TRACES OF NON-EQUILIBRIUM DYNAMICS IN RELATIVISTIC HEAVY-ION COLLISIONS

Pierre Moreau

In collaboration with: Y. Xu, T. Song, M. Nahrgang, S. A. Bass and E. Bratkovskaya

Based on: Phys. Rev. C, 2017 (arXiv:1703.09178 [nucl-th])

CPOD 2017 - Critical Point and Onset of Deconfinement,

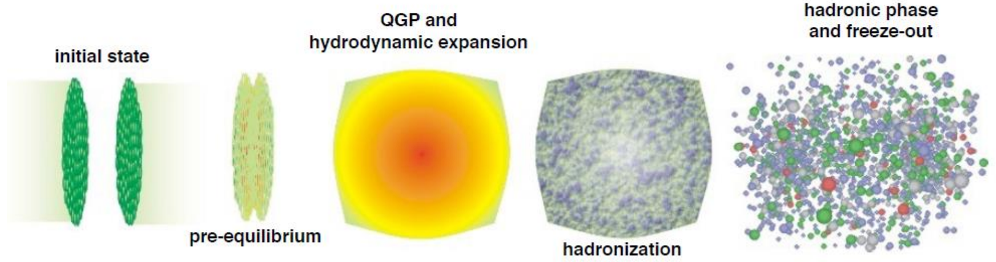
Stony Brook NY, USA

moreau@fias.uni-frankfurt.de

Description of relativistic heavy-ion collisions

Two types of model have been successful in describing relativistic heavy-ion collisions:

- **Hybrid approaches: Hydro description of the QGP + microscopic transport for the hadronic sector**



  - **Simplified dynamics, assumption of local equilibrium**
  - **Direct access to QGP properties in equilibrium**

- **Transport approaches: full microscopic description of heavy-ion collisions**
  - **No assumptions about local equilibrium**
  - **Access to the QGP degrees of freedom**

- **Can we identify which model features reflect the actual physical nature of the QGP ?**
- **Is there any differences in the dynamical evolution of the system that can be attribtuted to non-equilibrium effects ?**

2D+1 viscous hydrodynamics

- **Space-time evolution of the QGP via conservation equations:**

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = e\, u^\mu u^\nu - \Delta^{\mu\nu}(P + \Pi) + \pi^{\mu\nu}$$

- u^μ : cell 4-velocity
- e : local energy density
- P: local isotropic pressure
- $\pi^{\mu\nu}$: shear stress tensor
- Π: bulk viscous pressure
- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

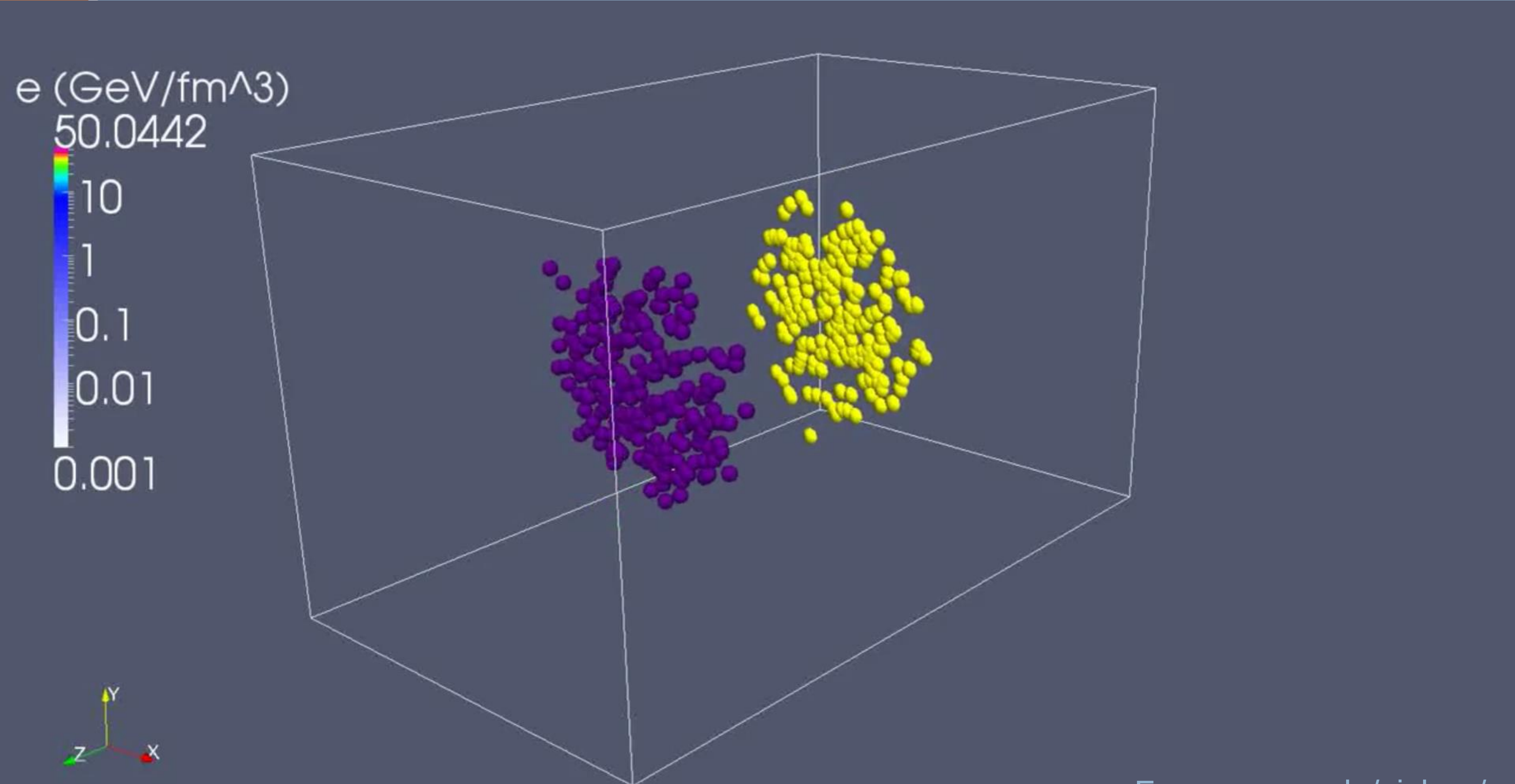
- **For this study we use VISH2+1: time evolution of the viscous corrections through the 2nd order Israel-Stewart equations**
  - η: shear viscosity
  - ζ : bulk viscosity

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \phi_1\Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \phi_3\pi^{\mu\nu}\pi_{\mu\nu}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}w^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \phi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \phi_6\Pi\pi^{\mu\nu}$$

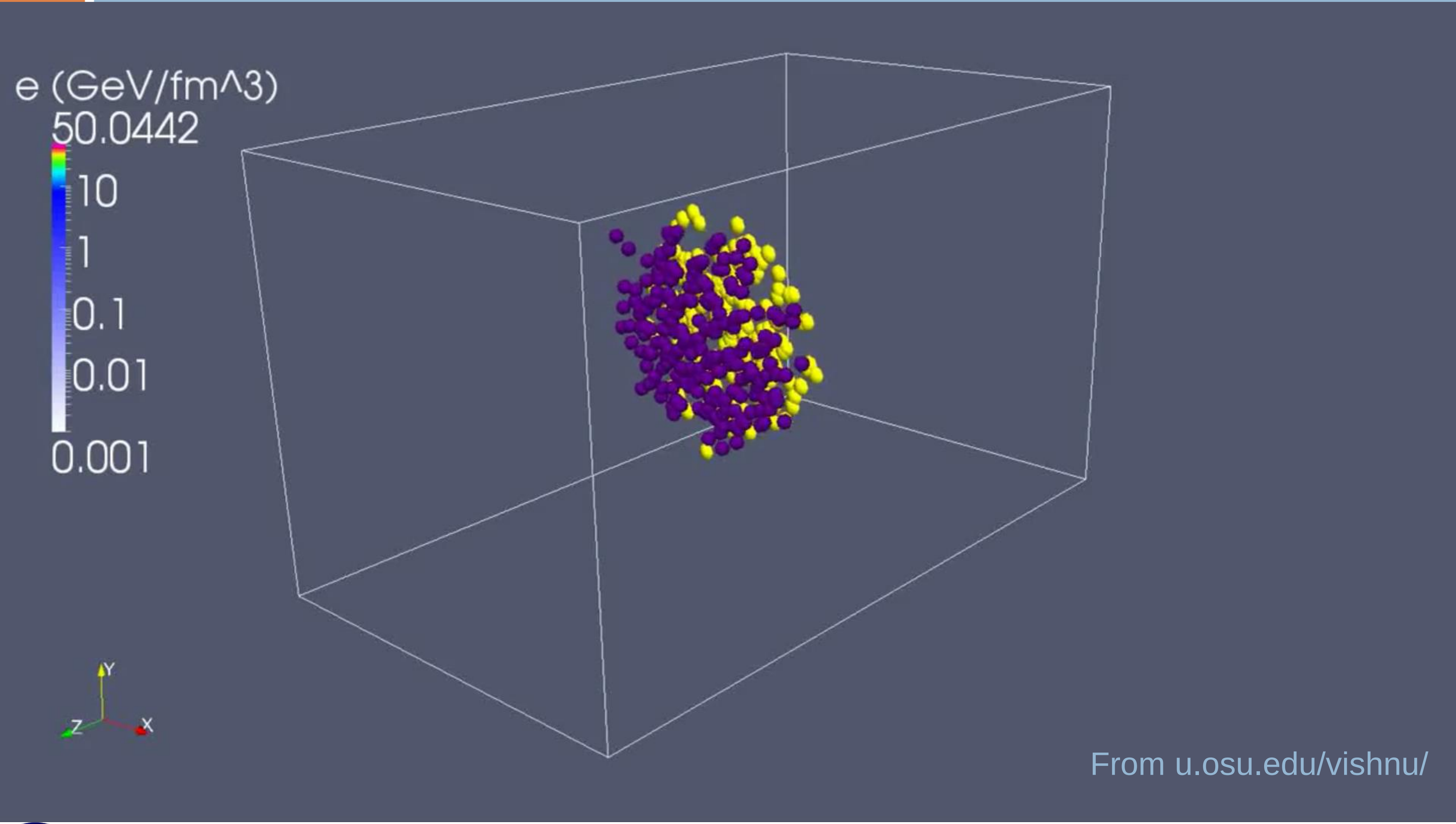
C. Shen, Z. Qiu, H. Song, J. Bernhard, S. Bass and U. Heinz, Comput.Phys.Commun. 199 (2016) 61-85

