CP & Particle Correlations under Thermal Stochastic Influence

/Random Fluctuating Walk to CP/

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CP & Critical Phenomena

Collider $\qquad$ ***Fixed target***

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\textit{strong interacting matter @ high } T \text{ \& } \mu_B}$$

In the proximity of *CP*:

- **Matter becomes weakly coupled**
- **Color is no more confined**
- **Chiral symmetry is restored**

➢ **Phase transition is associated with breaking of symmetry**

Instructive: ***CP* clarified through $(\mu_B - T)$ plane**

scanning of $(\mu_B - T)$ phase diagram

CP & Critical Phenomena

A few questions arise:

- ***CP* meaning?**
- **Basic observables to be measured when *CP* achieved?**
- **New knowledge if *CP* approached?**

Answer: in terms of QCD_T @ large distances

N/Perturbative phenomena: χSB & Confinement of color

$\downarrow$ *?relations?* $\downarrow$

Phase transition of χS Restoration Deconfinement

$\downarrow$ **correlations** $\downarrow$

important issue

NO correct solution (massless quarks in the theory)

Effective models, e.g., with topological defects

Phase transitions ⇔ Topological defects (TD's)

TD's exist only in phase with SSB where $\langle \phi \rangle_{vacuum}$ emerges

Non-broken symmetry phase: ***no solutions relevant to TD's***

Minimal model: TD's (strings) arise in Abelian Higgs-like model

(Nielsen, Olesen, 1973)

$$SU(N) \xrightarrow{reduction} \left[U(1)\right]^{N-1}$$ **dual scalar thery**

gauge symmetry breaking ↓ Higgs-like mechanism

- **MA Gauge suggests special properties of QCD vacuum**
  Abelian dominance
- **Condensation of scalar d.o.f.** **(Ezawa, Iwasaki, 1982)**
  which provides
- **Dual superconductor picture of QCD vacuum** **('t Hooft 1981)**

Effective model

CP • **Fluctuation measure** • **Observables**

may be visible ↓ **through**

Fluctuations of characteristic length ξ **of chiral end mode**

Model: effective dual approach to QCD.

Fluctuations based on the order parameter $m \sim \xi^{-1}$

- Deal with gauge-invariant quantities, TPCF as a function of $C_\mu(x)$

- Dual color string: $U_C(x,y) \sim exp\left[ig\int_y^x dz^\mu C_\mu(z)\right]$, C_μ^a *dual to* A_μ^a

- ***Particles:*** **Bound states in terms of flux tubes**

Flux tubes

Excitations above vacuum: narrow flux tubes, $r_s \sim \xi \sim m^{-1}$
(in the center, $r_s \to 0$, scalar condensate vanishes)

Ensemble of a single flux tube system, $N(R)$ configurations of f.t.'s

$$Z_{flux} = \sum_{\beta} \sum_{R} N(R) \exp\left[-\beta\, E(m,R)\right] D\left(|\vec{x}|, \beta; M\right)$$

effective energy: $E(m,R) \sim m^2 R\left[a + b \ln(\tilde{\mu} R)\right]$ ***GK, 2010***

Dual gauge field C_μ - critical end mode!

$$m^2(\beta) \sim g^2(\beta)\, \delta^{(2)}(0)$$

$$\downarrow$$

$$c / \left(\pi r_s^2\right), \quad c \sim O(1)$$

TPCF

At large distances for any correlator (observables)

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim A|\vec{x}|^{c} D(|\vec{x}|,\beta;M) \text{ as } |\vec{x}|\to\infty$$

$$D(|\vec{x}|,\beta;M)=\exp[-M(\beta)|\vec{x}|],\ \ D(|\vec{x}|,\beta;M)\neq 0 \text{ even at } \beta=\beta_c$$

$M^{-1}(\beta)$ is the measure of screening effect of color electric field

$$SU(N=2,3)\ \ M(\beta)=M^{LO}(\beta)+N\alpha T\ln\left(\frac{M^{LO}(\beta)}{4\pi\alpha T}\right)+4\pi\ \alpha\ T\ y_{n/p}(N)+O(\alpha^2 T)$$

$$M^{LO}(\beta)=\sqrt{4\pi\alpha\left(\frac{N}{3}+\frac{N_f}{6}\right)}\,T$$

Kajante et al. 1997

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim L_W^{-4}-\frac{T}{V}\sigma_0(\beta)\xi^2\left[\frac{1}{\xi}\sqrt{\frac{8\pi}{\sigma_0}}-N\ln\left(\xi\sqrt{2\pi\sigma_0}\right)+\dots\right]!$$

