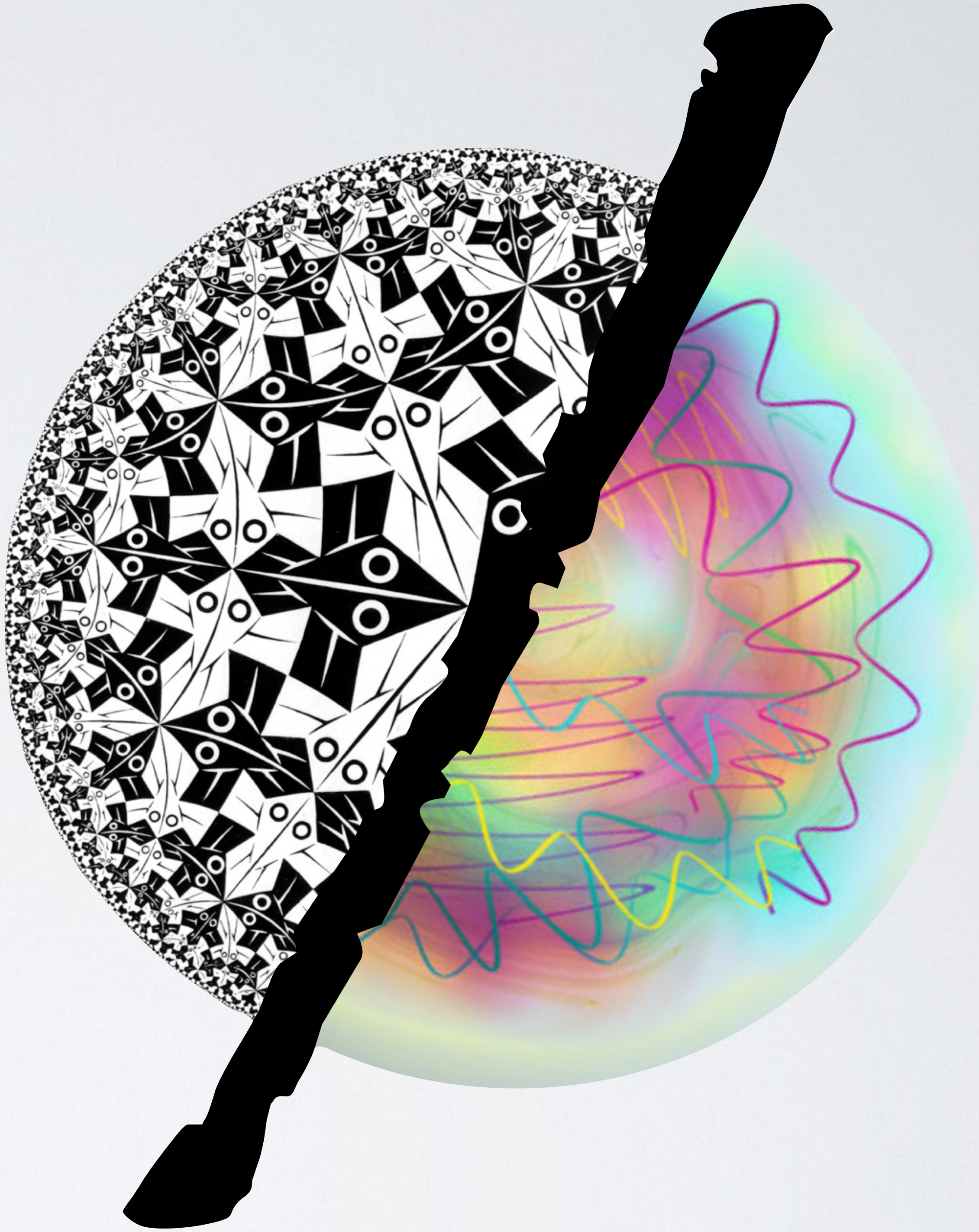
Chiral phase diagram in soft-wall AdS/QCD



Sean Bartz
Macalester College

Critical Point and Onset of Deconfinement
August 8, 2017

Overview

- Holographic duality
- High temperature/density
- Chiral phase transition
  - 2 flavors
  - 2+1 flavors

AdS/QCD Holography

4D Particle Field Theory	⟷	5D Gravity in curved Space
Strongly Coupled CFT	⟷	Weakly coupled Gravity
Operators	⟷	Fields
Global Symmetries	⟷	Gauged Symmetries

Contents of the model

Contents of the model

- Metric

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- Metric
- Dilaton

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- Metric
- Dilaton
- Chiral condensate

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- Meson Fields

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- T and **μ**

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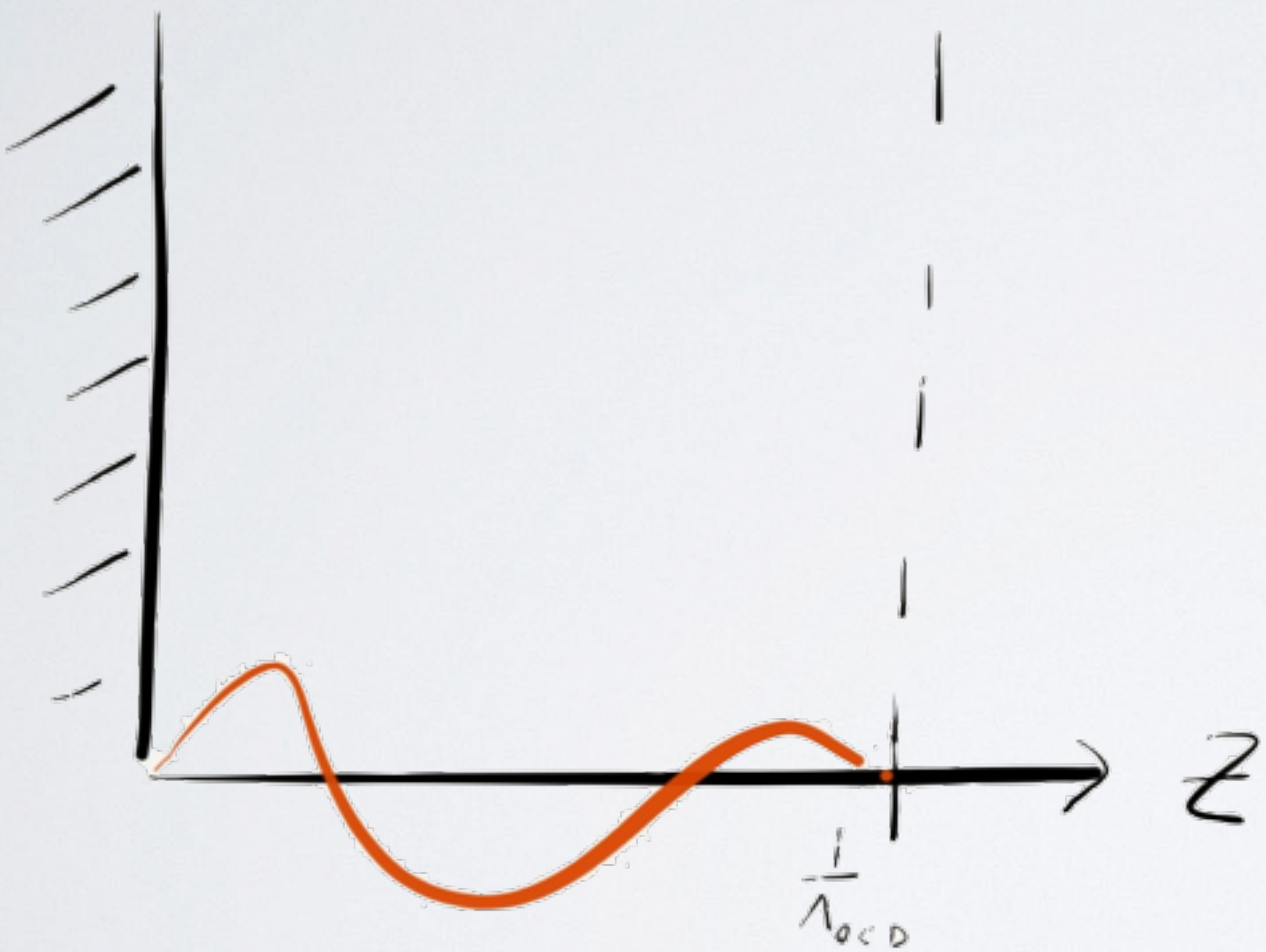
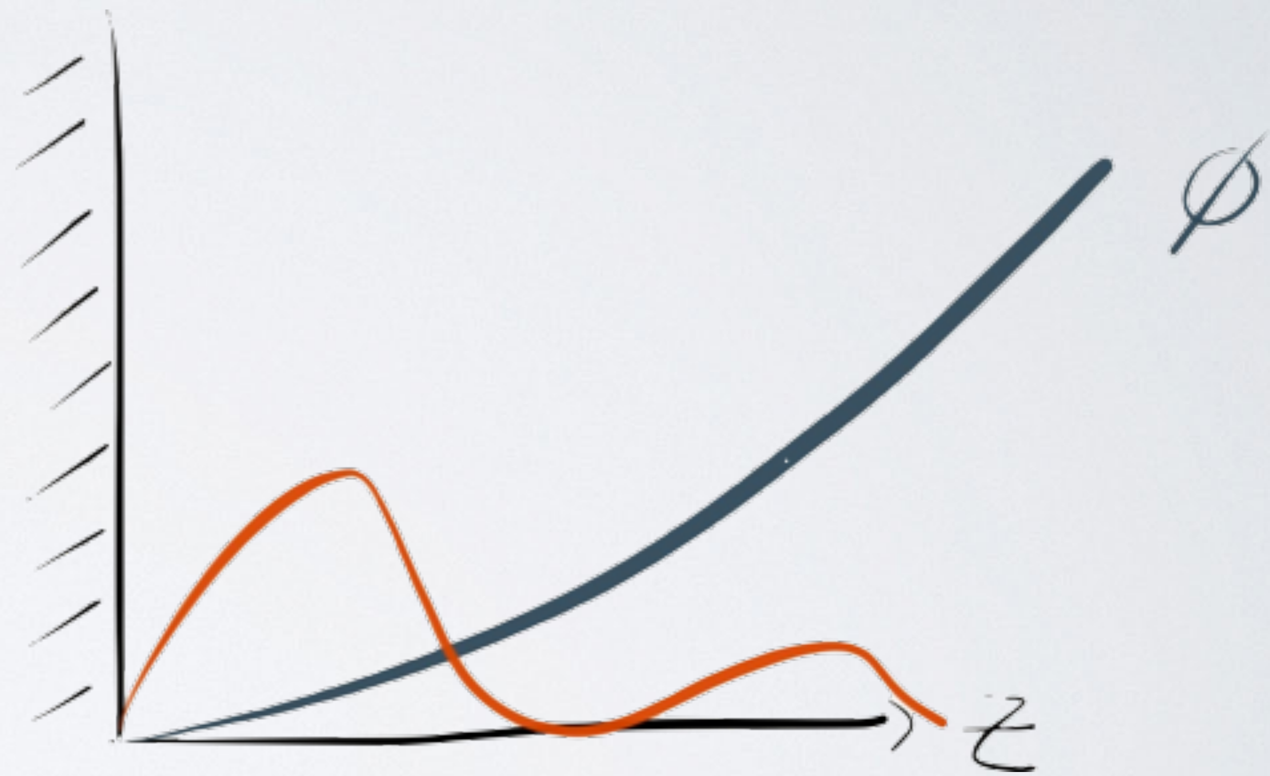
Confinement — Fields

