Bulk viscous corrections to Hard Thermal Loops in hot QCD

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CPOD 2017
Stony Brook Univ.
Aug. 7 – 11, 2017

- Bulk viscosity in QCD
- Hard thermal loops away from equilibrium
- Applications to static potential & heavy-Q diffusion

based on: Q. Du, A. Dumitru, Y. Guo and M. Strickland, arXiv:1611.08379

Viscous corrections :

local rest frame: $T_{0i}=0$

$$T_{ij} = P_{\rm eq}(\epsilon)\,\delta_{ij} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \zeta\delta_{ij}\nabla\cdot u$$

shear correction — bulk correction

$$P(\epsilon) = P_{\rm eq}(\epsilon) - \zeta\nabla\cdot u$$

[Note: non-conformal 2nd order viscous hydrodynamics features shear-bulk coupling which may reverse sign of bulk pressure.]

G. S. Denicol, S. Jeon and C. Gale,
Phys. Rev. C 90, 024912 (2014)

Parametric behavior:
(massless q; $\alpha_s \ll 1$)

$$\eta \sim \frac{T^3}{\alpha_s^2 \log\frac{1}{\alpha_s}}$$

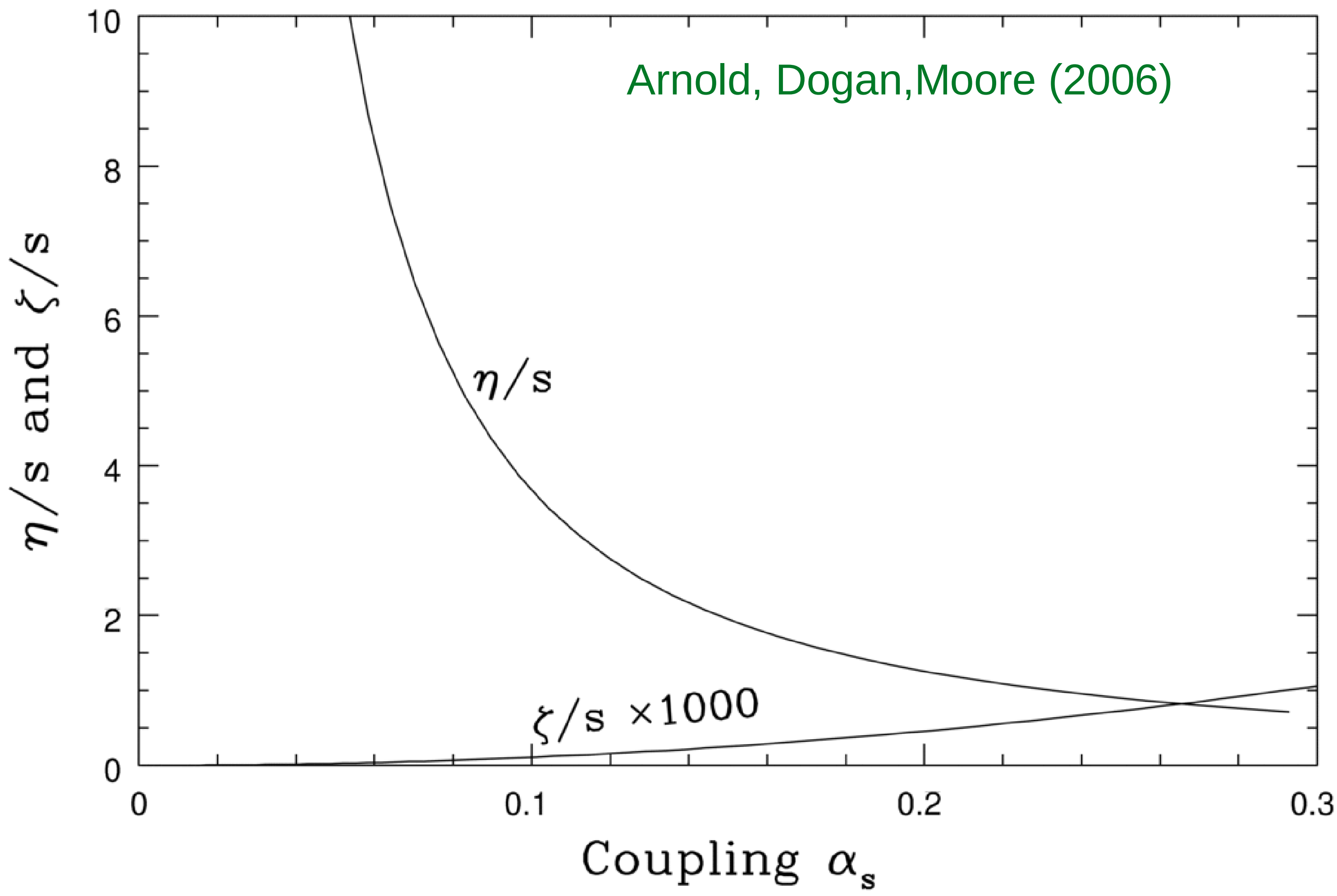
P. Arnold, G. D. Moore and L. G. Yaffe,
JHEP 0011, 001 (2000)
JHEP 0305, 051 (2003)

$$\zeta \sim \frac{\alpha_s^2 T^3}{\log\frac{1}{\alpha_s}} \sim \eta\left(1-3c_s^2\right)^2$$