GK, 2014

- ***Small $|\vec{x}|$, - singular behavior.***
- ***Large fluctuations ξ, - TPCF's disappear (CP does approach)***

Result: eff. theory in terms of non-perturbative TPCF describes the fluctuations at distances $g\xi/\sqrt{\pi} < |\vec{x}| < M^{-1}$ ***up to CP***

String tension $\sigma_0(\beta) \sim m^2(\beta)\alpha(\beta)$ *GK 2010*

Flux-tube scheme (with a scalar dilaton (monopole) condensate):

- $\xi \sim m^{-1}$ the penetration length of color-electric field
- $\xi \sim r_s$ "string"-like radius
- $l \sim m_\phi^{-1}$ coherent length of scalar (dilaton) condensate
- $\tau = \sqrt{4/(3\alpha)}\xi$ formation time of flux tube ($\to \infty$ @ CP)

➢ For SU(3), $m \approx 1.95\sqrt{\sigma_0}$ Baker et al., 1997

✓ **Lattice:** $T_c \approx 0.65\sqrt{\sigma_0}$ Lucini et al., 2002

Effective theory applicable in deconfined phase $T_c < T < 3T_c$!

Dual QCD vacuum.

In $SU(3)$ gluodynamics vacuum is characterized

$$k_{GL}=\frac{\xi}{l}\sim\frac{m_\phi}{m}\begin{array}{ll} <1 & (type\ I\ vacuum,\ \text{flux tubes}\ attracted) \\ >1 & (type\ II\ vacuum,\ \text{flux tubes}\ repel) \end{array}$$

Scalar fields, dilatons ϕ (condensate) remain massive up to the CP (1st order PT)

$k_{GL}\to\infty$ **Deconfinement!**

If $k_{GL}=1$ parallel strings (carry the same flux) do not interact each other. $T_c\approx 172\ MeV,\ N_c=3\ pions$ GK 2014

Singularity of $Z_{flux}\Rightarrow$ $$k_{GL}\geq\frac{3}{4}\frac{\alpha(\beta)}{\xi m_{q\bar q}}\left[1+\frac{4}{3}\frac{\xi^2}{\alpha(\beta)\beta}M(\beta)\frac{L_W}{R}\right]$$

$\downarrow$ **Type-I vac** $\downarrow$ **Type-II vac** $\to\infty\ \ as\ \ \xi\to\infty$

Observation of correlations between two bound states (strings) is rather useful & instructive to check the *CP* is approached!

Field theory $\Rightarrow$ RG $\Rightarrow$ Critical Behavior

Phase transitions $\Leftarrow$ presence and the properties of ***fixed points***

IR attracted Fixed Points $\left\{ \begin{array}{c} \mathit{Phase\ \ Transitions\ \ of\ \ 2nd\ \ kind} \\ \mathit{Critical\ \ scaling} \end{array} \right.$

RG Fluxes (solutions) may leave physical domain containing **IRFP** (even to ∞)

$\Downarrow$

Phase transition of the 1st kind

From theory to phenomenology: *Bose-Einstein Correlations @ finite T*

Def.:

BEC's are the quantum effect which enhances the probability that multiple bosons be found in the same state, same position, same momentum

Sample with production $\pi^+\pi^+, \pi^-\pi^-, \ \pi^0\pi^0$

$$AA\ (pp) \to high\ \ T\ \ quark - gluon\ \ bubble \to hadronization \to$$

$$\to chaotic\ \ pion's\ \ production\ \ with\ \ different\ \ directions,\ \ momenta,\ \ angles$$

What's happened once the critical *T is approached and above*

- **Shape of correlation behavior?**
- **Correlation radius size?**
- **Other characteristics to be measured?**

Size of the particle source

- Possible approach to ***CP*** study through spatial correlations of final state particles
- Size effect of space composed of "hot" particles $\Rightarrow$ derive theoretical formulas for $2-,\ldots,N-$ particle ***distribution-correlation functions*** (stochastic, chaotic behavior)

