VOLUME DEPENDENCE OF BARYON NUMBER CUMULANTS & THEIR RATIOS

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in collaboration with **Gabor Almasi** and Rob Pisarski

August 10, 2017

G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

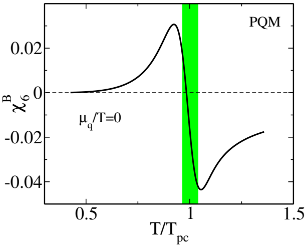
Outline

- Introduction: volume fluctuations
- Functional renormalization group approach
  & new numerical method
- Chiral model & finite volume
- Phase diagram and cumlants in finite volume
- Conclusions

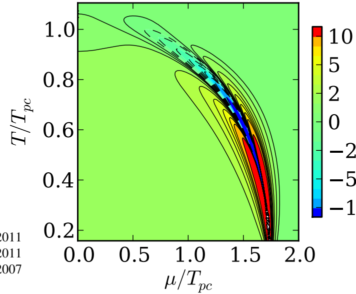
Signatures of phase transition and critical point

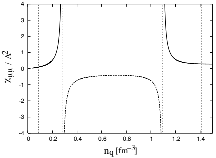
- O(4) crossover: universal sign structure of higher order cumulants, χ_6
- Divergent cumulants of baryon number fluctuations at CP

$$\chi_n \propto \xi^{3\left(\frac{n\beta\gamma}{2-\alpha}-1\right)}$$



- In vicinity of CP: universal sign structure of cumulants
- Divergent baryon number susceptibility at off-equilibrium first order phase transition



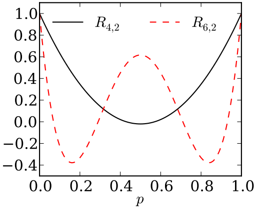


B. Friman, F. Karsch, K. Redlich, V. S. 2011
M. Stephanov 2011
C. Sasaki, K. Redlich, B. Friman 2007

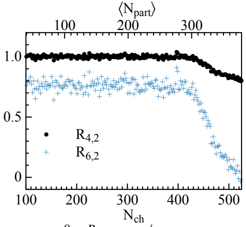
Previous slide: listed predictions for infinite static medium

In Heavy Ion Collisions:

- Finite lifetime
  S. Mukherjee, R. Venugopalan, Y. Yin 2015/2016
- Finite size and anisotropy
  this talk
- Conservation laws
  A. Bzdak, V. Koch, V.S. 2011
- Fluctuations not related to critical, e.g. V-fluctuations
  V.S., B. Friman, K. Redlich 2012
- Stopping
  A. Bzdak, V. Koch, V.S. 2017
- …

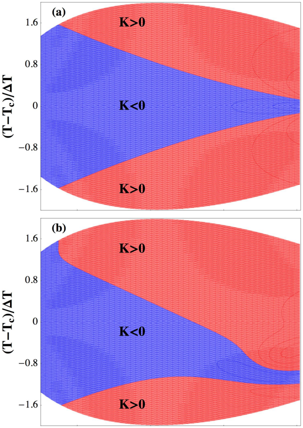


p is fraction of measured baryons



$\mu = 0 \quad R_{n,m} = \chi_n/\chi_m$

Modification due to finite life-time:



S. Mukherjee, R. Venugopalan, Y. Yin 2015

Volume fluctuations: introduction

- Usually it is assumed that cumulants are independent of volume
- Even in this case, volume fluctuations give non-trivial contribution to observable cumulants

$$c_1 = \kappa_1, \qquad c_2 = \kappa_2 + \kappa_1^2 v_2,$$
$$c_3 = \kappa_3 + 3\kappa_2\kappa_1 v_2 + \kappa_1^3 v_3, \quad c_4 = \kappa_4 + (4\kappa_3\kappa_1 + 3\kappa_2^2) v_2 + 6\kappa_2\kappa_1^2 v_3 + \kappa_1^4 v_4$$

$$c_6 = \kappa_6 + \kappa_1 (60\kappa_2\kappa_3 v_3 + 6\kappa_5 v_2) + \kappa_1^2 \left(45\kappa_2^2 v_4 + 15\kappa_4 v_3\right) + 15\kappa_2\kappa_4 v_2 + 10\kappa_3^2 v_2 + 20\kappa_3\kappa_1^3 v_4 + 15\kappa_2^3 v_3 + 15\kappa_2\kappa_1^4 v_5 + \kappa_1^6 v_6$$

where $\kappa_n = T^3 \chi_n$ and v_n are reduced volume cumulants, e. g. $v_2 = (\langle V^2 \rangle - \langle V \rangle^2)/\langle V \rangle$.