Hard Wall

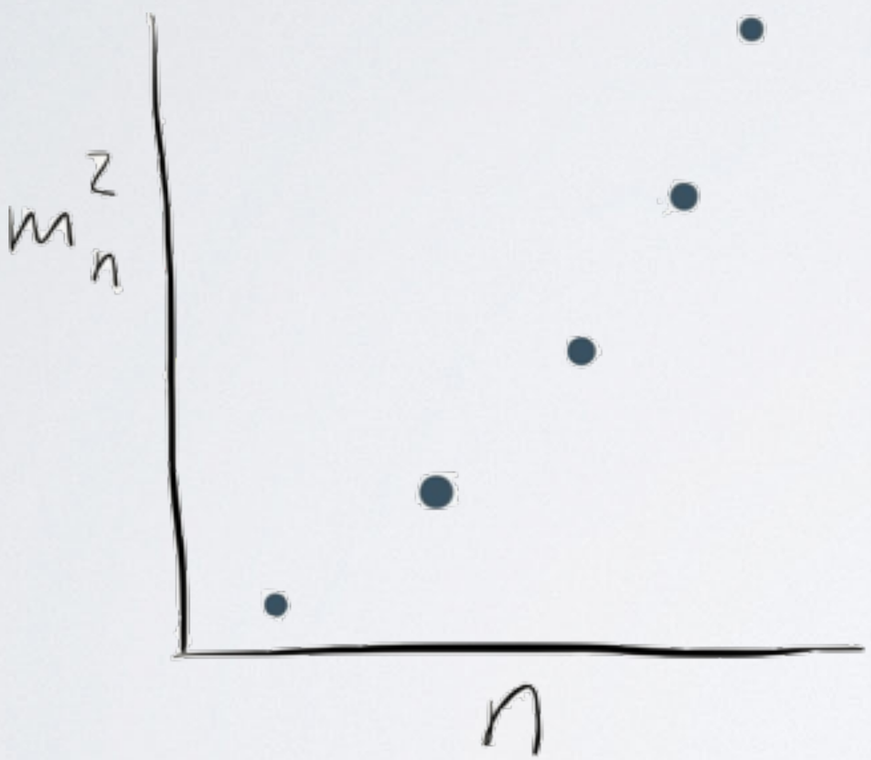


Soft Wall

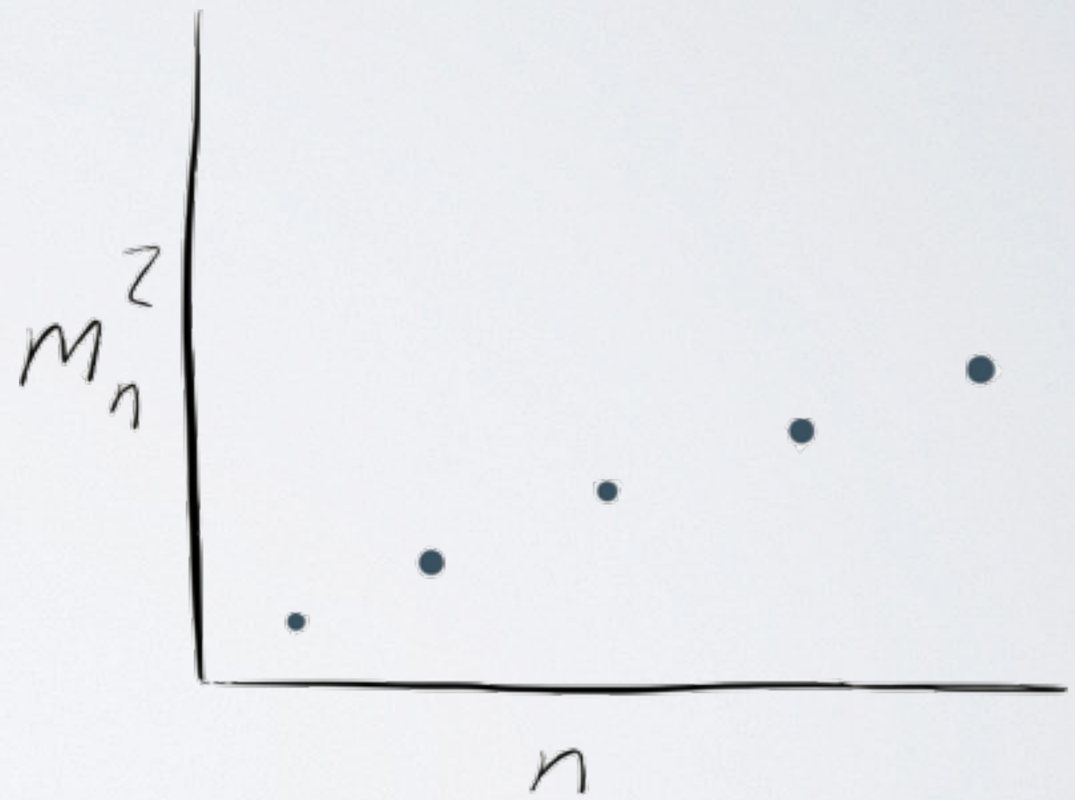


Confinement — Spectra

Soft Wall



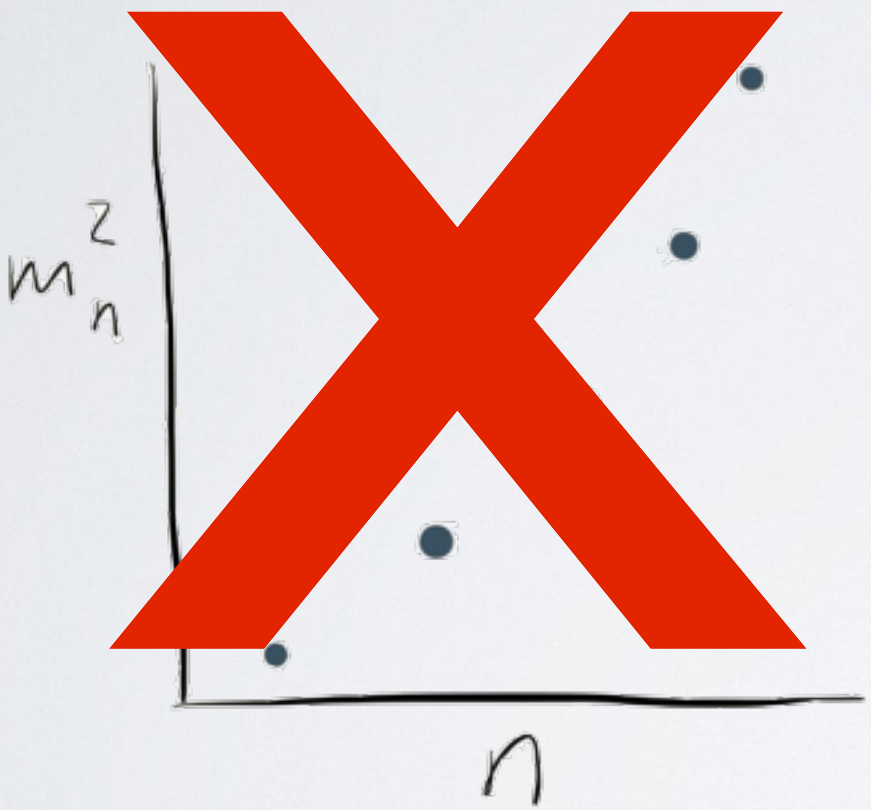
Quadratic

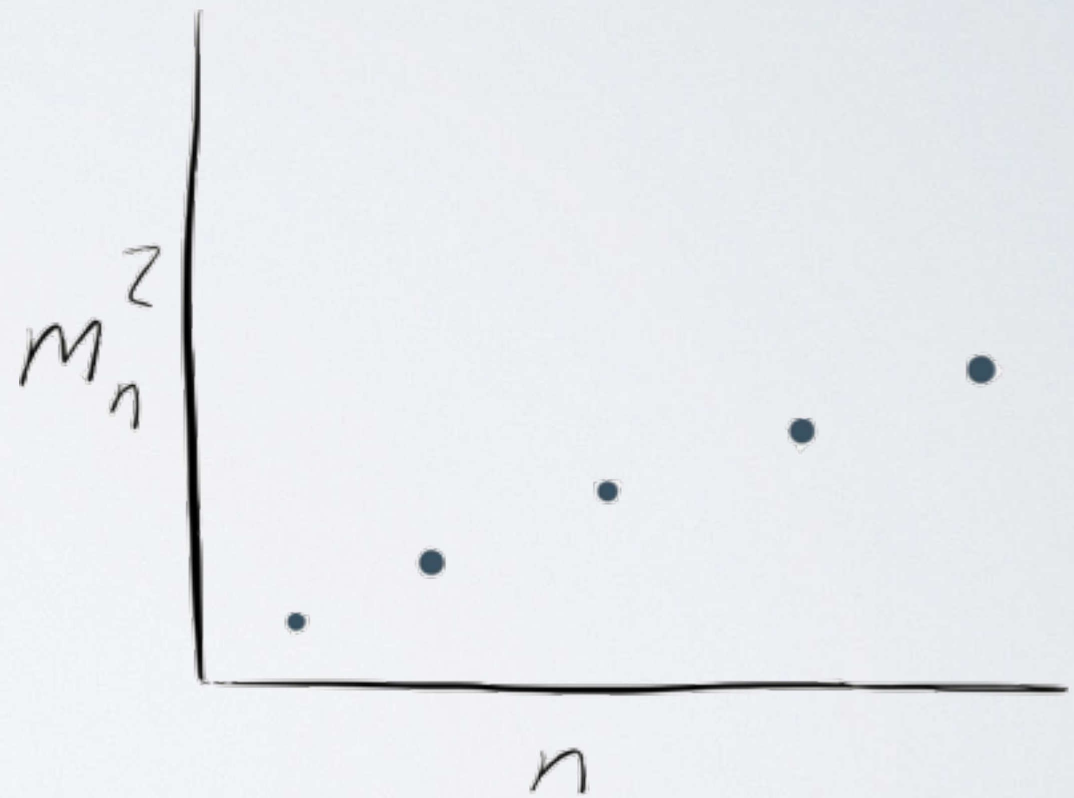


Linear

Confinement — Spectra

Soft Wall

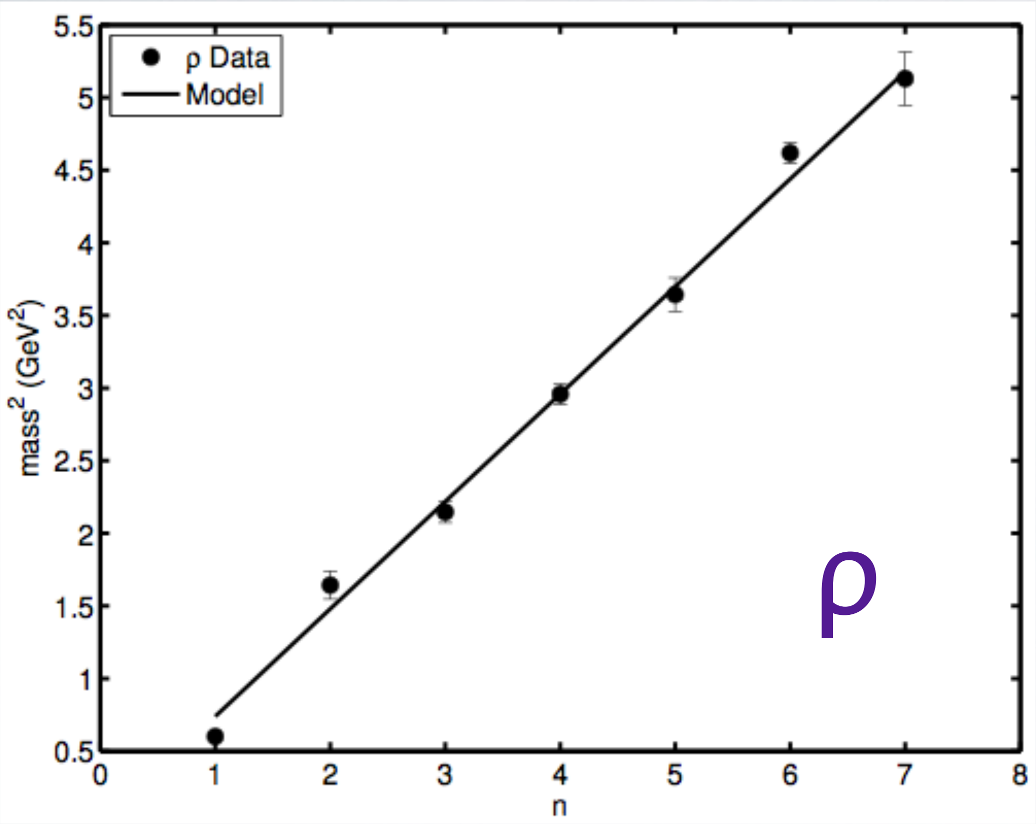


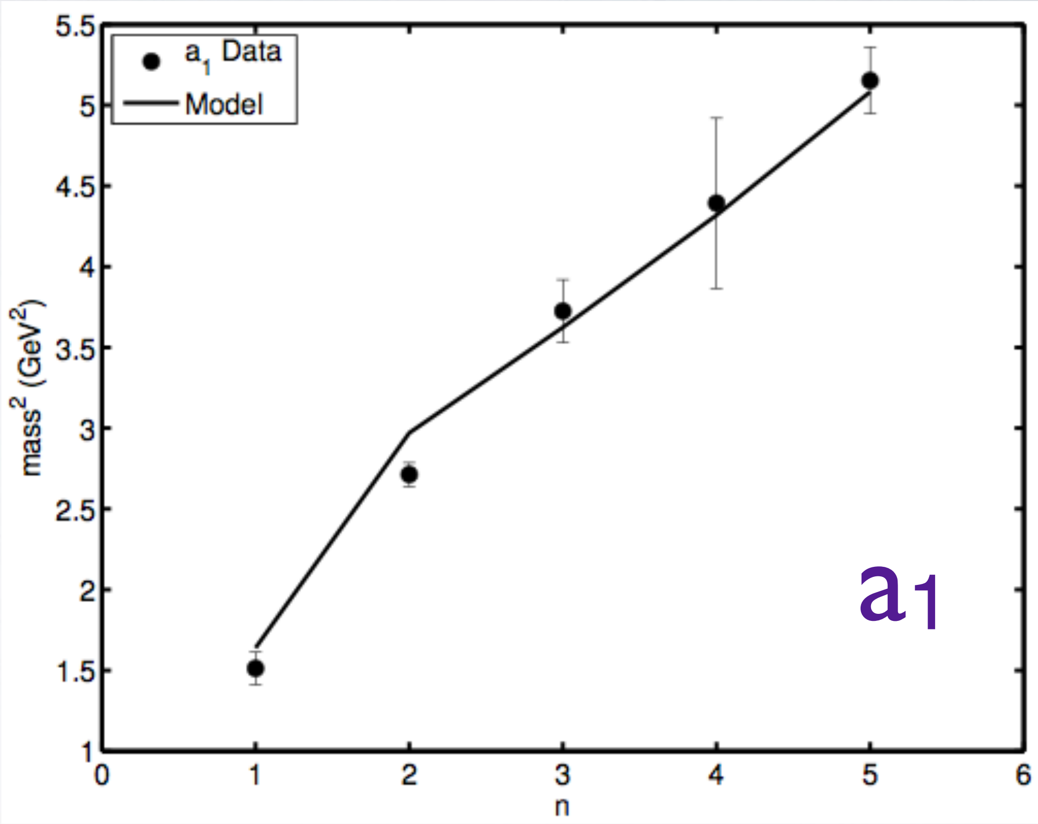


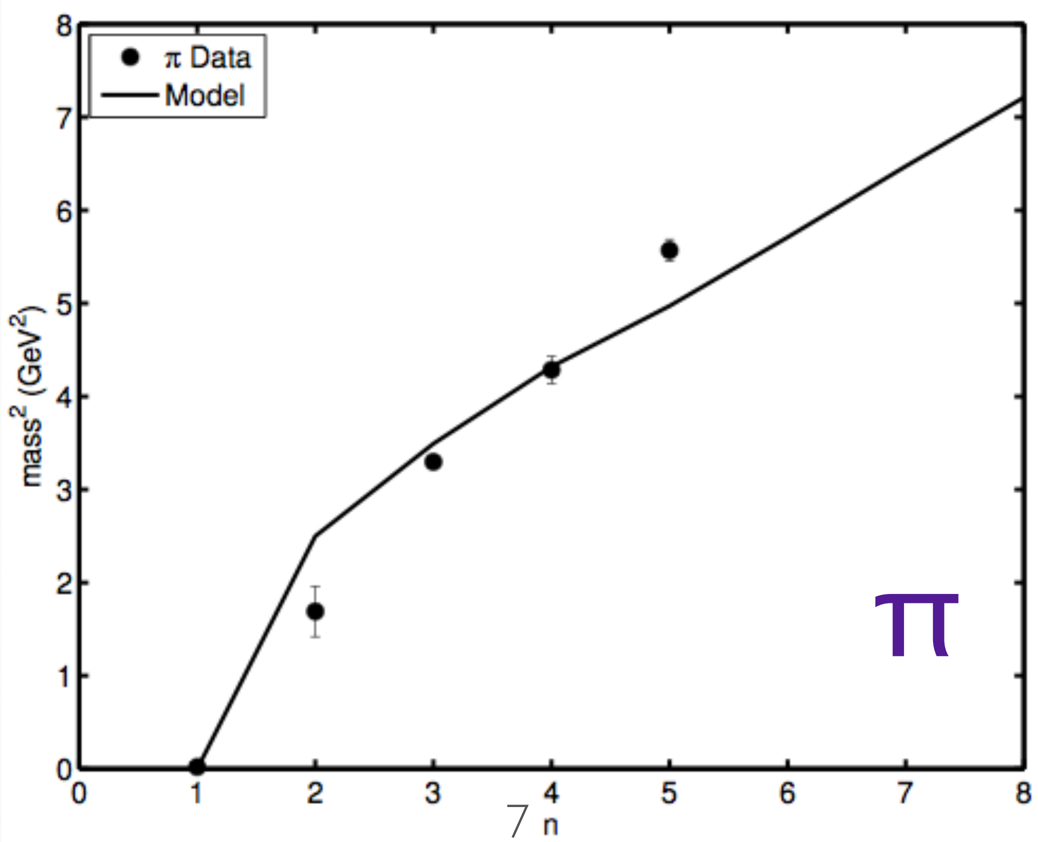
Quadratic

Linear

Zero Temperature







SB, Kapusta 2014

Confinement — Dilaton

$$\Phi = -\mu_1^2 z^2 + \left(\mu_0^2 + \mu_1^2\right) z^2 \left[1 - \exp\left(-\mu_2^2 z^2\right)\right]$$