Stochastic scale (size) in C's Bose-Einstein GK (2008-2010)

$$C_2(q,\lambda) \approx \eta(N)\left[1+\lambda(\nu)e^{-\Delta_{qL}}\right], \quad \Delta_{qL} = q^{\mu}\Re_{\mu\nu}q^{\nu}, \quad \eta(N) = \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}$$

event-to-event fluctuations

Chaoticity function:

$$\lambda(\nu) = 1/(1+\nu)^2, \; 0<\nu<\infty, \; N \sim V\int d\omega\, m^2 \frac{1}{e^{(\omega-\mu)\beta}-1}$$

$$b(x) = a(x)+R(x), \; \tilde{R}(p_{\mu}) = \sqrt{\nu\, \Xi(p,p)}, \; \Xi = \langle a^{+}(p)a(p)\rangle$$ GK'98-02

Strength of BE correlations $\tilde{\lambda}(k_T,\beta)$

(almost immediate emission of the particle's pairs from a source,)

$$C_2(q,\beta) \approx \eta(N)\left\{1+\tilde{\lambda}(\beta)e^{-q^2L_{st}^2}\left[1+\lambda_1(\beta)e^{+q^2L_{st}^2/2}\right]\right\}(1+\delta\cdot q+\ldots)$$

$$\tilde{\lambda}(\beta)=\frac{\gamma(\omega,\beta)}{\left[1+\nu(N)\right]^2},\ \lambda_1\approx 2\nu,\ \nu\sim\frac{1}{n}\frac{1}{k_{GL}^2},\ \text{as}\ \ T\to T_c,\ \ \gamma(\omega,\beta)\sim O(1)$$

$$L_0\sim 10^{-24}\sec\sim\tau_{Z^0};\ HIColl.\ L_0\sim\frac{ma^2}{\hbar c^2}A^{2/3}\sim 10^{-23}\sec,\ \ a\sim O(1.0\,fm)$$

Proposal: *GK 2009-2014*

- $\tilde{\lambda}(k_T,\beta)$ decreases with k_T far away from the *CP*
  - ✓ $\tilde{\lambda}(k_T,\beta)\to 0$ **as *CP* approached,** $k_{GL}\to\infty$ ***DECONFINEMENT***

Origin: infinite fluctuation length $\xi\to\infty$

- When ***CP*** approached:
  - **NO** signal of enhancement of pairs of same-sign charge particles is observed! C_2 ***-function does not deviate from 1.***

OBSERVABLES?

The scaling form C_2 is useful to predict behavior of observables @ **CP**

$L_{st} \to \infty \; as \; T \to T_c, \; \mu \to \mu_c$ indicate the vicinity of **CP**

- Observable, e.g., $k_T^2 = \dfrac{1}{\nu(N)\; T^3\; L_{st}^5}, \quad k_T = \left| \vec{p}_{T_1} + \vec{p}_{T_2} \right|$ GK(2011)

Exp: L3/CMS(2011)/ATLAS(2015)/ALICE L decreasing (smooth) with k_T

- Chaoticity λ measured. *(Most important theor. study)*

$$0 < \lambda[\nu(N)] \leq 1$$

$$\Downarrow \qquad\qquad \Downarrow$$

fully coherent phase chaotic *(critical behavior from BM to AM)*

Chiral restoration & Particle emission size

Theory: $L_{st} = L_{st}\left(\beta, k_T, m, \nu(N)!\right) \sim \dfrac{1}{\nu^{1/5}(N) m_h^{\alpha} T^{\gamma}}$ *GK, 2009-2010*

$$\nu(N) = \frac{2-\tilde{C}_2(0) + \sqrt{2-\tilde{C}_2(0)}}{\tilde{C}_2(0)-1}, \qquad \tilde{C}_2(0) = \frac{C_2(q=0)}{\dfrac{\langle N^2\rangle}{\langle N\rangle^2} - \dfrac{1}{\langle N\rangle}}$$

$$\langle N\rangle \geq 1 + C_2(0)/2, \quad C_2(0) \leq 2$$

CMS (2011): $\sqrt{s}$ =0.9 TeV; 7 TeV,- L_{st} increases with $\langle N\rangle$