Stages of a collision in VISHNU



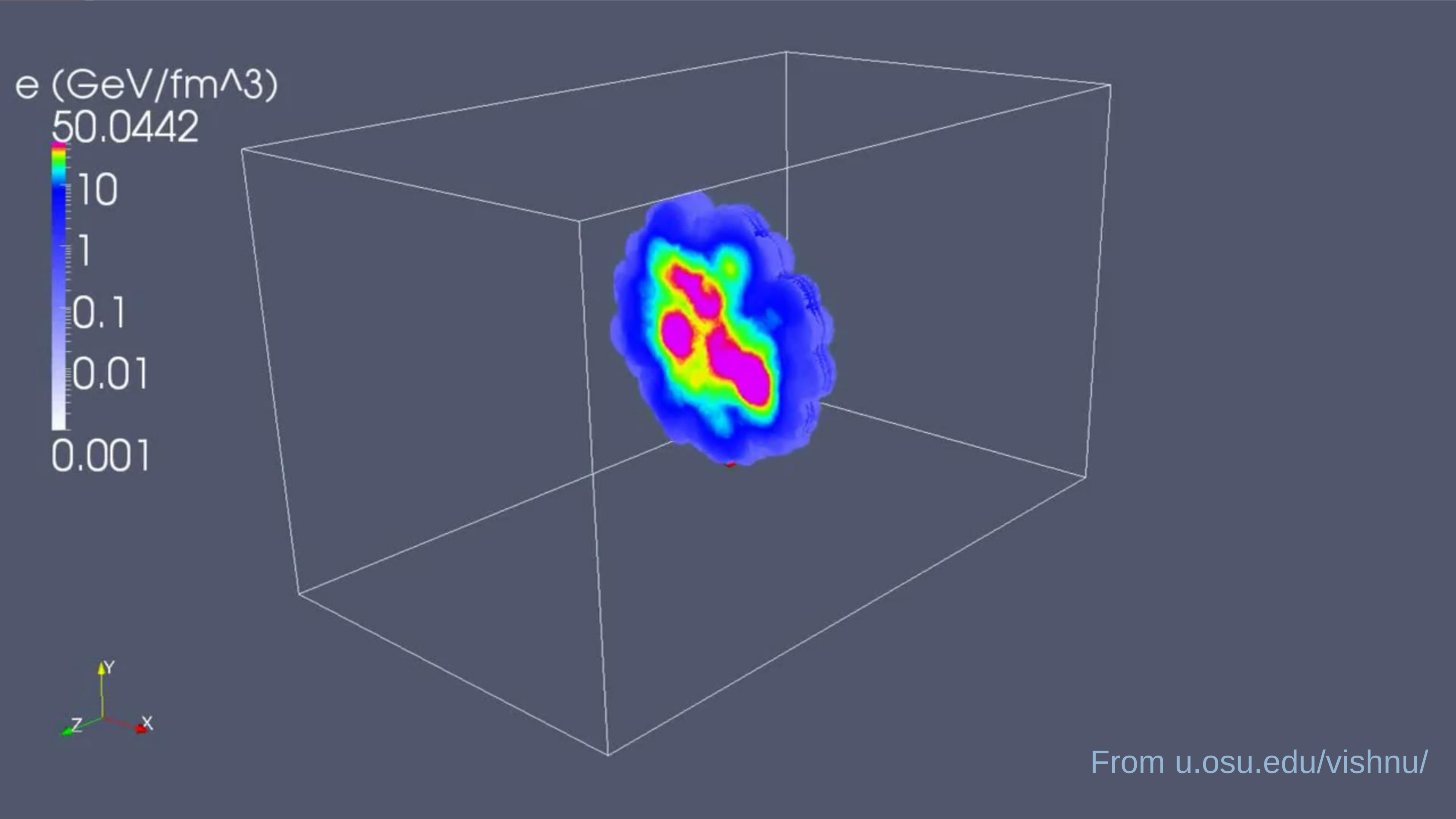
From u.osu.edu/vishnu/

Stages of a collision in VISHNU



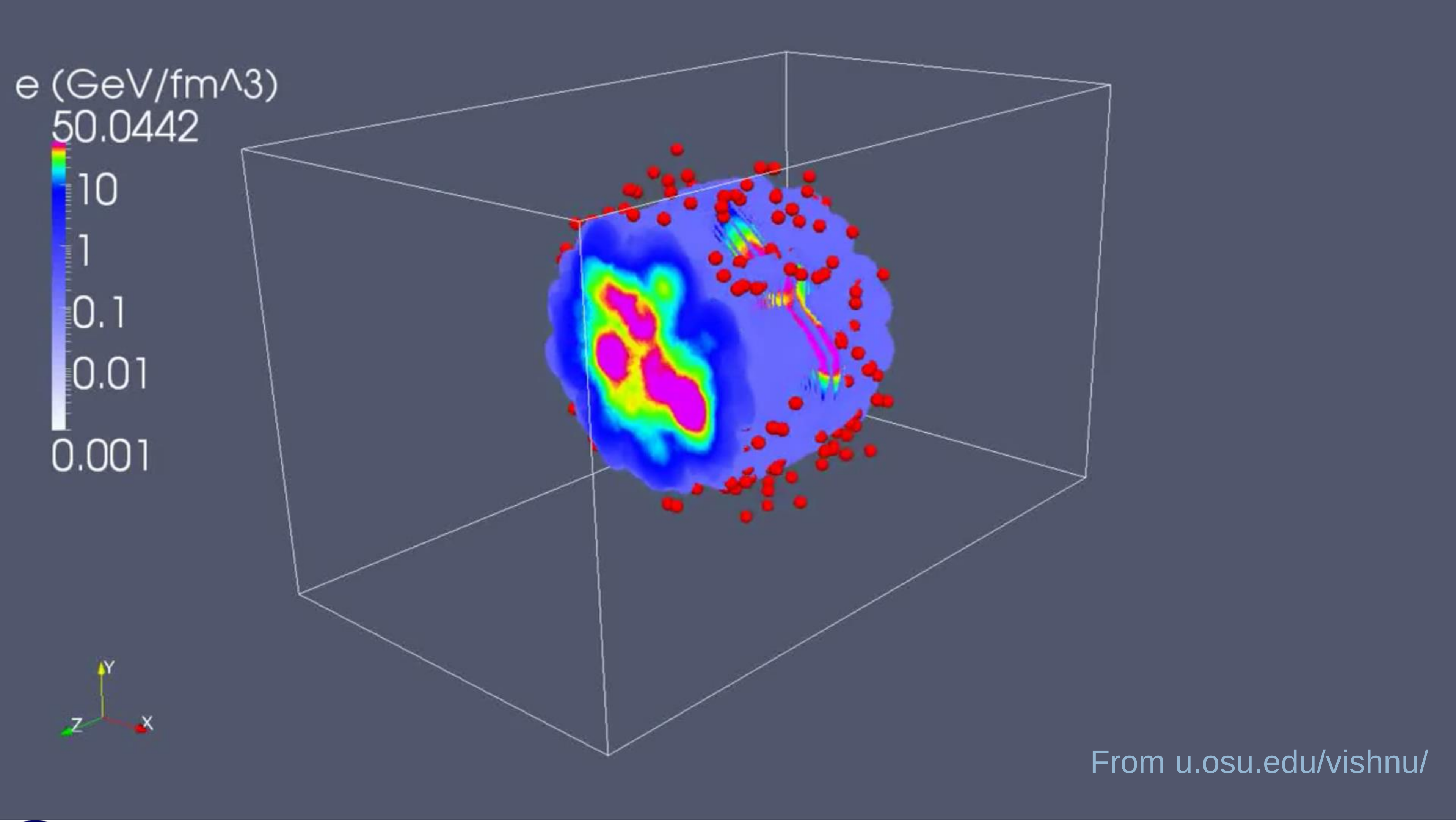
From u.osu.edu/vishnu/

Stages of a collision in VISHNU



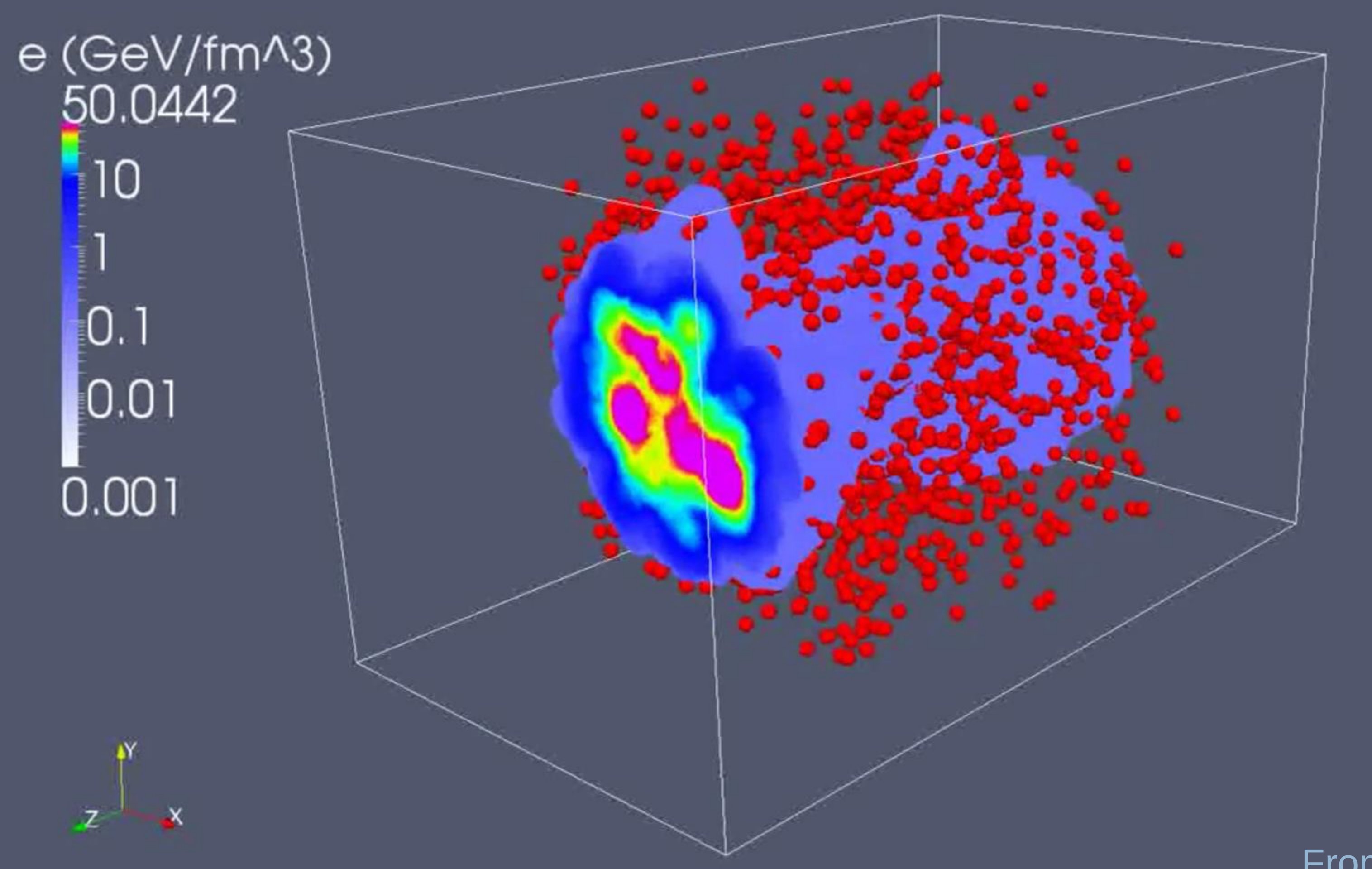
From u.osu.edu/vishnu/

Stages of a collision in VISHNU



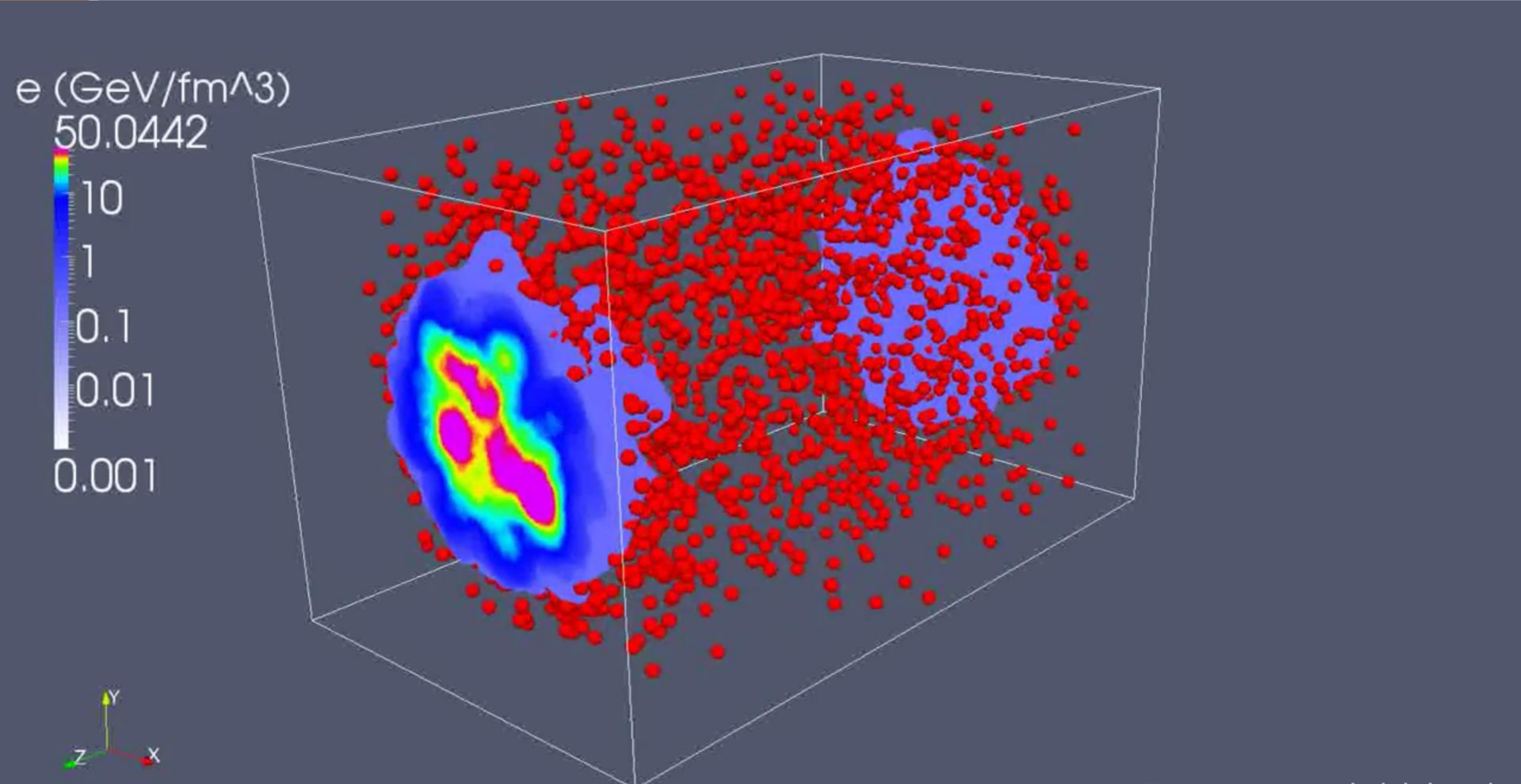
From u.osu.edu/vishnu/

Stages of a collision in VISHNU



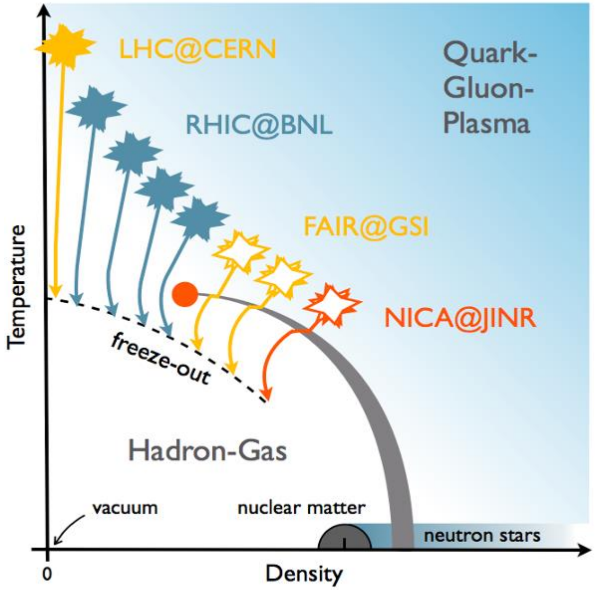
From u.osu.edu/vishnu/

Stages of a collision in VISHNU



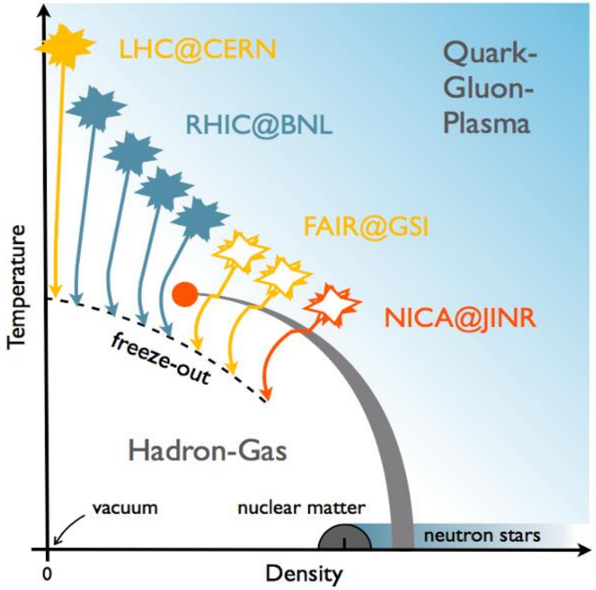
From u.osu.edu/vishnu/

The PHSD transport approach



- **Goal: Study the properties of strongly interacting matter under extreme conditions from a microscopic point of view**
- **Realization: dynamical many-body transport approach**

The PHSD transport approach



- **Goal:** Study the properties of **strongly interacting matter under extreme conditions from a microscopic point of view**
- **Realization: dynamical many-body transport approach**

Parton-Hadron-String-Dynamics (PHSD)

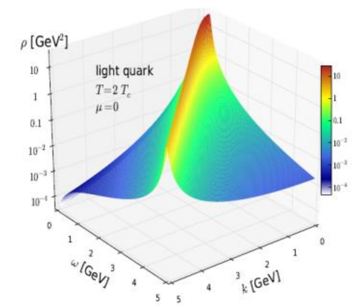
- **Explicit parton-parton interactions, explicit phase transiton from hadronic to partonic degrees of freedom**
- **Transport theory: off-shell transport equations in phase-space representation based on Kadanoff-Baym equations for the partonic and hadronic phase**

W.Cassing, E.Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W.Cassing, EPJ ST 168 (2009) 3

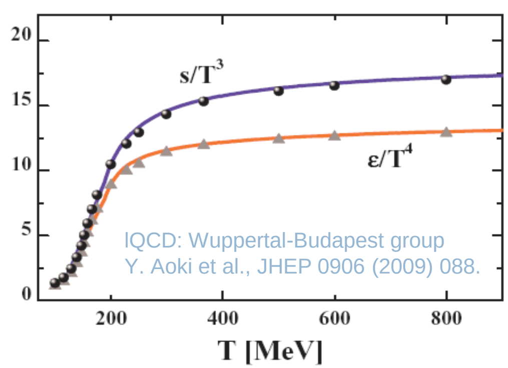
Dynamical Quasi-Particle Model (DQPM)

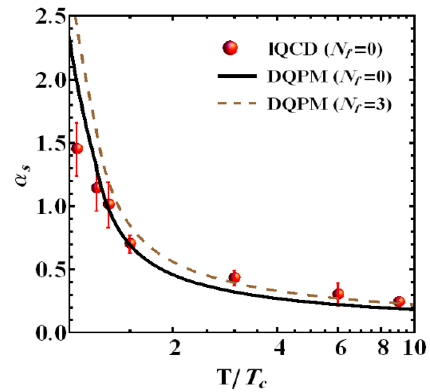
- **The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:**