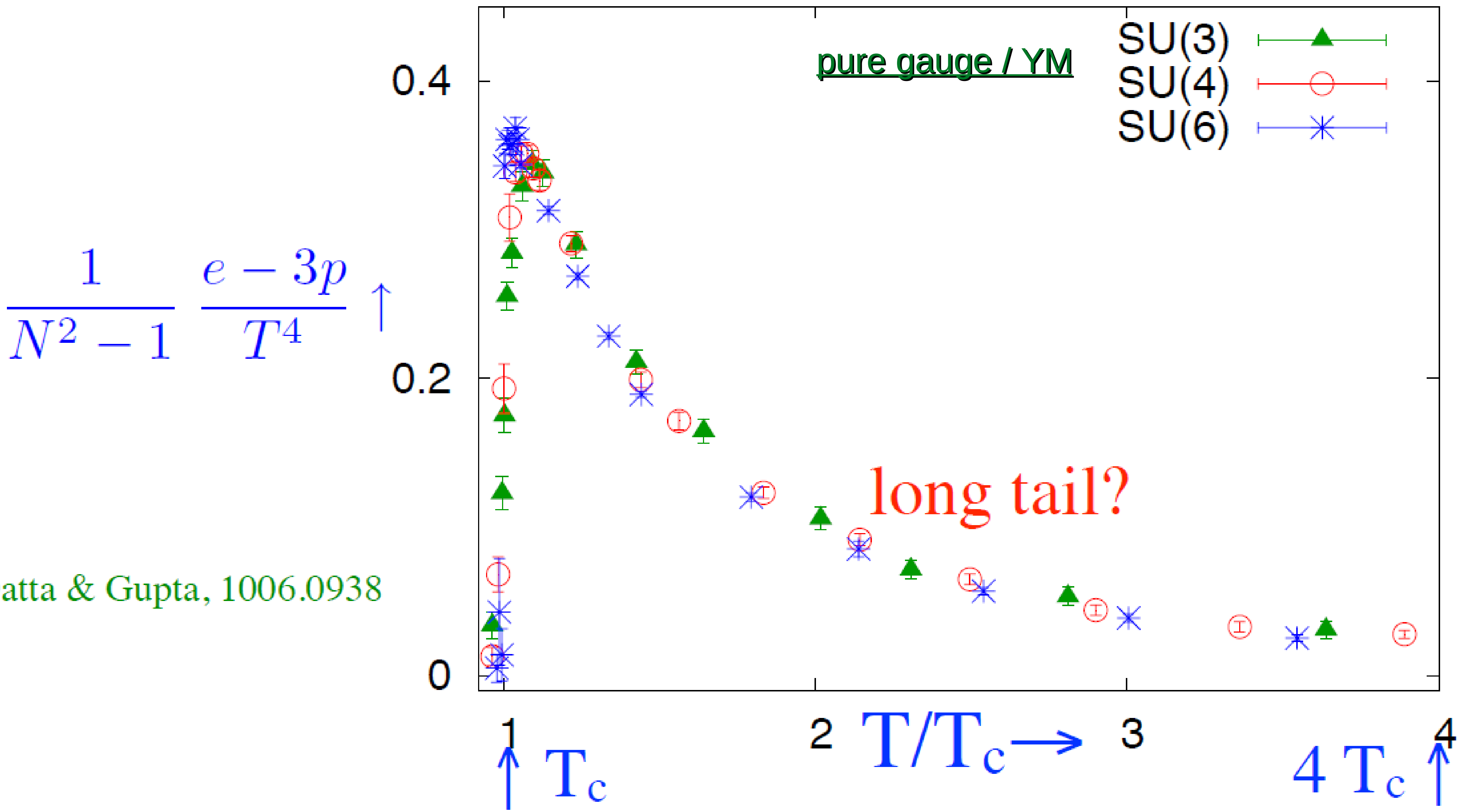
P. Arnold, C. Dogan and G. D. Moore
Phys. Rev. D 74, 085021 (2006)

ζ suppressed by **two** powers of $\beta(\alpha_s) \sim -\alpha_s^2$ since:

- uniform compression / rarefaction ~ dilatation;
  does not bring scale invariant theory out of equilibr.
- away from equilibrium still $p(\varepsilon) = p_{id}(\varepsilon) = \varepsilon/3$ because
  $T^\mu_\mu = \varepsilon - 3p = 0$ in conformal theory