ATLAS (2015): $\sqrt{s}$ =0.9 TeV; 7 TeV,- L_{st} increases with $\langle N\rangle$ as well

High $T \to T_c$: $\quad L_{st} \sim \left[\nu^{\delta}(N) m_h^{\alpha}\ T^{\gamma}\right]^{-1} \to \infty\ as\ m_h \to 0\ \ \chi SR$

L_{st} vs R_{HBT} radius

$$R_{HBT} = \sqrt{\frac{3}{2}}\, q_{rms}^{-1} \approx \sqrt{3}\left(1 - \frac{\sqrt{2}\nu}{1+2\sqrt{2}\nu}\right) L_{st}$$

root-mean-squared momentum

$$q_{rms}^{2} = \left\langle \vec{q}^{\,2} \right\rangle = \frac{\int_0^\infty dq\, q^2 \left[\tilde{C}_2(q;\nu) - 1\right]}{\int_0^\infty dq \left[\tilde{C}_2(q;\nu) - 1\right]}, \quad \tilde{C}_2(q;\nu) = \frac{C_2(q;\nu)}{\eta(N)}$$

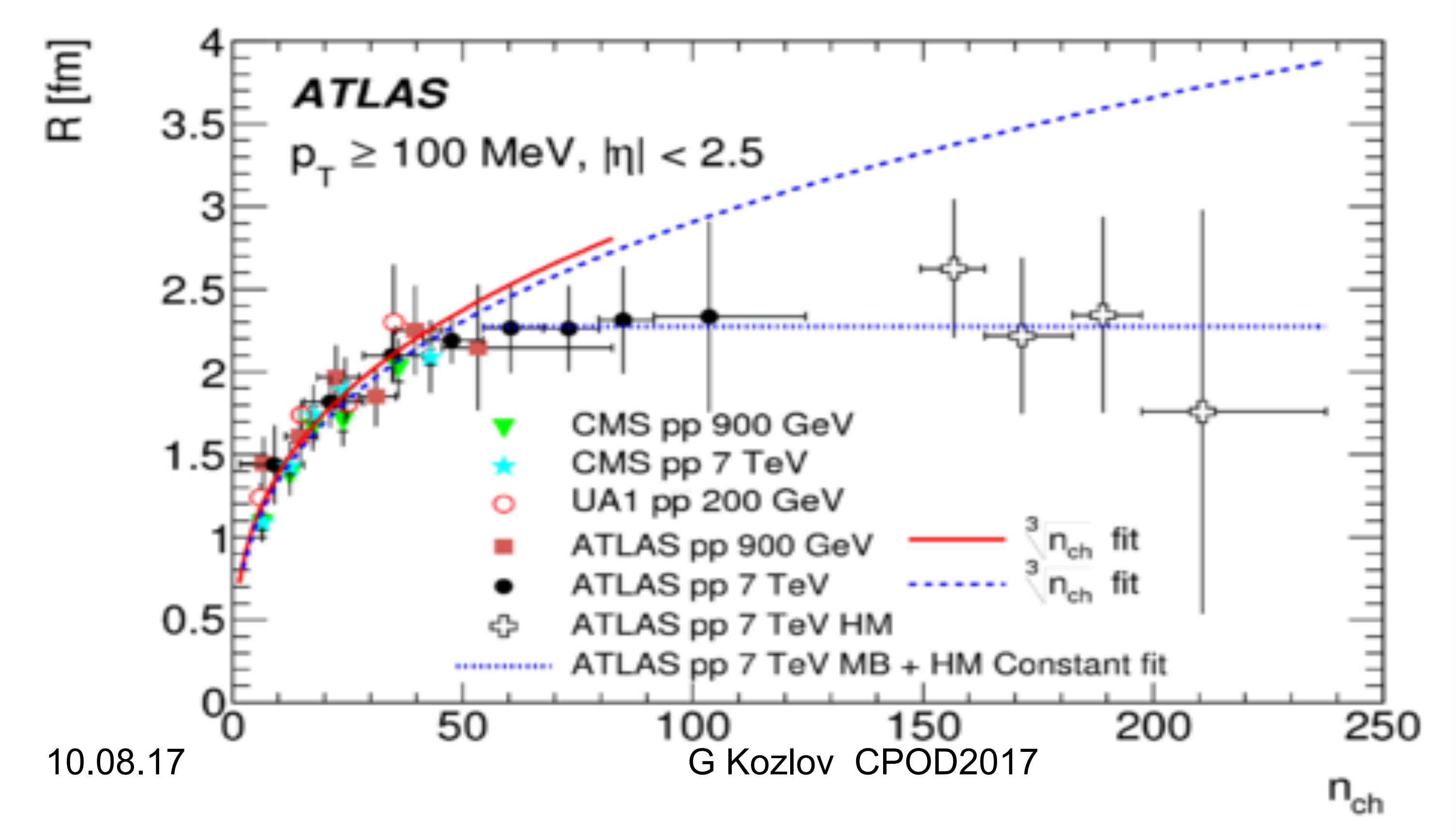
- In the vicinity of CP $(\nu \approx 0)$ R_{HBT} is $\sim L_{st}$ itself which increases very rapidly.
- ✓ On the other hand, for strong external forces (fields) influence $(\nu \to \infty)$ R_{HBT} approaches the point-like measure.