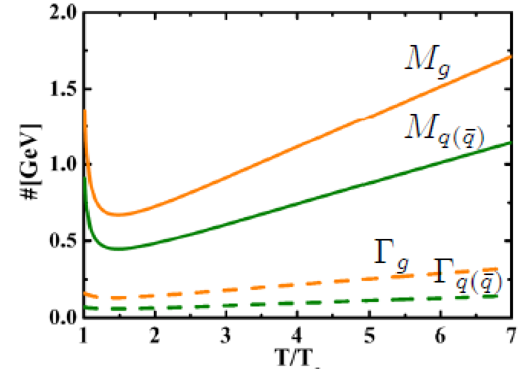
$$\rho_i(\omega, T) = \frac{4\omega\Gamma_i(T)}{(\omega^2 - \mathbf{p}^2 - M_i^2(T))^2 + 4\omega^2\Gamma_i^2(T)} \qquad (i = q, \bar{q}, g)$$



- **Properties of quasiparticles (large widths and masses) are fitted to the lattice QCD results**





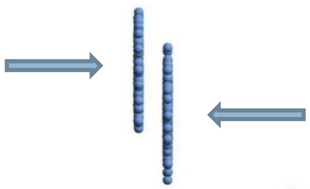


- **DQPM provides mean-fields (1P1) for quarks and gluons as well as effective 2-body interactions (2P1)**

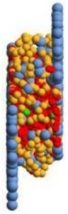
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Stages of a collision in PHSD

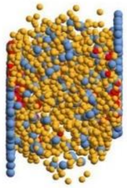
Initial A+A collision



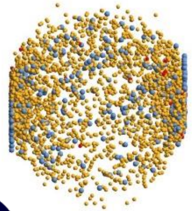
Partonic phase



Hadronization

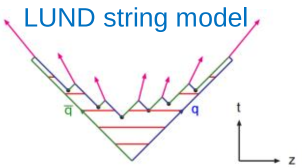


Hadronic phase



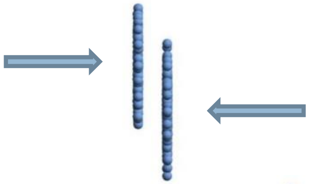
- **String formation in primary NN collisions**

→ **decays to pre-hadrons (baryons and mesons)**

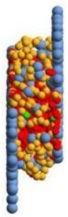


Stages of a collision in PHSD

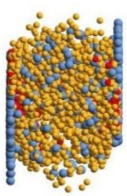
Initial A+A collision



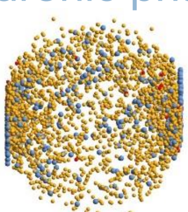
Partonic phase



Hadronization



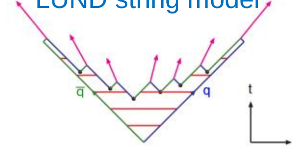
Hadronic phase



- **String formation in primary NN collisions**

→ decays to pre-hadrons (baryons and mesons)

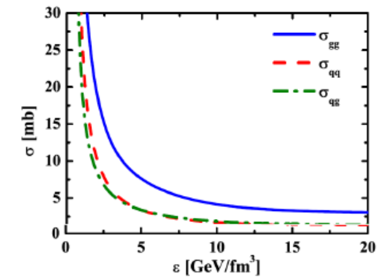
LUND string model



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**

Dissolution of pre-hadrons → DQPM

→ massive quarks/gluons and mean-field energy



(quasi-)elastic collisions :

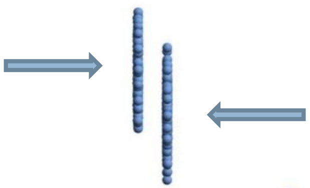
$$q+q \to q+q \qquad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \qquad g+g \to g+g$$

inelastic collisions :

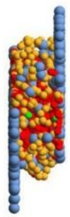
$$q+\bar{q} \to g \qquad q+\bar{q} \to g+g$$
$$g \to q+\bar{q} \qquad g \to g+g$$

Stages of a collision in PHSD

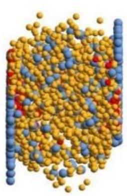
Initial A+A collision



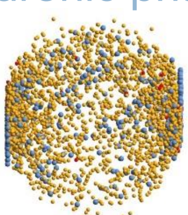
Partonic phase



Hadronization

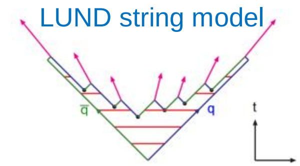


Hadronic phase



- **String formation in primary NN collisions**

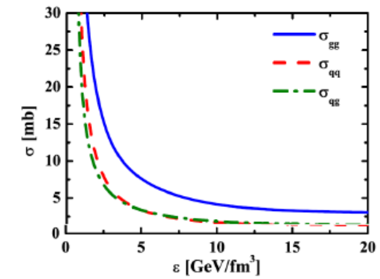
  → decays to pre-hadrons (baryons and mesons)



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**

  Dissolution of pre-hadrons → DQPM

  → massive quarks/gluons and mean-field energy



(quasi-)elastic collisions :

$$q+q \to q+q \qquad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \qquad g+g \to g+g$$

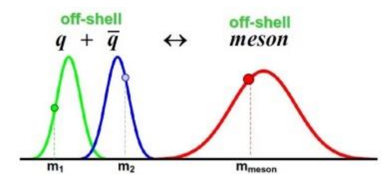
inelastic collisions :

$$q+\bar{q} \to g \qquad q+\bar{q} \to g+g$$
$$g \to q+\bar{q} \qquad g \to g+g$$

- **Hadronization to colorless off-shell mesons and baryons**

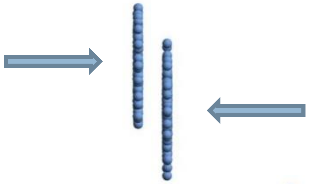
$$g \to q+\bar{q}, \quad q+\bar{q} \leftrightarrow meson\ ('\,string\,')$$
$$q+q+q \leftrightarrow baryon\ ('\,string\,')$$

Strict 4-momentum and quantum number conservation

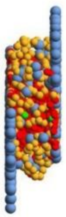


Stages of a collision in PHSD

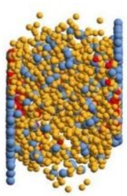
Initial A+A collision



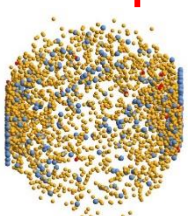
Partonic phase



Hadronization

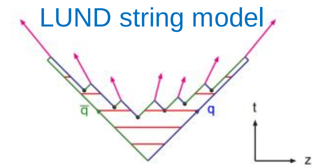


Hadronic phase



- **String formation in primary NN collisions**

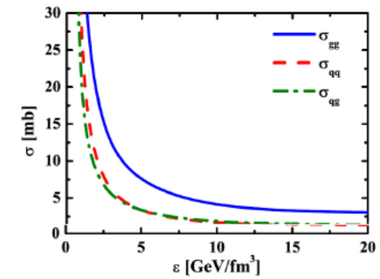
→ decays to pre-hadrons (baryons and mesons)



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**

Dissolution of pre-hadrons → DQPM

→ massive quarks/gluons and mean-field energy



(quasi-)elastic collisions :

$$q+q \rightarrow q+q \qquad g+q \rightarrow g+q$$
$$q+\bar{q} \rightarrow q+\bar{q} \qquad g+\bar{q} \rightarrow g+\bar{q}$$
$$\bar{q}+\bar{q} \rightarrow \bar{q}+\bar{q} \qquad g+g \rightarrow g+g$$

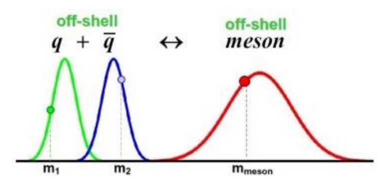
inelastic collisions :

$$q+\bar{q} \rightarrow g \qquad q+\bar{q} \rightarrow g+g$$
$$g \rightarrow q+\bar{q} \qquad g \rightarrow g+g$$

- **Hadronization to colorless off-shell mesons and baryons**

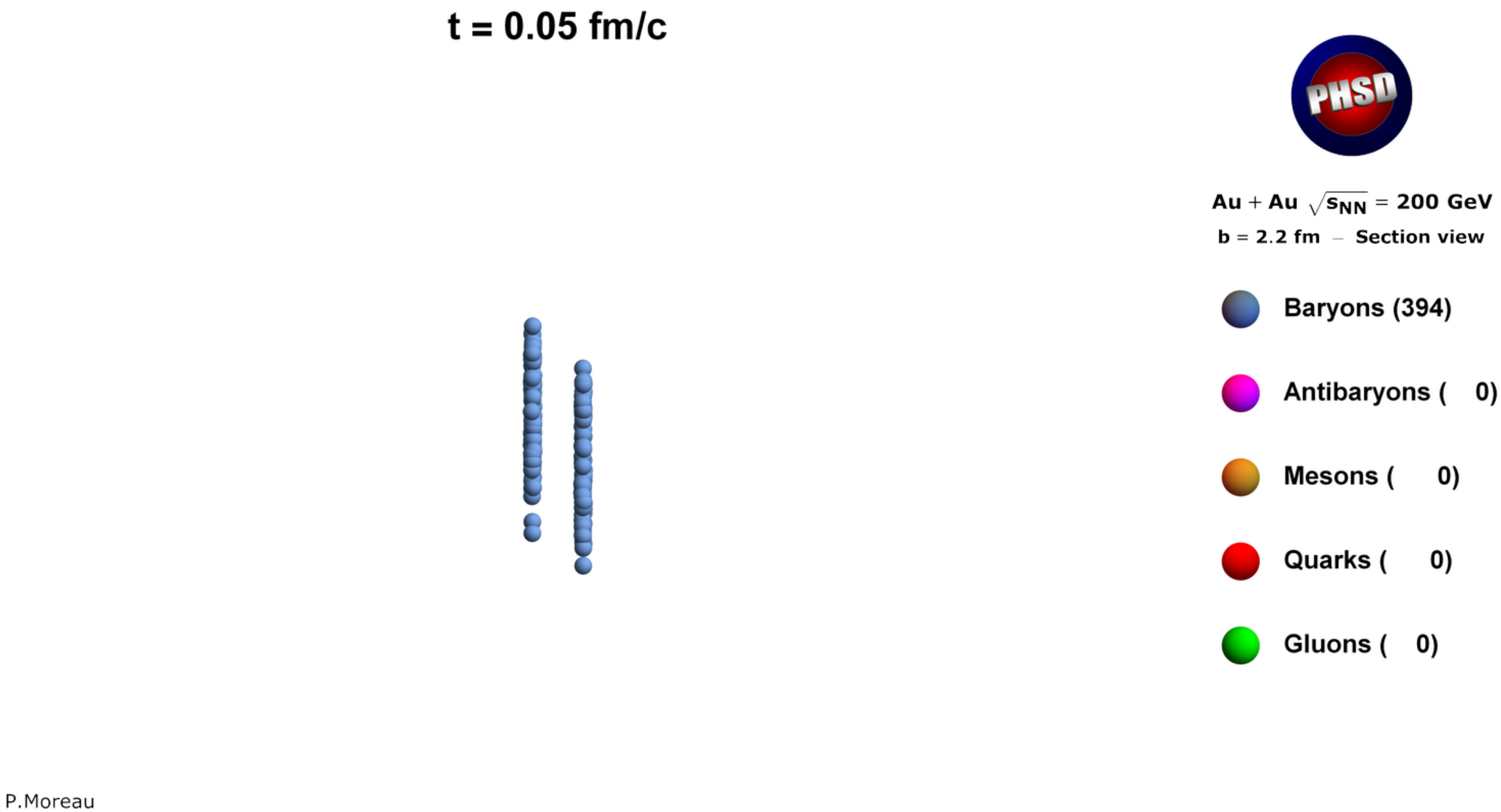
$$g \rightarrow q+\bar{q}, \qquad q+\bar{q} \leftrightarrow meson \ ('string')$$
$$q+q+q \leftrightarrow baryon \ ('string')$$

