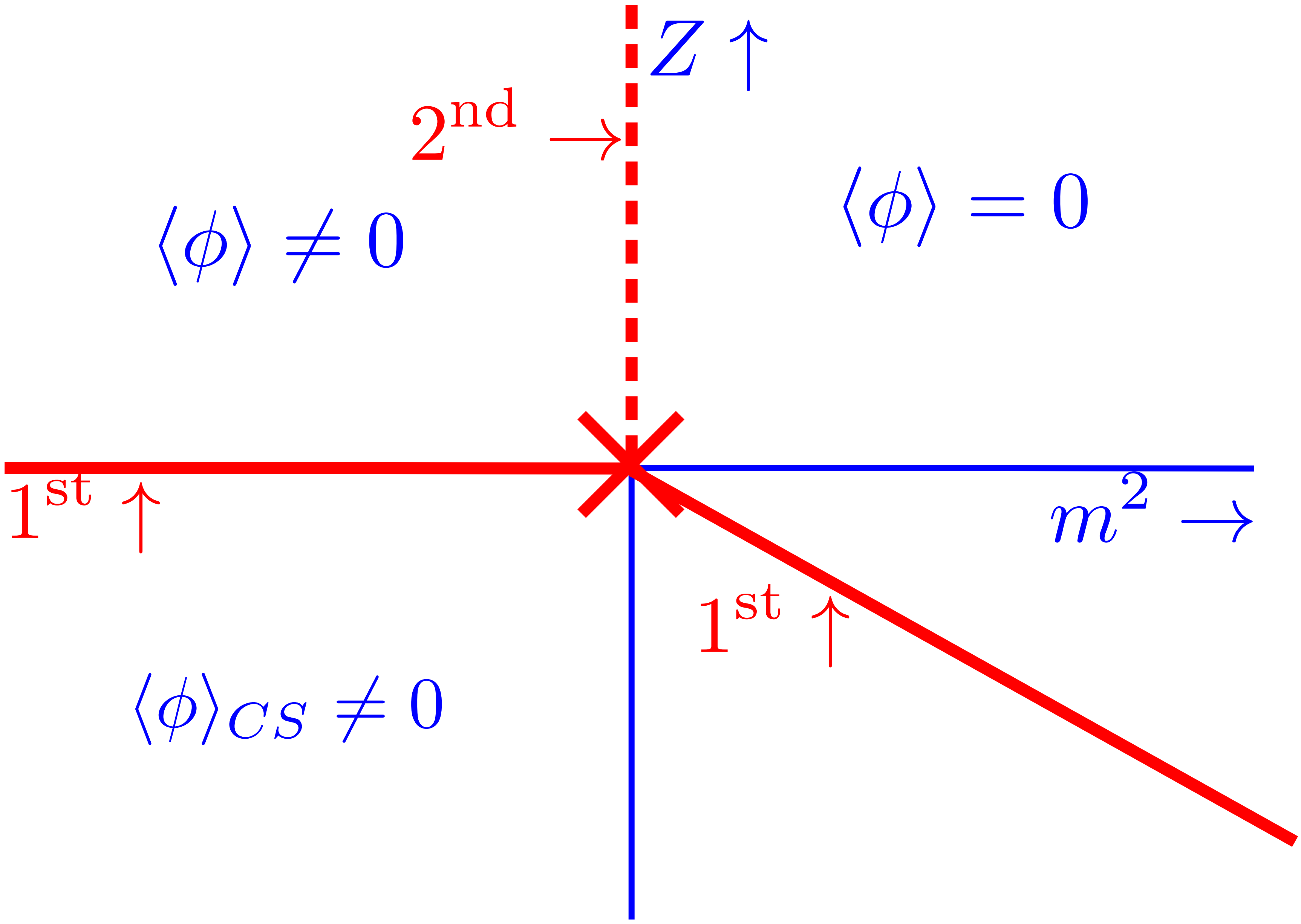
$$\Delta\lambda \sim (-)\lambda^2 \int \frac{d^3k}{((k-k_0)^2 + m_{\rm eff}^2)^2} \sim (-)\lambda^2 \int \frac{d(k-k_0)}{(k-k_0)^4} \sim (-)\frac{\lambda}{m_{\rm eff}^3}$$

mass dimensions made up by ~ Λ^2

Can*not* tune m^2 to 0, λ goes negative, so 1^{st} order trans. induced by fluctuations

Not like other 1st order fluc-ind'd trans's: just that in 1-d, $m_{\rm eff}^2 \neq 0$

Lifshitz phase diagram, with eff. 1-D fluc.'s



What about fluctuations at the Lifshitz point?