Confinement — Dilaton

$$\Phi = -\mu_1^2 z^2 + \underline{\left(\mu_0^2 + \mu_1^2\right) z^2} \left[1 - \exp\left(-\mu_2^2 z^2\right)\right]$$

Quadratic at large z

Linear confinement

at zero temperature

Confinement — Dilaton

$$\Phi = -\underline{\mu_1^2 z^2} + \underline{\left(\mu_0^2 + \mu_1^2\right) z^2} \left[1 - \exp\left(-\mu_2^2 z^2\right)\right]$$

Quadratic at large z

Linear confinement at zero temperature

Negative quadratic at small z

Needed to solve for chiral condensate

Finite Temperature

- Black hole metric $ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z=z_h}$$

- Schwarzschild: $f(z) = 1 - \left(\frac{z}{z_h}\right)^4, \quad T = (\pi z_h)^{-1}$

Finite Temperature and Density

Reissner-Nordstrom metric

$$
\begin{aligned}
f(z) &= 1-\left(1+Q^2\right)\left(\frac{z}{z_h}\right)^4+Q^2\left(\frac{z}{z_h}\right)^6 \\
\mu &= \kappa\frac{Q}{z_h} \\
T &= \frac{1}{\pi z_h}\left(1-\frac{Q^2}{2}\right)
\end{aligned}
$$

Meson action

$$\mathcal{S} = \frac{1}{2k}\int d^5x\sqrt{-g}e^{-\Phi(z)}\ \mathrm{Tr}\left[|DX|^2 + V_m(X) + \frac{1}{2g_5^2}\left(F_A^2 + F_V^2\right)\right]$$

$$V(\chi) = \left\langle \mathrm{Tr}\, V_m(X)\right\rangle = \frac{m_5^2}{2}\chi^2 + v_4\chi^4$$

SB, Jacobson 2016

Meson action

$$\mathcal{S} = \frac{1}{2k}\int d^5x\sqrt{-g}e^{-\Phi(z)}\ \mathrm{Tr}\left[|DX|^2 + V_m(X) + \frac{1}{2g_5^2}\left(F_A^2 + F_V^2\right)\right]$$

Vector and Axial-Vector

$$V(\chi) = \left\langle \mathrm{Tr}\, V_m(X)\right\rangle = \frac{m_5^2}{2}\chi^2 + v_4\chi^4$$

SB, Jacobson 2016

Meson action

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Scalar

Vector and Axial-Vector

$$V(\chi) = \left\langle \mathrm{Tr}\, V_m(X)\right\rangle = \frac{m_5^2}{2}\chi^2 + v_4\chi^4$$

SB, Jacobson 2016

Meson action

$$\mathcal{S} = \frac{1}{2k}\int d^5x\sqrt{-g}e^{-\Phi(z)}\ \mathrm{Tr}\left[|DX|^2 + V_m(X) + \frac{1}{2g_5^2}\left(F_A^2 + F_V^2\right)\right]$$

Scalar

Vector and Axial-Vector

$$V(\chi) = \left\langle \mathrm{Tr}\, V_m(X) \right\rangle = \frac{m_5^2}{2}\chi^2 + v_4\chi^4$$

SB, Jacobson 2016

Chiral Field

- Solved dynamically

$$\chi''(u) - \left(\frac{4 - f(u) + uf(u)\Phi'(u)}{uf(u)}\right)\chi'(u) - \frac{1}{u^2 f(u)}\frac{\partial V}{\partial \chi} = 0$$

$$u = \frac{z}{z_h}$$

- AdS/CFT dictionary at small z

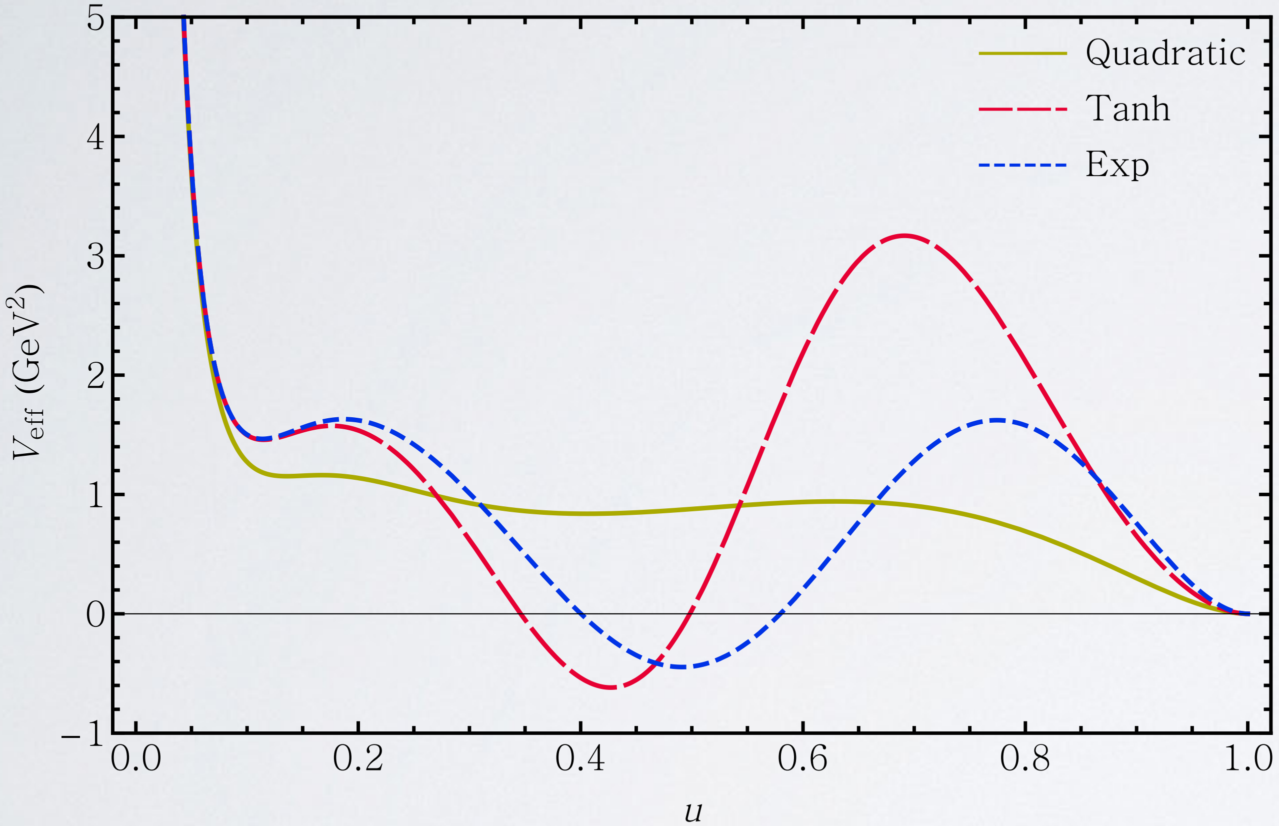
$$\chi(z \to 0) = m_q z + \sigma z^3$$

- Regular at horizon

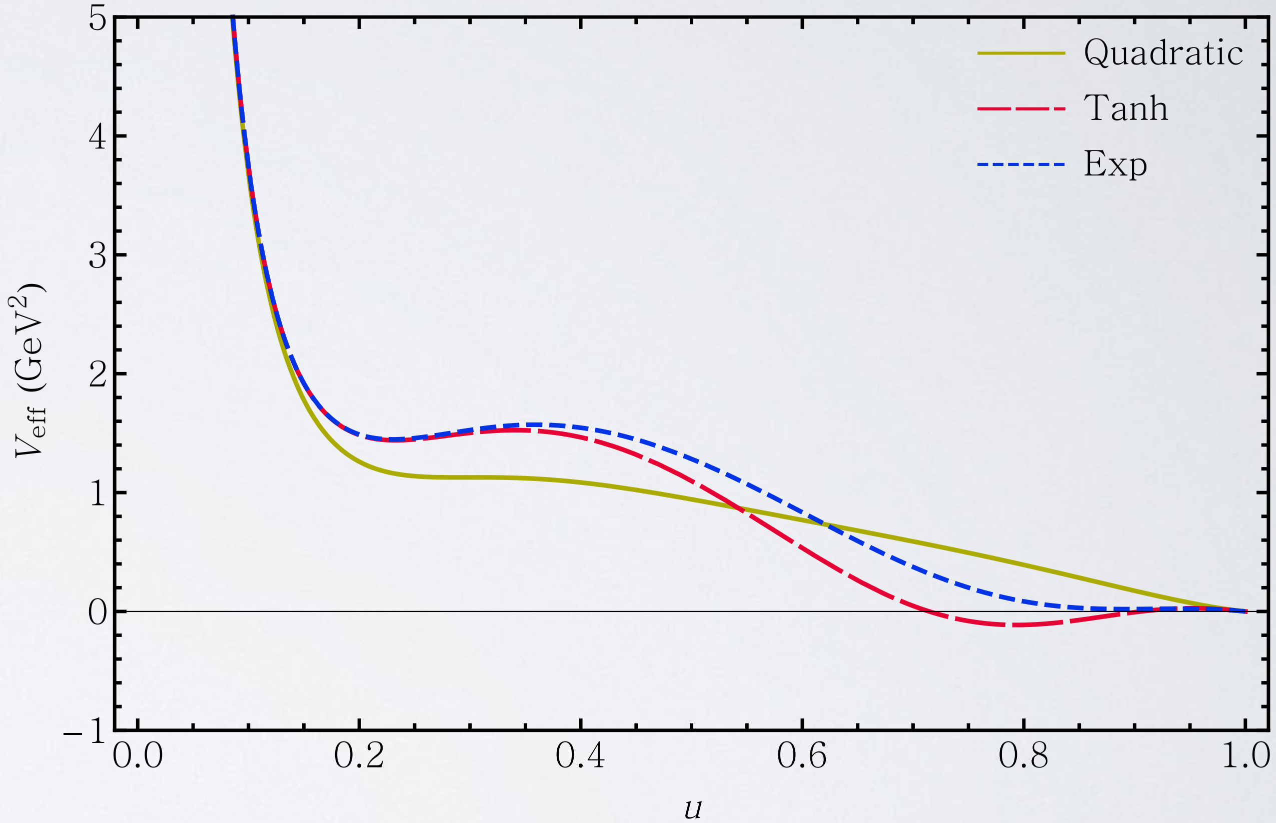
- No solution → $\sigma = 0$

Effective Potentials

T = 35 MeV



T = 70 MeV



- Visualization of bound states → no bound states
- Quadratic dilaton: linear trajectories, but no chiral condensate
- Exponential parameterization: best trajectories

Spectral Method

- On-shell action

$$\mathcal{S}_0 = -\lambda \int d^4x \sqrt{-g} e^{-\Phi} g^{zz} \, \tilde{S}_0(-q) \tilde{s}(q,z) \partial_z \tilde{s}(q,z) \tilde{S}_0(q) \Big|_{z=0}$$

- Differentiate to find Green's Function

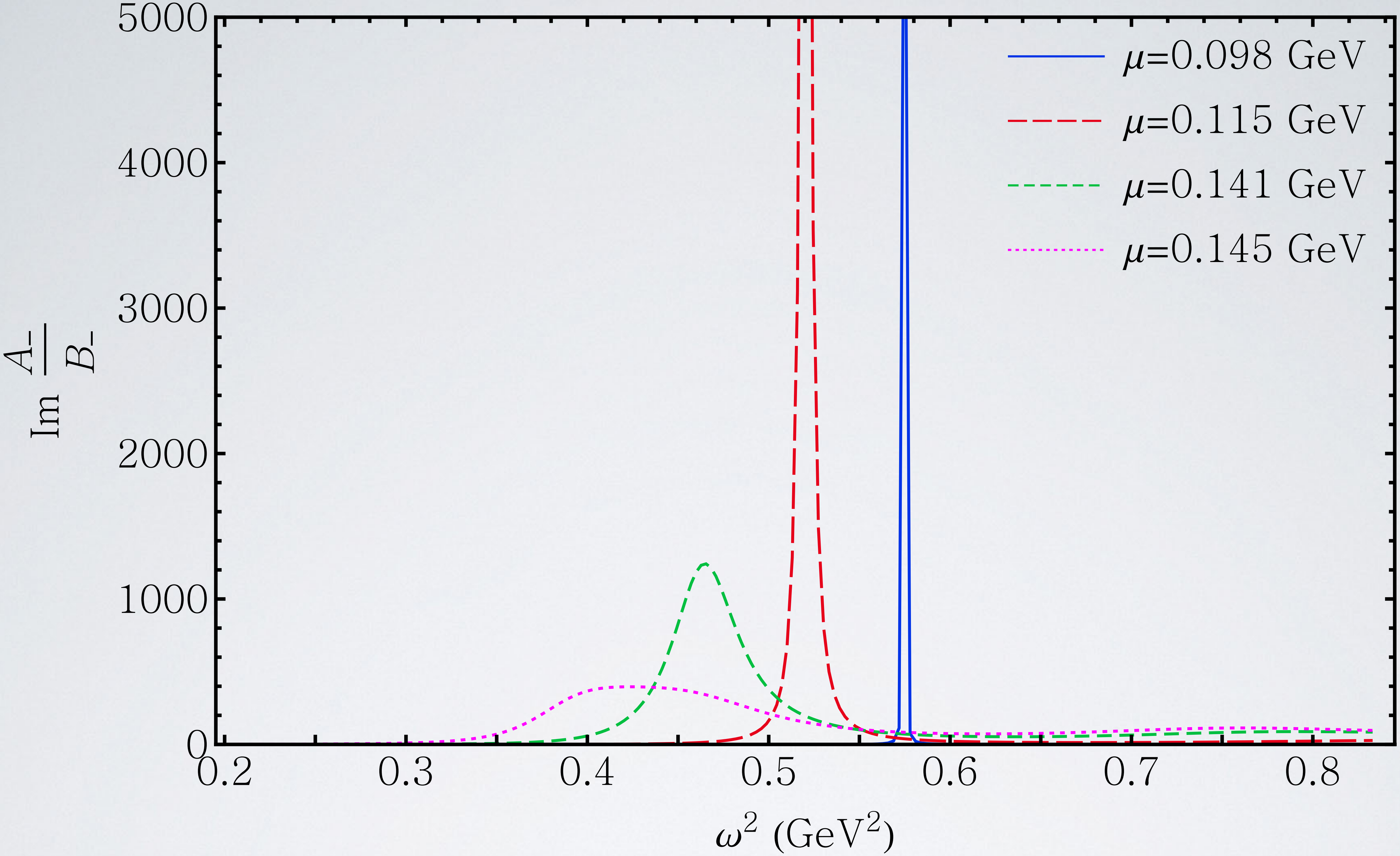
$$\Pi^R(\omega^2) = \sqrt{-g} e^{-\Phi} g^{zz} \tilde{s}(\omega^2, z) \partial_z \tilde{s}(\omega^2, z) \Big|_{z=0}$$

Spectral Method

- Solutions at horizon (in-falling or out-going waves)