V.S., B. Friman and K. Redlich, 1205.4756

- Measured cumulants: c_n
- Equilibrium theory: κ_n

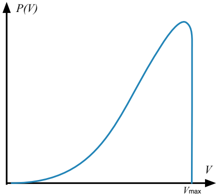
Volume fluctuations: general properties

$$c_1 = \kappa_1, \qquad c_2 = \kappa_2 + \kappa_1^2 v_2,$$

$$c_3 = \kappa_3 + 3\kappa_2\kappa_1 v_2 + \kappa_1^3 v_3, \qquad c_4 = \kappa_4 + (4\kappa_3\kappa_1 + 3\kappa_2^2) v_2 + 6\kappa_2\kappa_1^2 v_3 + \kappa_1^4 v_4$$

$$c_6 = \kappa_6 + \kappa_1 (60\kappa_2\kappa_3 v_3 + 6\kappa_5 v_2) + \kappa_1^2 \left(45\kappa_2^2 v_4 + 15\kappa_4 v_3\right) + 15\kappa_2\kappa_4 v_2 + 10\kappa_3^2 v_2 + 20\kappa_3\kappa_1^3 v_4 + 15\kappa_2^3 v_3 + 15\kappa_2\kappa_1^4 v_5 + \kappa_1^6 v_6$$

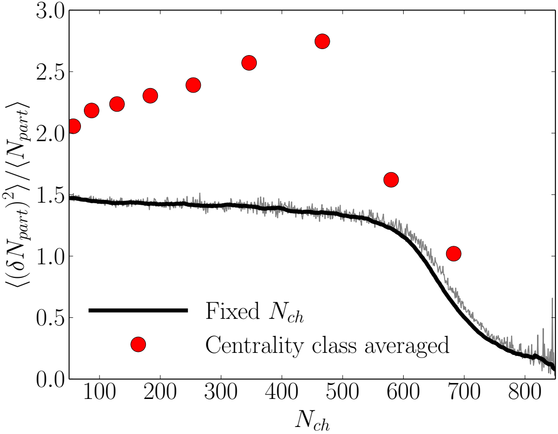
- $v_2 \geq 0 \rightsquigarrow \boxed{c_2 \geq \kappa_2}$
- c_3 receives contribution from v_3.
  For very central events (defined by N_{ch}) v_3 is negative because volume (or $S_\perp$) has an upper bound $V_{\max}$, while systems with lower volume may produce large N_{ch} owing to fluctuations.
  $\boxed{c_3 < \kappa_3 \text{ if } v_3 < -3 v_2 \kappa_2/\kappa_1^2}$ Large μ?!



- At nonzero μ, c_4: competing contributions with opposite signs
- At zero μ only c_n with $n \geq 4$ are modified, i.e. $c_4 = \kappa_4 + 3\kappa_2^2 v_2$. Thus $c_4 \geq \kappa_4$.
  Competing contributions to $c_6 = \kappa_6 + 15\kappa_2\kappa_4 v_2 + 15 v_3 \kappa_2^3$

Volume fluctuations: illustrations for v_2

- N_{part} as proxy for V
- Glauber + Negative binomial; Au-Au $\sqrt{s} = 200$



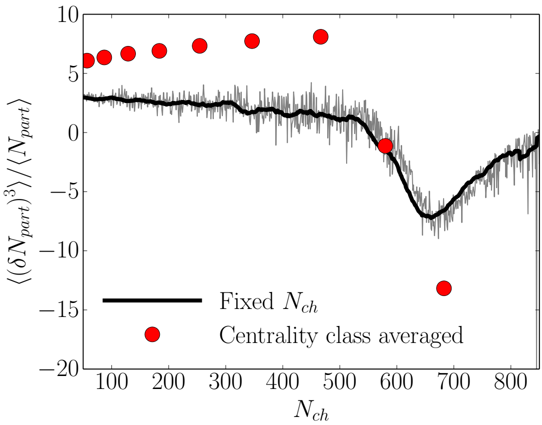
- Suppression of fluctuations at very large and very small (not shown) N_{ch}
- Centrality binning increases volume fluctuations and introduces non-monotonicity