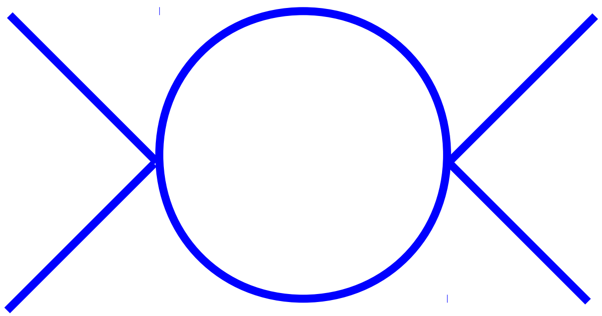
Critical dimensions at the Lifshitz point

At the Lifshitz point, Z=m=0,
massless propagator ~ 1/k^4

$$\mathcal{L}_{\mathrm{Lifshitz}} = (\partial^2\phi)^2 + \lambda\phi^4$$

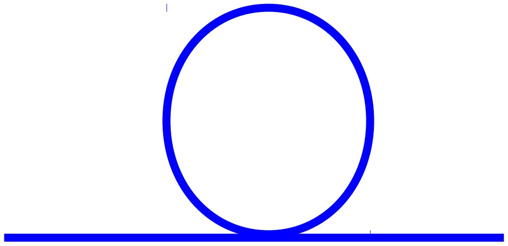
d_{upper} = 8 : expand in d = 8 – ε dimensions

$$\int d^8k \, \frac{1}{(k^4)^2} \sim \log \Lambda_{\mathrm{UV}}$$



d_{lower} = 4 : expand in d = 4 + ε dimensions

$$\int d^4k \, \frac{1}{k^4} \sim \log \Lambda_{\mathrm{IR}}$$



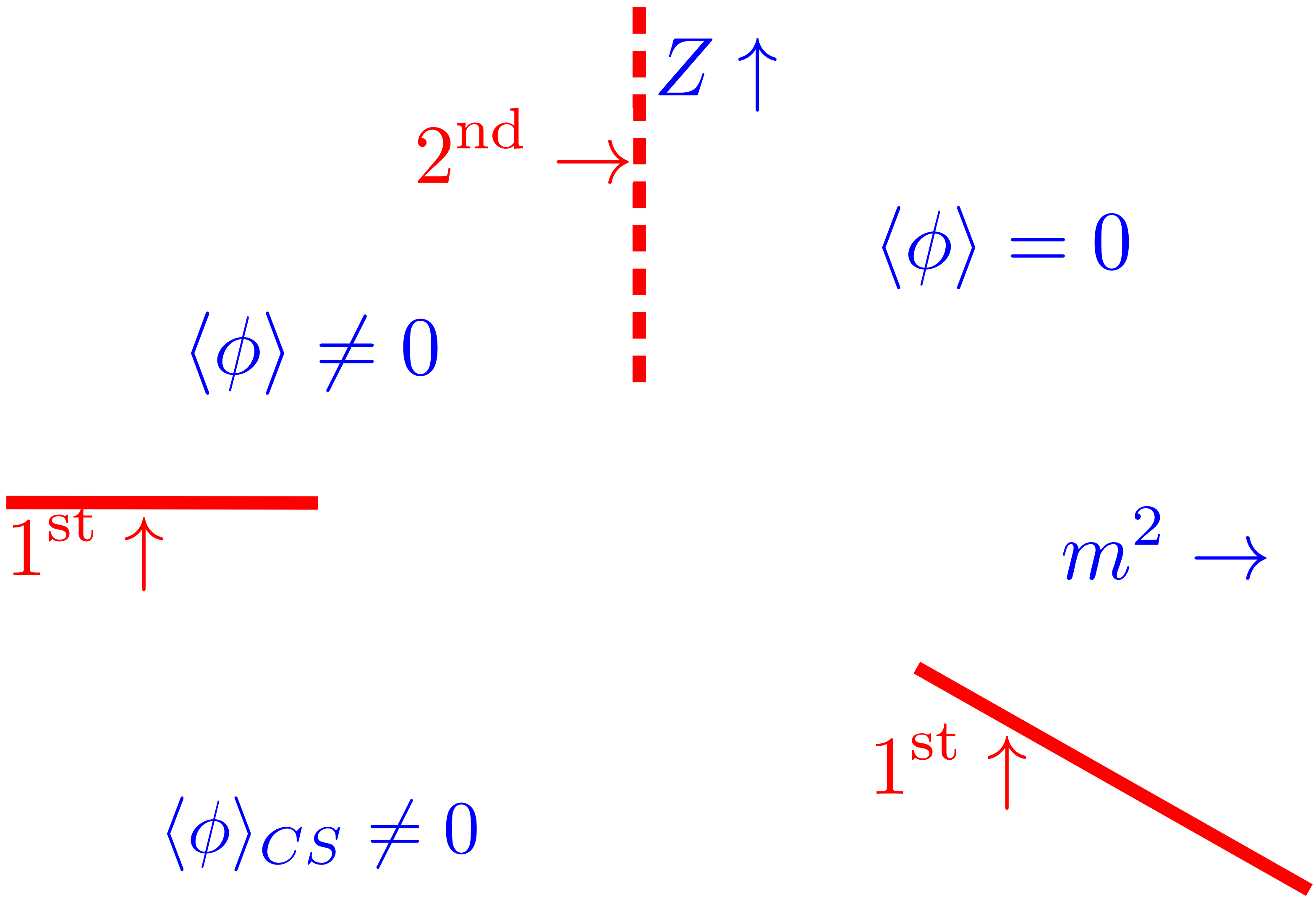
d = 3 < d_{lower}: there is *NO* (isotropic) Lifshitz point in *three* dimensions

...+ Bonanno & Zappala, 1412.7046; Zappala, 1703.00791

Infrared fluctuations *always* generate a mass gap *dynamically*.