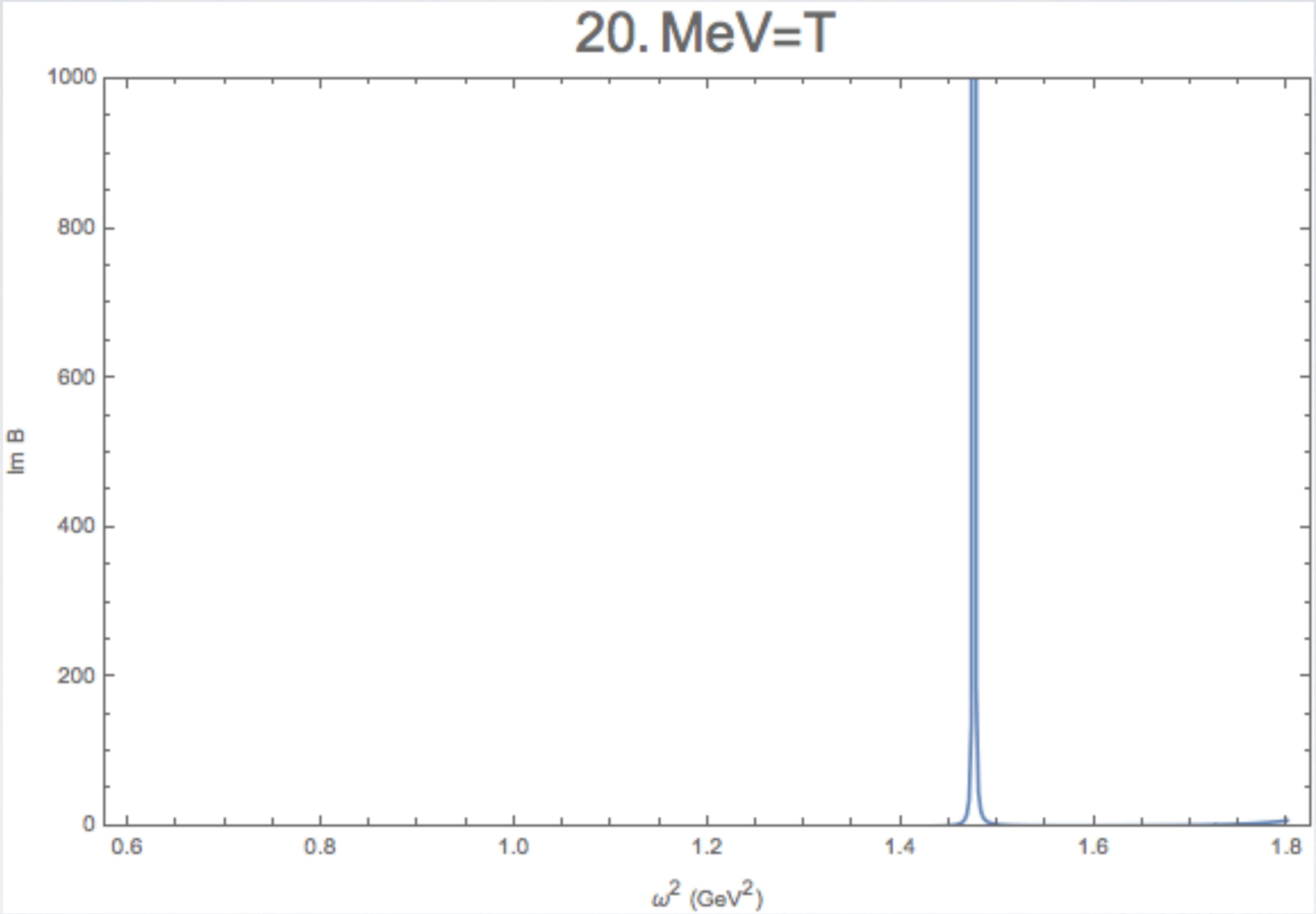
$$\psi_\pm = (1-u)^{\pm i \frac{\omega z_h}{4}}$$

- Solutions in UV (Method of Frobenius)
- Find values of **ω** that link normalizable solutions



Scalar meson melting

Spectral function for scalar mesons



Scalar meson melting

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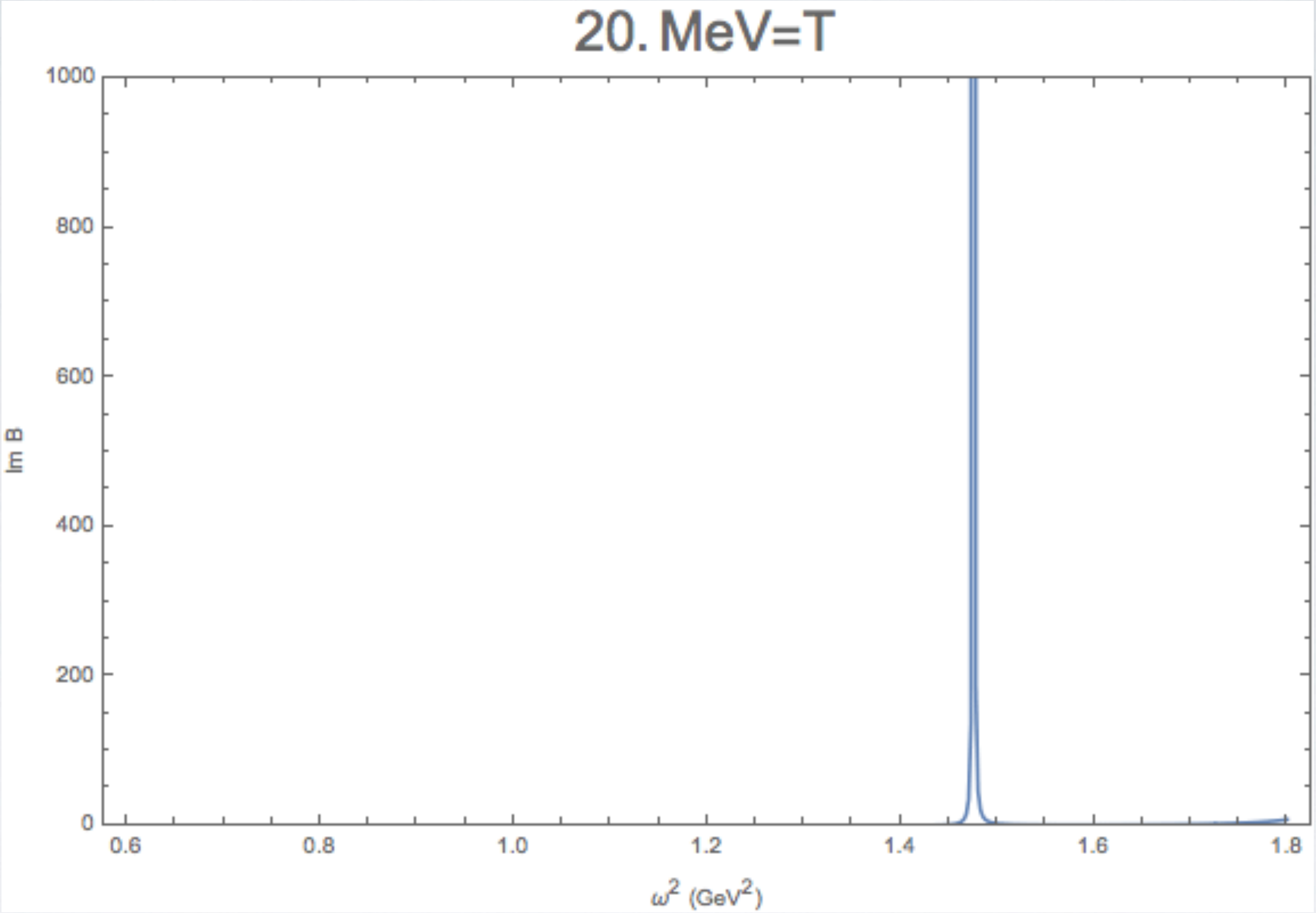
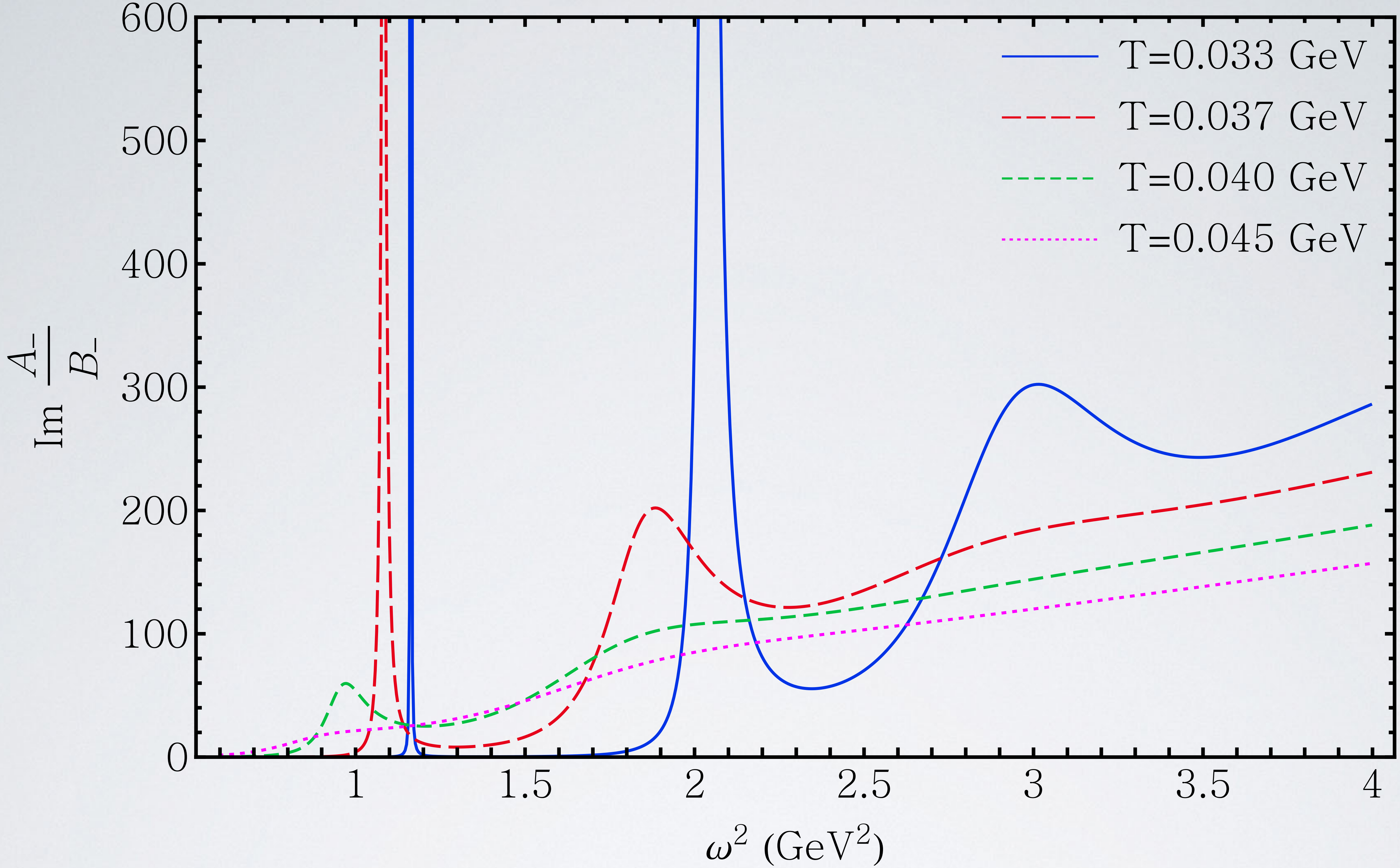


