

Quasiparticle anisotropic hydrodynamics for heavy-ion collisions

Based on PRL 119, 042301, and arXiv:1705.10191

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Motivation

- Viscous hydrodynamics is derived by linearization around an isotropic equilibrium distribution function

$$f(x, p) = f_{\text{eq}} \left(\frac{p^\mu u_\mu}{T} \right) [1 + \delta f(x, p)]$$

- However, QGP in the local rest frame is a highly anisotropic plasma.
 - At early times, $P_L/PT \ll 1$ which relaxes slowly towards isotropy at late times.
 - At all times, large momentum-space anisotropies at the system edges.
- To take this into account in anisotropic hydrodynamics (aHydro), momentum-space anisotropies are included from the beginning

$$f(x, p) = f_{\text{eq}} \left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)} \right) + \delta f(x, p)$$

Motivation

- Anisotropic hydrodynamics is expected to do better at
 - Early time dynamics and near the system edges
 - Small systems (p+A, p+p)
 - Temperature dependent $\frac{\eta}{s}(T)$ See , M. Strickland, arXiv:1410.5786
- Some situations where aHydro does well compared with other methods:
 - Exact Boltzmann solutions in 0+1d systems (more on coming slides)
 - Gubser flow in 1+1d systems (more on coming slides)
 - Cold atoms. See: M. Bluhm and T. Schaefer, arXiv:1512.00862
- The ultimate goal: To extract transport coefficients from data: the shear viscosity to entropy density ratio (η/s) and the bulk viscosity (ζ/s) in a more reliable way, more accurately than other methods.
- As we will see, we were able to fit the data with a much smaller bulk viscosity than other methods suggest.

Anisotropic hydro distribution function

- The distribution function in aHydro is

$$f(x, p) = f_{\text{eq}} \left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)} \right) + \delta f(x, p)$$

- Here $\Xi_{\mu\nu}$ is the anisotropy tensor which in aHydro replaces the viscous stress tensor in viscous hydro.