Phase diagram with*out* a Lifshitz point?

Have three phases, three lines of phase transition far from the would be Lifshitz point. *How can they connect?*

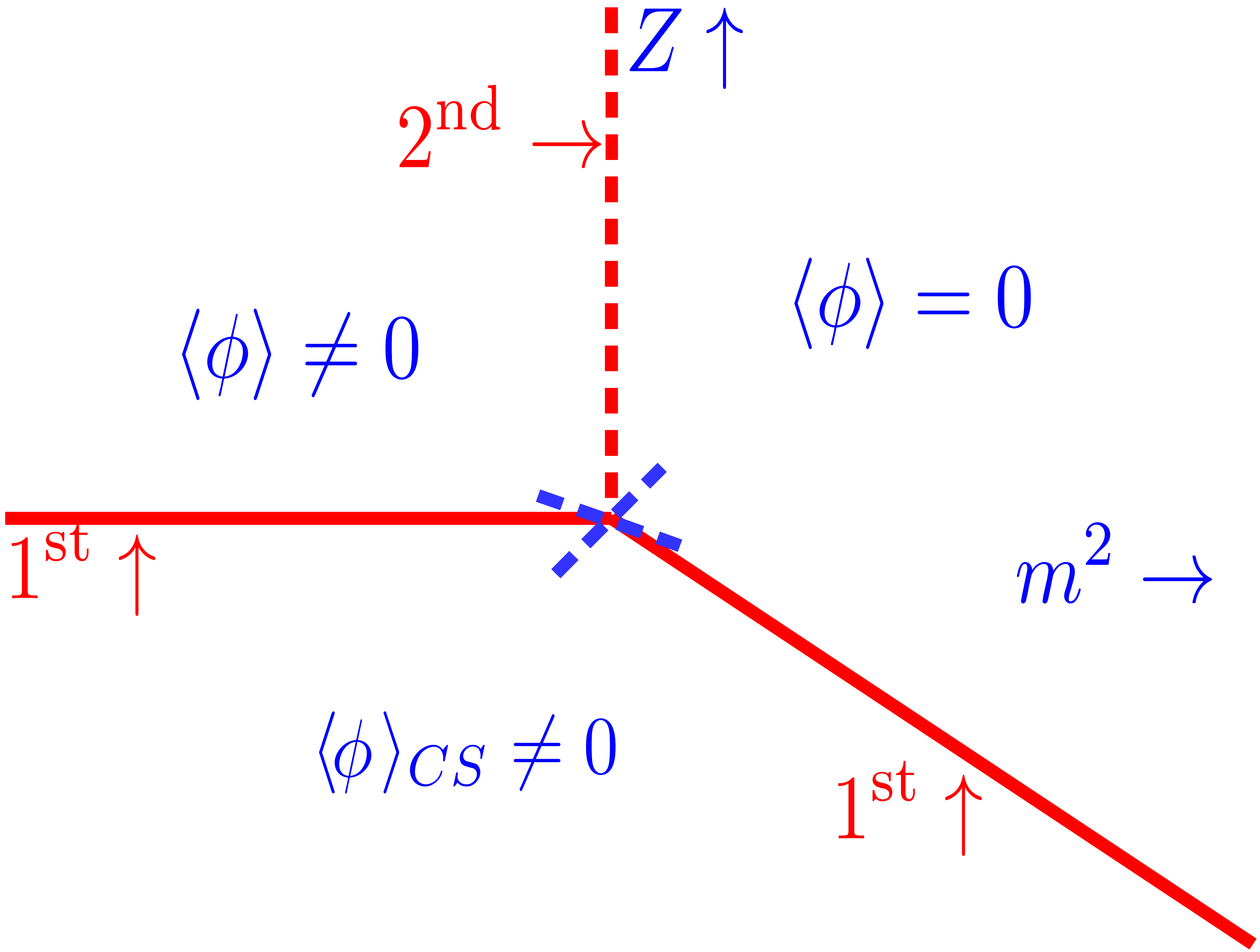


A: looks like Lifshitz point, but isn't

All three lines connect at a "pseudo"-Lifshitz point.