Charactersitic size of the Correlation source

In terms of Ginzburg-Landau criterium k_{GL}

«Radius» increases with $n \sim n_{ch}$ $R \sim L_{st} \sim \left(\frac{n\ k_{GL}^2}{k_T^2\ T^3} \right)^{1/5}$ *GK (2009)*

ATLAS Coll., Eur. Phys. J.C75 (2015) 466



➢ Expansion of particle emission size

$$L_{st}(\beta) \sim \left[\nu(N) k_T^2 \; T^3\right]^{-1/5} \to \infty \; \boldsymbol{as} \; \nu(N) \to 0 \text{ at } T \to T_c$$ ***KG 2010***

The temperature at which the signal of two-particles correlations disappears is the critical temperature at *CP* : $C_2(q, T_c) = 1$

! Too rapid phase transition can include the ***explosion*** of a "hadronic fireball" just after a phase transition

➢ *Dip-effect*

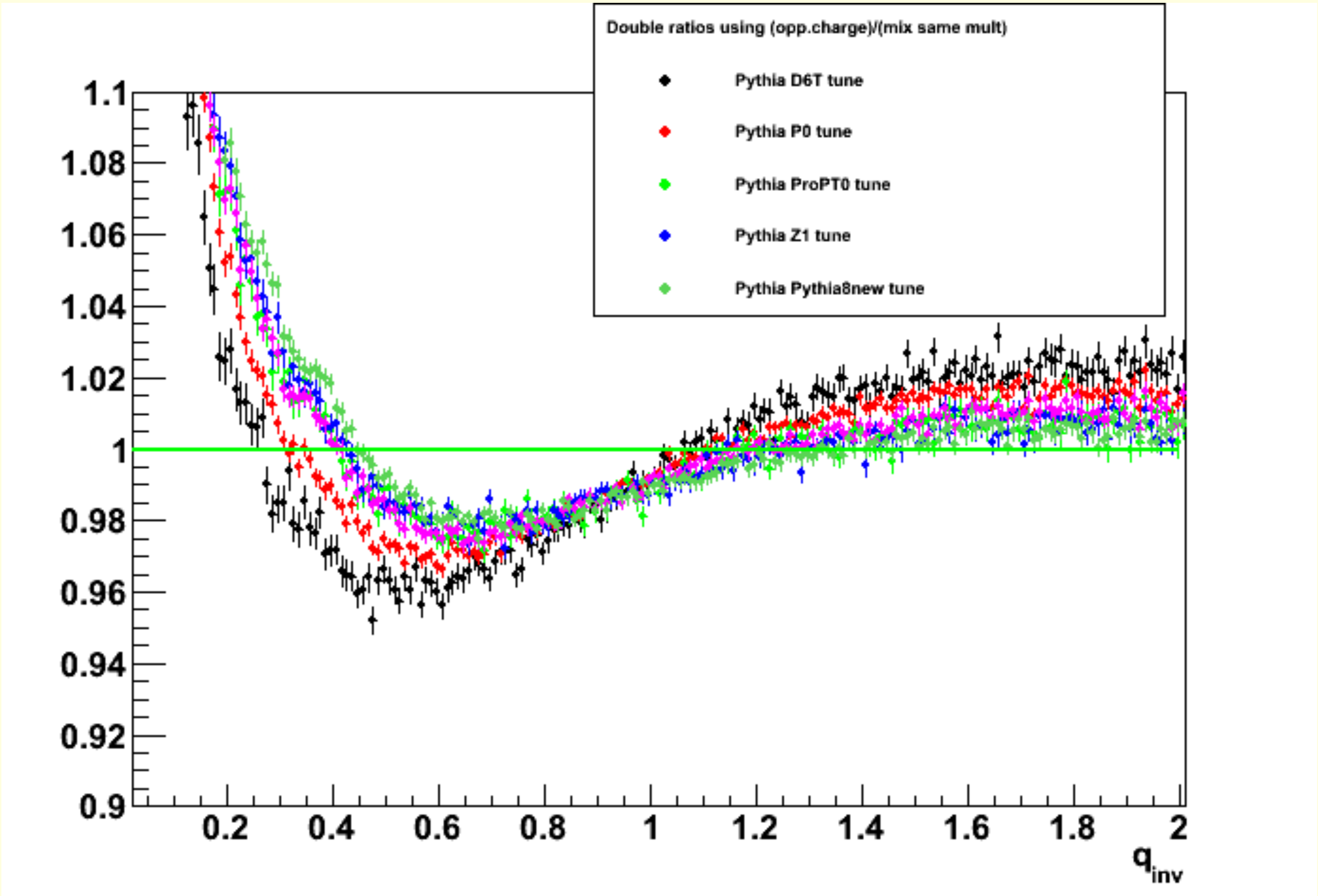
The effect of anti-correlations (the dip-effect) is predicted at low charged-particle multiplicity N **in the event:** $C_2(\{q\}, N) < 1$ **?!** ***KG 2010***