Strict 4-momentum and quantum number conservation

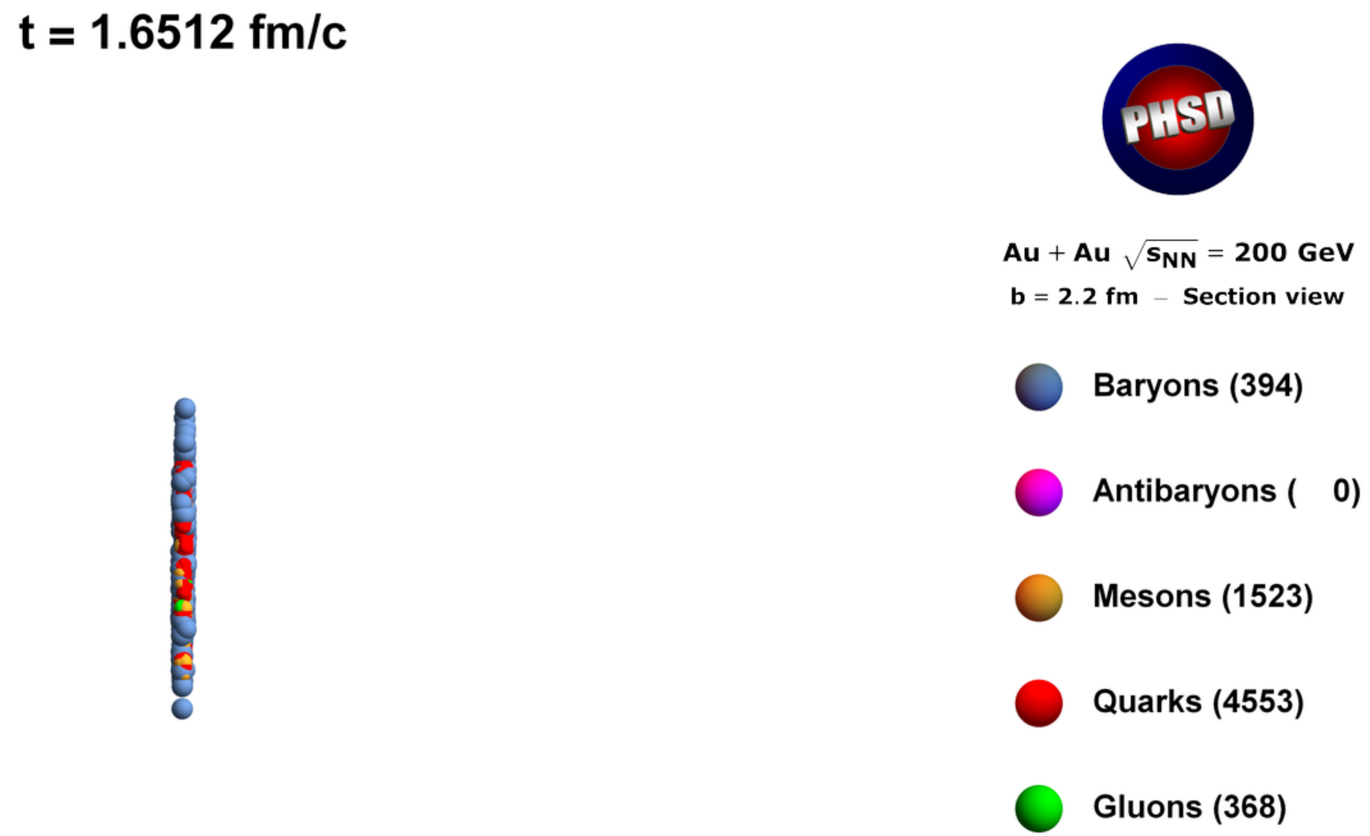


- **Hadron-string interactions – off-shell HSD**

Stages of a collision in PHSD

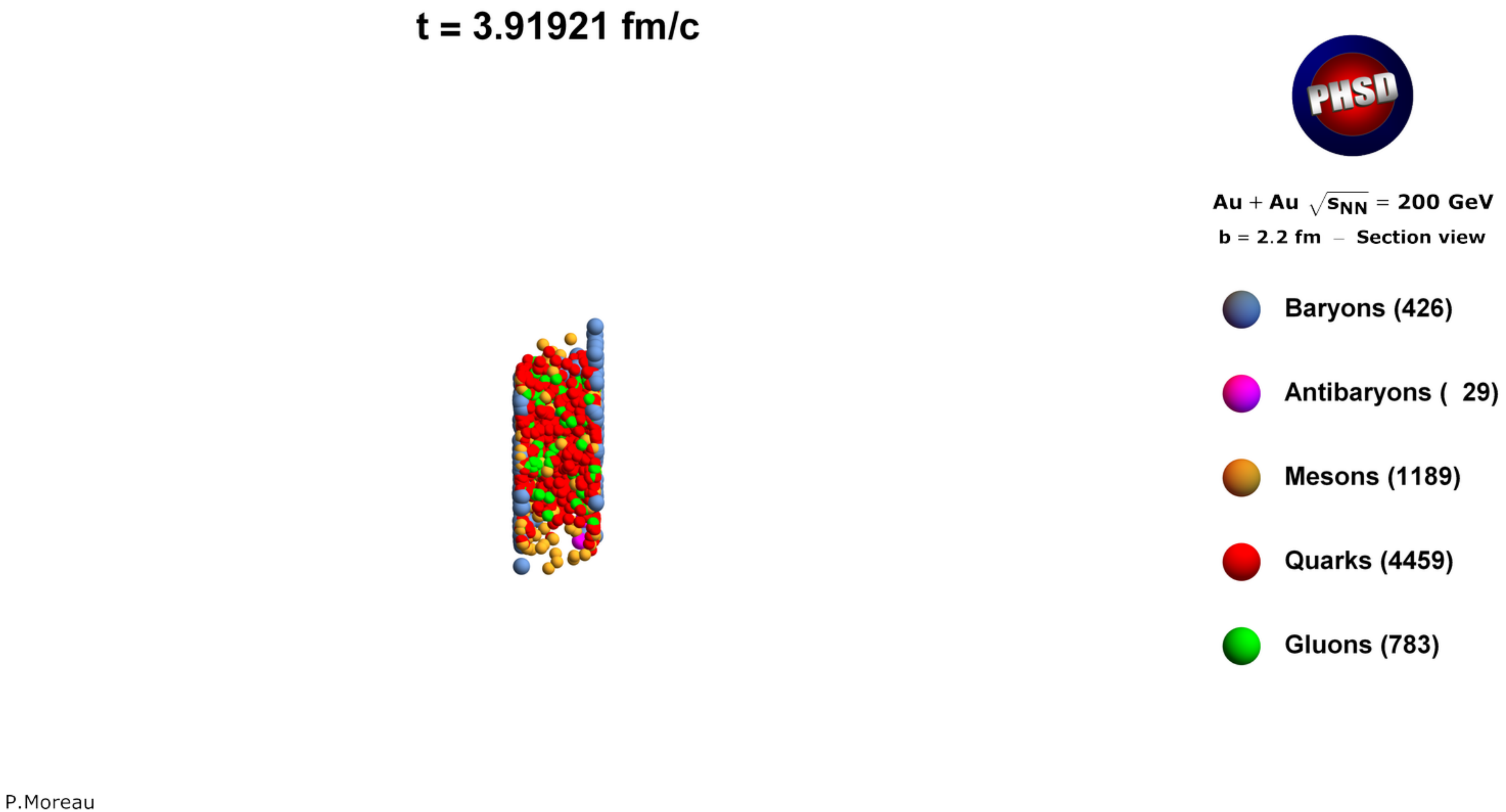


Stages of a collision in PHSD

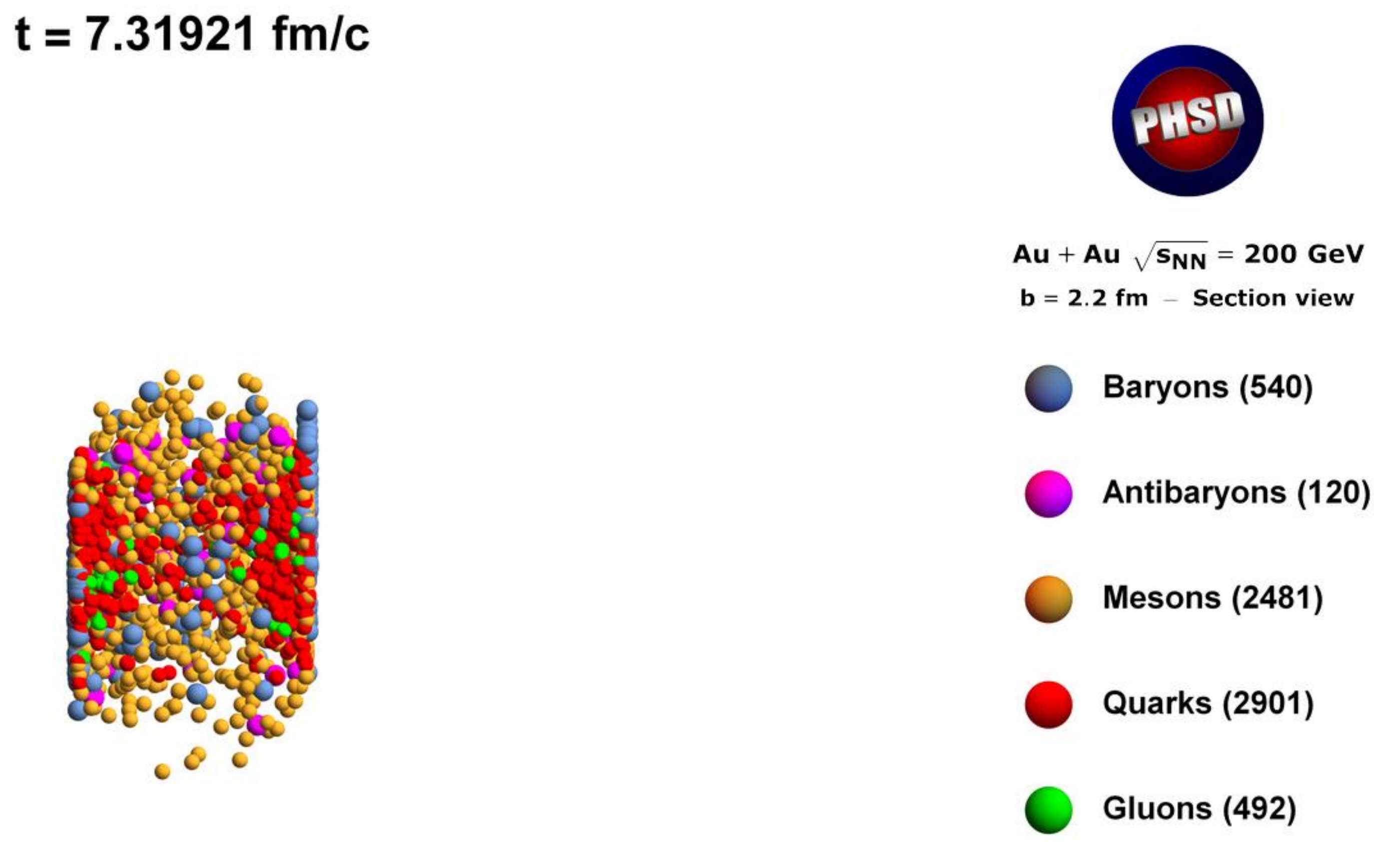


P.Moreau

Stages of a collision in PHSD

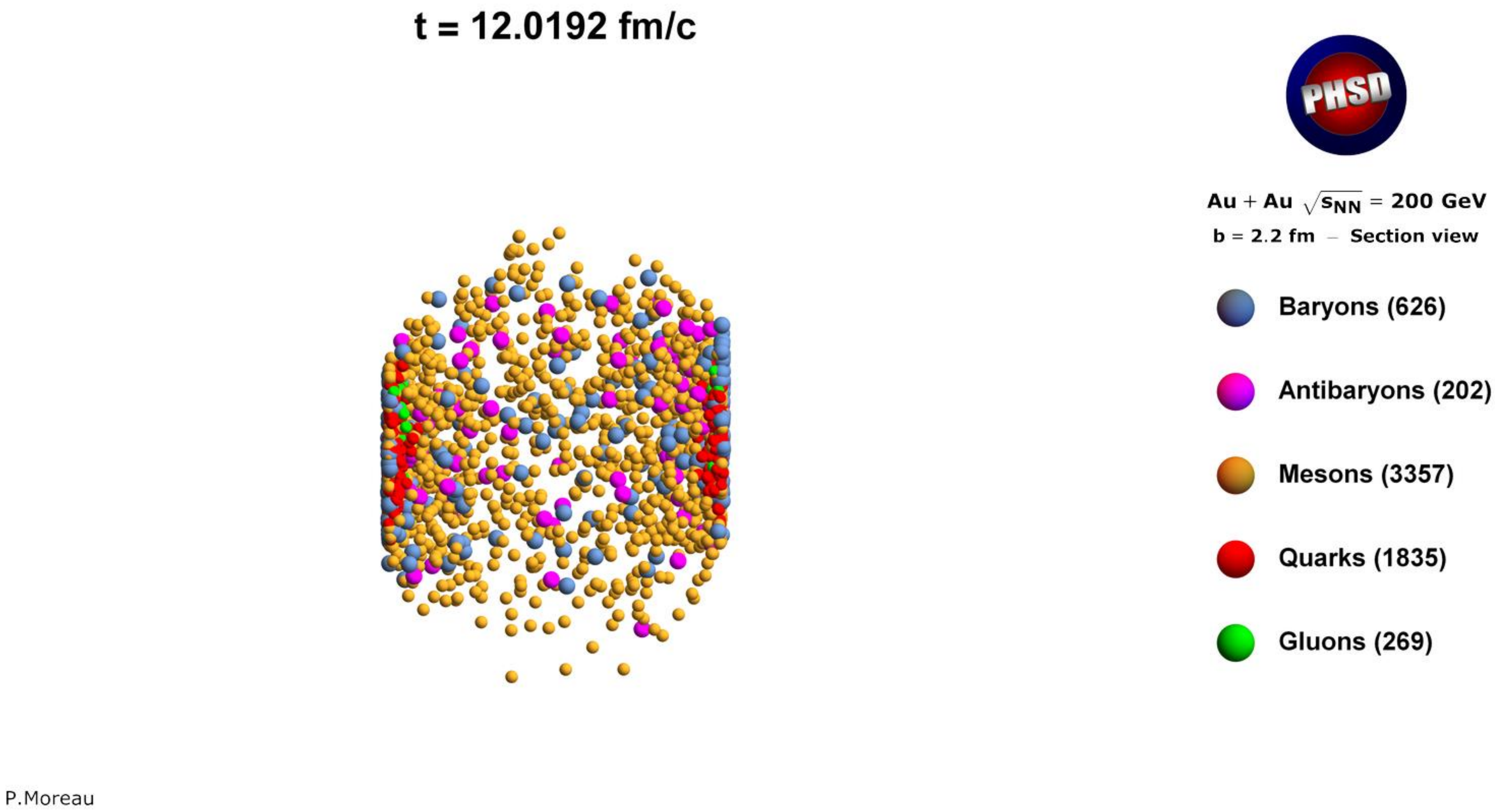


Stages of a collision in PHSD

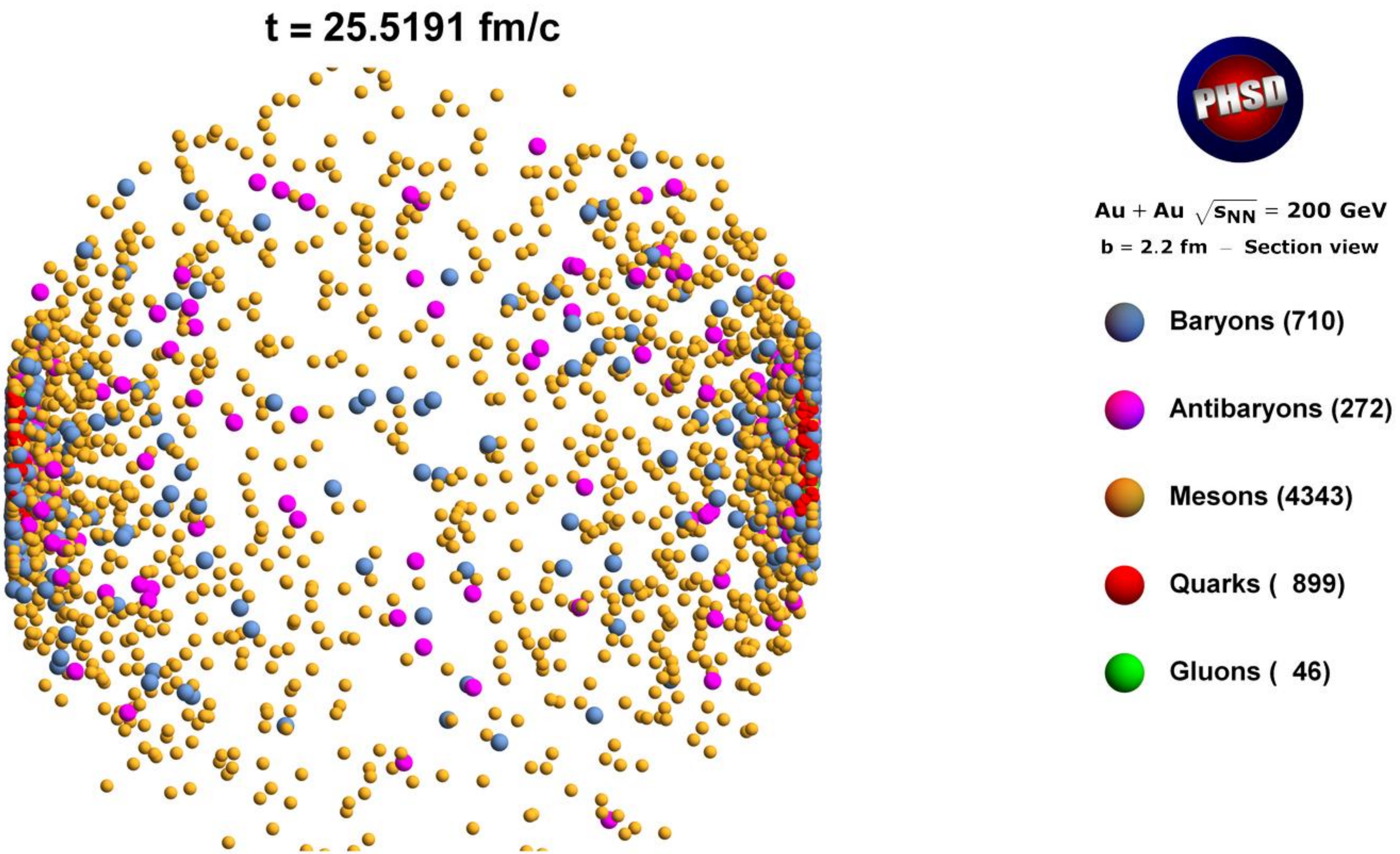


P.Moreau

Stages of a collision in PHSD



Stages of a collision in PHSD



P.Moreau

Coarse graining of PHSD

- **Goal: Initialize the hydro with a non-equilibrium profile from PHSD to compare the both evolutions**
- **Energy-momentum tensor should have the form:**

$$T^{\mu\nu} = e\, u^{\mu} u^{\nu} - \Delta^{\mu\nu}(P + \Pi) + \pi^{\mu\nu}$$

  - **The viscous corrections should be small**
  - **Spatial gradients should not be too large**