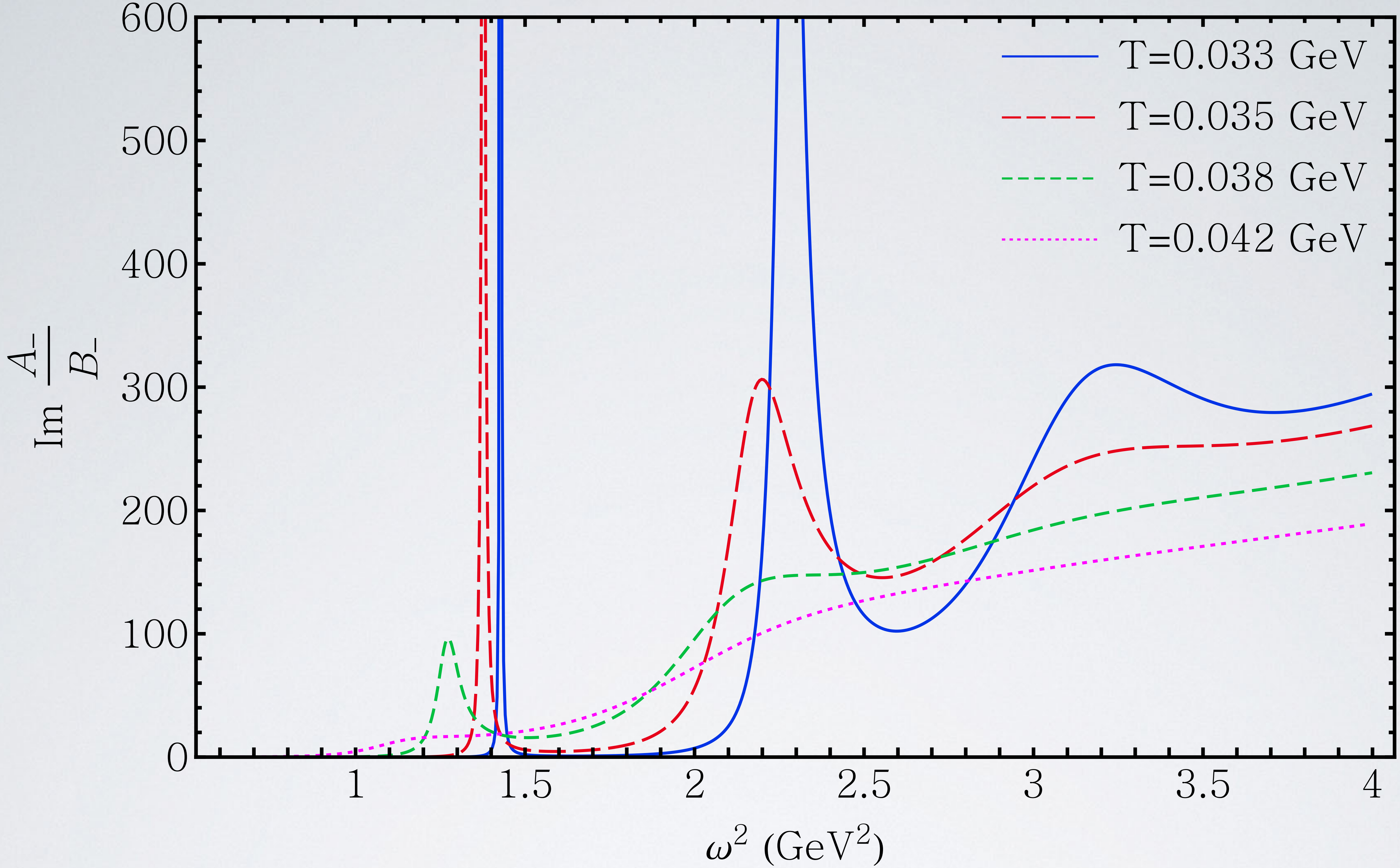
Scalar meson melting

Spectral function for scalar mesons



Vector meson melting

Spectral function for vector mesons

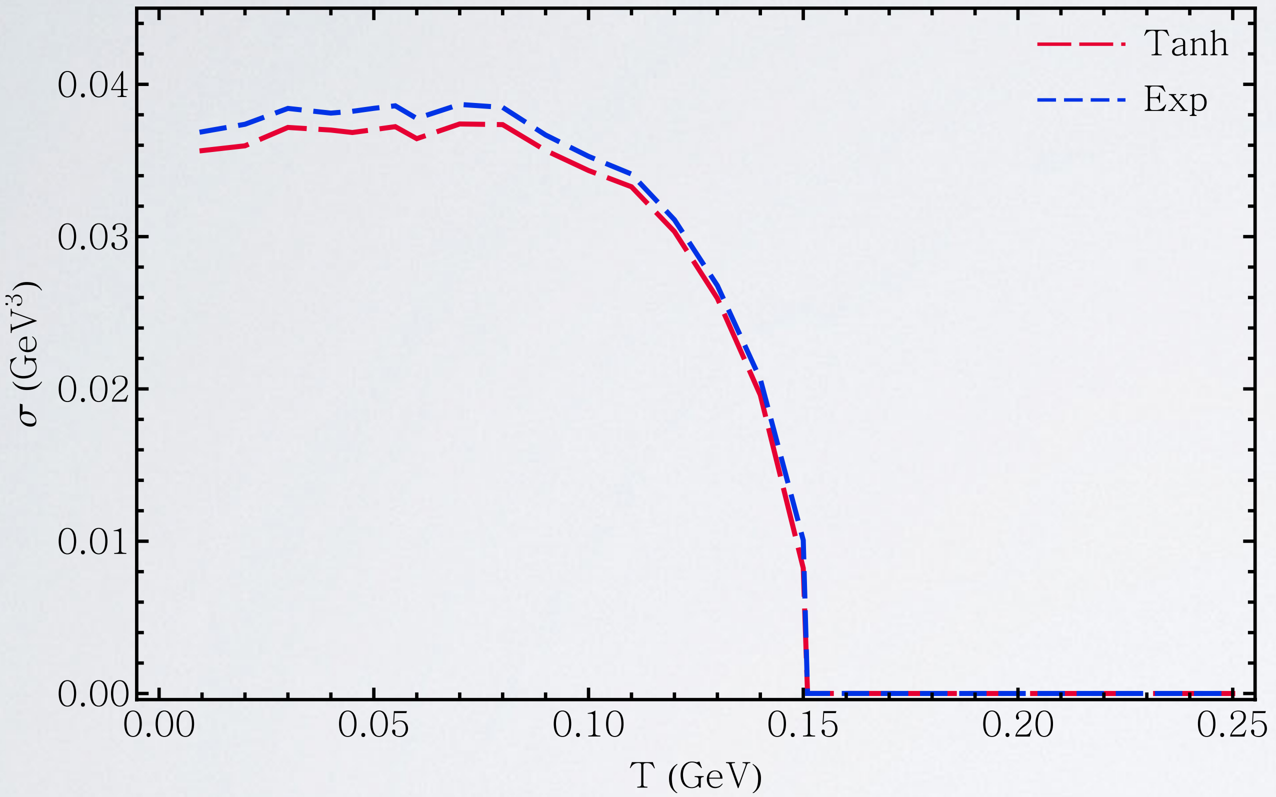


Axial meson melting

Spectral function for axial mesons

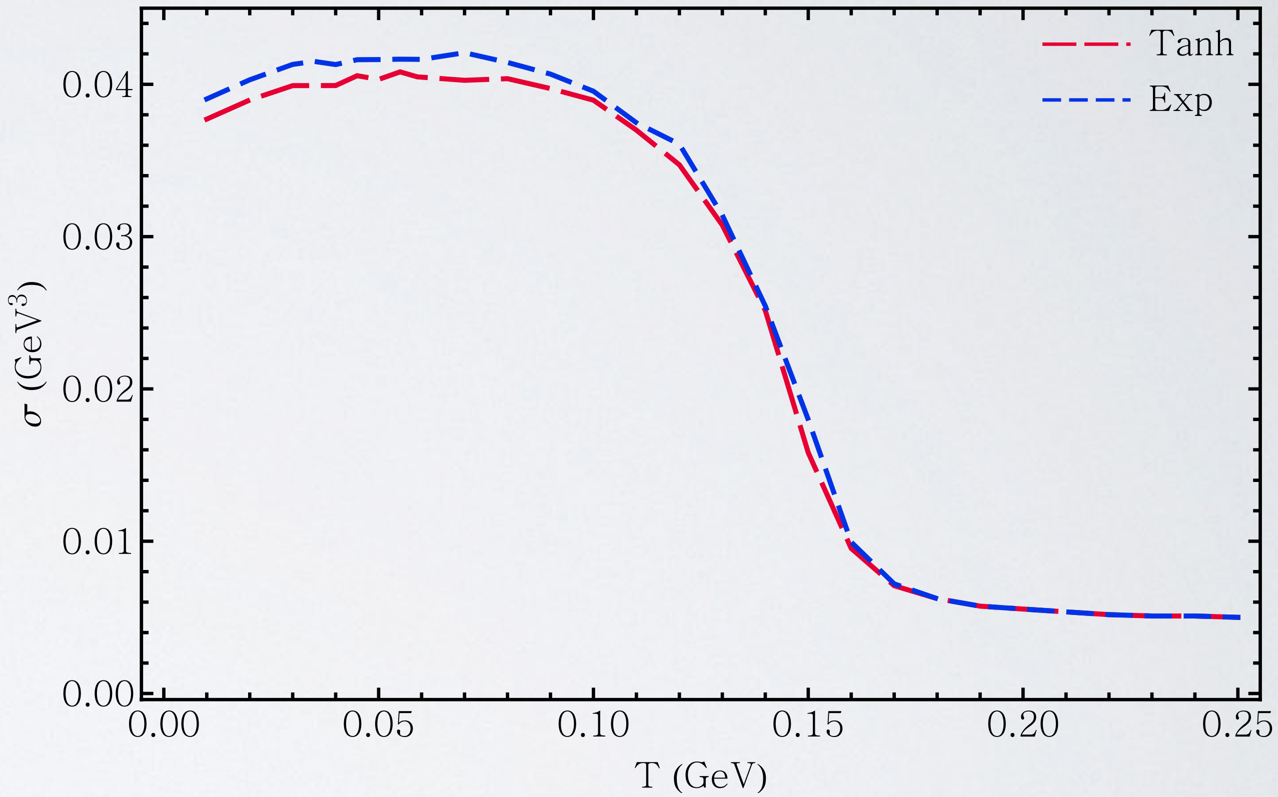
Chiral Transition — Temperature

Massless Quarks



- Second-order
- T_c = 155 MeV

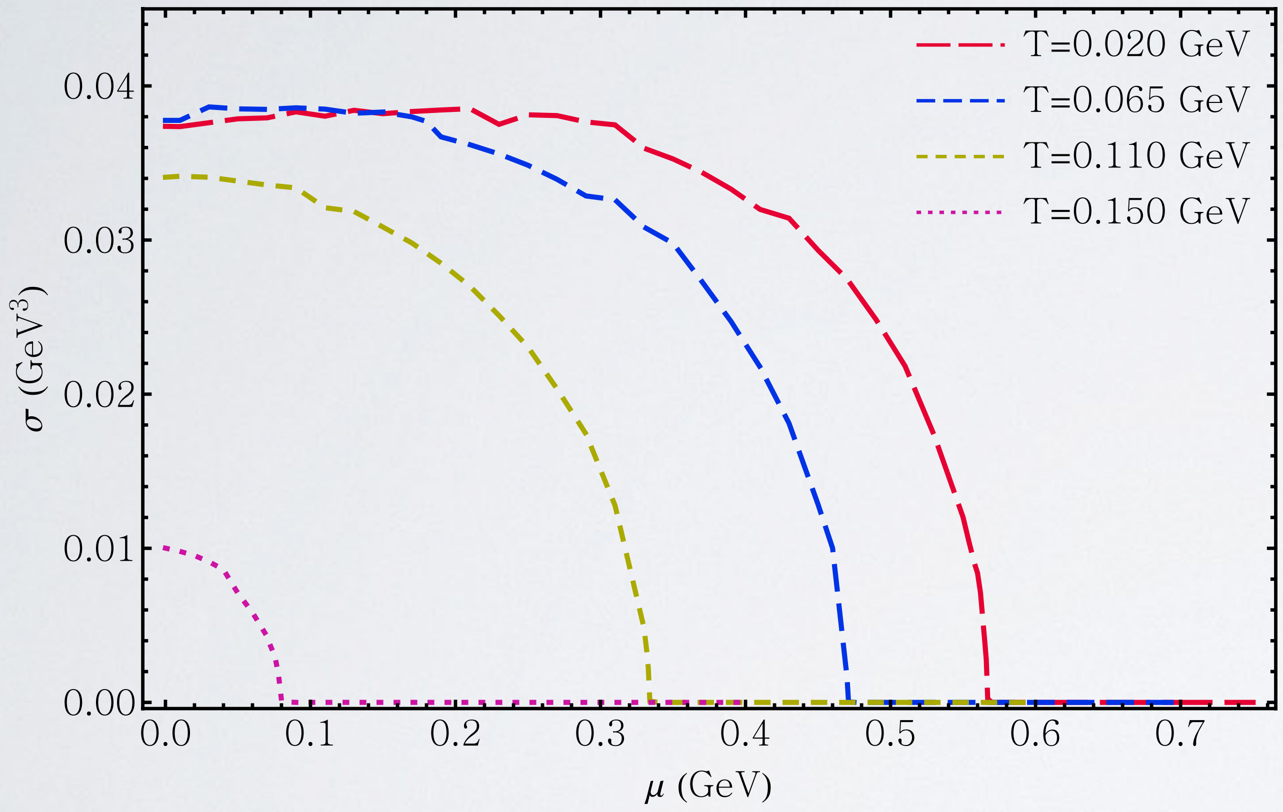
Physical Quark Mass



- Rapid crossover
- T_c ~ 151 MeV

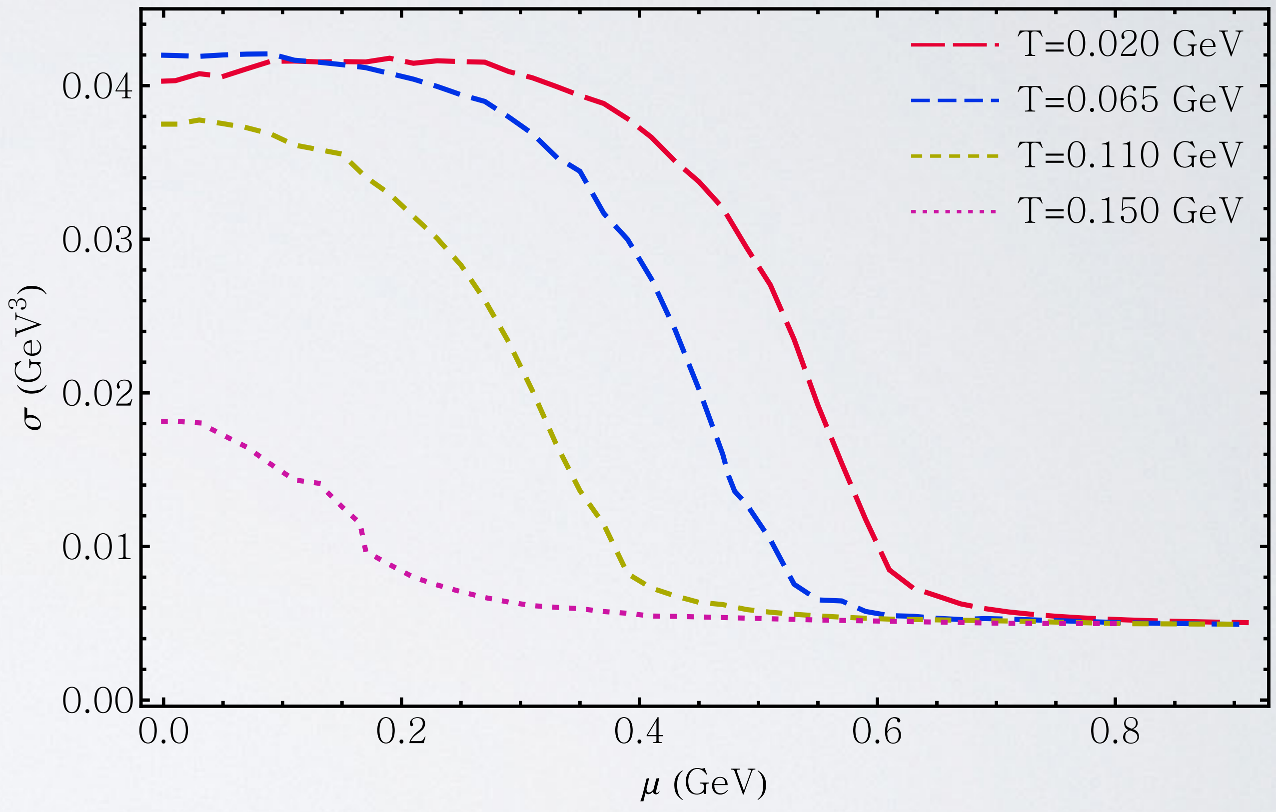
Chiral Transition — Baryon Chemical Potential