V.S., B. Friman and K. Redlich, 1205.4756

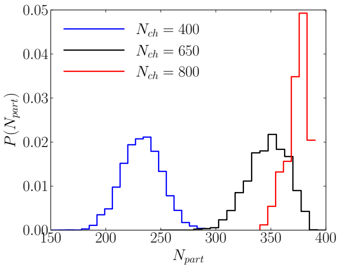
P. Braun-Munzinger, A. Rustamov, J. Stachel, 1612.00702

Points: centrality classes from right to left 0-5%, 5-10%, 10-20%, 20-30%, . . .

Volume fluctuations: illustrations for ν_3



- Negative skewness for central collisions; negative and large skewness for 0-5% centrality bin.
- Skewness changes sign at about $N_{ch} \sim 500$



Points: centrality classes from right to left 0-5%, 5-10%, 10-20%, 20-30%, …

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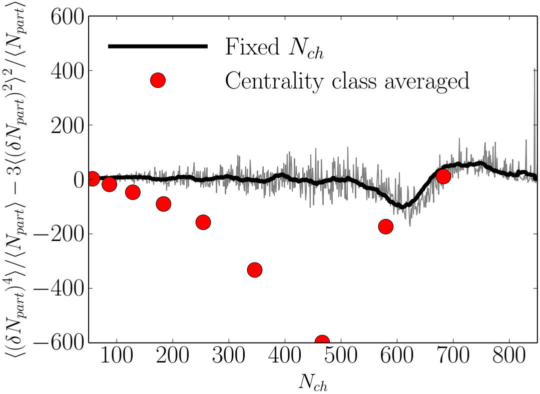
Volume fluctuations: illustrations for ν_4



- Changes sign
- Centrality binning effect is strong

Points: centrality classes from right to left 0-5%, 5-10%, 10-20%, 20-30%, ...

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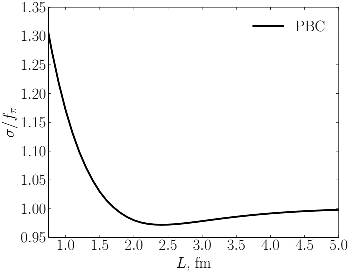
Volume fluctuations: are χ_n volume independent?

- Key assumption: cumulants are volume independent
- Generally this is true if surface effects can be neglected
- This is violated by long-range interactions
- Volume dependence of chiral condensate at $T = 0$, physical pion mass



Main ingredients:

- Functional Renormalization Group + Quark-meson model
- Equilibrium calculations in box L^3 for physical pion mass

- Expectation: susceptibilities are more sensitive to system size

G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

On chiral model

- Quark Meson model

(talk by Jochen Wambach)

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\tau\cdot\pi)\right]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2 - U(\sigma,\pi)$$

- Mean-field approximation: integrate out fermion fields & disregard fluctuations of mesonic field, $\Omega = \Omega_q - U(\sigma,\pi)$

$$\Omega_q = -\frac{1}{V}\mathrm{Tr}\log\left(i\gamma^\mu D_\mu - g\sigma\right)$$

- Beyond mean-field approximation: Functional Renormalization Group

V. S. and B. Friman et al, 1005.3166; R. Pisarski and V. S., 1604.00022

Functional renormalization group

General flow equation for scale-dependent effective action Γ_k. Γ_k: loosely speaking modes with momenta greater than k are integrated out, k is IR cut-off.

$$\partial_k \Gamma_k[\Phi,\psi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_{kB}\left(\Gamma_k^{(2,0)}[\Phi,\psi] + R_{kB}\right)^{-1}\right\} - \mathrm{Tr}\left\{\partial_k R_{kF}\left(\Gamma_k^{(0,2)}[\Phi,\psi] + R_{kF}\right)^{-1}\right\}$$