- **Coarse graining of PHSD medium in the transverse plane at the midrapidity region ($z \approx 0.$ fm):**

$$T^{\mu\nu}(x) = \sum_i \int_0^{\infty} \frac{d^3 p_i}{(2\pi)^3} f_i(E_i) \frac{p_i^{\mu} p_i^{\nu}}{E_i} = \frac{1}{V} \sum_i \frac{p_i^{\mu} p_i^{\nu}}{E_i}$$

$$\Delta x = \Delta y = 1 \text{ fm}$$

Evaluation of the energy momentum tensor

- **Diagonalization of the energy-momentum tensor**

$$T^{\mu\nu}\ (x_\nu)_i = \lambda_i\ (x^\mu)_i = \lambda_i\ g^{\mu\nu}\ (x_\nu)_i$$

- **Landau-matching condition:**

$$T^{\mu\nu}u_\nu = eu^\mu = (eg^{\mu\nu})u_\nu$$

- **Evaluation of the characteristic polynomial:**

$$P(\lambda) = \begin{vmatrix} T^{00}-\lambda & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11}+\lambda & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22}+\lambda & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33}+\lambda \end{vmatrix}$$

- **The four solutions λ_i are identified to $(e, -P_1, -P_2, -P_3)$**

The pressure components P_i do not necessarily correspond to (P_x, P_y, P_z)

Evaluation of the energy momentum tensor

- **Using the Landau-matching condition we have :**

$$\begin{cases} (T^{00} - e) + T^{01}X + T^{02}Y + T^{03}Z = 0 \\ T^{10} + (T^{11} + e)X + T^{12}Y + T^{13}Z = 0 \\ T^{20} + T^{21}X + (T^{22} + e)Y + T^{23}Z = 0 \\ T^{30} + T^{31}X + T^{32}Y + (T^{33} + e)Z = 0 \end{cases}$$

- **With the 4-velocity** $u_\nu = \gamma\,(1, X, Y, Z) = \gamma\,(1, -\beta_x, -\beta_y, -\beta_z)$

- **Evaluation of viscous corrections:**

$$\Pi = -\frac{1}{3}\,\Delta_{\mu\nu}T^{\mu\nu} - \mathcal{P}_{\mathrm{EoS}}$$

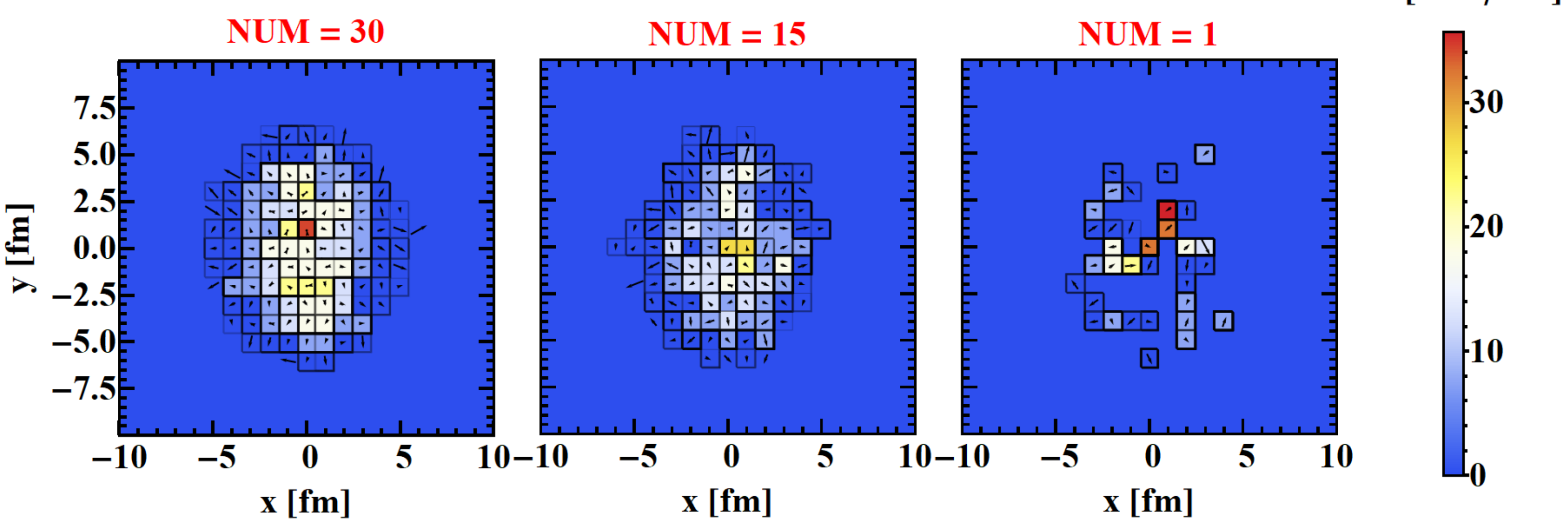
$$\pi^{\mu\nu} = T^{\mu\nu} - eu^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P}_{\mathrm{EoS}} + \Pi)$$

Liu et al., **Phys. Rev. C 91, 064906 (2015)**

Dependance on the parallel ensembles

- In order to have a smooth mean-field potential to propagate the particles and to initialize the hydro evolution, we need to average PHSD events over *N* parallel ensembles.
- The more parallel ensembles (NUM) are considered, the smoother the obtained profile is:

PHSD: Au+Au @ 200 GeV with b = 6 fm: t = 0.6 fm/c

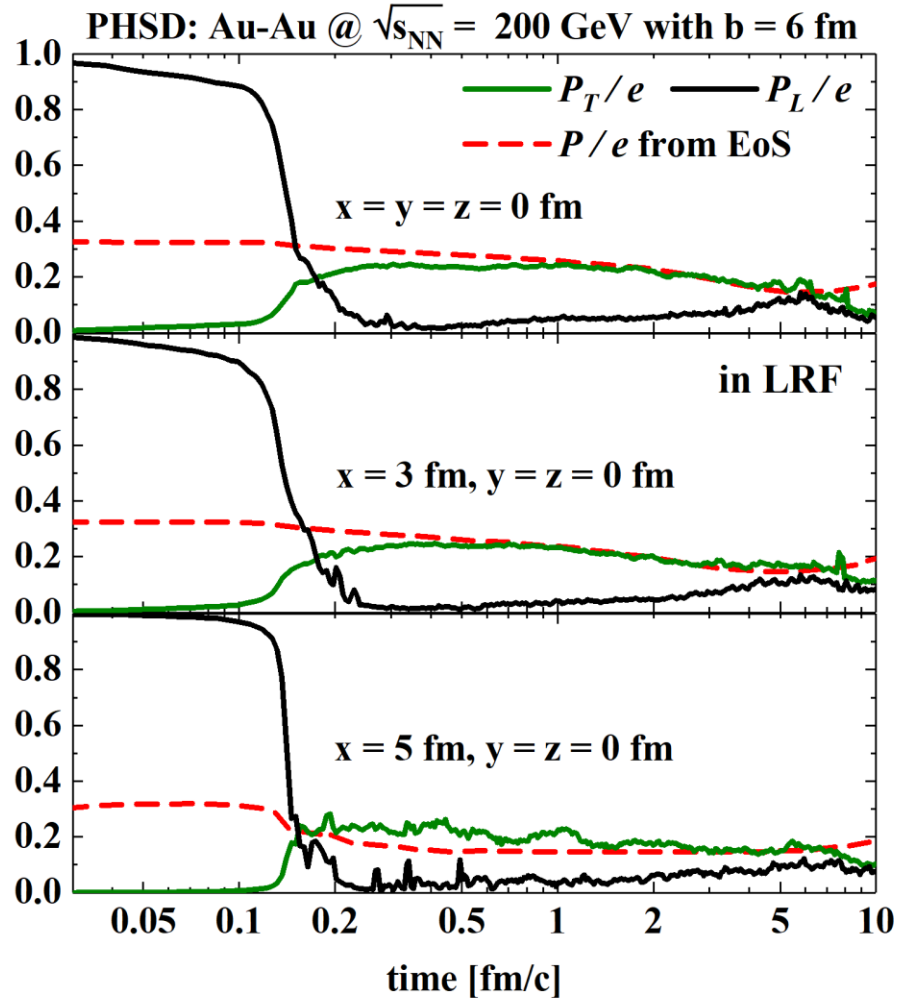


- **For this study we take NUM = 30 parralel events**

Pressure isotropization

- **We look at the pressure components P_i of several cells along the x-axis**
- **The longitudinal pressure dominates when the collisions happens and then decrease rapidly towards 0**
- **The transverse pressure increases with time and reach the equilibrated pressure at 0.5 – 1. fm/c**

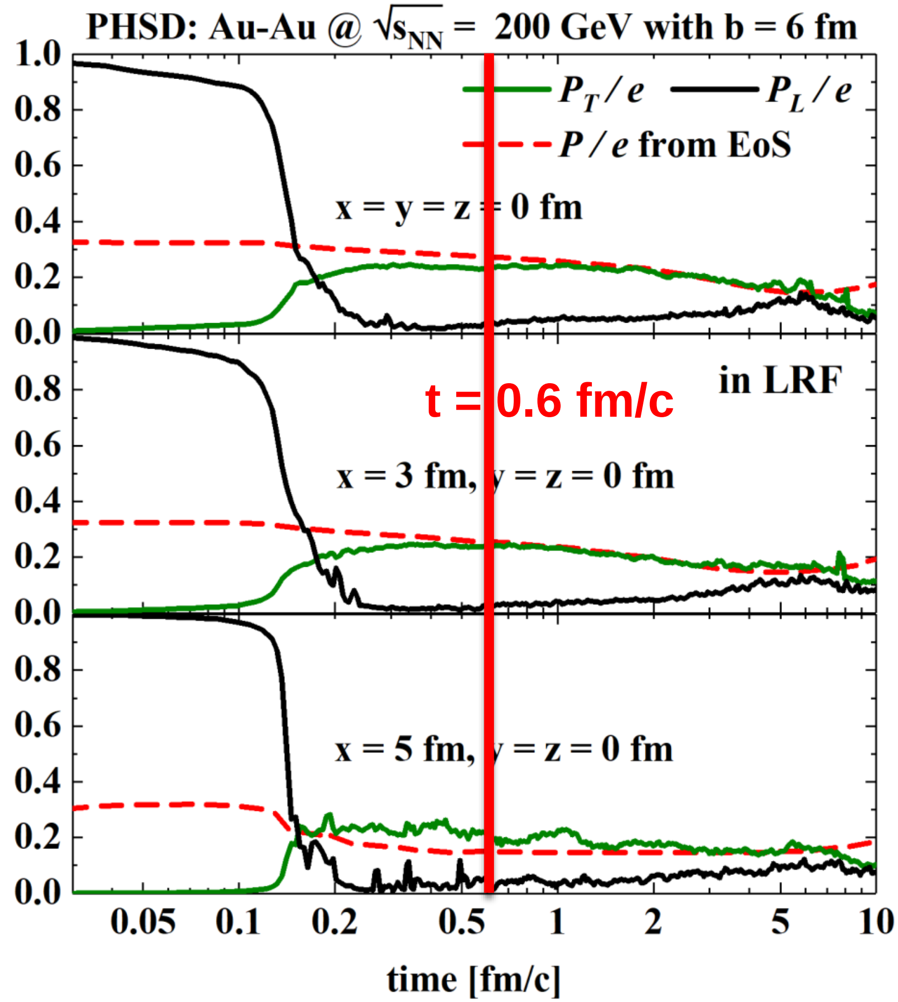
PHSD profile averaged over 100*NUM events



Pressure isotropization

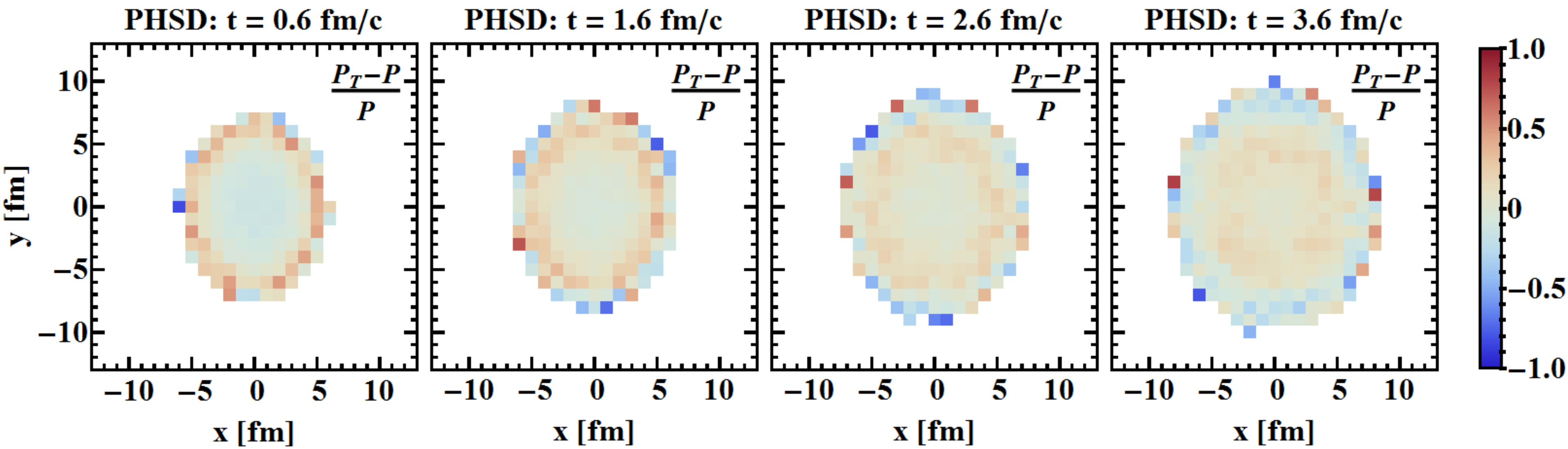
- We look at the **pressure components P_i** of several cells along the x-axis
- The **longitudinal pressure dominates when the collisions happens** and then decrease rapidly towards 0
- The **transverse pressure** increases with time and **reach the equilibrated pressure at 0.5 – 1. fm/c**
- We choose **t_0 = 0.6 fm/c** to initialize the hydro evolution

PHSD profile averaged over 100*NUM events



Pressure isotropization

- Evaluation of the **relative value between the transverse pressure and the one given by the QCD EoS**