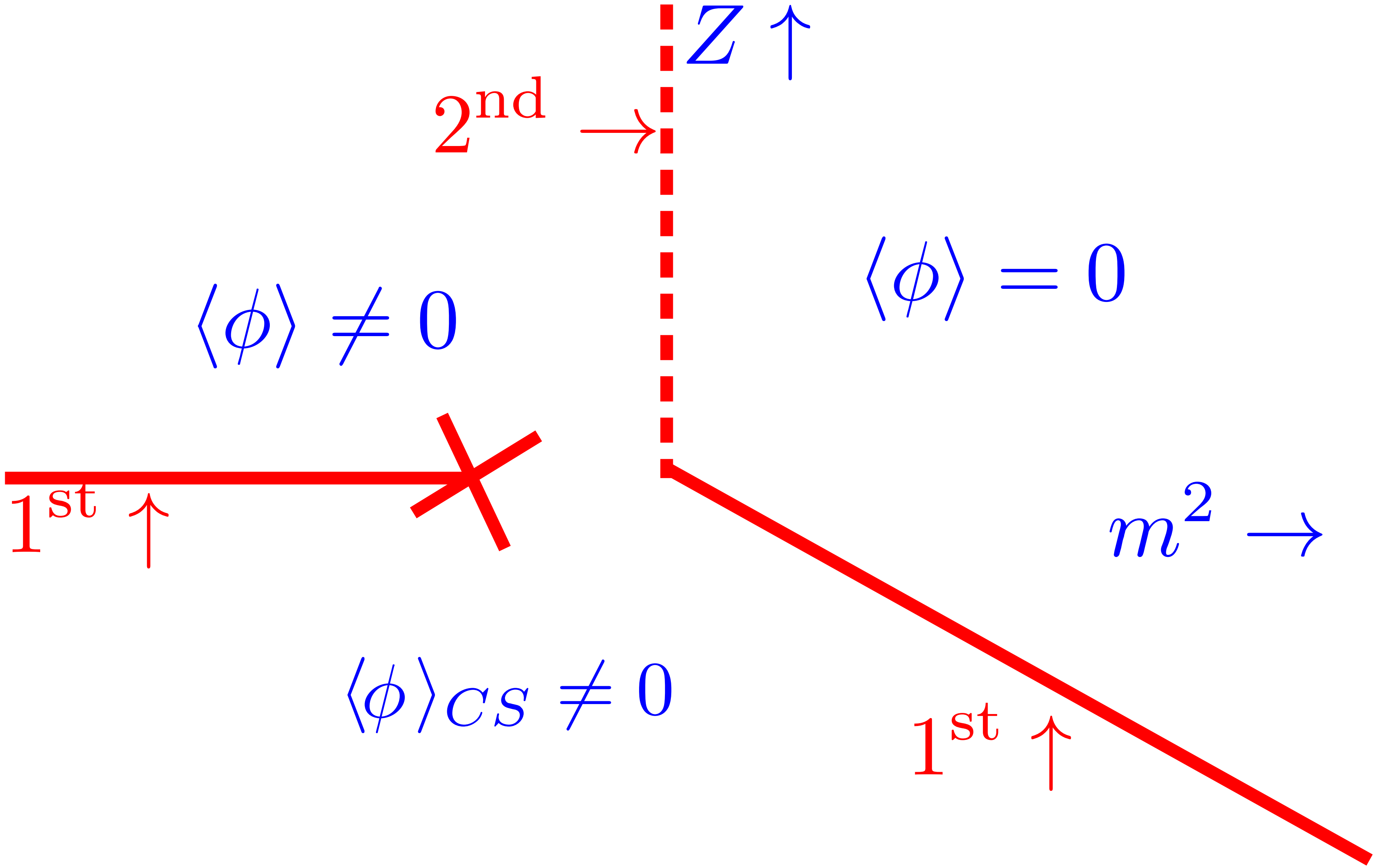
As terminus of 2nd order line, $m^2 = 0$. So at pseudo-Lifshitz point, $Z \neq 0$

Why do fluctuations drive symmetric-CS transition 1st order if $Z \neq 0$?



B: 1st order line between broken/CS phases ends

Crossover between broken and CS phases? But <φ>≠0 in the broken phase, and <φ>=0 for a Chiral Spiral. Crossover seems unlikely, unless fluctuations are *small* (so long range order in CS phase)