The depth of the dip in the anti-correlation region decreases as N **increases.**

Observed by CMS at LHC [CMS Coll., JHEP 5 (2011) 029]

Proposal: dip-effect disappears at *CP*

Dip-effect @ 7 TeV CMS (2011)



Random Fluctuation Walk $(BM \Rightarrow AM)$

Random stochastic (chaotic) walk with respect to quantum correlations of identical particles. Cross-over walk.

Model 1D x-oriented $(-\infty < x < \infty)$

$$P\left(x;\bar{\lambda},\mu_c\right) = p\sum_{j=0}^{\infty}\bar{\lambda}^j \frac{1}{2}\sqrt{\frac{\pi}{t}}\left[e^{-y_-^{2j}/4t} + e^{-y_+^{2j}/4t}\right]$$

$$y_\pm^j = x\mu_c \pm a^j,\ a = \left(\mu / \mu_c\right) > 1,\ \ t = l\mu_c \text{ (lattice spacing)}$$

$$\underbrace{\lim}_{\substack{l\to 0\\ \mu_c\neq 0}} P\left(x;\bar{\lambda},\mu_c\right) = p\left(\bar{\lambda}\right)\sum_{j=0}^{\infty}\bar{\lambda}^j\pi\left[\delta\left(x\mu_c - a^j\right) + \delta\left(x\mu_c + a^j\right)\right]$$

$$p\left(\bar{\lambda}\right) = \frac{1}{2\pi}\left(1-\bar{\lambda}\right),\ \ 0 < \bar{\lambda} \le 1,\ \ \text{NC: } 2\left(p + \bar{\lambda}p + \ldots + \bar{\lambda}^j p + \ldots\right) = 1$$

The limit $\bar{\lambda} \to 1 \Rightarrow$ broad behavior of P: *vicinity of* ***CP*** *is approached*

$$\bar{\lambda} \to 0 \ \ \Rightarrow \ \ P\left(x;\bar{\lambda}\to 0,\mu_c\right) \to 1/\left(2\pi\right) \quad \text{trivial}$$