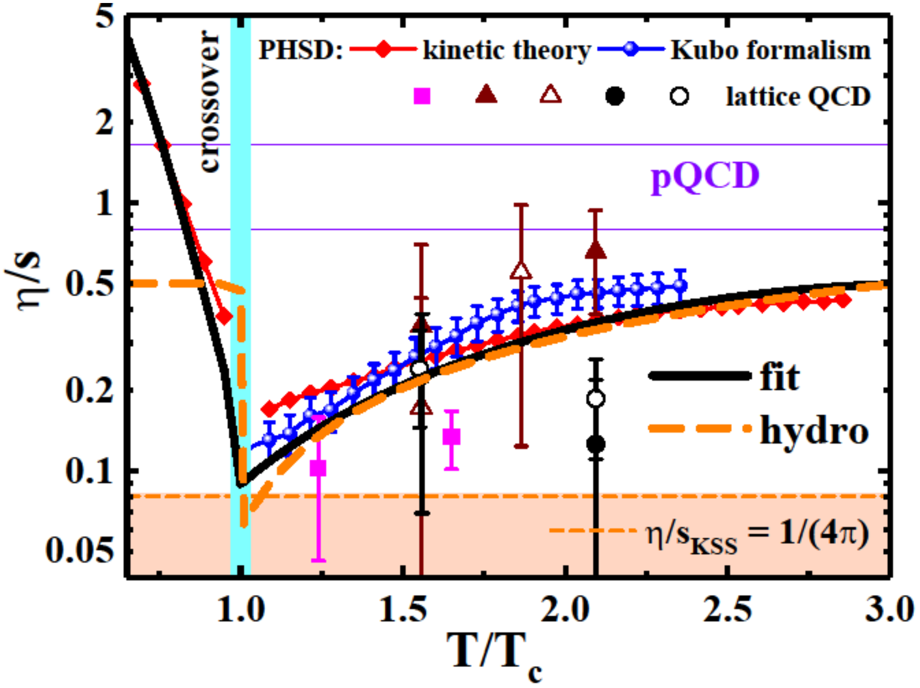


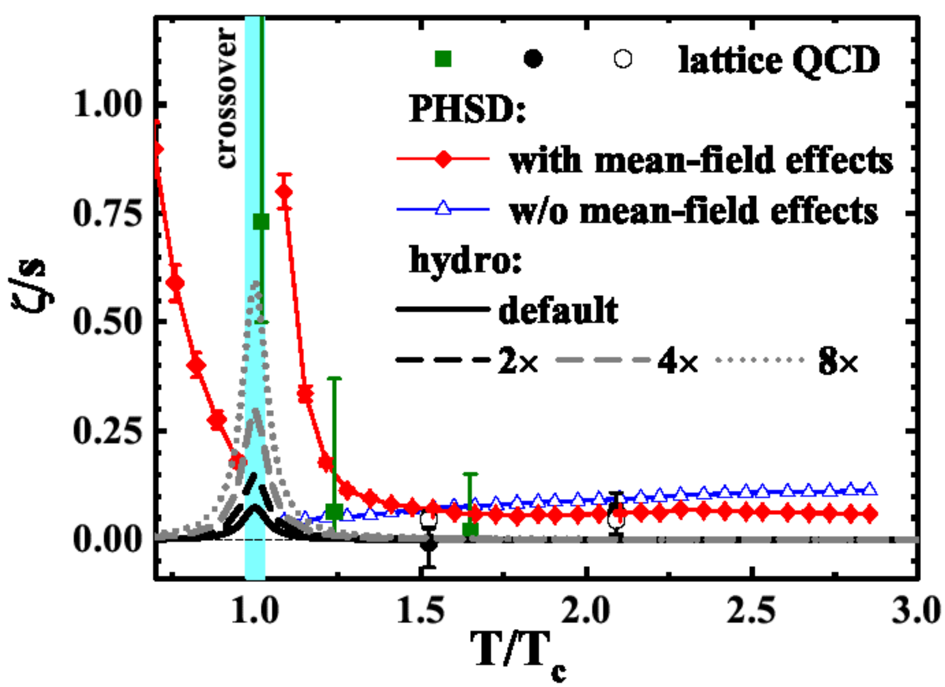
PHSD profile averaged over 100*NUM events

- **In averaged, we can see that the PHSD medium reach the EoS pressure in the transverse direction after a time of 0.5 – 1. fm/c**

Adjustment of hydro parameters

- In order to compare the two models, the **temperature-dependant shear viscosity from PHSD** is used in the hydro code

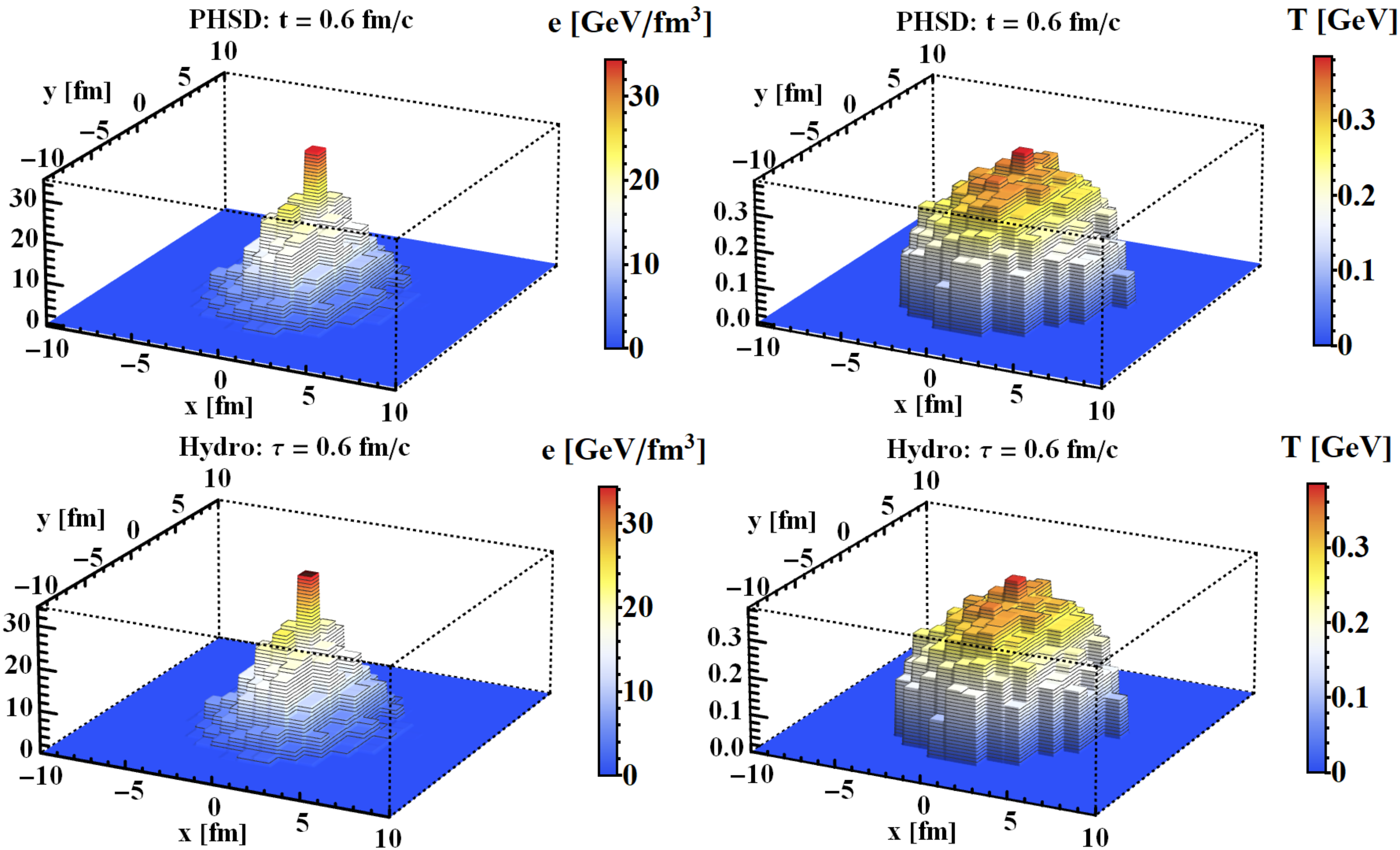




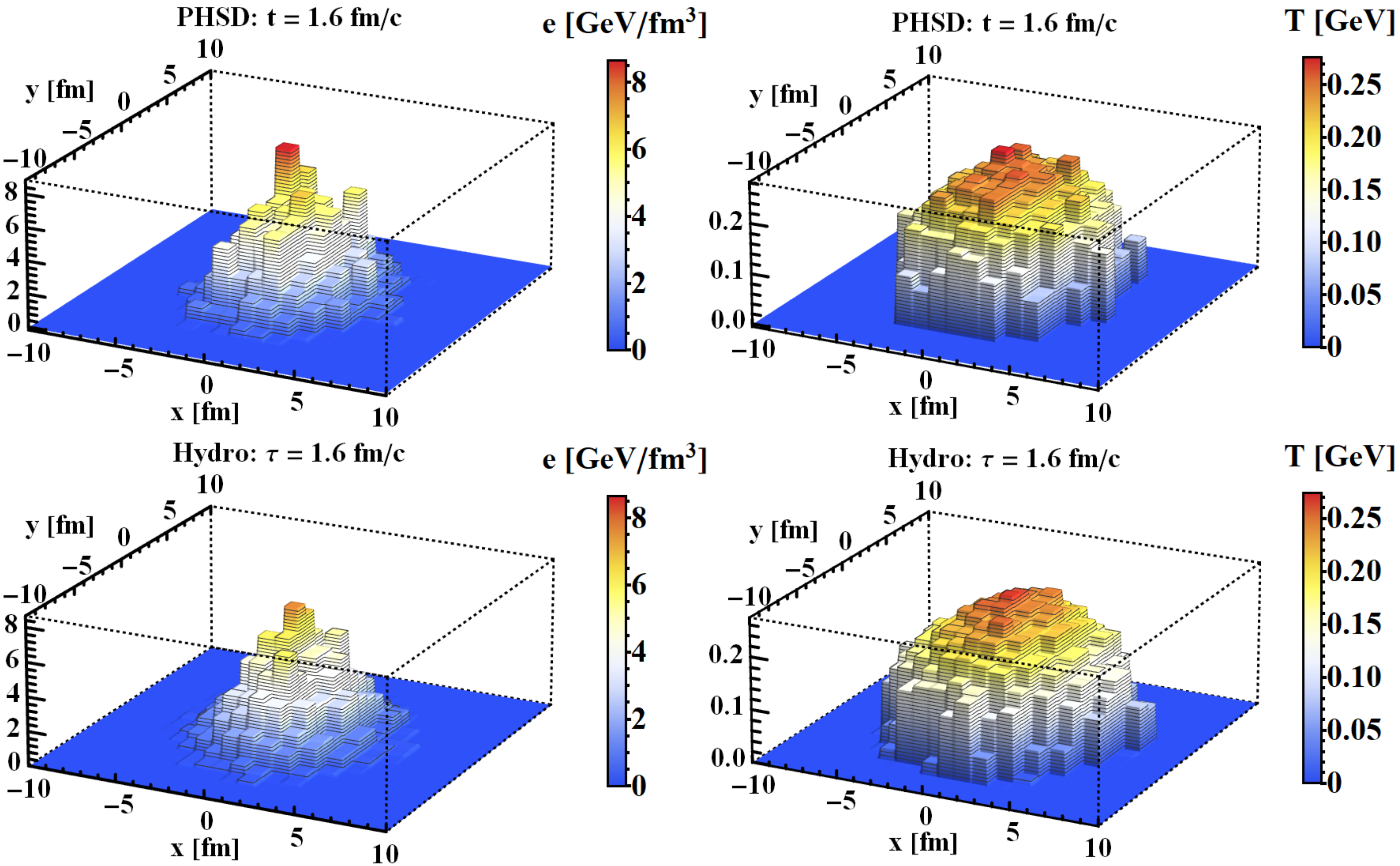
PHSD results from: Ozvenchuk et al., **Phys. Rev. C 89, 064903 (2013)**

Duke Bayesian analysis for the bulk viscosity: Bernhard et al, **arXiv:1704.04462**

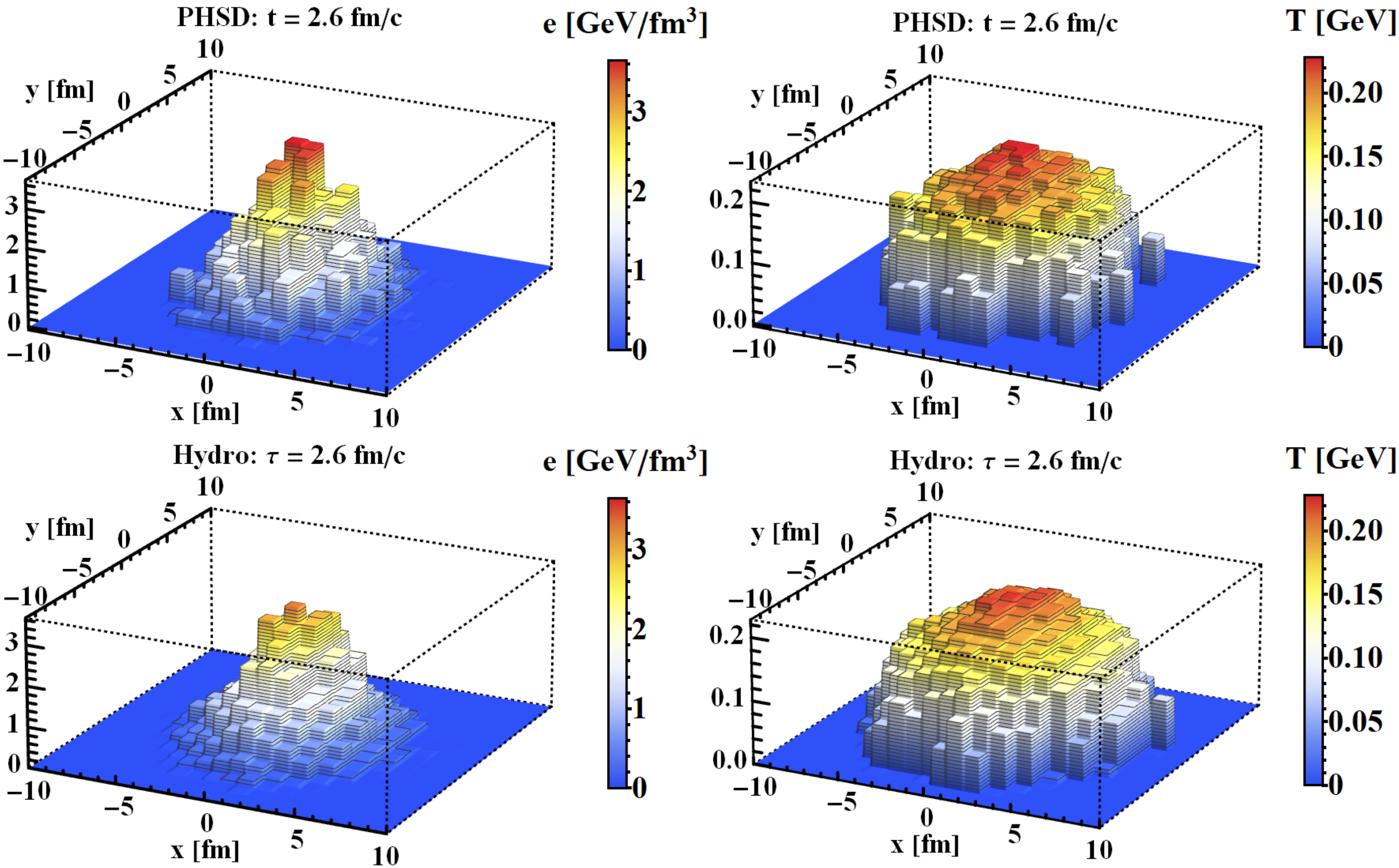
Space-time evolution (e and T)