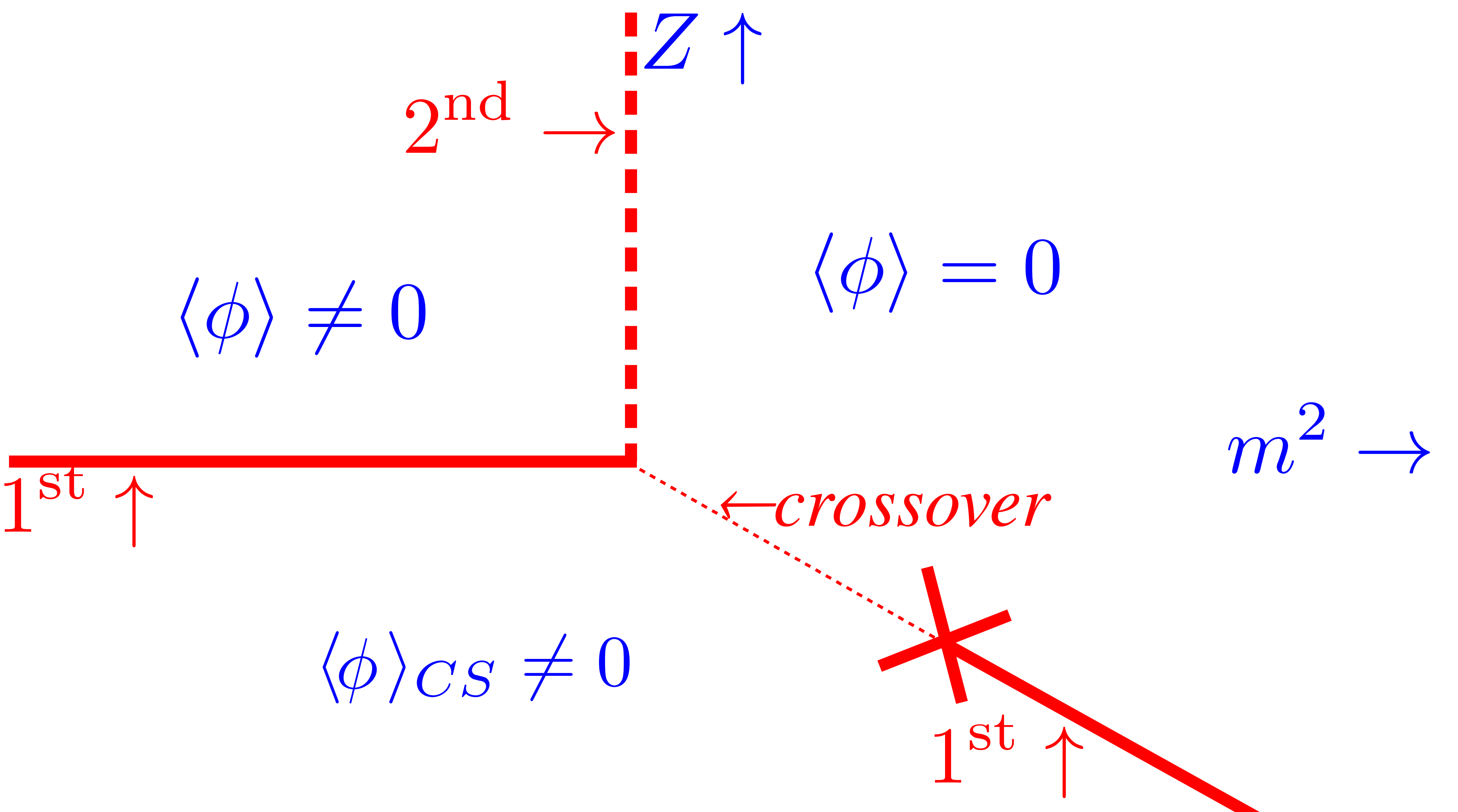


C: Brazovski 1st order CS/sym. line ends

Chiral spiral has *no* long range order, so *when* fluctuations are large, possible to have just *crossover* between CS & symmetric phases.

Brazikovski 1st order line ends in critical endpoint.

Novel tricritical point where 2nd order line joins to 1st order, at small Z.