But: conformal anomaly substantial up to ~ $2T_C$



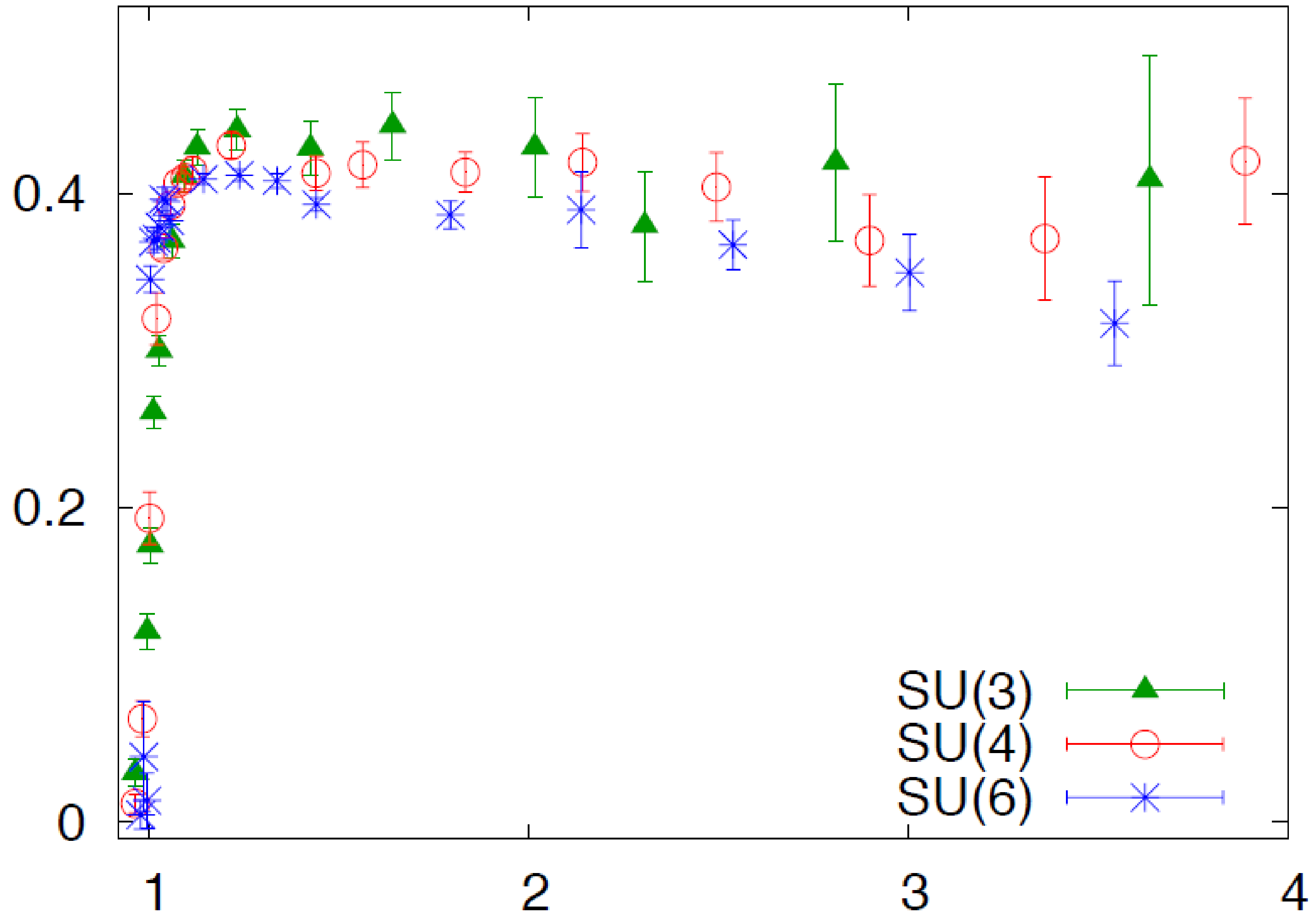
Datta & Gupta, 1006.0938

Lattice: scaling of the conformal anomaly

Scaling: $(e-3p)/T^2$ *approximately* constant near T_c: MMO '01; RDP, ph/0608242

Only true near T_c ; eventually, $(e-3p)/T^4 \sim g^4(T)$

$$\frac{1}{N^2-1}\;\frac{e-3p}{T^2\,T_c^2}\uparrow$$



$\uparrow T_c$ $\quad T/T_c \rightarrow$ $\quad 4\,T_c \uparrow$

Datta & Gupta, 1006.0938

One-loop pert. theory, conformal at UV fixed point (g → 0, asymptotically short scales)

$$e - 3p \equiv T^4\,\Delta(T)\;, \qquad \Delta(T) = -\frac{N(N^2-1)}{72}\,\beta(g(T))\,g(T)$$

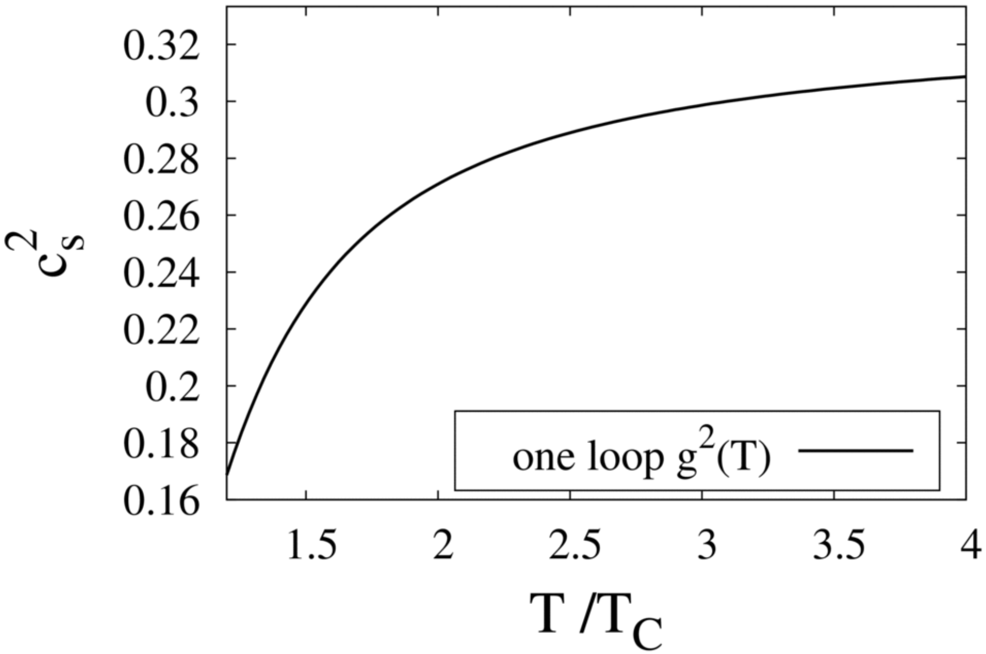
$$= \frac{\beta(g)}{2g}\,\langle F^2\rangle$$

(pure glue)