Massless Quarks



- Second-order
- μ_c = 566 MeV at T = 0

Physical Quark Mass



- Rapid crossover
- μ_c ~ 559 MeV at T = 0

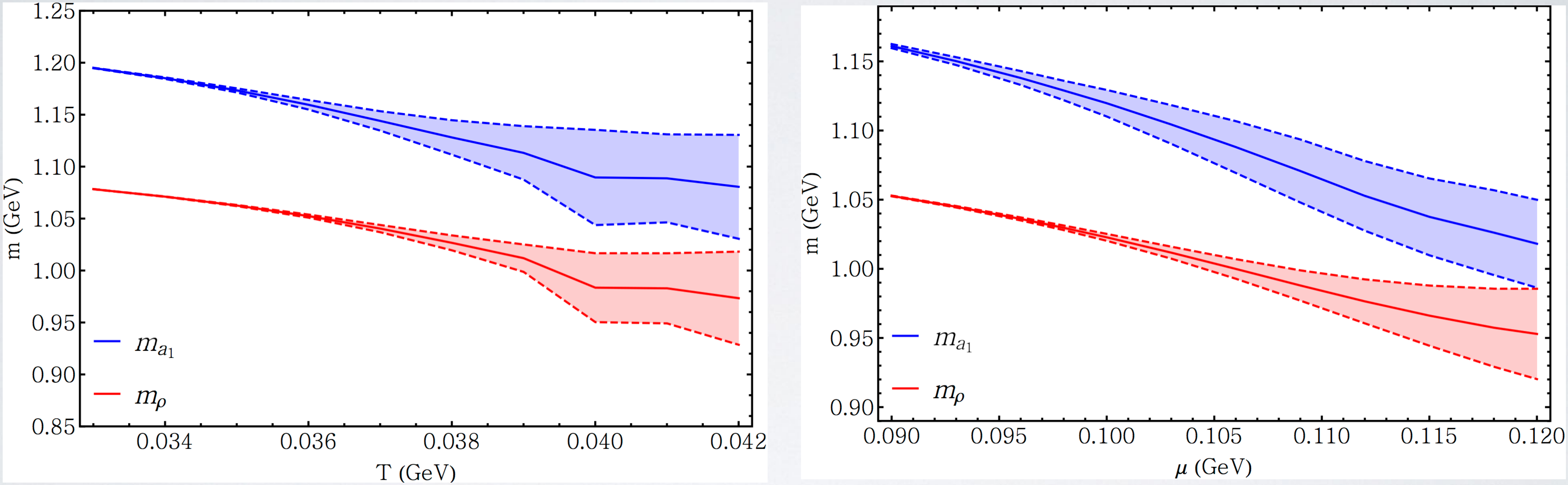
Critical Behavior

$$\frac{\sigma(T)}{\sigma_0} = 1 - \left(\frac{T}{T_c}\right)^{\alpha}, \qquad \frac{\sigma(\mu)}{\sigma_0} = 1 - \left(\frac{\mu}{\mu_c}\right)^{\beta}$$

$$\alpha = 7.3 \pm 0.9, \quad \beta = 7.8 \pm 0.6$$

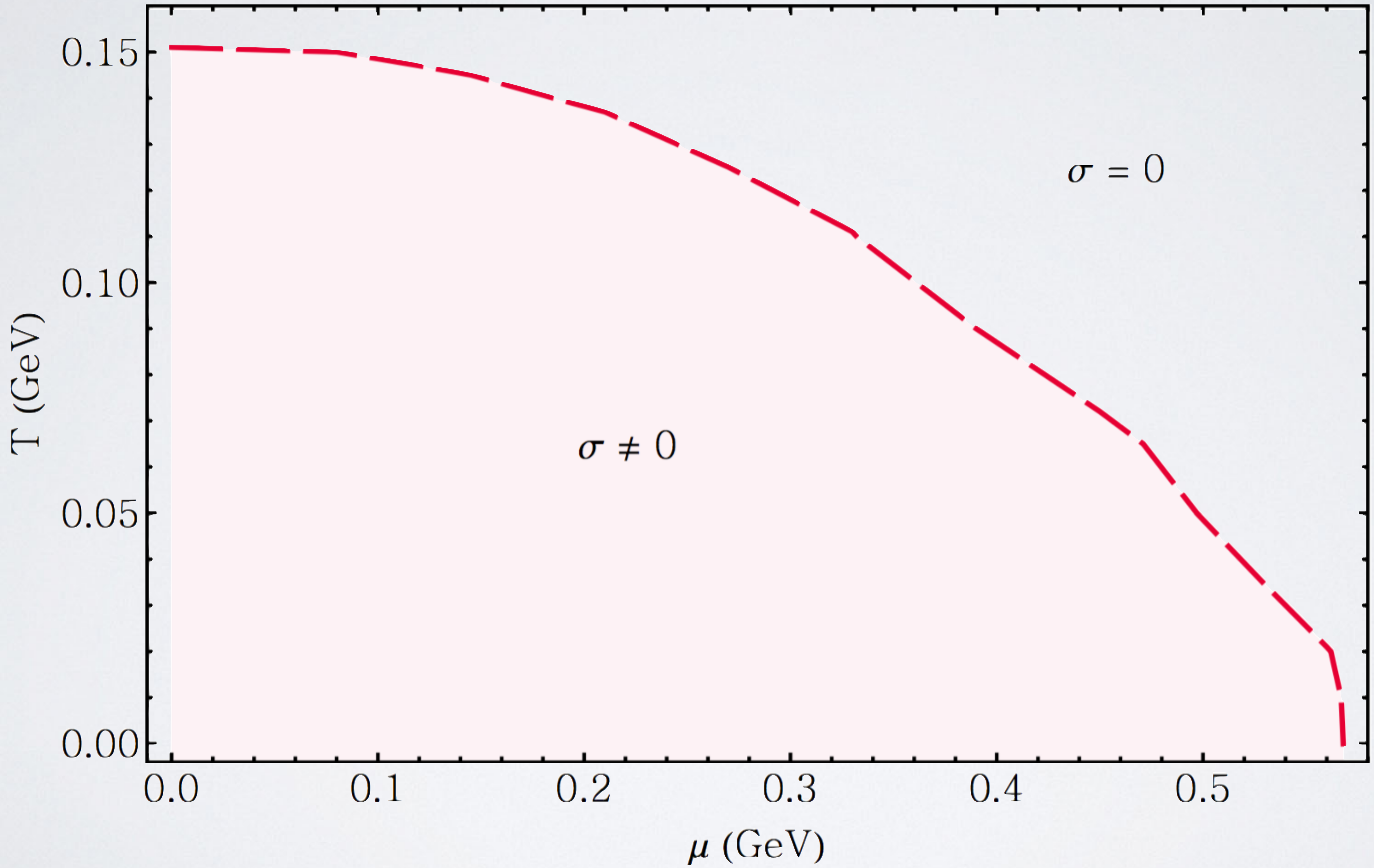
Suggests universal critical behavior

Vector-Axial Splitting

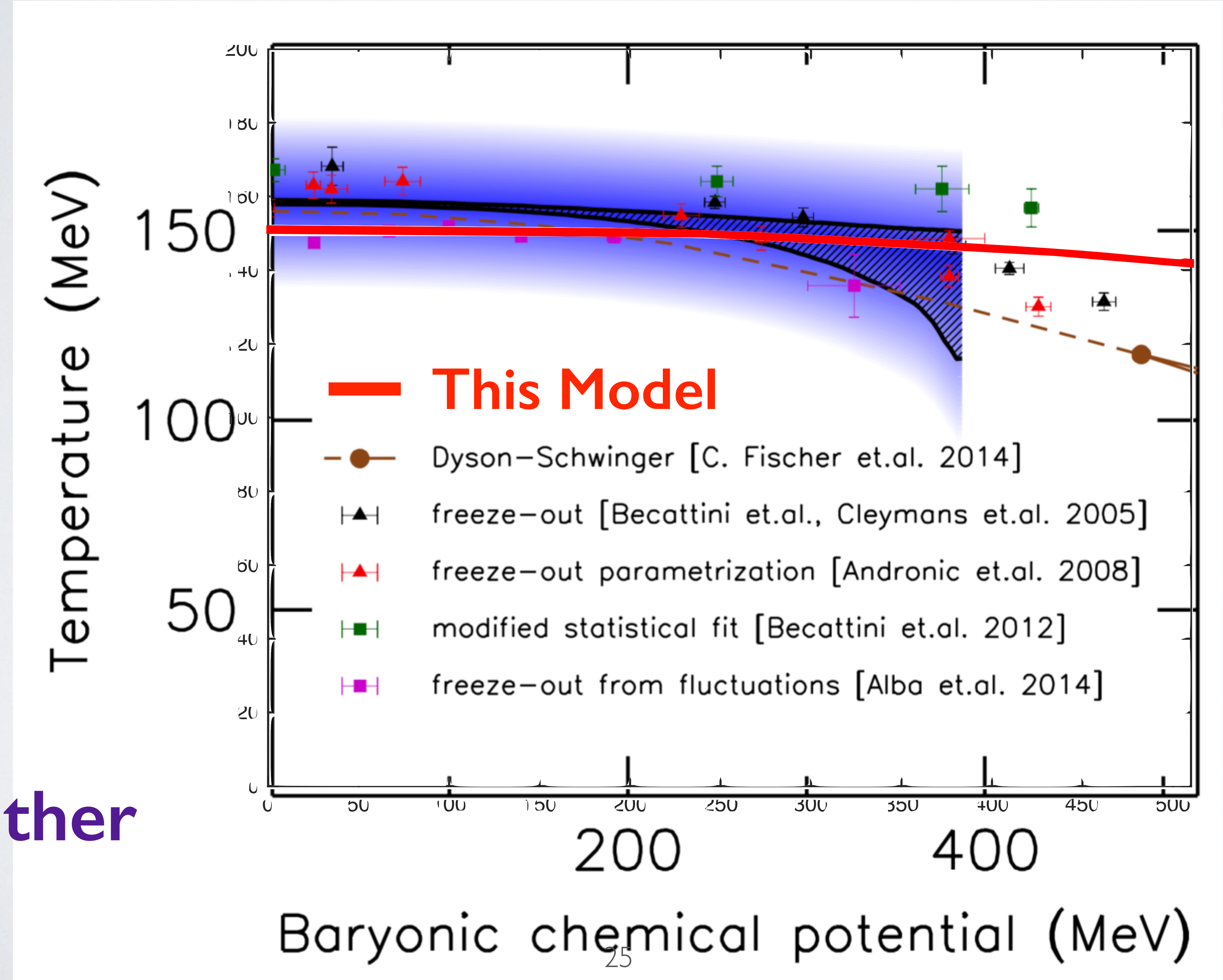


- Gap roughly constant until melting
- Physical m_q shown

Chiral Phase Diagram



Comparison to Lattice



J Günther

2+1 Flavors

- Vacuum expectation value becomes

$$\langle X \rangle = \begin{pmatrix} \frac{\chi_l(z)}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\chi_l(z)}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{\chi_s(z)}{\sqrt{2}} \end{pmatrix}$$

- With scalar potential

$$V(\chi) = \langle \mathrm{Tr}[V_m(X)] \rangle = m_5^2 \left(\chi_l^2 + \frac{1}{2}\chi_s^2 \right) + 3v_3\chi_l^2\chi_s + v_4(2\chi_l^4 + \chi_s^4)$$

2+1 Flavors

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$$\langle X \rangle = \begin{pmatrix} \frac{\chi_l(z)}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\chi_l(z)}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{\chi_s(z)}{\sqrt{2}} \end{pmatrix}$$

- With scalar potential

Mixing

$$V(\chi) = \langle \mathrm{Tr}[V_m(X)] \rangle = m_5^2 \left(\chi_l^2 + \frac{1}{2}\chi_s^2 \right) + \underline{3v_3 \chi_l^2 \chi_s} + v_4(2\chi_l^4 + \chi_s^4)$$

Flavor Asymmetric Case

- Simultaneous solution for condensates

$$\chi_l'' - \left(\frac{3f(u) - uf'(u) + uf(u)\Phi'(u)}{uf(u)}\right)\chi_l' + \frac{1}{u^2 f(u)}\left(3\chi_l - 3v_3\chi_l\chi_s - 4v_4\chi_l^3\right) = 0$$

$$\chi_s'' - \left(\frac{3f(u) - uf'(u) + uf(u)\Phi'(u)}{uf(u)}\right)\chi_s' + \frac{1}{u^2 f(u)}\left(3\chi_s - 3v_3\chi_l^2 - 4v_4\chi_s^3\right) = 0$$

- Shooting method

Flavor Asymmetric Case

- Simultaneous solution for condensates

$$\chi_l'' - \left(\frac{3f(u) - uf'(u) + uf(u)\Phi'(u)}{uf(u)}\right)\chi_l' + \frac{1}{u^2 f(u)}\left(3\chi_l - \underline{3v_3\chi_l\chi_s} - 4v_4\chi_l^3\right) = 0$$

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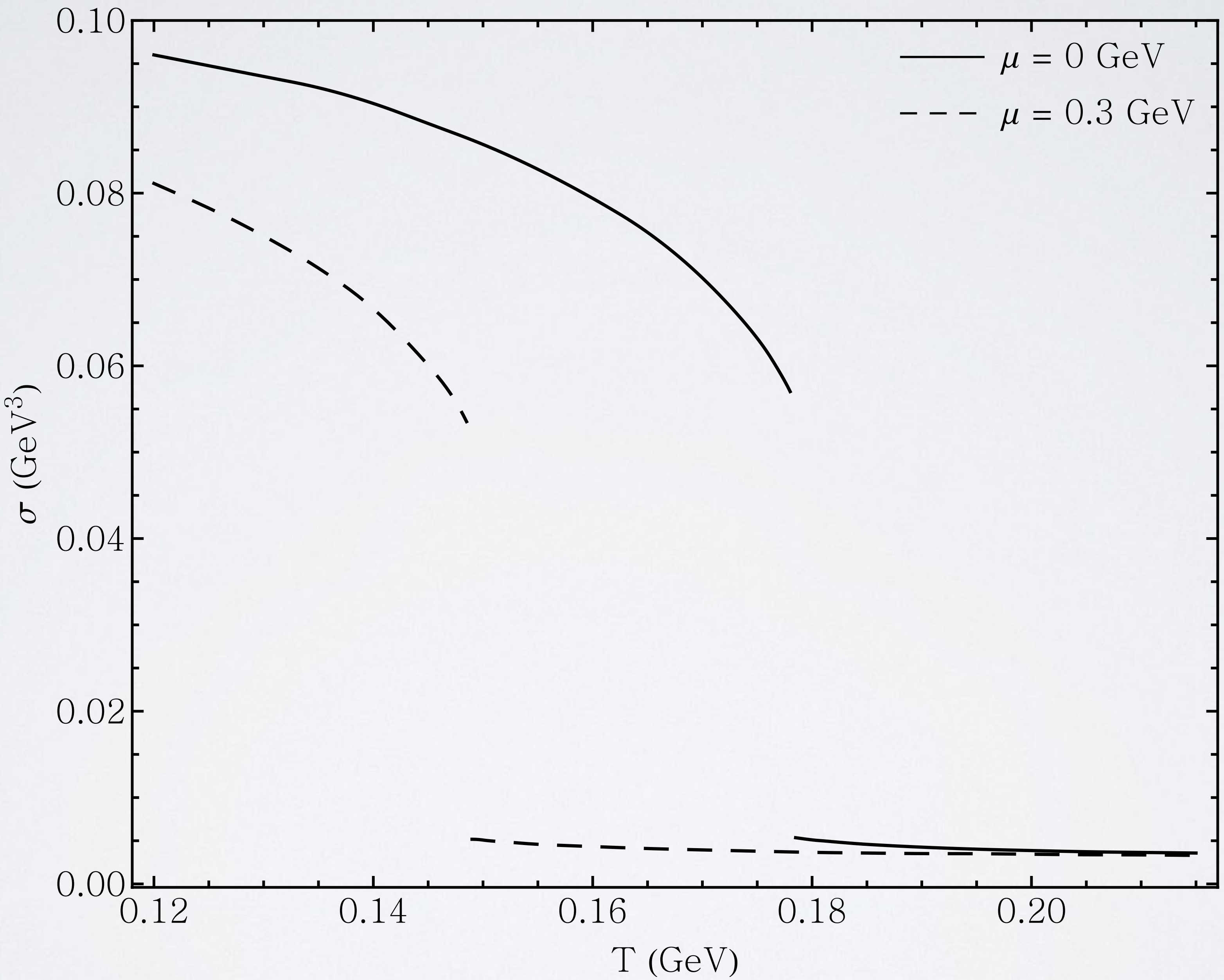
Mixing