Flow equation for QM model in infinite volume (local potential approximation & sharp regulator)

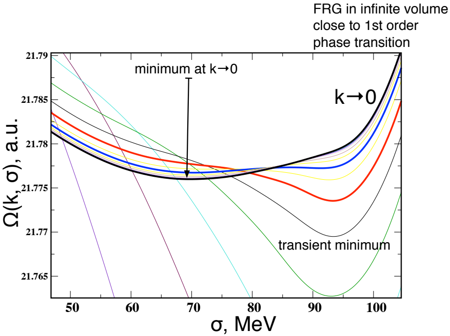
$$\partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \pi^2]) = \frac{k^4}{12\pi^2}\left\{\frac{3}{E_\pi}\left[1 + 2n_B(E_\pi; T)\right] + \frac{1}{E_\sigma}\left[1 + 2n_B(E_\sigma; T)\right] - \frac{4N_f N_c}{E_q}\left[1 - n_F(T,\mu_q) - \bar{n}_F(T,\mu_q)\right]\right\}$$

$n_B(E;T)$ is boson distribution functions
$N(T,\mu_q)$ are fermion distribution function modified owing to coupling to gluons
E_σ and E_π are functions of k, $\partial\Omega/\partial\rho$ and $\rho\partial^2\Omega/\partial\rho^2$
$E_q = \sqrt{k^2 + 2g^2\rho}$

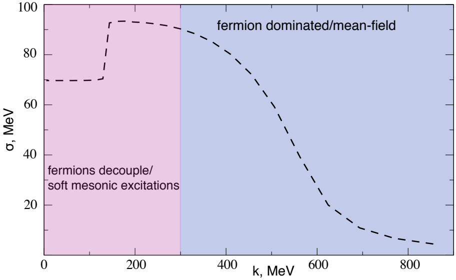
FRG defines $\Omega(k, \rho; T, \mu_Q, \mu_B)$
Physically relevant quantity is thermodynamic potential
$\bar{\Omega}(T,\mu_Q,\mu_B) \equiv \Omega(k \to 0, \rho \to \rho_0; T, \mu_Q, \mu_B)$, where ρ_0 is minimum of Ω V.S. et al 1004.2665

Functional renormalization group: evolution

Scale-dependent effective potential:



Minimum of scale-dependent effective potential:



- Soft mesonic excitations drive evolution at very small k

Functional renormalization group in finite volume

General flow equation for effective action

$$\partial_k \Gamma_k[\Phi,\psi] = \underbrace{\frac{1}{2}\mathrm{Tr}\left\{\partial_k R_{kB}\left(\Gamma_k^{(2,0)}[\Phi,\psi]+R_{kB}\right)^{-1}\right\}}_{\text{bosons}} - \underbrace{\mathrm{Tr}\left\{\partial_k R_{kF}\left(\Gamma_k^{(0,2)}[\Phi,\psi]+R_{kF}\right)^{-1}\right\}}_{\text{fermions}}$$

- Flow equation involve summation over modes $\sum_{n_1,n_2,n_3}$. For box with dimensions $L_1 \times L_2 \times L_3$ and periodic boundary conditions $q^2 = \sum_{i=1}^{3}\left(\frac{2\pi n_i}{L_i}\right)^2$
- In local potential approximation $[E_x = \sqrt{m_x^2+q^2+R_k(q)}$, e.g. $m_\sigma^2 = \partial^2\Omega/\partial\sigma^2]$

$$\partial_k\Omega = \frac{1}{4L^3}\underbrace{\sum_{n_x,n_y,n_z}}_{!!!}\left(\frac{1+2n_B(E_\sigma)}{E_\sigma}+3\frac{1+2n_B(E_\pi)}{E_\pi}-4N_fN_c\frac{1-n_F(E_q)-\bar{n}_F(E_q)}{E_q}\right)\partial_k R_k(q).$$

- Commonly applied sharp cut-off function $R_k^{\text{sharp}}(q) = (k^2-q^2)\theta(k^2-q^2)$ ✗
This is illustrated best with Wegner-Houghton functional RG.