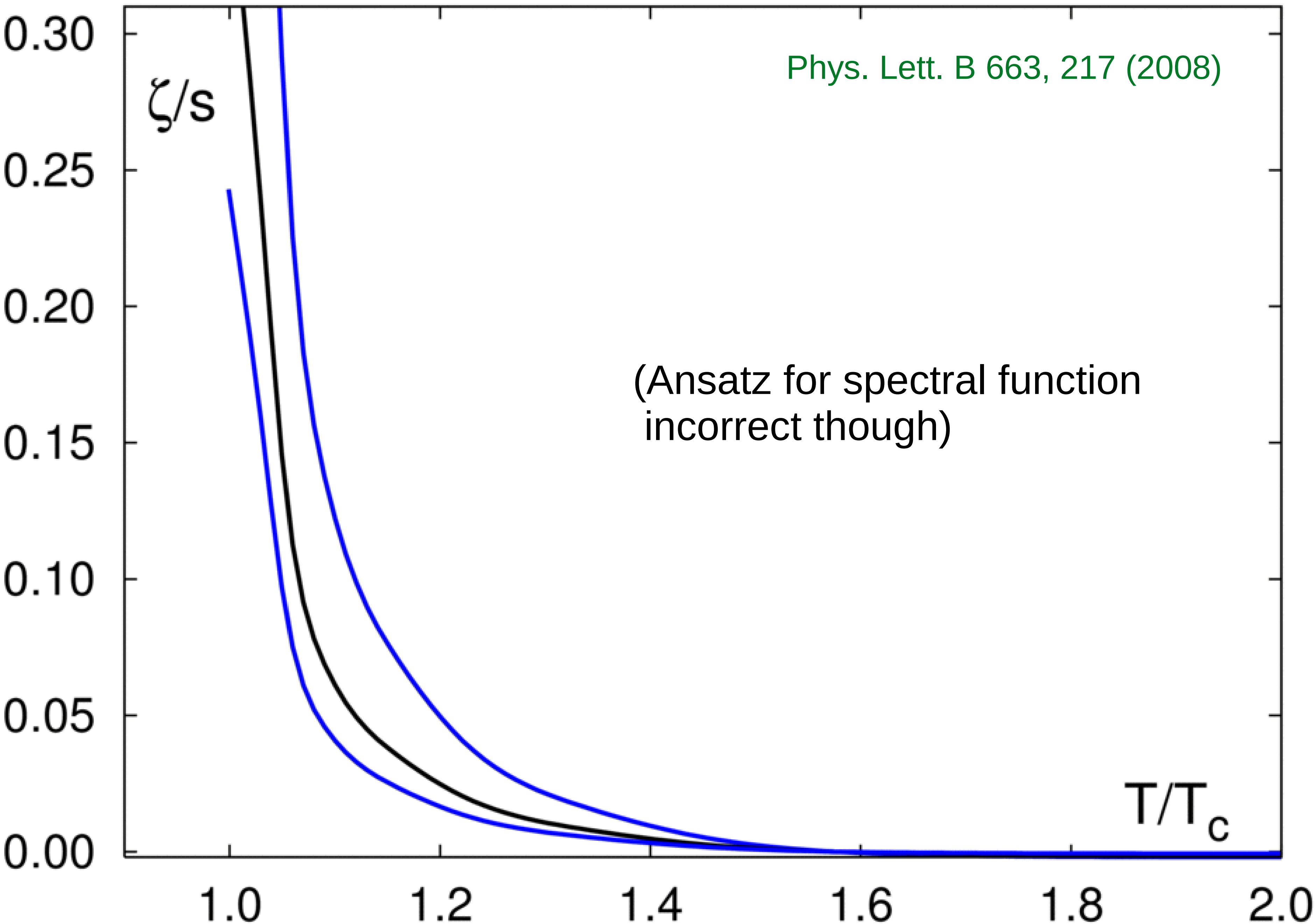
$$\frac{1}{c_s^2} - 3 = \left(\frac{\partial p}{\partial T}\right)^{-1}\frac{\partial}{\partial T}\left(T^4\Delta(T)\right) \approx \frac{\Delta(T)}{p/T^4}$$

(p/T^4 = const here)

$$= -\#\beta(g)\,g(T) = \frac{\#}{\log^2 T/\Lambda}$$



Bulk viscosity estimate by Karsch, Kharzeev, Tuchin



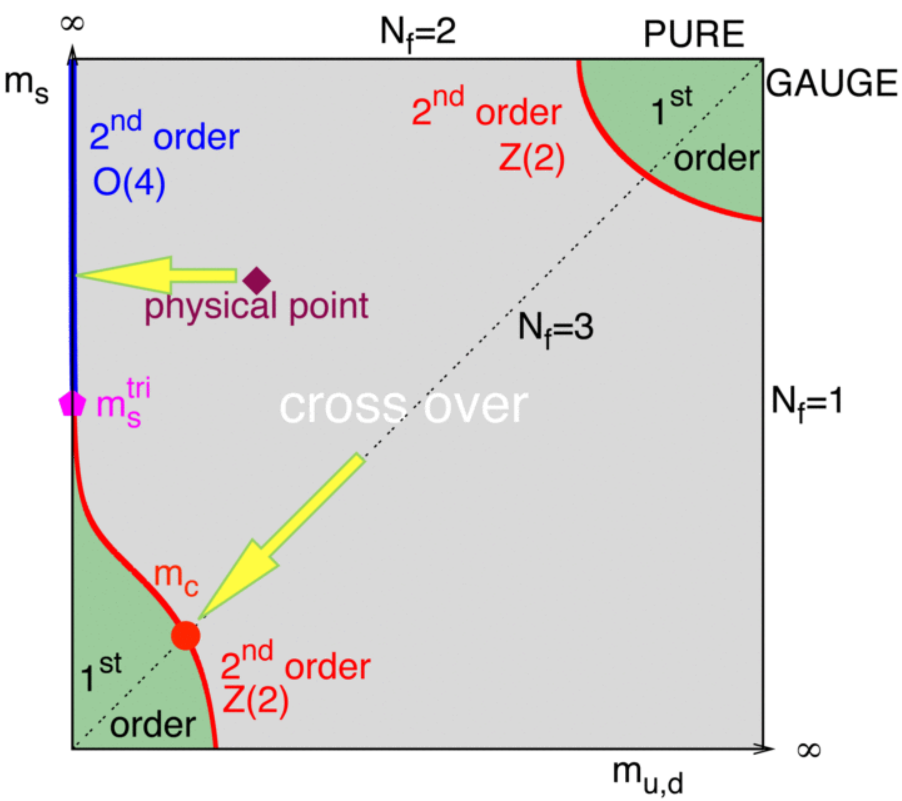
Behavior of ζ near a 2nd order critical point

bulk viscosity diverges:

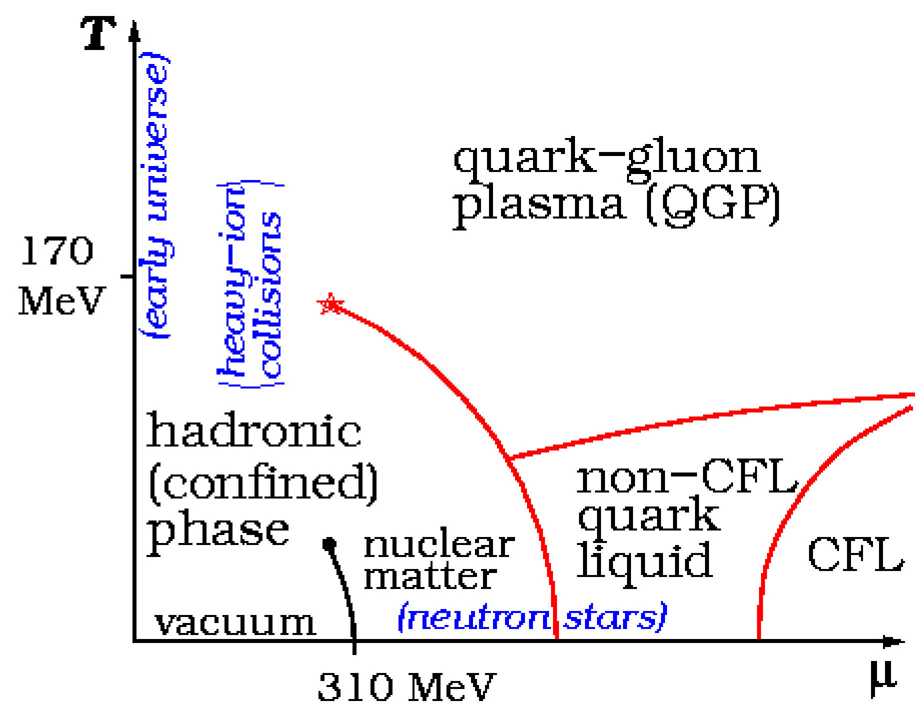
$$\zeta \sim \xi^{z-\alpha/\nu}$$

A. Onuki, PRD 55, 403 (1997);
G.D. Moore and O. Saremi, JHEP 09, 015 (2008)

$\xi \longrightarrow \infty$: correlation length
z: dynamical critical exponent
(=3 for model H w/o fluctuations)
α,ν: equilibrium critical exponents
(α=0 for model H)



F.R. Brown et al, Phys.Rev.Lett. 65 (1990) 2491



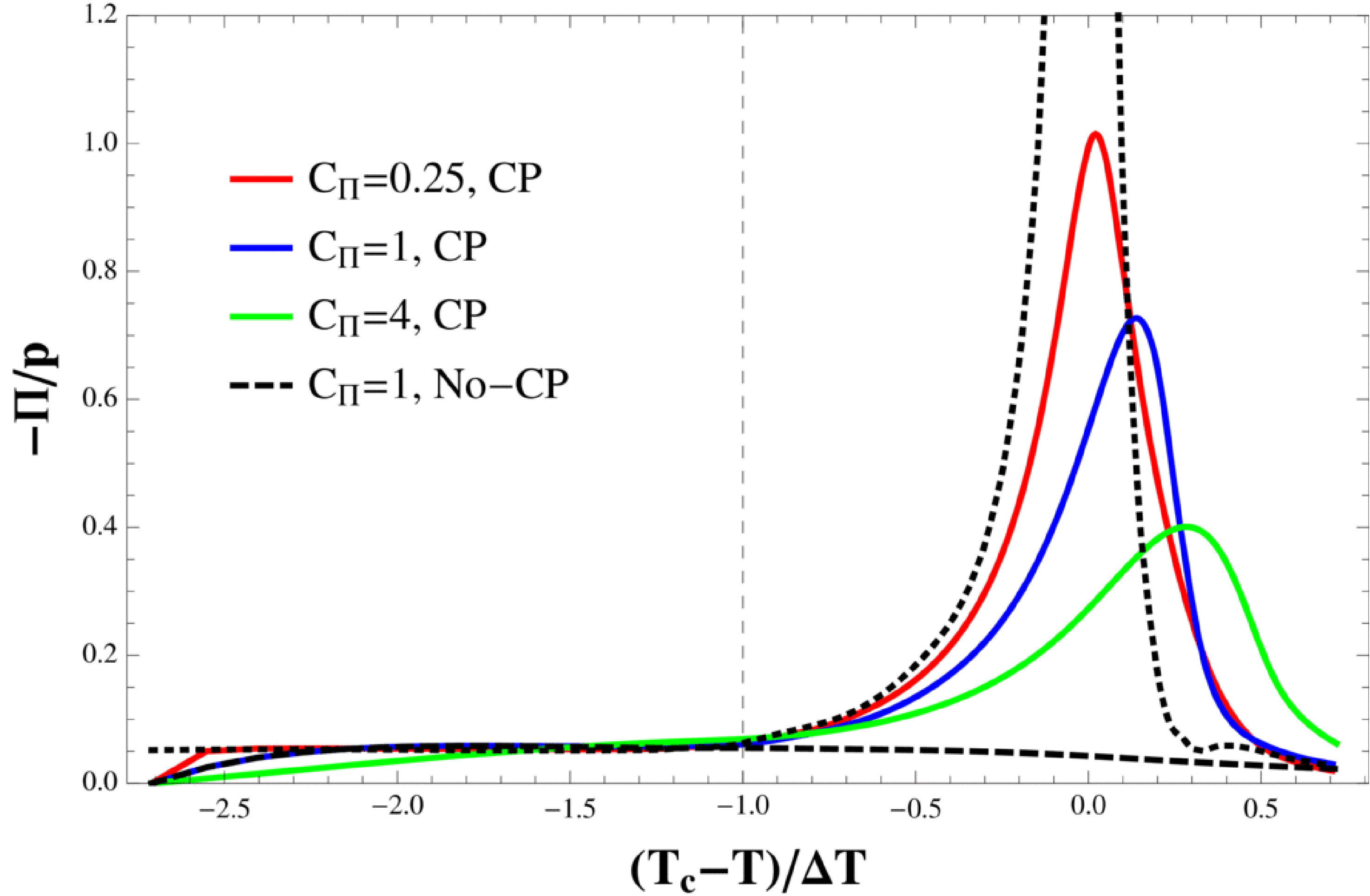
(Wikipedia)

relaxation of bulk pressure in causal relativ. hydrodynamics:

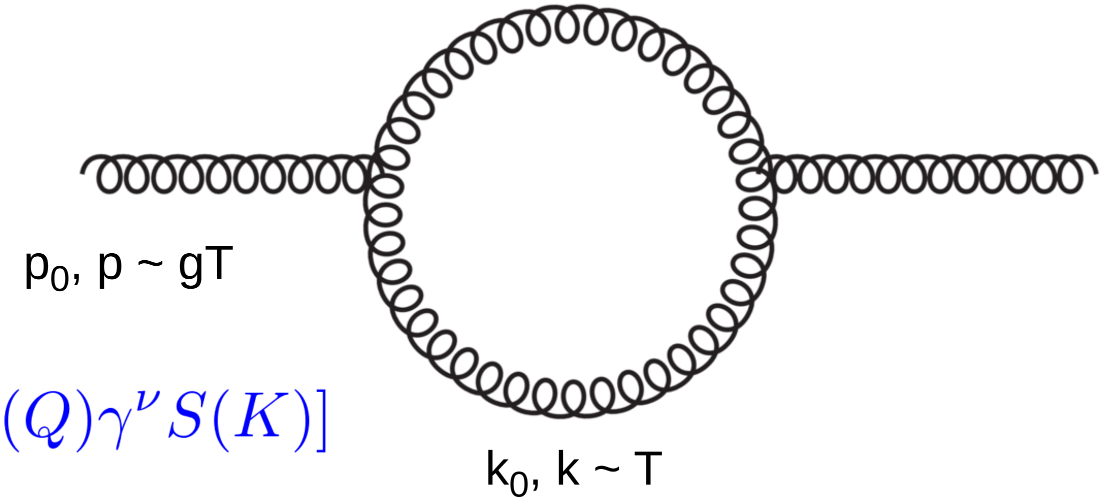
$$u^\mu \partial_\mu \Pi = -\frac{1}{\tau_\Pi}[\,\zeta \partial_\mu u^\mu + \Pi\,] \qquad \Pi \equiv \delta_{\rm bulk} p$$

$$\tau_\Pi \sim \xi^z$$

A. Monnai, S. Mukherjee, Y. Yin, arXiv:1606.00771

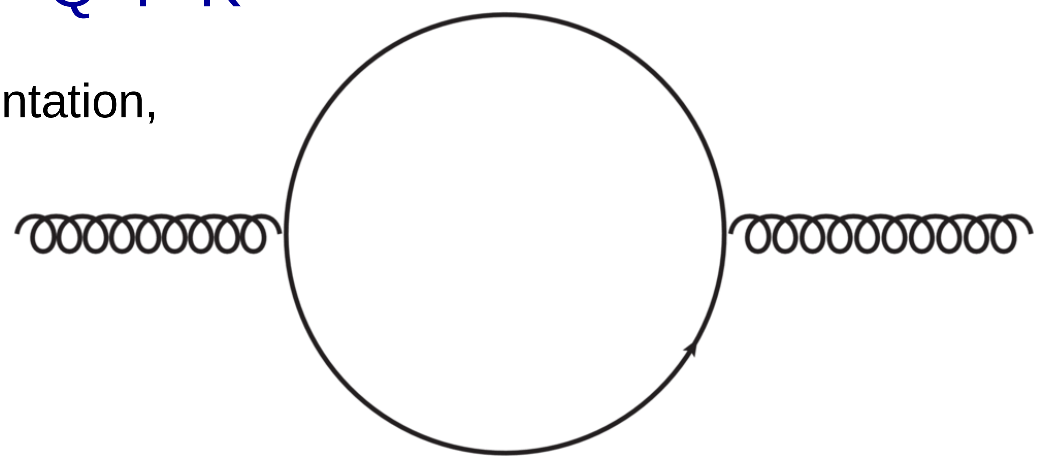


Hard thermal loops:
(real time formalism)



$$\Pi^{\mu\nu}(P) = -\frac{i}{2} N_f g^2 \int \frac{d^4K}{(2\pi)^4} tr\left[\gamma^\mu S(Q)\gamma^\nu S(K)\right]$$

Q=P-K



sum 11 and 12 components of Keldysh representation,
write $S(K) \equiv \not{K}\tilde{\Delta}(K)$:

$$\tilde{\Delta}_{R,A}(P) = \frac{1}{P^2 \pm i\mathrm{sgn}(p_0)\epsilon}$$

$$\tilde{\Delta}_F(P) = -2\pi i\,(1 - 2f_F(p))\,\delta(P^2)$$

$$\Pi^L_R(P) = \frac{4\pi N_f g^2}{(2\pi)^4} \int kdkd\Omega\, f_F(\mathbf{k}) \frac{1 - (\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2}{(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}} + \frac{p_0 + i\,\epsilon}{p})^2}$$

Gauge boson loop: replace fermion by boson distribution in equilibrium (thermal fixed point):

$$\Pi_R^{\rm id}(P) = \left(2N_c + N_f\left(1 + \frac{3\mu^2}{\pi^2T^2}\right)\right)\frac{g^2T^2}{6}\left(\frac{p_0}{2p}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon} - 1\right)$$

$$\equiv m_R^2\left(\frac{p_0}{2p}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon} - 1\right)$$