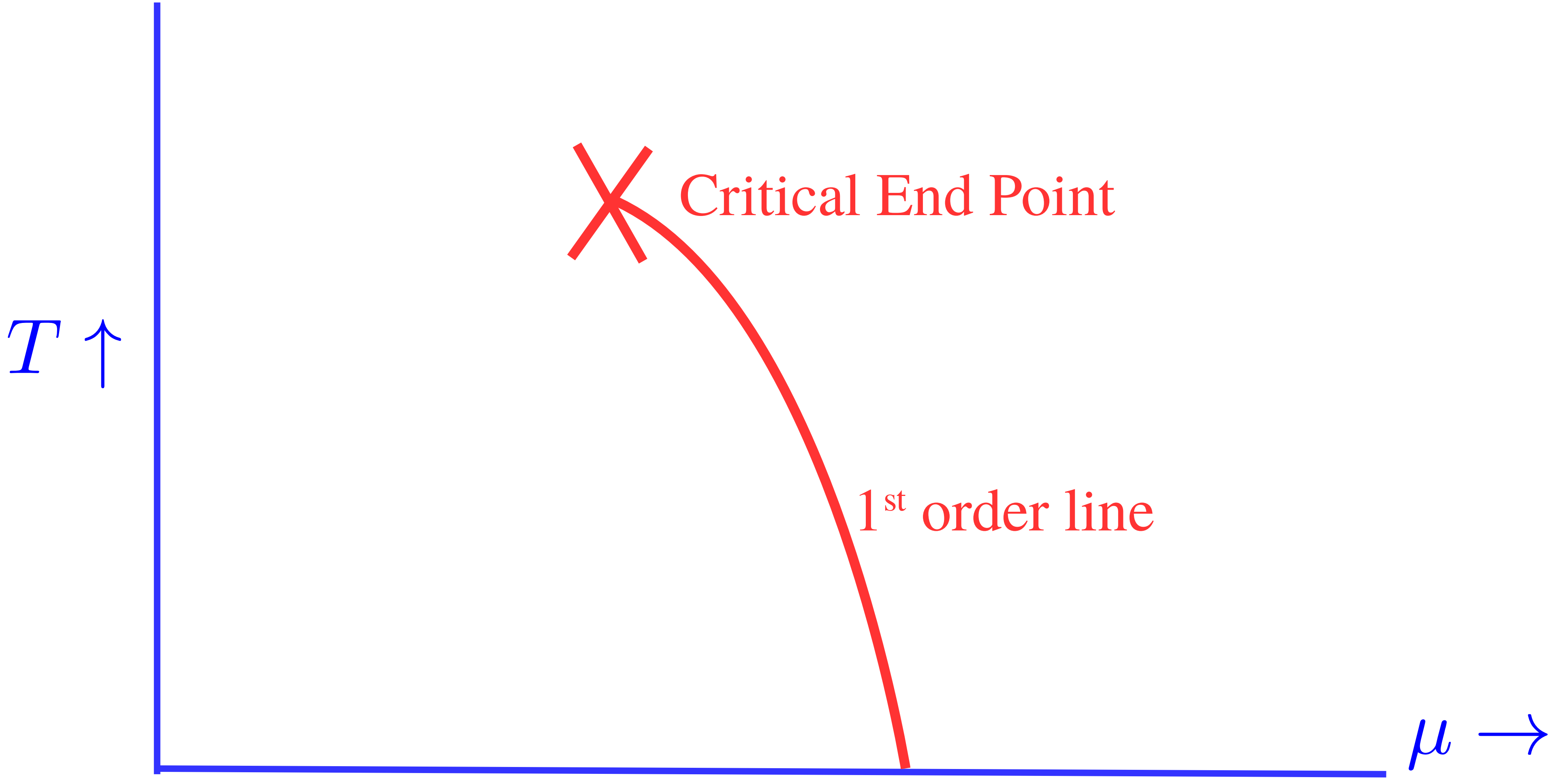


Phase diagram for QCD in T & μ: usual picture

Two phases, one Critical End Point (CEP)

between crossover and line of 1st order transitions

Ising fixed point, dominated by *massless* fluctuations at CEP

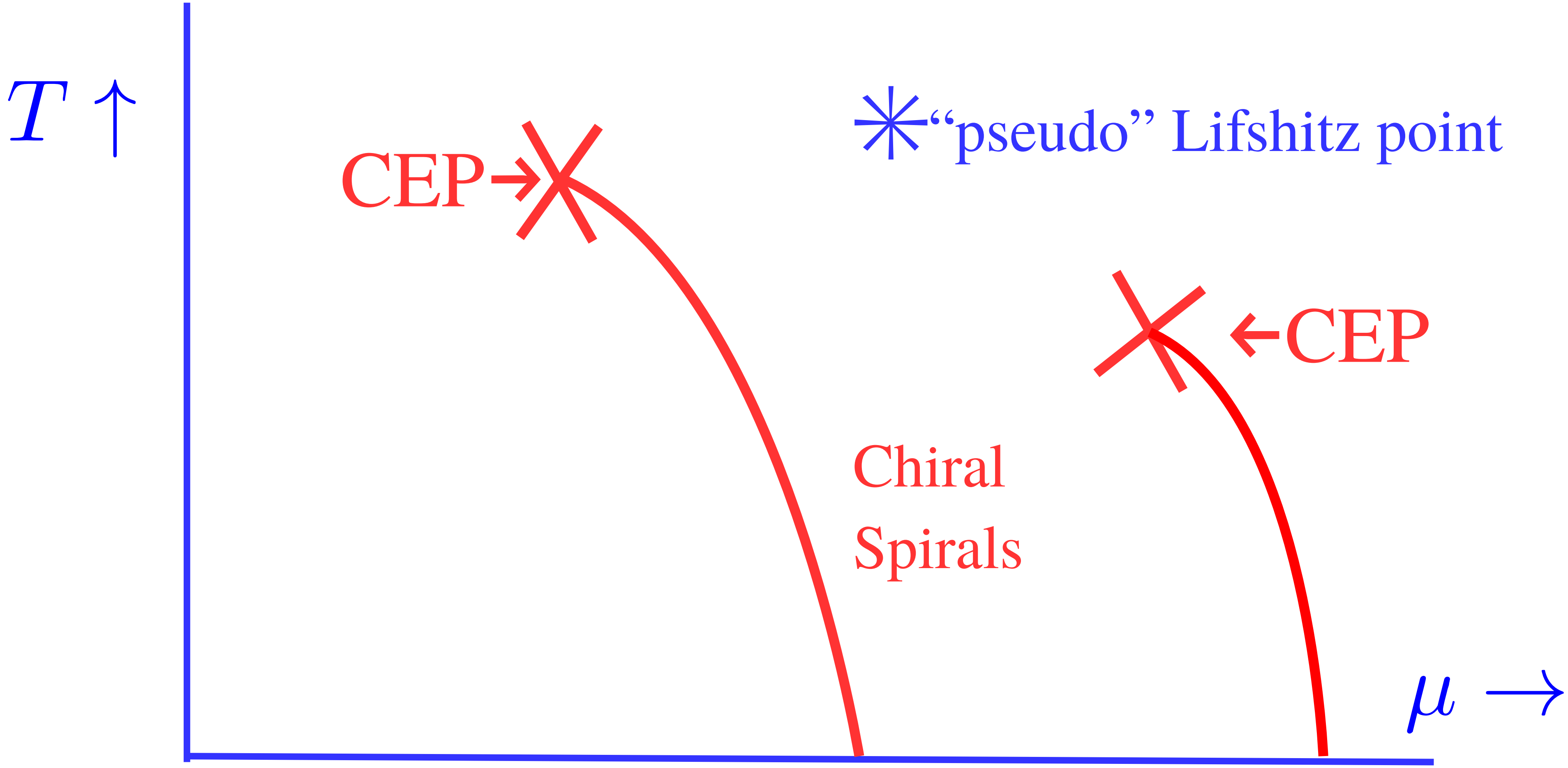


Phase Diagram with Chiral Spirals

Now *three* phases. If model "C", *two* 1st order lines and *two* CEP's

"Pseudo" Lifshitz point with large fluc.'s.