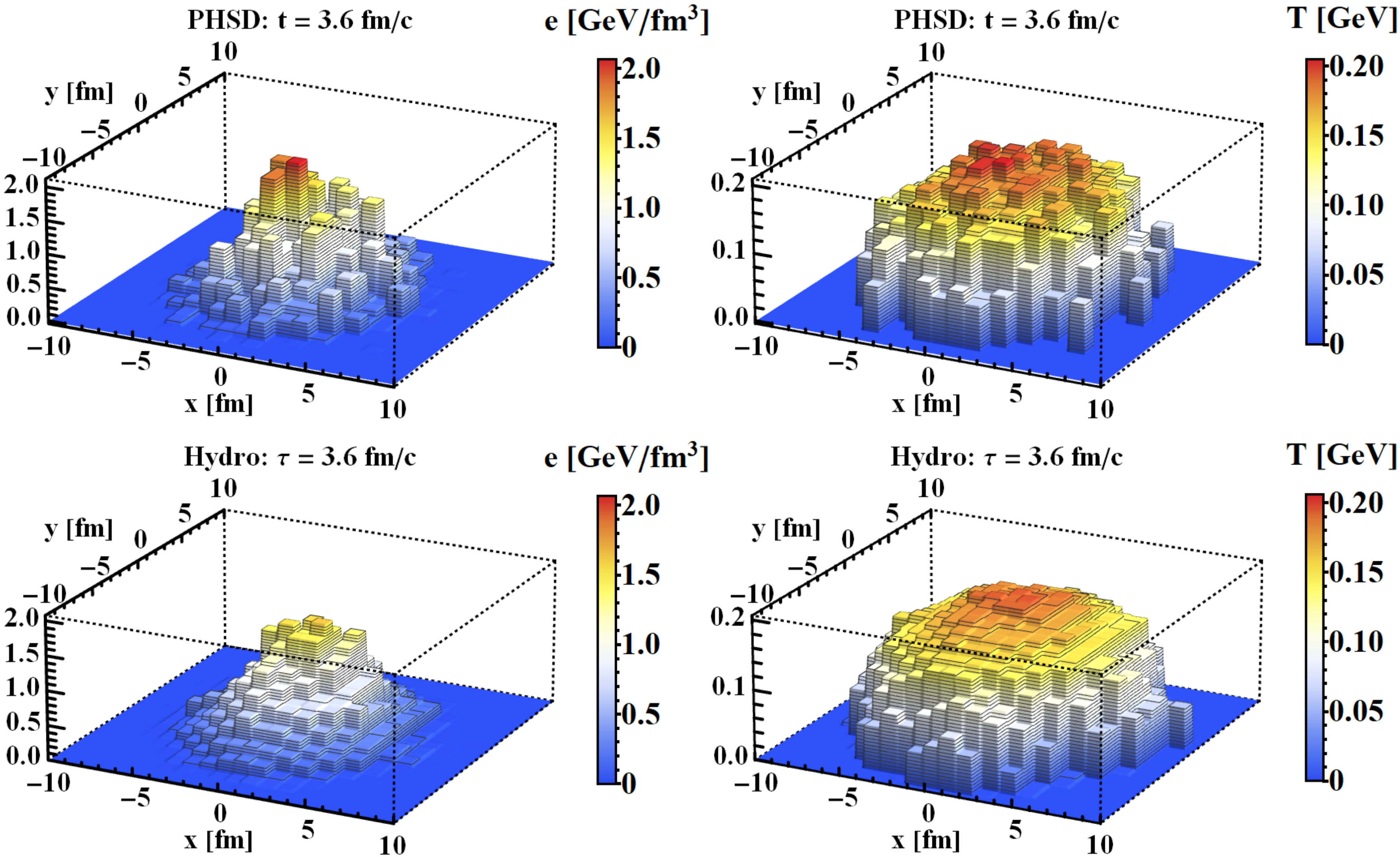
Space-time evolution (e and T)



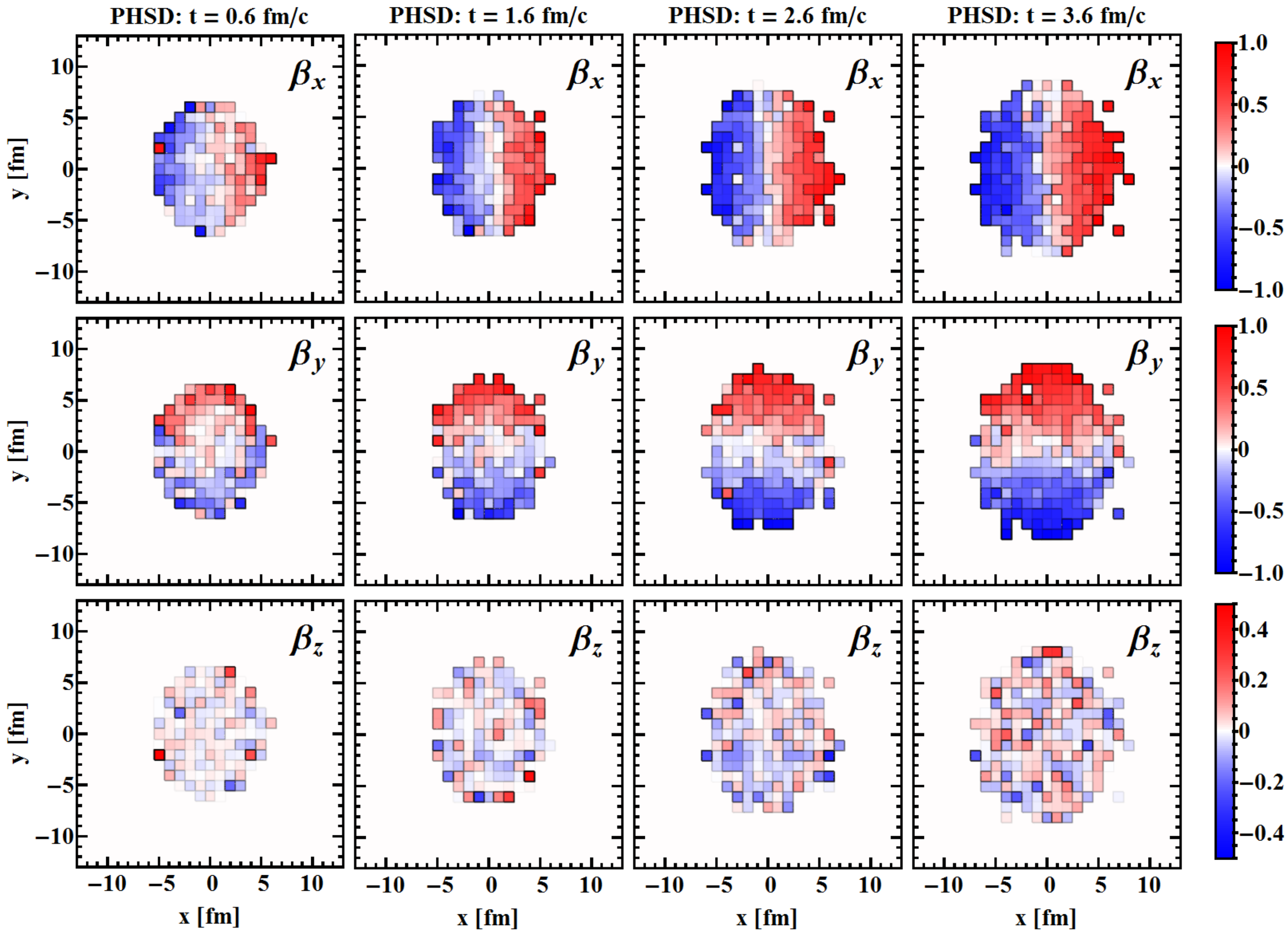
Space-time evolution (e and T)



Space-time evolution (e and T)

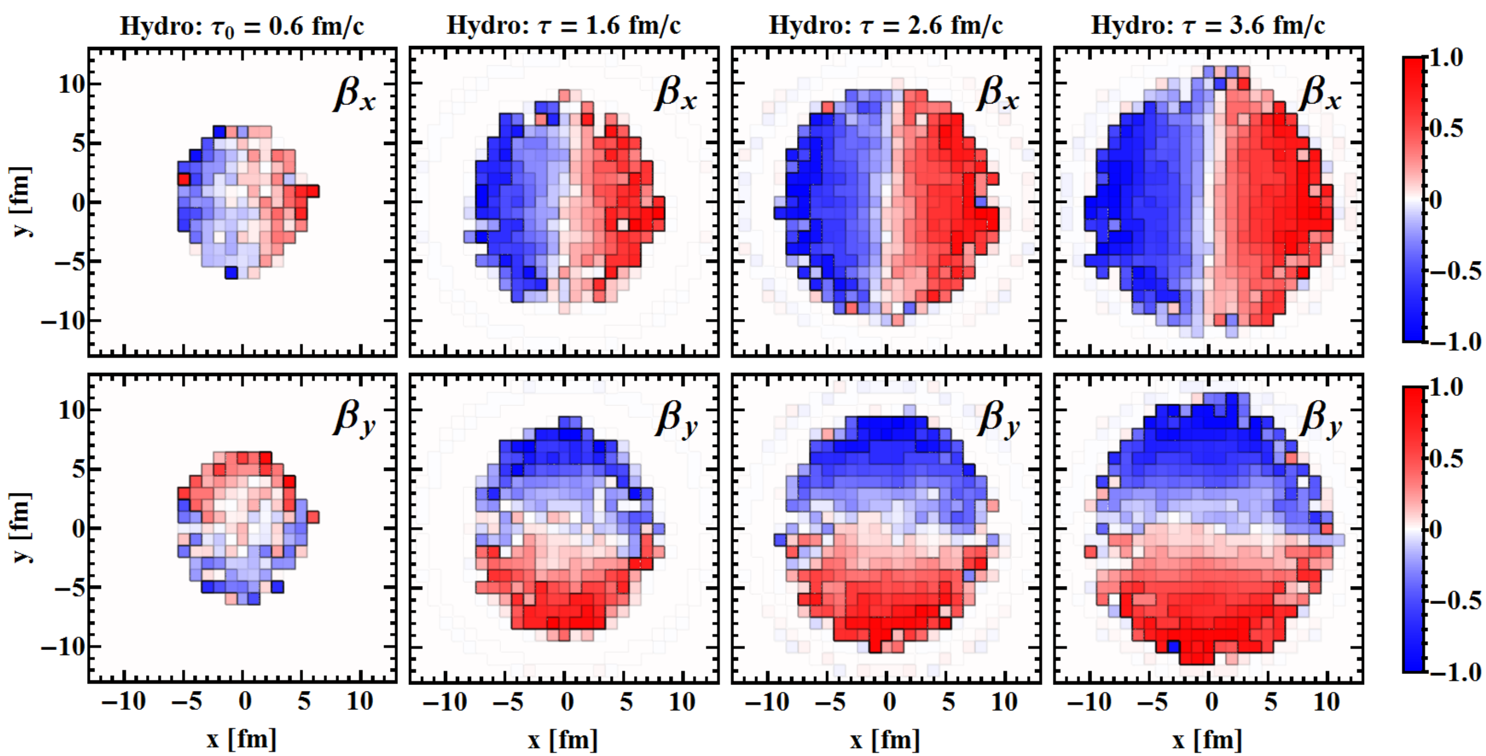


Space-time evolution (cell velocity β)



Space-time evolution (cell velocity β)

- The PHSD evolution remains chaotic for all times
- The hydro code seems to smooth the initial PHSD profile during the evolution in time

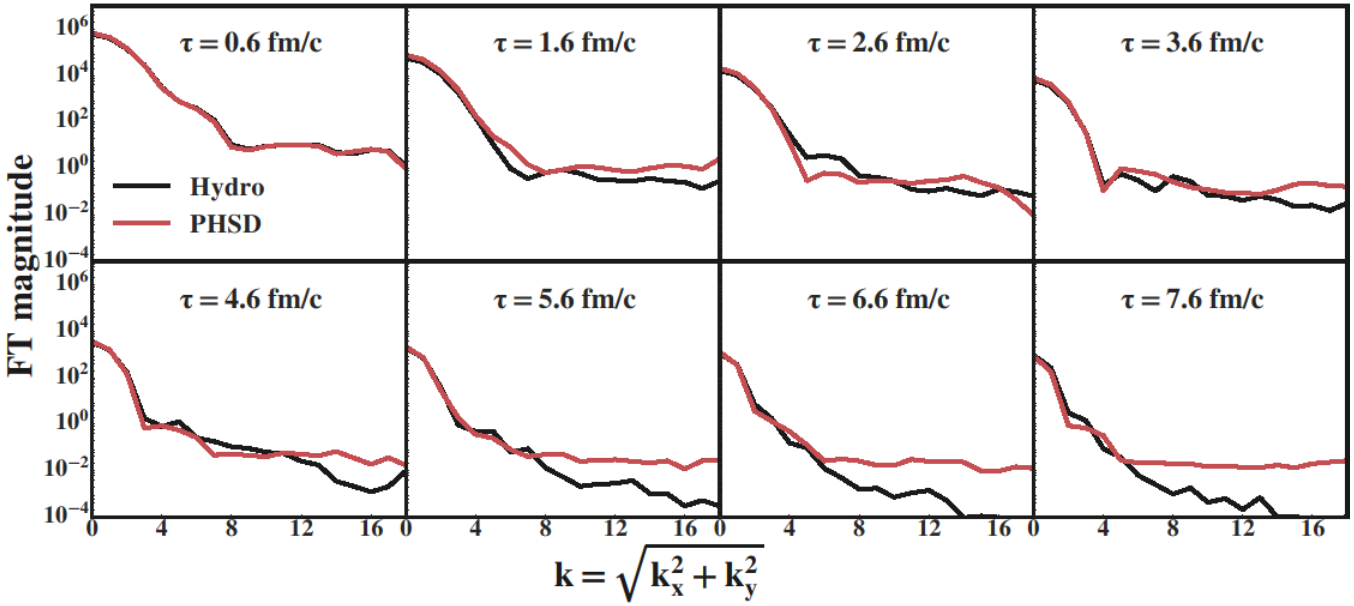


Fourier images of energy density

- **How to quantify this change in the hydro evolution ?**
- **Fourier transformation of the energy density profile:**

$$\tilde{e}(k_x, k_y) = \frac{1}{m}\frac{1}{n}\sum_{x=0}^{m-1}\sum_{y=0}^{n-1} e(x,y)e^{2\pi i\left(\frac{xk_x}{m}+\frac{yk_y}{n}\right)}$$