Similarly, the symmetric self energy is given by:

$$\Pi_F(P) = -4iN_fg^2\pi^2\int\frac{k^2dk}{(2\pi)^3}f_F(k)(1-f_F(k))\frac{2}{p}\Theta(p^2-p_0^2)$$

Gauge boson loop: replace $f_F(1\text{-}f_F)$ by $f_B(1+f_B)$

in equilibrium:

$$\Pi_F^{\rm id}(P) = -2\pi i\left(2N_c + N_f\left(1 + \frac{3\mu^2}{\pi^2T^2}\right)\right)\frac{g^2T^2}{6}\frac{T}{p}\Theta(p^2-p_0^2)$$

$$\equiv -2\pi i\; m_F^2\frac{T}{p}\Theta(p^2-p_0^2)$$

Bulk viscous corrections

- unlike the distributions at the thermal fixed point, non-equilibrium corrections are not universal
- work out specific examples such as

$$\delta_{\rm bulk} f(k) = \left(\frac{k}{T}\right)^a \Phi\, f_{\rm id}(k)(1 \pm f_{\rm id}(k))$$

$$\Phi \sim \frac{\delta_{\rm bulk} p}{p_{\rm eq}}$$ (may depend on T, N_c, N_f, ...)

$$|\Phi| \gg g^2$$ (neglect 2-loop corrections)

$$a > 0$$ (required for gluon loop to be a HTL)

retarded self energy Π_R(P): modified screening mass

$$\left(2N_c + N_f\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right)\right)\frac{g^2T^2}{6} \to$$

$$m_R^2 + \delta m_R^2 = \left(2N_c\left(1+c_R^{(g)}(a)\Phi\right)\right.$$
$$\left. + N_f\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right)\left(1+c_R^{(q)}(a,\tilde{\mu})\Phi\right)\right)\frac{g^2T^2}{6}$$

$$\tilde{\mu} \equiv \mu/T$$

$$c_R^{(g)}(a) = \frac{6}{\pi^2}\Gamma(2+a)\,\zeta(1+a)$$

$$c_R^{(q)}(a,\tilde{\mu}) = \frac{-6\Gamma(2+a)[\mathrm{Li}_{(1+a)}(-e^{-\tilde{\mu}}) + \mathrm{Li}_{(1+a)}(-e^{\tilde{\mu}})]}{\pi^2+3\tilde{\mu}^2}$$
$$\to \frac{12}{\pi^2}(1-2^{-a})\Gamma(2+a)\zeta(1+a) \quad (\text{as } \mu \to 0)$$

symmetric self energy Π_F(P):

$$\left(2N_c + N_f\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right)\right)\frac{g^2T^2}{6} \to$$

$$m_F^2 + \delta m_F^2 = \left(2N_c\left(1 + c_F^{(g)}(a)\Phi\right)\right.$$
$$\left. + N_f\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right)\left(1 + c_F^{(q)}(a,\tilde{\mu})\Phi\right)\right)\frac{g^2T^2}{6}$$

$$c_F^{(g)}(a) = \frac{3}{\pi^2}\Gamma(3+a)\,\zeta(1+a) = \frac{1}{2}(2+a)\,c_R^{(g)}(a)$$

$$c_F^{(q)}(a,\tilde{\mu}) = \frac{-3\Gamma(3+a)[\mathrm{Li}_{(1+a)}(-e^{-\tilde{\mu}}) + \mathrm{Li}_{(1+a)}(-e^{\tilde{\mu}})]}{\pi^2+3\tilde{\mu}^2}$$
$$= \frac{1}{2}(2+a)\,c_R^{(q)}(a,\tilde{\mu})$$

Notes:

- in equilibrium, $m_R^2 = m_F^2$ *increase* with the baryon-chemical potential $\mu_B = 3\mu$
- (negative) bulk pressure acts oppositely, $\Phi \sim O(1)$ may even “short out” self energies: $m_R^2, m_F^2 \sim 0$

Resummed propagators (for long. gluon):

- employ Coulomb gauge: $D^{0i}=0$, D^{00} independent of Π^{ij}
- Schwinger-Dyson equation, retarded propagator

$$
\begin{aligned}
\tilde{D}^*_R(P) &= D_R(P) + D_R(P)\,\tilde{\Pi}_R(P)\,\tilde{D}^*_R(P) \rightarrow \\
\tilde{D}^*_R(P) &= \frac{1}{p^2 - \tilde{\Pi}_R(P)}
\end{aligned}
$$

- symmetric (time ordered) propagator:

$$
\begin{aligned}
\tilde{D}^*_F(P) &= (1+2\tilde{f}(p_0))\,\mathrm{sgn}(p_0)\,[\tilde{D}^*_R(P) - \tilde{D}^*_A(P)] \\
&+ \tilde{D}^*_R(P) \\
&\quad \{\tilde{\Pi}_F(P) - [1+2\tilde{f}(p_0)]\,\mathrm{sgn}(p_0)\,[\tilde{\Pi}_R(P) - \tilde{\Pi}_A(P)]\} \\
&\quad \tilde{D}^*_A(P) \\
&= \tilde{D}^*_R(P)\,\tilde{\Pi}_F(P)\,\tilde{D}^*_A(P)
\end{aligned}
$$