G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

Wegner-Houghton equation

- Kadanoff-Wilson blocking in Fourier space for partition function

$$Z_k = \int \mathcal{D}\phi \exp\left(-S_k[\phi]\right) = \underbrace{\int \mathcal{D}\phi_<}_{|p|\le k-\delta k} \underbrace{\int \mathcal{D}\phi_>}_{k-\delta k<|p|\le k} \exp\left(-S_k[\phi_< + \phi_>]\right) = \underbrace{\int \mathcal{D}\phi_<}_{|p|\le k-\delta k} \exp\left(-S_{k-\delta k}[\phi_<]\right)$$

- Eliminating δk modes using saddle point approximation $\delta S_k[\phi_< + \phi^0_>]/\delta\phi_< = 0$ (shell by shell momentum integration)

$$\exp\left(-S_{k-\delta k}[\phi_<]\right) = \underbrace{\int \mathcal{D}\phi_>}_{k-\delta k<|p|\le k} \exp\left(-S_k[\phi_< + \phi_>]\right)$$

$$\overset{\delta k/k\ll 1}{\approx} \exp\left(-S_k[\phi_< + \phi^0_>]\right) \int \mathcal{D}\phi_> \exp\left(-\frac{1}{2}\int d^dk \frac{\delta^2}{\delta\phi_<\delta\phi_<} S_k[\phi_< + \phi^0_>](\phi_> - \phi^0_>)^2\right)$$

or

$$S_{k-\delta k}[\phi_<] = S_k[\phi_< + \phi^0_>] + \frac{1}{2}\mathrm{Tr}\ln\left(\frac{\delta^2}{\delta\phi_<\delta\phi_<} S_k[\phi_< + \phi^0_>]\right)$$

- For trivial saddle, taking $\delta k \to 0$, one recovers usual FRG equation with sharp cut-off function (modulo surface term) $\frac{\partial S_k[\phi<]}{\partial k} = -\lim_{\delta k\to 0} \frac{1}{2\delta k}\mathrm{Tr}\ln\left(\frac{\delta^2}{\delta\phi_<\delta\phi_<} S_k[\phi_<]\right)$.
Key point: $\boxed{\delta k \to 0 \text{ must exist}}$.

FRG in finite volume: smooth cut-off regulator

- We have to use smooth regulator $R_k(q) = \frac{q^2}{\exp(q^2/k^2)-1}$ ✓
- …and thus deal with summation over modes on each step of FRG evolution and for each value of σ

reminder Ω is function of two variables: k and σ

$$\partial_k \Omega = \frac{1}{4L^3} \underbrace{\sum_{n_x,n_y,n_z}}_{!!!} \left(\frac{1+2n_B(E_\sigma)}{E_\sigma} + 3\frac{1+2n_B(E_\pi)}{E_\pi} - 4N_f N_c \frac{1-n_F(E_q)-\bar{n}_F(E_q)}{E_q} \right) \partial_k R_k(q)$$

- Need for efficient algorithm to solve stiff partial differential equation which involves mode summation
  - Optimizing mode summation: One-particle energies depend on magnitude q^2. This symmetry can be taken into account by introducing multiplicity of states with given magnitude $G(m)$

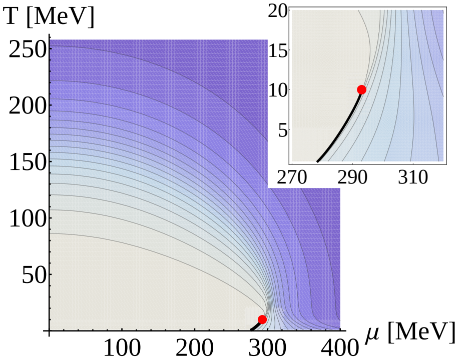
$$\sum_{\mathbf{n}} f(\mathbf{n}^2) = \sum_{m=0}^{\infty} \left(\sum_{\mathbf{n}} \delta_{m,\mathbf{n}^2} \right) f(m) = \sum_{m=0}^{\infty} G(m) f(m)$$

  - Usually applied Taylor and grid methods for solving FRG cannot reliably applied for first-order PT or slow/unstable at $k \to 0$.
    Instead we developed pseudo-spectral Chebyshev collocation algorithm for FRG.