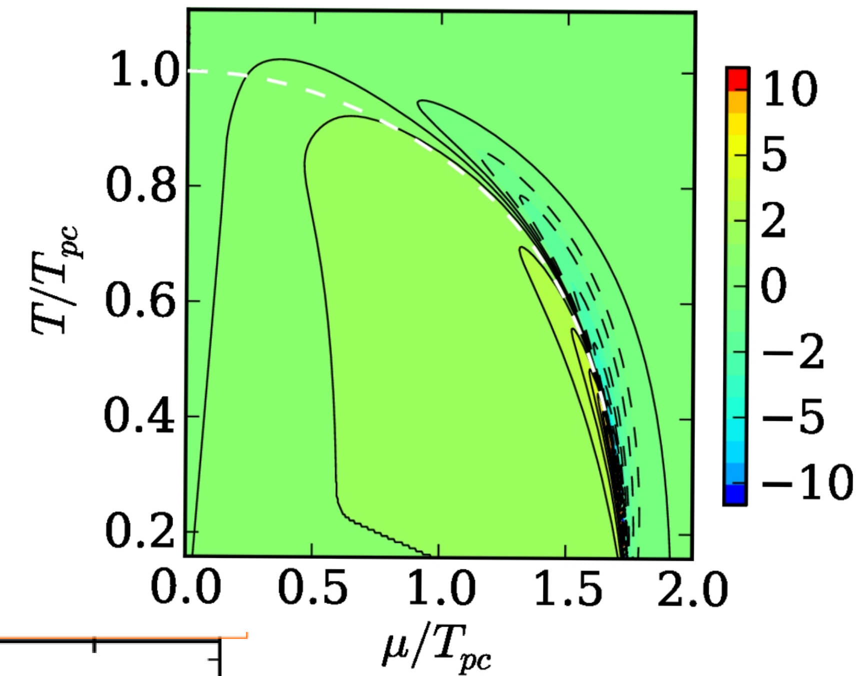
In CS, large fluc.'s at *non*zero momenta, ~ k_0 .



Beam Energy Scan and cumulants

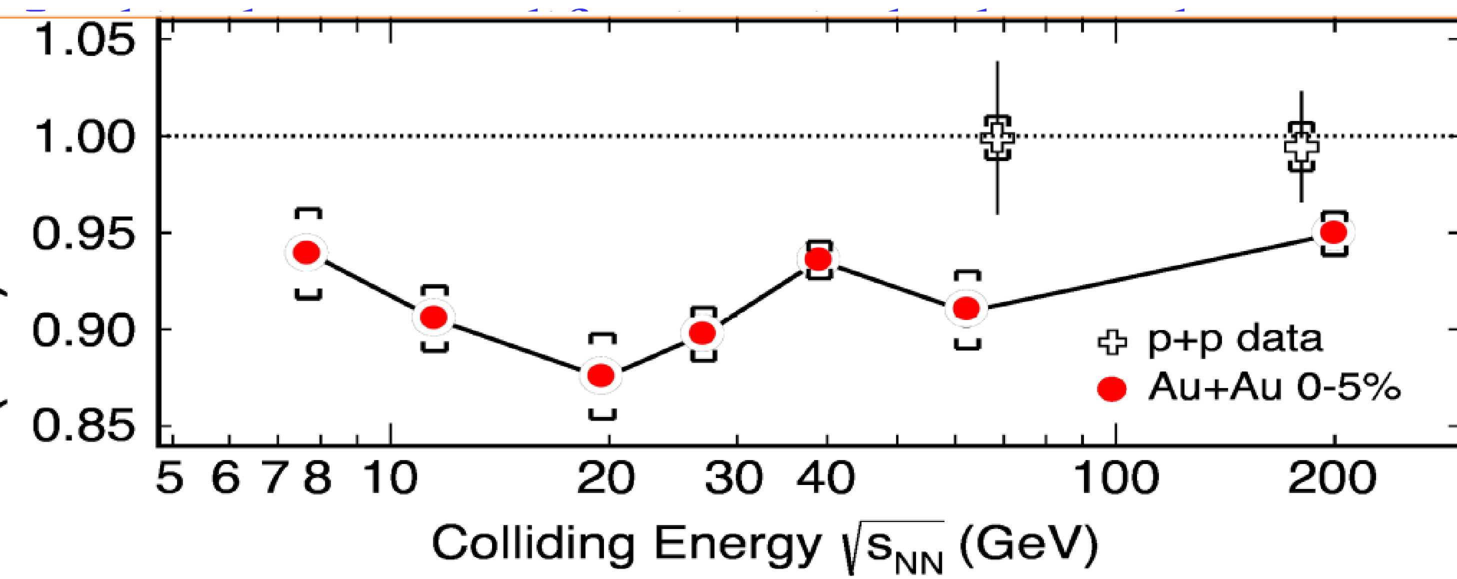
To look for Critical End Point, typically compute cumulants