Applications I: static potential

$$V(\mathbf{r}) = (ig)^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \left(\tilde{D}^*(p_0 = 0, \mathbf{p})\right)_{11}$$

$$= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{1}{2}\left(\tilde{D}^*_R + \tilde{D}^*_A + \tilde{D}^*_F\right)$$

$$\mathrm{Re}\, V(r) = -\frac{g^2 C_F}{4\pi r} e^{-\hat{r}}$$

$$\hat{r} \equiv r\sqrt{m_R^2 + \delta m_R^2}$$

(assuming $m_R^2+\delta m_R^2 > 0$)

$$\mathrm{Im}\, V(r) = -\frac{g^2 C_F T}{4\pi} \frac{m_F^2 + \delta m_F^2}{m_R^2 + \delta m_R^2} \phi(\hat{r})$$

$$\Gamma/2 = -\langle\psi|\mathrm{Im}\, V|\psi\rangle$$

Landau damping,
M. Laine et al: hep-ph/0611300
N. Brambilla et al: 0804.0993

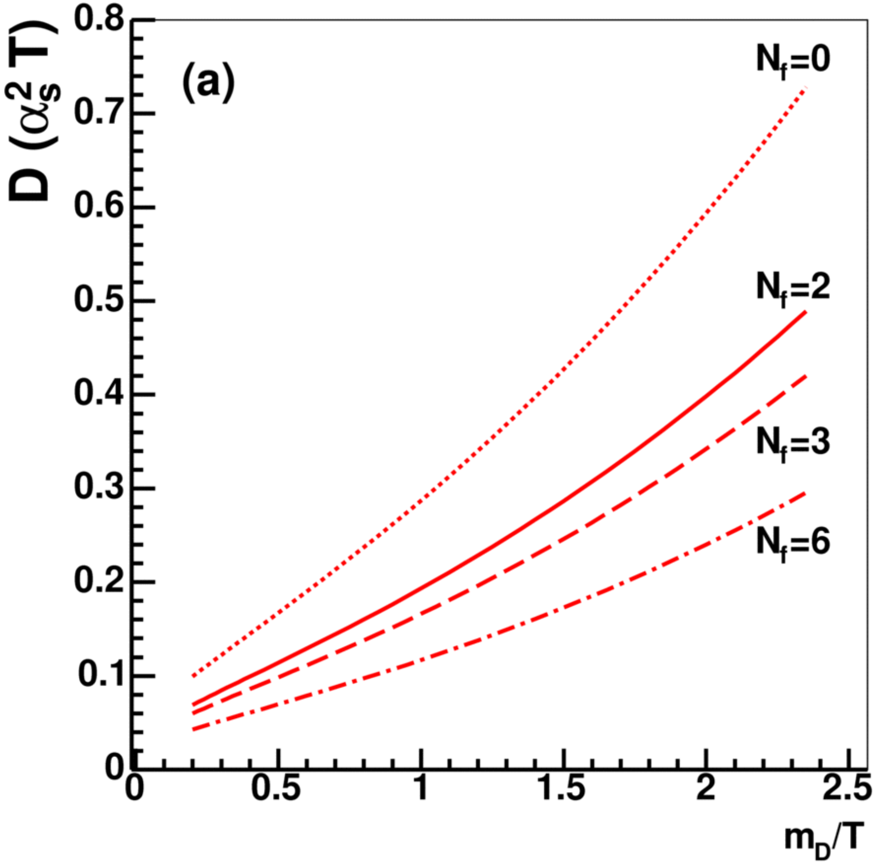
$$\phi(\hat{r}) \sim \hat{r}^2 \ln\frac{1}{\hat{r}} \quad \text{for } \hat{r} \ll 1$$

Applications II: heavy-quark diffusion

$$D = \frac{36\pi}{C_F\, g^4 T} \frac{1}{N_c \left(\log \frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right) + \frac{1}{2} N_f \left(\log \frac{4T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right)}$$

(in weak coupling limit, $m_D/T << 1$)

G. Moore and D. Teaney, hep-ph/0412346



- drop in screening mass → smaller diffusion constant D

Summary

- bulk viscosity is small at very high T: $\zeta/\eta \sim \alpha_s^4$ (up to logs)
- may be large near T_C though, especially near a critical point where $\zeta \sim \xi^{z-\alpha/\nu}$ diverges with some power of the correl. length
- $\rightarrow$ non-equilibrium corrections to Hard Thermal Loops (screening and damping)
- m_R^2 and m_F^2 sensitive to critical behavior of ζ
- affects static potential, heavy-Q diffusion, ...

Thank you !

Backup Slides

Real-time finite-T propagators (free):

(diagonal in color, color indices omitted)

$$D(P) = \begin{pmatrix} \frac{1}{P^2-m^2+i\epsilon} & 0 \\ 0 & \frac{-1}{P^2-m^2-i\epsilon} \end{pmatrix}$$

$$-2\pi i\,\delta(P^2-m^2)\begin{pmatrix} f_B & \Theta(-p_0)+f_B \\ \Theta(p_0)+f_B & f_B \end{pmatrix}$$

$$D_R(P) = D_{11}(P) - D_{12}(P) \quad , \quad D_A(P) = D_{11}(P) - D_{21}(P)$$

$$D_F(P) = D_{11}(P) + D_{22}(P)$$

$$S(P) = (\not{P}+m)\left[\begin{pmatrix} \frac{1}{P^2-m^2+i\epsilon} & 0 \\ 0 & \frac{-1}{P^2-m^2-i\epsilon} \end{pmatrix}\right.$$

$$\left.+2\pi i\,\delta(P^2-m^2)\begin{pmatrix} f_F & -\Theta(-p_0)+f_F \\ -\Theta(p_0)+f_F & f_F \end{pmatrix}\right]$$

Kubo formula:

$$\zeta = \frac{1}{2} \lim_{\omega \to 0^+} \frac{1}{\omega} \int_{-\infty}^{\infty} dt e^{-i\omega t} \int d^3x \left\langle \left[\tfrac{1}{3} T_i^i(x,t), \tfrac{1}{3} T_i^i(0,0) \right] \right\rangle$$

spectral function:

$$\rho(\omega) \equiv \frac{1}{9} \int dt e^{-i\omega t} \int d^3x \left\langle \left[T_\mu^\mu(x,t), T_\nu^\nu(0,0) \right] \right\rangle$$

$$\zeta = \frac{1}{2} \lim_{\omega \to 0^+} \rho(\omega)/\omega$$

$$T_\mu^\mu \to \frac{\beta}{g^4} \mathrm{tr}\, F^2$$

see G.D. Moore and O. Saremi, JHEP 09, 015 (2008)