Random fluctuation weight $\bar{\lambda}$

Vicinity of **CP**: theory conformal, scalar dilaton field ϕ

$\bar{\lambda} \to \lambda(\nu) = \left[1+\nu(N)\right]^{-2}$ stochastic (external) influence strength

$$\nu(N) \approx \frac{1}{n\, k_{GL}^2} O\left(\frac{m_\phi^2}{m^2}\right)$$

Dual Higgs-Abelian gauge model ($A_\mu \to B_\mu$)

dual superconductor QCD vacuum

$$k_{GL} \sim \frac{m_\phi}{m_B} \begin{cases} <1 \quad \textit{vacuum} \quad \textit{type} - I\textit{, two flux tubes attracted} \\ >1 \quad \textit{vacuum} \quad \textit{type} - II\textit{, two flux tubes repel} \end{cases}$$

CP: $k_{GL} \to \infty$ as $\xi \sim m_B^{-1} \to \infty$ fluctuation length/
penetration depth of color-electric field/radius of the flux tube

Analyticity of probability $P(x;\lambda,\mu_c)$

Large x (sharp increasing of L_{st}) / or $k \to 0$ (IR analogue)

To smooth the particularity (speciality) of $P(x;\lambda,\mu_c)$

$$P(x;\lambda,\mu_c) \to G(k;\lambda,\mu_c) = p(\lambda)\sum_{j=0}^{\infty}\lambda^j \boldsymbol{cos}\left(\frac{k}{\mu_c}a^j\right), \quad G(0;\lambda) = \frac{1}{2\pi}$$

Fluctuation length through the even moments of the order $2s$:

$$\xi^2_{(2s)}(\lambda) \sim m_{(2s)}(\lambda) = \frac{\partial^{2s} G(k;\lambda,\mu_c)}{\partial k^{2s}} \Big|_{k=0}$$

- Finite ξ will provide analytical form of G, however large $\xi \to$ non-analytical behavior of G @ $k \to 0$

The dual QCD vacuum will influence *(through k_{GL})* ξ up to cross-over: unified process of phase transition between BM and AM

Analyticity of probability $P(x;\lambda,\mu_c)$ cont'd I

$G(k;\lambda,\mu_c)$ in terms of CBE function $C_2(q,\lambda)$

$$G(k;\lambda,\mu_c) \sim \frac{const}{\bar{C}_2(q;\lambda)-1}, \quad \bar{C}_2(q;\lambda) \equiv \frac{C_2(q;\lambda)}{\eta(N)}, \quad 0<\lambda\le 1$$

CP: $\underbrace{lim}_{\lambda\to 1}\, l.h.s. \to \infty$ and $\underbrace{lim}_{\lambda\to 1}\, r.h.s. \to \infty$

Fluctuation length result: $\left|\xi^2_{(2s)}(\lambda)\right| = p(\lambda)\sum_{j=0}^{\infty}\left(\frac{a^j}{\mu_c}\right)^{2s}\lambda^j, \; a=\frac{\mu}{\mu_c}>1$

Converged @ $(a/\mu_c)^{2s}\lambda<1, \left|\xi^2_{(2s)}(\lambda)\right| \approx p(\lambda)\mu_c^{-2s}\left(1+a^{2s}\lambda\right)$ finite

If $\mu\approx\mu_c$ and $\lambda\to 1, \; \xi\to\infty$ divergence $/\xi \Leftrightarrow$ **CP/**

Analyticity of probability $P(x;\lambda,\mu_c)$ cont'd II

- Infinite # of divergent (singular) terms in $G(k;\lambda,\mu_c)$

  Why?

Because wide range of λ, μ; singularity @ $k \ll \mu_c$ $(k \to 0)$

- To find non-analytical part @ $k \approx 0$

$G(k;\lambda,\mu_c) = G_{BM}(k;\lambda,\mu_c) + G_{AM}(ak;\lambda,\mu_c)$ linear non-homog. eq.