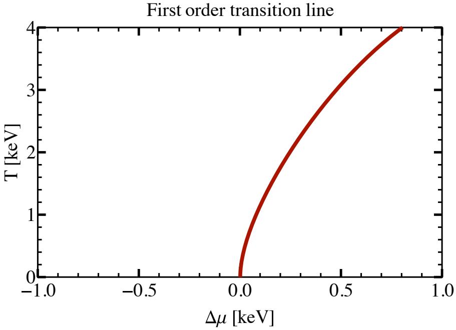
G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

Chebyshev collocation: demonstration

- First-order phase transition line must be perpendicular to μ axis due to Clausis-Clapeyron relation $d\mu/dT|_{\text{along 1st order tran.}} = -\Delta s/\Delta n$. At zero T, $\Delta s = 0$.



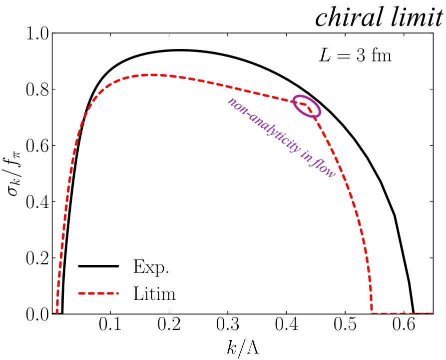
R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, 1311.0630
Talk by Jochen Wambach

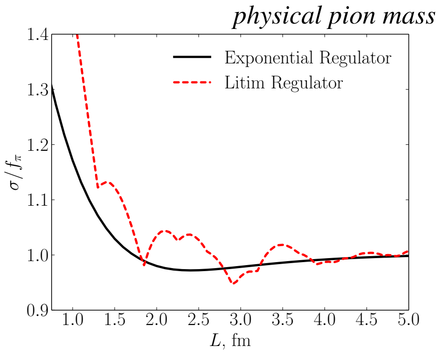


Gabor Almasi, V. S., 2016

- Algorithm allows to resolve this old puzzle

Litim (sharp) regulator vs Exponential

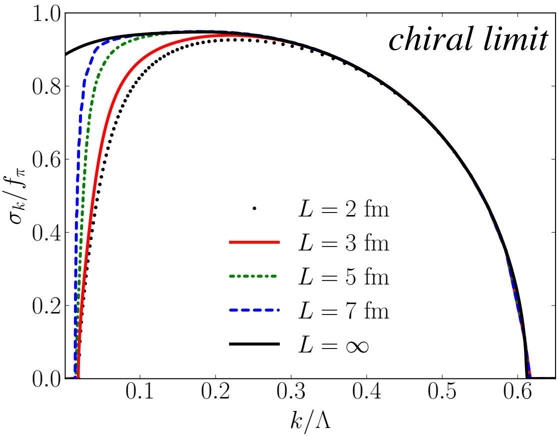




- Litim regulator $\rightsquigarrow$ non-analyticity in evolution
- Most importantly: artifacts in order parameter dependence on system size

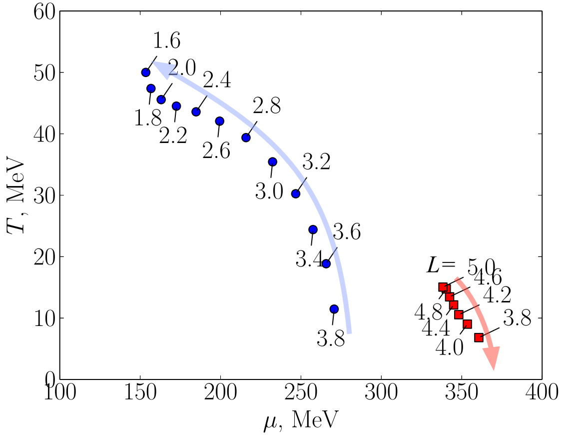
G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

Absence of spontaneous χSB in finite volume



- Cf. mean-field approximation resulting in non-zero expectation value in finite volume
- Fluctuations are important!