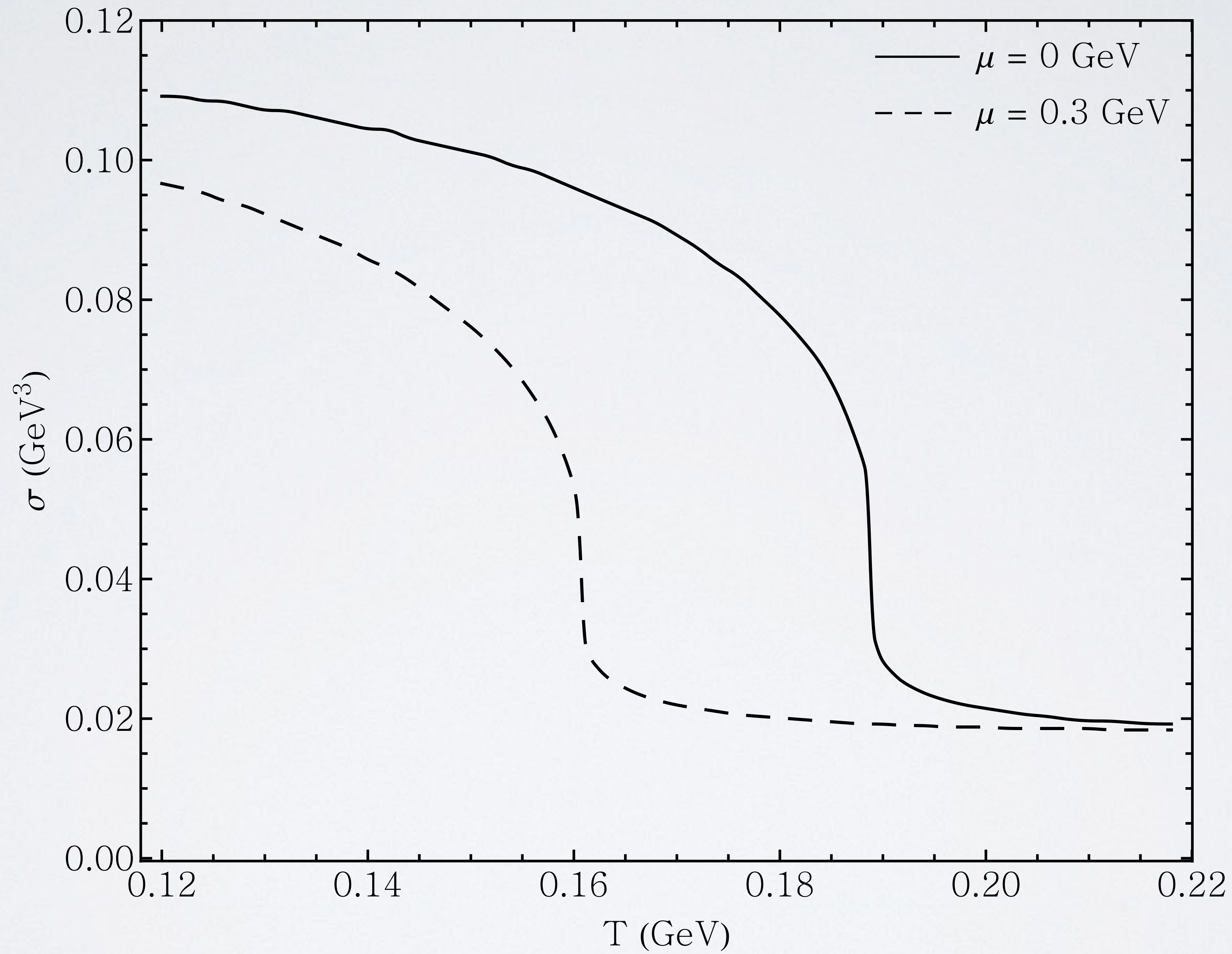
- Shooting method

1st-Order (Flavor Symmetric)



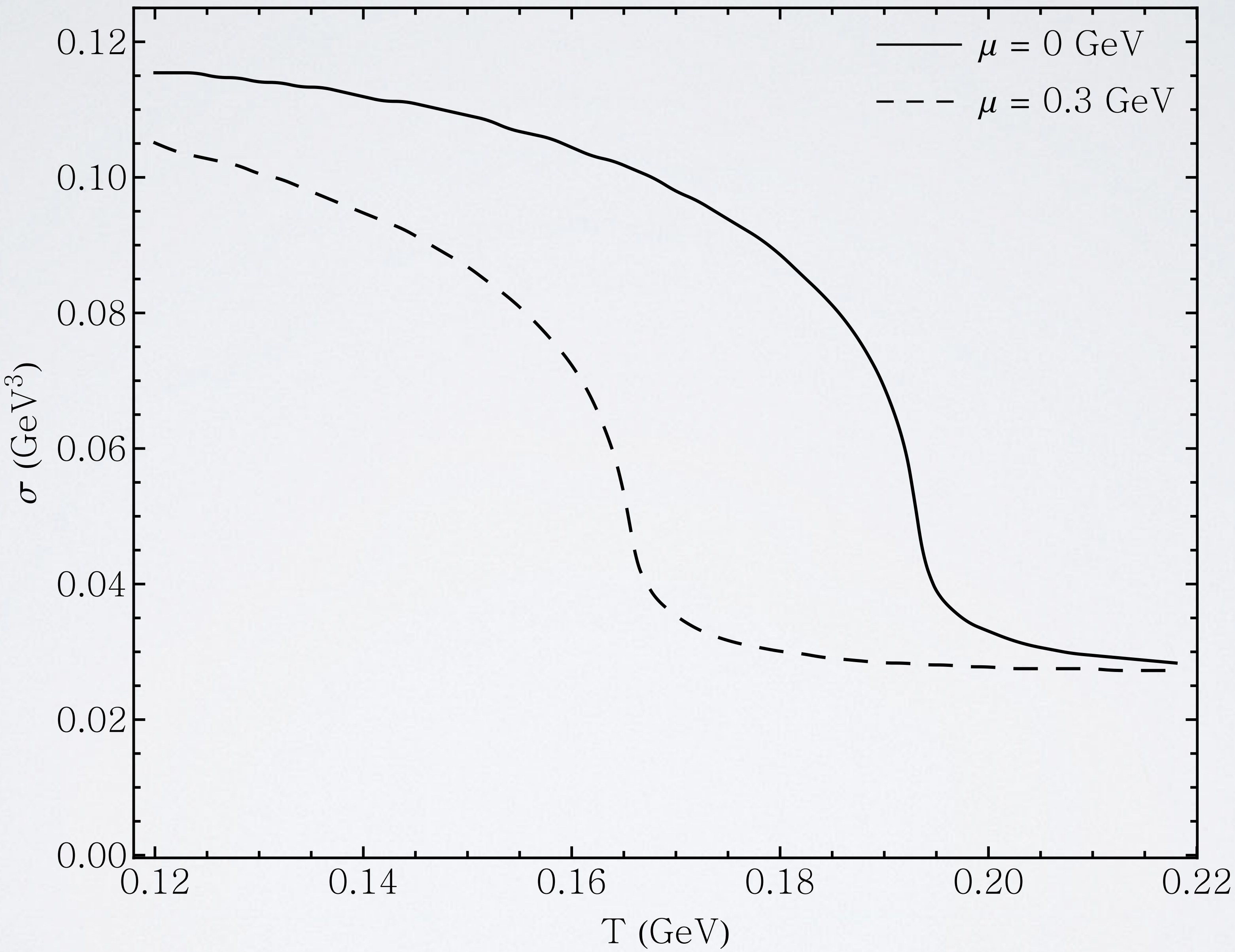
$$m_l = m_s = 10\,\mathrm{MeV}$$

2nd-Order (Flavor Symmetric)



$$m_l = m_s = 35\,\text{MeV}$$

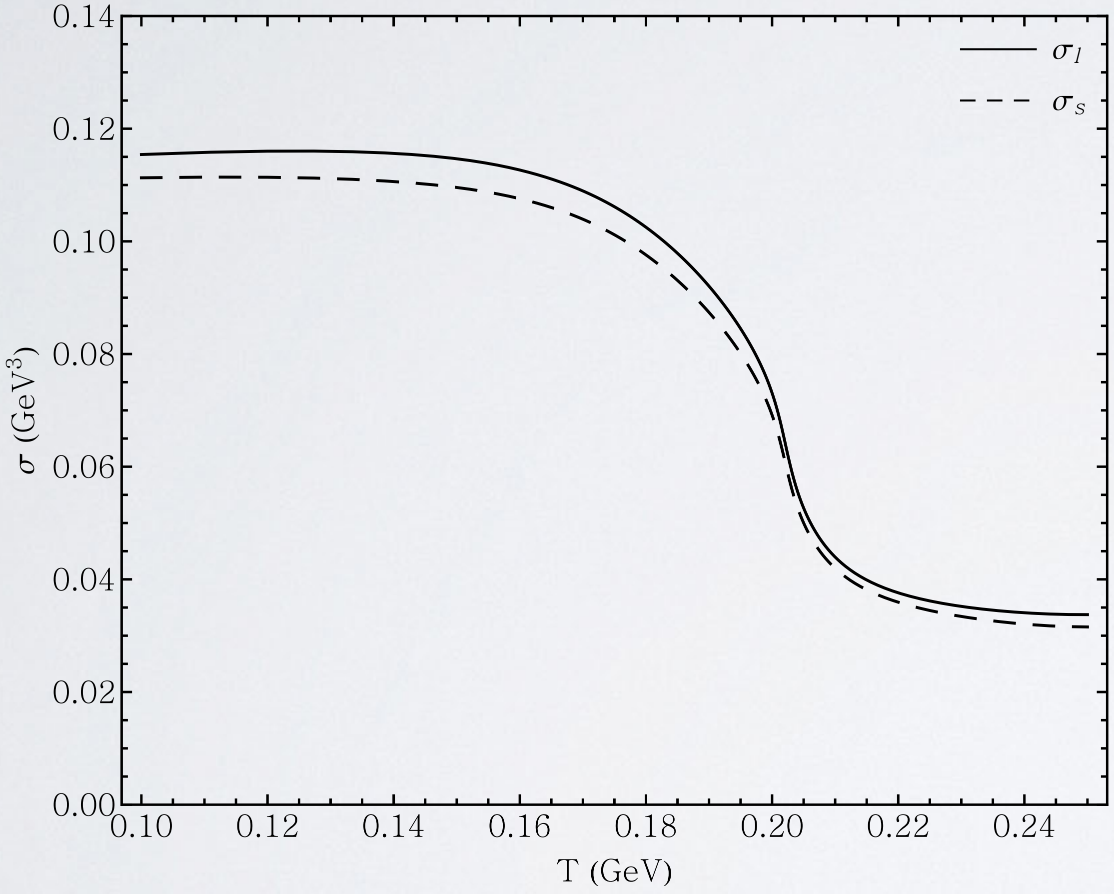
Crossover (Flavor-Symmetric)