BM $G_{BM}(k;\lambda,\mu_c) = p(\lambda)\boldsymbol{cos}(k/\mu_c)$ regular if $k \approx 0$, for all λ

AM $G_{AM}(k;\lambda,\mu_c) = \lambda G(ak;\lambda,\mu_c)$, for $a^{-2} < \lambda < 1$

$G_{AM}(k;\lambda,\mu_c) \to 0$, *if* $\lambda \to 0$ The phase with **BM** does exist only

While $G(ak;\lambda \approx 0,\mu_c) \approx (1/2\pi)\boldsymbol{cos}(k\mu/\mu_c^2)$ Finite

Special solution for AM

$$G_{AM}\left(k;\lambda(S),\mu_c\right)=C\left(\lambda(S)\right)\left(k/\mu_c\right)^{\alpha(\lambda(S))}Q(k)$$

$$\alpha\left(\lambda(S)\right)=\ln\left[\frac{Q(k)}{Q(ak)\cdot\lambda(S)}\right]/\ln a,\quad a>1$$

$\downarrow$ associated function of the 1st order to the measure γ with the proper function of the operator of dilatation transformation u: $uQ(k)=Q(ak)$

For any $a=(\mu/\mu_c)>1$: $\quad Q(ak)=a^{\gamma}Q(k)+a^{\gamma}\,\boldsymbol{ln}(a)\cdot Q_0(k)$

Vicinity of CP (a close to 1 from above) $\quad Q(ak)\approx Q(k)$

Finally, $\alpha\left(\lambda(S)\right)\approx-\left[\dfrac{\ln\lambda(S)}{\ln a}+\gamma\right]$, $2(S+1)-\gamma\ln a\geq\alpha(S)>2S-\gamma\ln a$

Solution for AM phase

$$G_{AM}(k;\lambda,\mu_c) \sim \frac{i^{\alpha-2}}{\Gamma(\alpha-1)}\xi^2_{(\alpha-2)}(\lambda)\left|\frac{k}{\mu_c}\right|^{\alpha}\frac{1}{\log a}\sum_{m=0}^{\infty} b_m \cos\left[2\pi m\frac{\log(|k|)}{\log a}+\vartheta_m\right]$$

Divergent if a) $\lambda \to 1$ because of $\xi(\lambda)$

b) $a = \frac{\mu}{\mu_c} \to 1$ from above

- $C[\lambda(S)] = \frac{i^{\alpha-2}}{\Gamma(\alpha-1)}\xi^2_{(\alpha-2)}(\lambda)$ *used within calculation*

The **AM** disappears if $\frac{\alpha(S)}{S} \to 2$ as $S \to \infty$ at $\lambda(S) \to 0$

Close to **AM:** $k_{GL} \geq \frac{1}{n}\frac{2}{(\mu/\mu_c)^2-1}$

CP (cross-over) $k_{GL} \to \infty$ at $\mu = \mu_c$.

Results

1. Theoretical search for **CP,** cross-over between **BM** and **AM** in 1D random fluctuation walk accompanied by the **CBE.**

2. Main points: $\bar{\lambda}$, $\lambda(\nu)$, $m_{(2s)}(\lambda) \sim \xi^2_{(2s)}(\lambda)$, $a = (\mu / \mu_c) > 1$, k_{GL}.

3. Source size L_{st} increases (smoothly) with N at low T.
   L_{st} blows up as $T \to T_c$ due to $\nu(N) \to 0$, $m_h \to 0$;
   L_{st} singular @ transition point, CP.

4. RFW solution: **BM** (regular) + **AM** (singular).

5. Smooth $\lambda(\nu) \Rightarrow$ **BM**
   $\lambda(\nu) \to 1$ (strong particle chaoticity) $\Rightarrow$ **AM**
   asymp. behavior at $k \to 0$ (**IR**), large x is approached
   $\Downarrow$
   infinite size of particle source.

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Solution for $Q(|k|)$

Solution with smoothly changing function, e.g., the log-periodic one with the period $\boldsymbol{log}(a)$

Method: $|k| \to \boldsymbol{log}(|k|),\ \ a \cdot |k| \to \boldsymbol{log}(a) + \boldsymbol{log}(|k|)$

$$Q(|k|) \sim \frac{1}{\boldsymbol{log}\, a} \sum_{m=0}^{\infty} b_m\, \boldsymbol{cos}\left[2\pi m \frac{\boldsymbol{log}|k|}{\boldsymbol{log}\, a} + \vartheta_m\right]$$

If $b_m = 1,\ \ \vartheta_m = 0 \Rightarrow Q(|k|) \sim \frac{1}{2\,\boldsymbol{log}\, a}\left[1 + \sum_{m=-\infty}^{+\infty} \delta\left(\frac{\boldsymbol{log}(|k|)}{\boldsymbol{log}\, a} - m\right)\right]$