Radial distribution of the Fourier modes of the energy density

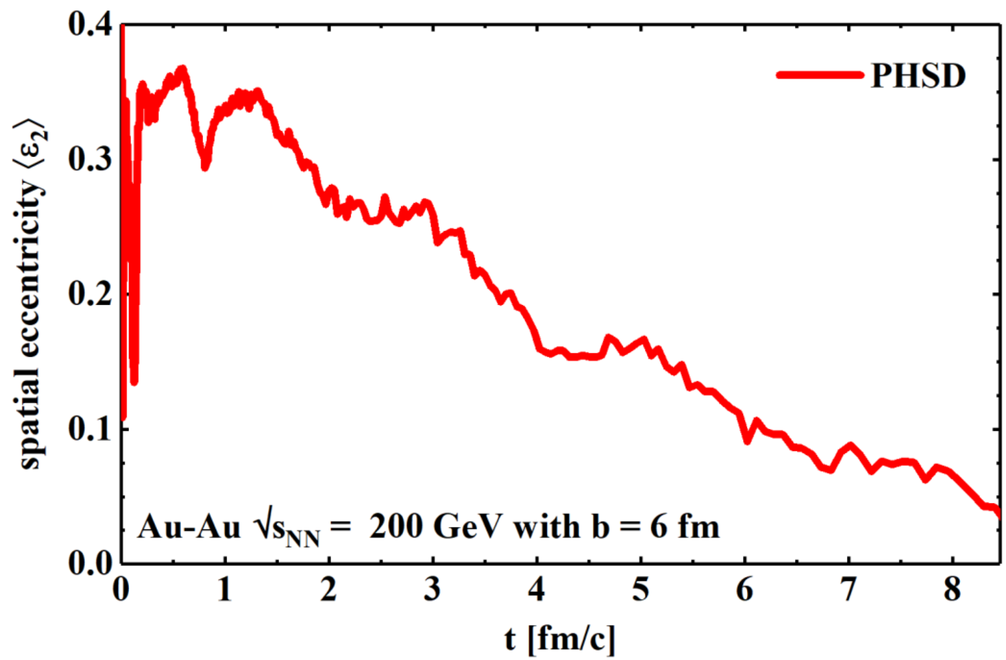


- **Shorter wavelength modes survive only in PHSD which indicates the constant inhomogeneity of the QGP medium**

Spatial eccentricity

- **We compare 100 events from PHSD and the event-by-event hydro code**
- **The spatial eccentricity is calculated as a function of time by the formula:**

$$\epsilon_2 = \frac{\sqrt{\{r^2\cos(2\phi)\}^2 + \{r^2\sin(n\phi)\}^2}}{\{r^2\}}$$



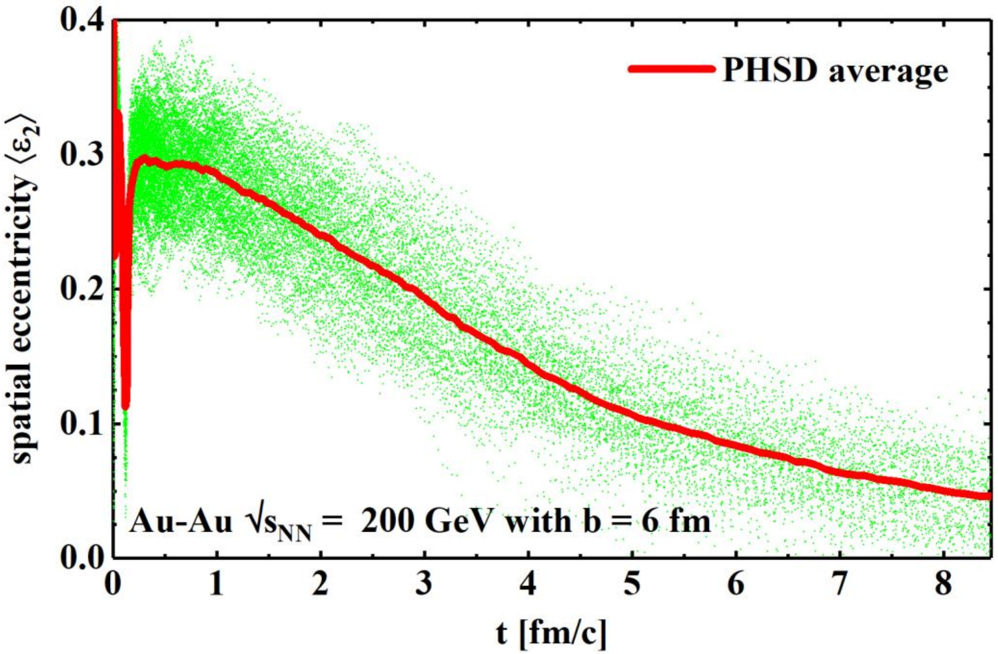
Spatial eccentricity

- **We compare 100 events from PHSD and the event-by-event hydro code**
- **The spatial eccentricity is calculated as a function of time by the formula:**

$$\epsilon_2 = \frac{\sqrt{\{r^2\cos(2\phi)\}^2 + \{r^2\sin(n\phi)\}^2}}{\{r^2\}}$$

PHSD profile averaged over 100*NUM events

- **The green dots represent each PHSD event: large fluctuations**



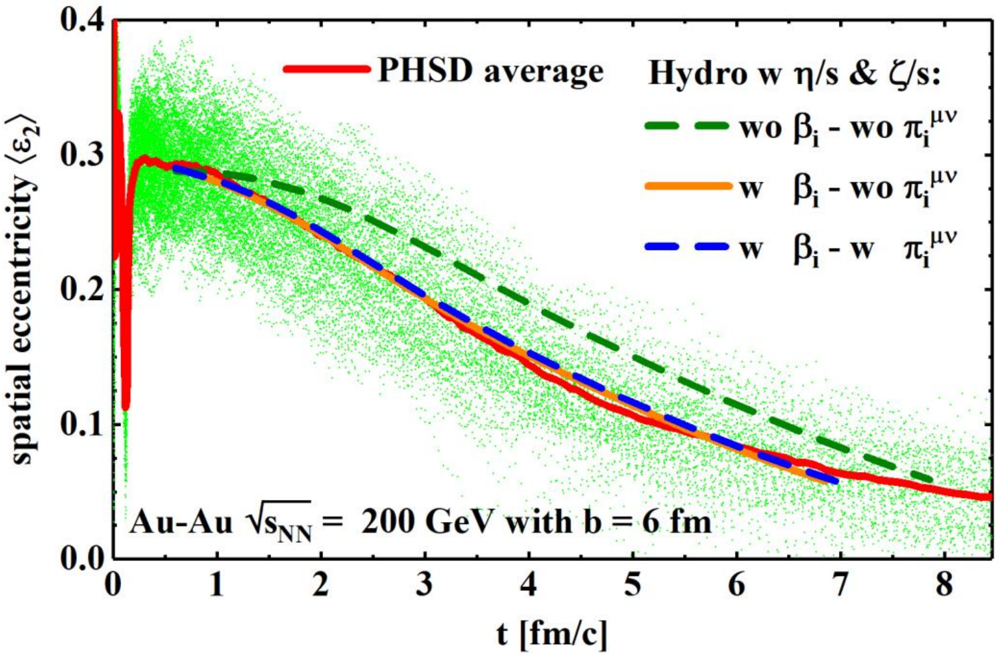
Spatial eccentricity

- **We compare 100 events from PHSD and the event-by-event hydro code**
- **The spatial eccentricity is calculated as a function of time by the formula:**

$$\epsilon_2 = \frac{\sqrt{\{r^2\cos(2\phi)\}^2 + \{r^2\sin(n\phi)\}^2}}{\{r^2\}}$$

PHSD profile averaged over 100*NUM events

- **The green dots represent each PHSD event: large fluctuations**
- **Each hydro event has a very smooth evolution contrary to PHSD**
- **Both PHSD and hydro agree with each other when the full initial state information is taken into account (initial velocity β_i + viscous corrections $\pi_i^{\mu\nu}$)**

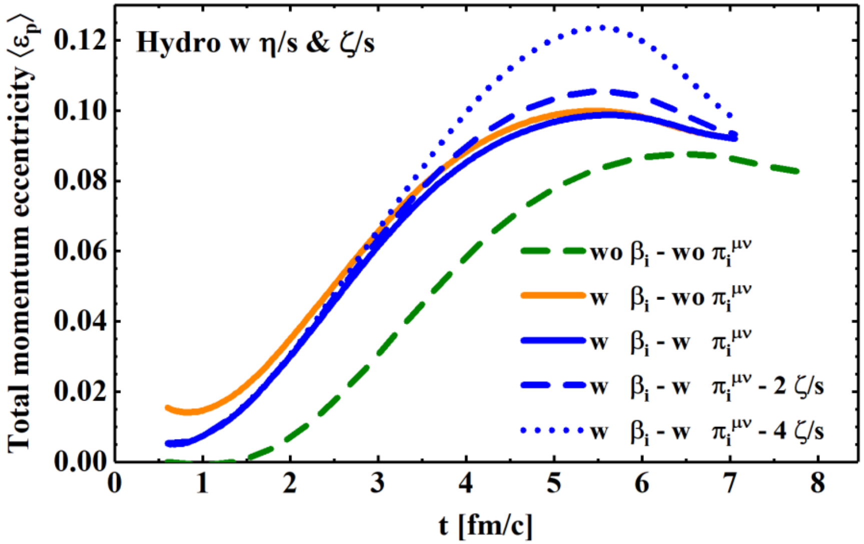


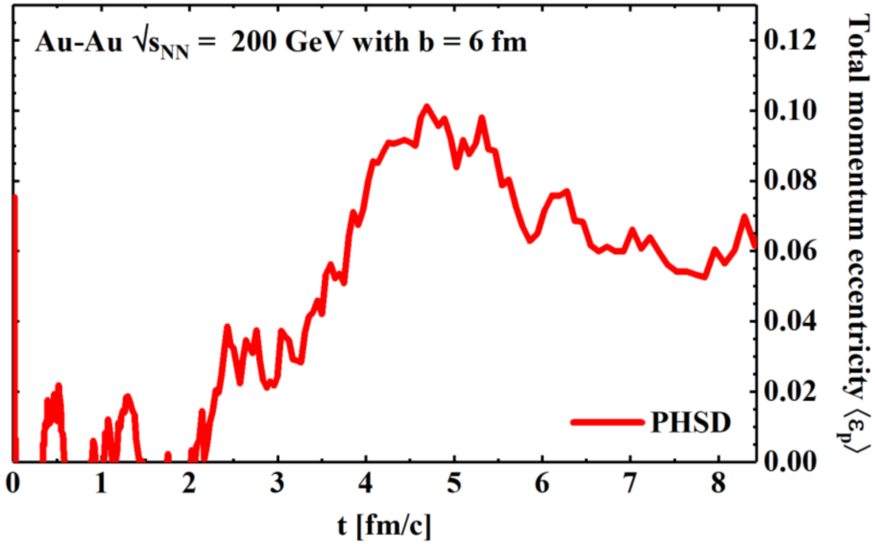
Momentum eccentricity

- **Momentum anisotropies reflect the medium's response to initial spatial anisotropies:**

$$\epsilon_p = \frac{\int dxdy\,(T^{xx} - T^{yy})}{\int dxdy\,(T^{xx} + T^{yy})}$$

- **Including the initial flow velocity β_i in the hydro has a huge effect in ϵ_p**
- **The initial viscous corrections $\pi_i^{\mu\nu}$ play only a role at small times**
- **The increase of the bulk viscosity ζ produces a bump during the hadronization process**





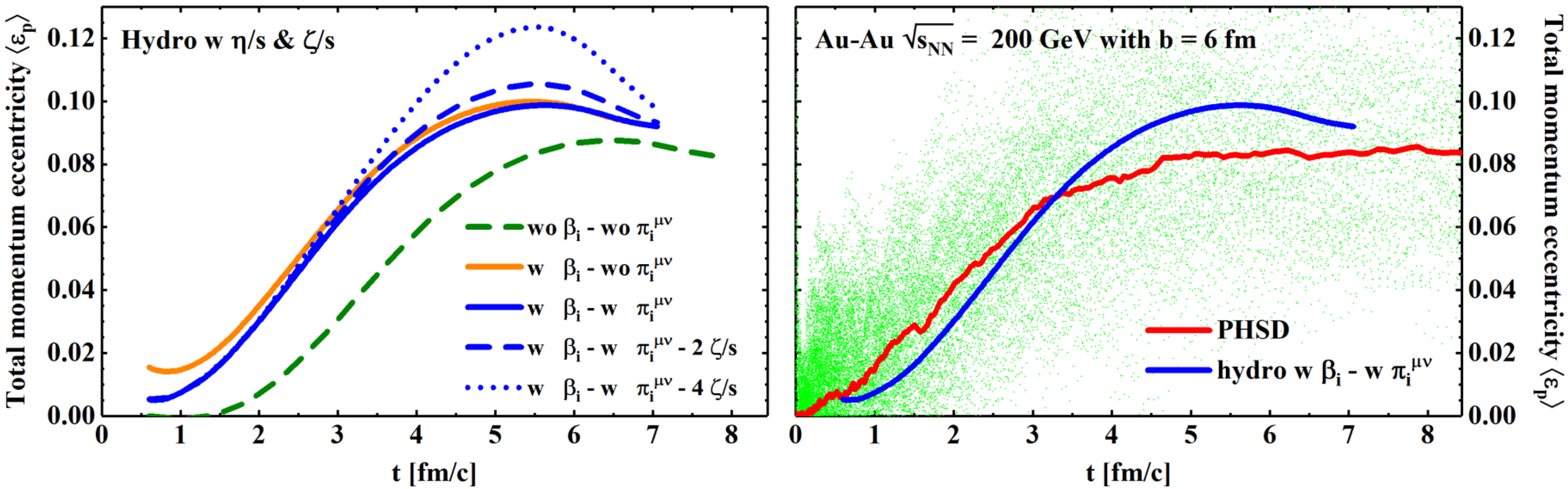
Momentum eccentricity

- **Momentum anisotropies reflect the medium's response to initial spatial anisotropies:**

$$\epsilon_p = \frac{\int dxdy\,(T^{xx} - T^{yy})}{\int dxdy\,(T^{xx} + T^{yy})}$$

- **Including the initial flow velocity β_i in the hydro has a huge effect in ϵ_p**
- **The initial viscous corrections $\pi_i^{\mu\nu}$ play only a role at small times**
- **The increase of the bulk viscosity ζ produces a bump during the hadronization process**

PHSD profile averaged over 100*NUM events



Summary

- **We have compared two descriptions of the QGP medium evolution in heavy-ion collisions: PHSD and 2+1D hydro**
- **We matched the hydrodynamical evolution as closely as possible with the PHSD medium**

- **Similar QCD EoS**
- **Same shear viscosity η/s**
- **Flexible bulk viscosity ζ/s**

In average, both QGP mediums evolve in a similar way, but:

- **Very strong fluctuations are observed in the PHSD medium at any time of the evolution while the hydrodynamical medium evolves smoothly**
- **Strong response of the hydrodynamical medium to transport coefficients**

- **In the future, non-equilibrium traces on other observables (hard probes…)**

Thank you for your attention!

PHSD group

GSI - Frankfurt University - FIAS
Elena Bratkovskaya
Taesoo Song
Pierre Moreau
Andrej Ilner
Hamza Berrehrah

Giessen University
Wolfgang Cassing
Thorsten Steinert
Alessia Palmese
Eduard Seifert
Olena Linnyk

External PHSD Collaborations

SUBATECH, Nantes University:
Jörg Aichelin
Christoph Hartnack
Pol-Bernard Gossiaux
Marlene Nahrgang

Texas A&M University:
Che-Ming Ko

JINR, Dubna:
Viacheslav Toneev
Vadim Voronyuk

Valencia University:
Daniel Cabrera

Barcelona University:
Laura Tolos
Angel Ramos

Duke University:
Steffen Bass
Yingru Xu