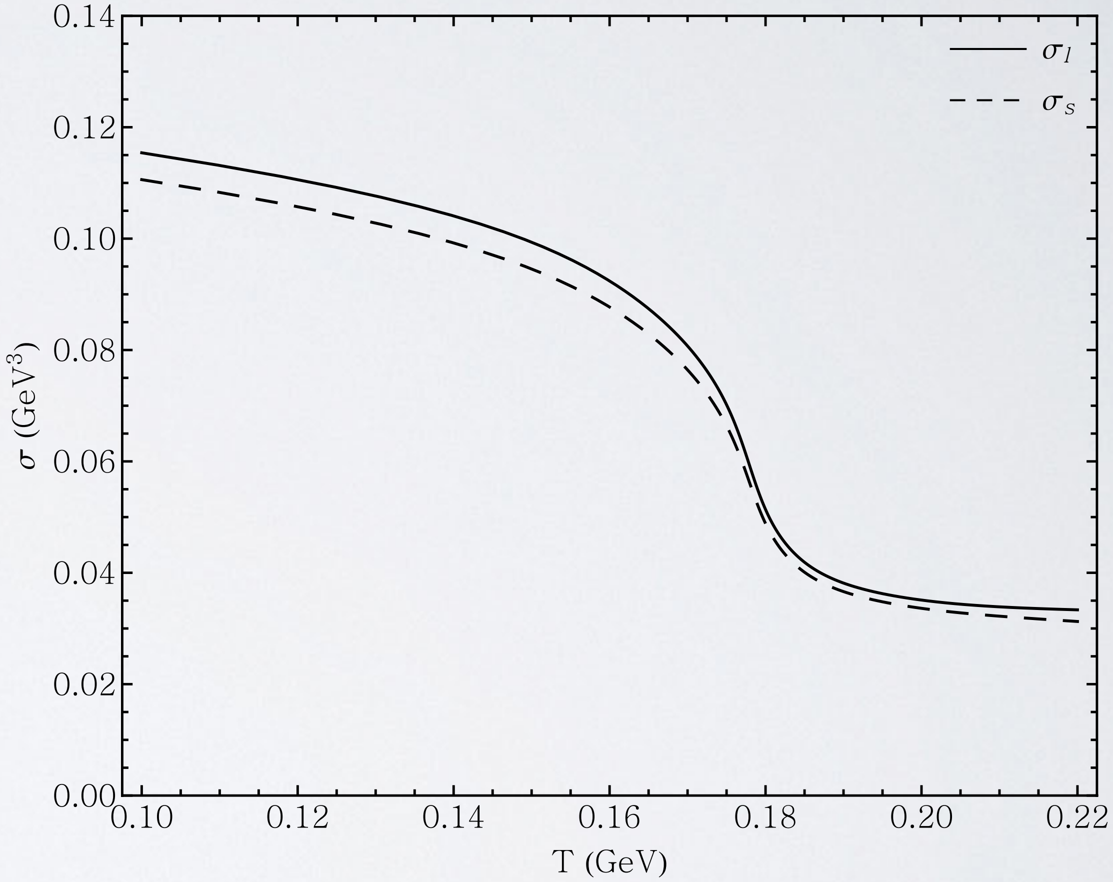


$$m_l = m_s = 45\,\mathrm{MeV}$$

Flavor Asymmetric Case

$$m_l = 40,\ \mathrm{MeV}\ m_s = 70\,\mathrm{MeV}$$

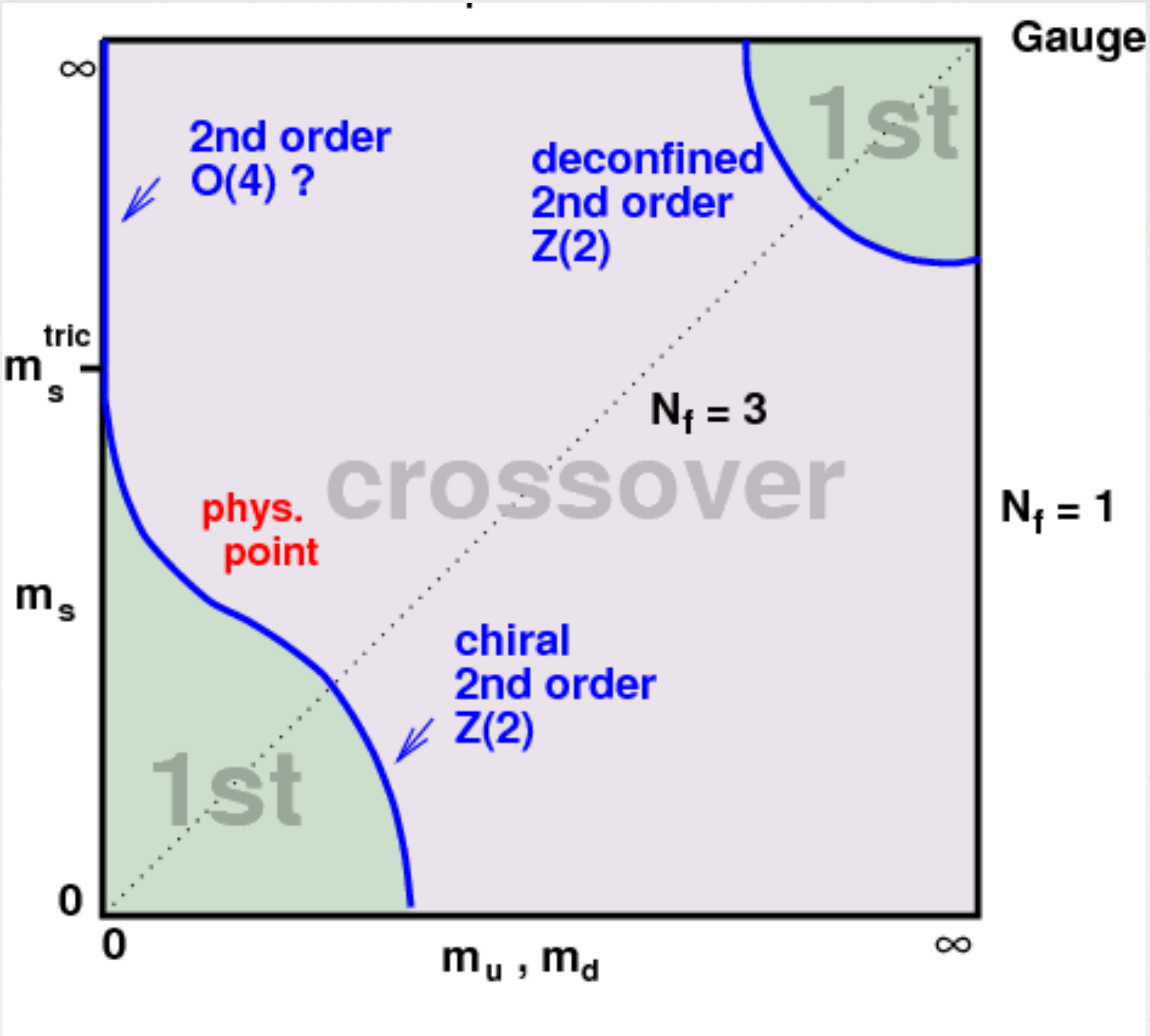




μ = 0

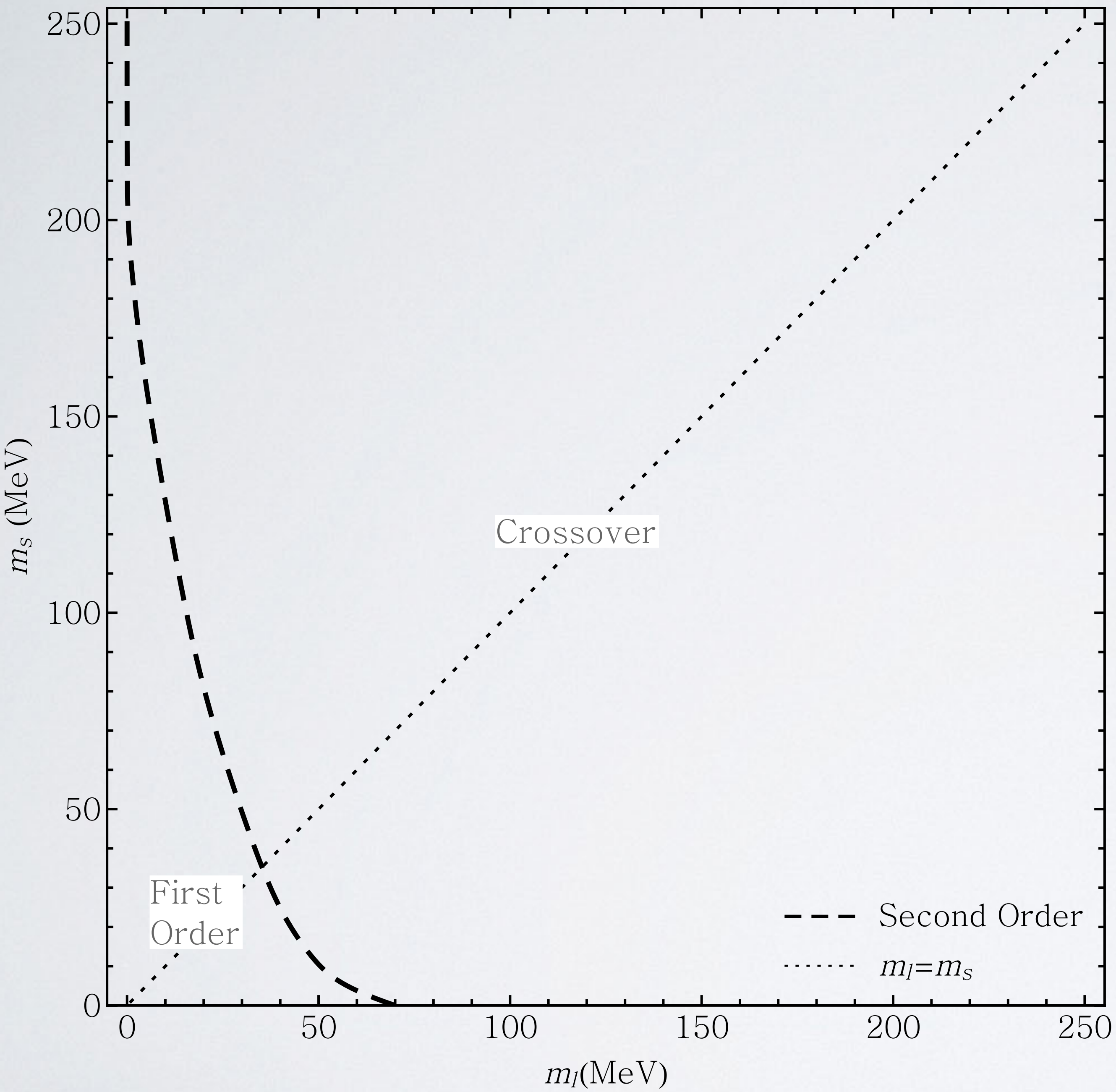
μ = 300 MeV

2+1 Flavors

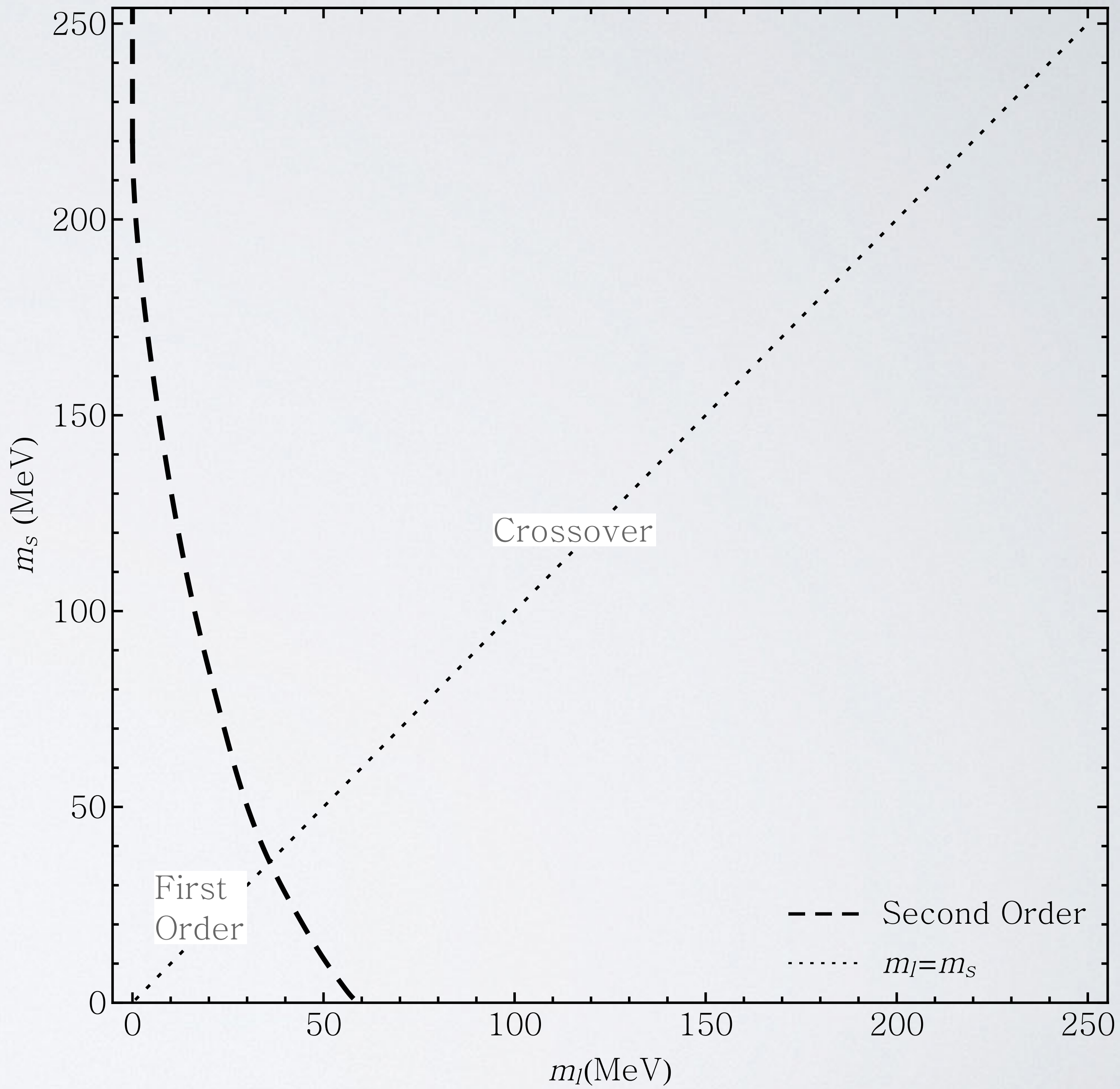


Columbia Plot

Columbia Plot



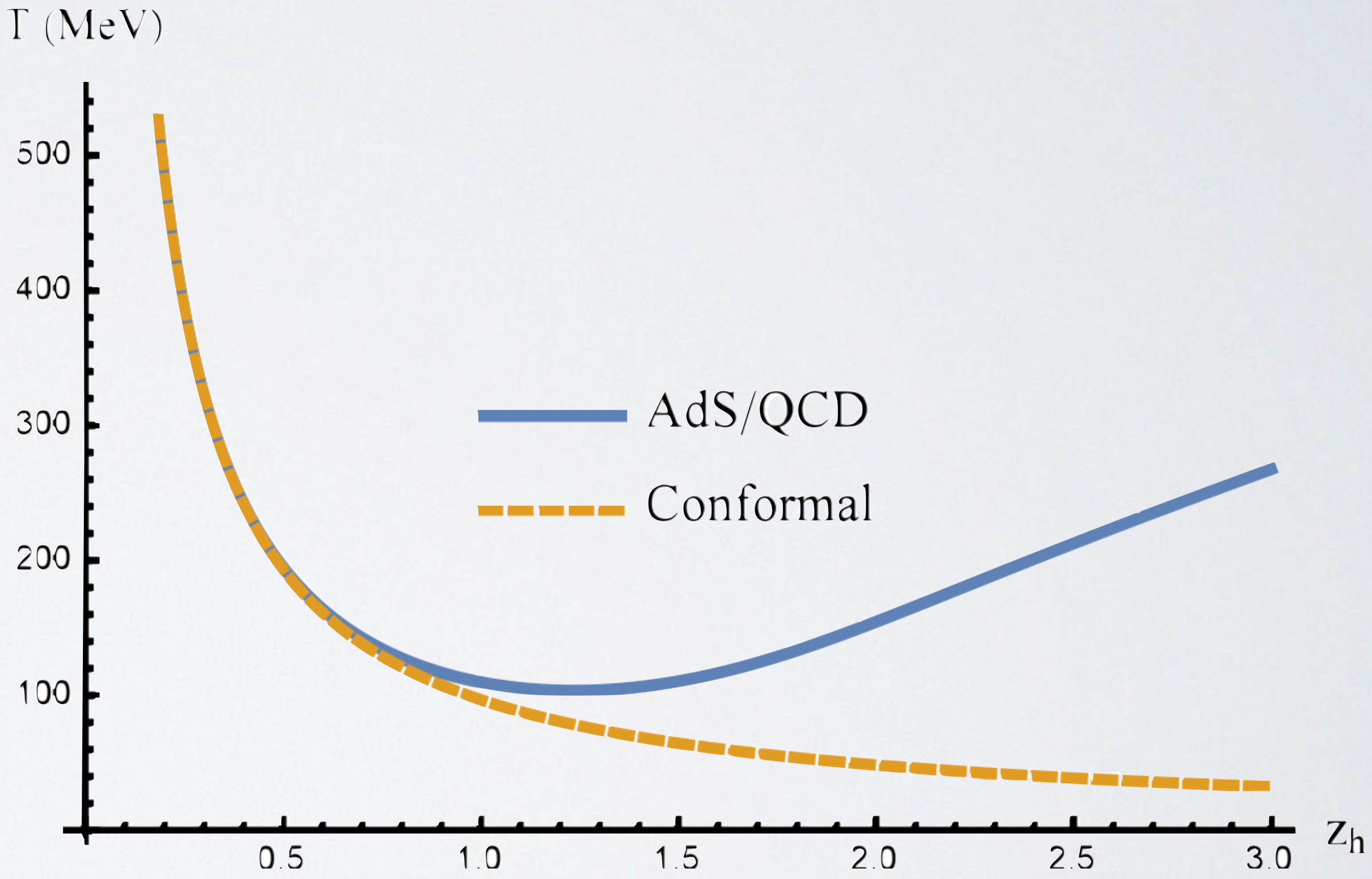
μ = 0



μ = 300 MeV

Deconfinement Temperature

- Chiral transition occurs before Tc from lattice QCD
- Mesons melt before chiral transition
- Require Hawking-Page transition
- Suggests non-conformal metric Zöllner, Kämpfer, 2016



Future work

- Pseudoscalar sector
  - Gell-Mann—Oakes—Renner relation
- Hawking-Page transition
  - Vanishing axial-vector mass gap?
- Dynamical model

Conclusions

- Reproduce Columbia Plot
- Chemical potential does not affect transition order
- Critical point missing

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