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T, \mu)$$



Expectation from theory, to right: corrections to c_3 are *positive*

But STAR finds that the corrections to c_3, below, are *negative*



$$\frac{c_3}{c_2} \uparrow$$

Divided by Skellam

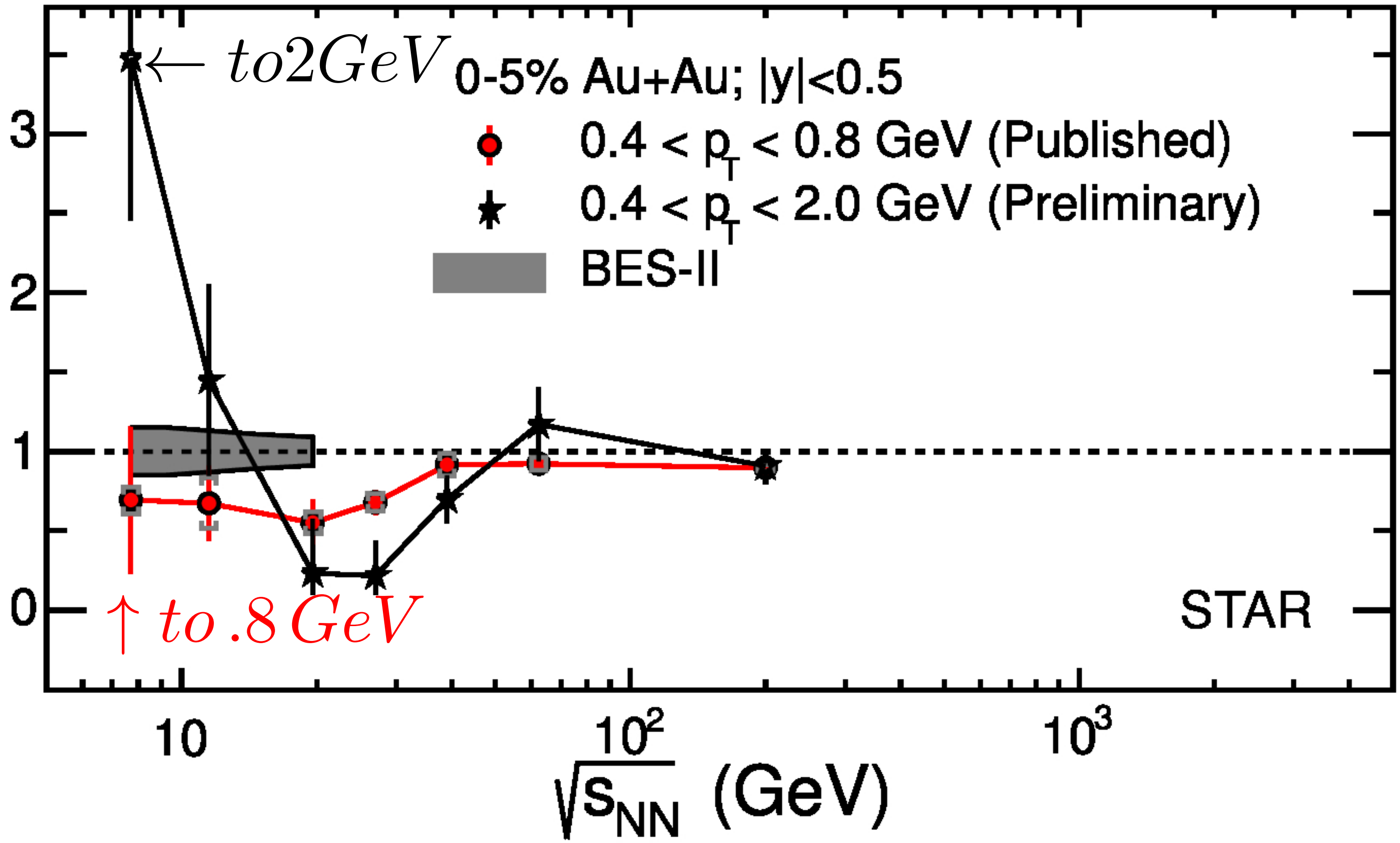
Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.

Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV

Trivial multiplicity scaling? ... or first evidence for a Chiral Spiral?!

$\frac{c_4}{c_2} \uparrow$



STAR: fig. 14,https://drupal.star.bnl.gov/STAR/files/STAR_iTPC_proposal_06_09_2015.pdf

Suggestion for experiment

For *any* sort of periodic structure (1D, 2D, 3D...),

fluctuations concentrated about some characteristic momentum k_0

So "slice and dice": bin in intervals, 0 to .5 GeV, .5 to 1., etc.

If peak in fluctuations in a bin not including zero, *may* be evidence for $k_0 \neq 0$.

If periodic structure, fluctuations must go *up* as $\sqrt{s}$ goes *down,* since μ increases

NJL models and Lifshitz points

Consider Nambu-Jona-Lasino models.

Nickel, 0902,1778 & 0906.5295 + ... + Buballa & Carignano 1406.1367

$$\mathcal{L}_{\rm NJL} = \overline{\psi}(\partial\!\!\!/ + g\sigma)\psi + \sigma^2$$

Integrating over ψ,

$$\log(\partial\!\!\!/ + g\sigma) \sim \ldots + \kappa_1((\partial\sigma)^2 + \sigma^4) \\ + \kappa_2((\partial^2\sigma)^2 + \sigma^2(\partial\sigma)^2 + \sigma^6) + \ldots$$

Consequently, in NJL @ 1-loop, *tricritical = Lifshitz point.*

Above due to scaling $\partial \to \xi\partial$, $\sigma \to \xi\sigma$.

Special to including only σ at one loop.

Not generic: violated by the inclusion of more fields, to two loop order, etc.