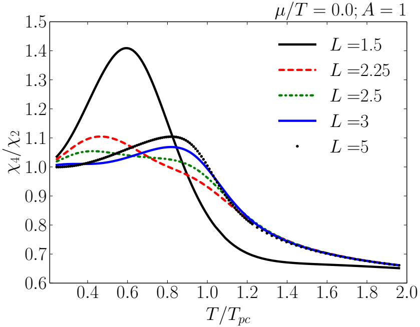
G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

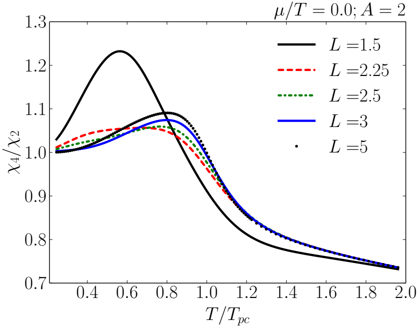


- There is true critical point for $V \to \infty$. In finite volume, CP $\rightsquigarrow$ apparent CP.
- There is some degree of arbitrariness in defining ACP.
- In this work: maximum of chiral susceptibility.

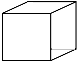
G. Almasi, R. Pisarski and V. Skokov, Phys. Rev. D **95**, 056015 (2017); arXiv:1612.04416.

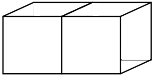
$$\chi_n = \partial^n (p/T^4)/(\partial \mu/T)^n$$



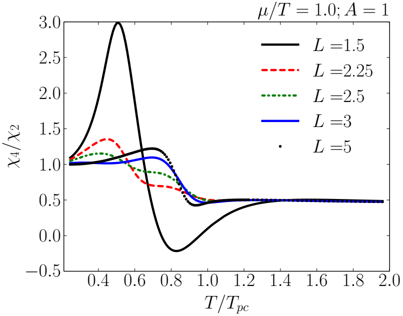


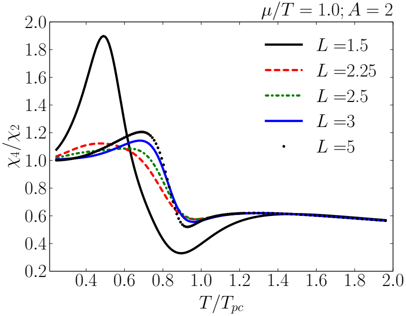
- Anisotropy coefficient $A = L_{\parallel}/L_{\perp}$



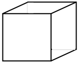


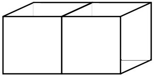
Results: QM + FRG in finite model $\mu \neq 0$



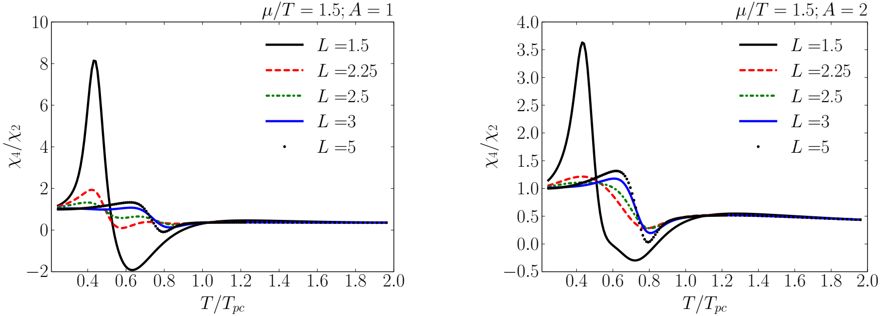


- Anisotropy coefficient $A = L_{\|}/L_{\perp}$





RESULTS: QM + FRG IN FINITE MODEL $\mu \neq 0$



- Anisotropy coefficient $A = L_{\parallel}/L_{\perp}$

- Approximation of volume independence of χ_n breaks down at about $L = 3 - 4$ fm: this makes analysis of volume fluctuations significantly more complicated and tractable only in fully dynamical model.

Conclusions

- Volume fluctuations: some properties are model independent, i.e. negative v_3
- In general, v_n are non-monotonic functions of centrality and energy
- Chebyshev collocation is powerful and fast method for solving FRG
  Future applications include inhomogeneous phases, talk by Pisarski
- Drastic shift of apparent CP as function of system size
- Ratios of cumulants depend on volume for $L \sim 3$ fm