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu} \Phi$$

u^μ LRF four velocity

$\xi^{\mu\nu}$ the traceless anisotropy tensor

$\Delta^{\mu\nu}$ the transverse projector

Φ the degree of freedom associated with bulk

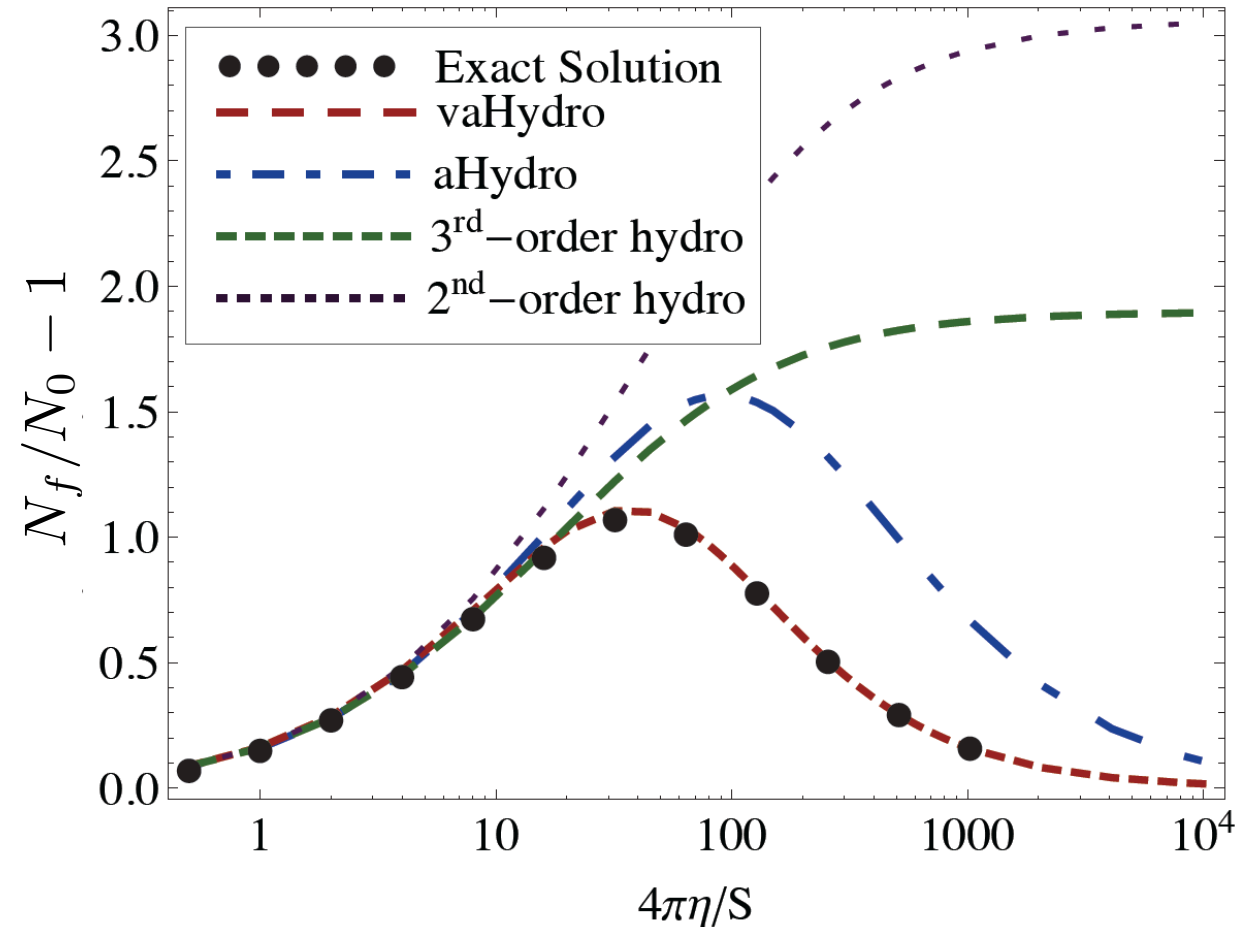
- In leading-order aHydro, δf is assumed to be zero. In this case, the distribution function is positive definite in both the hydro stage and at freeze-out.

Some Examples where aHydro does better

Example 1: The entropy production in the conformal limit

D. Bazow, U. Heinz, and M. Strickland, arXiv: 1311.6720

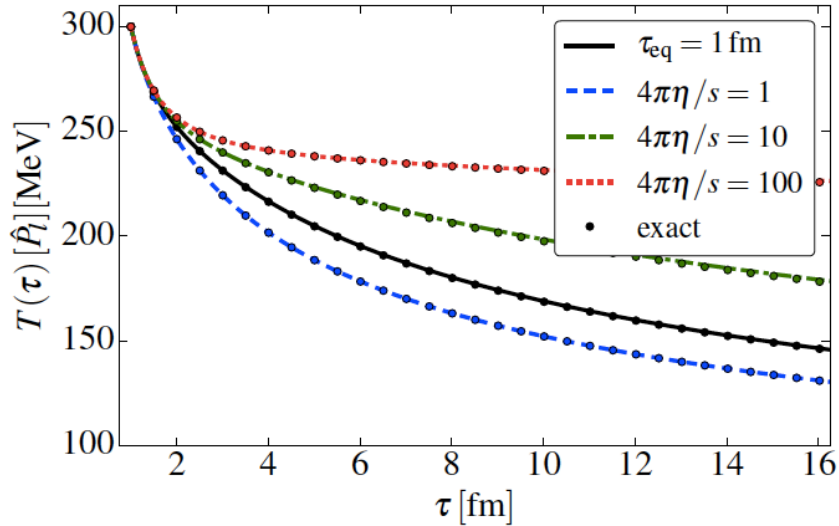
- The number production vanishes in two limits: ideal hydrodynamic and free streaming limits.
- The only methods that capture this behavior are leading aHydro (quite good agreement) and NLO aHydro ($\delta f \neq 0$) which almost reproduces the exact solutions.



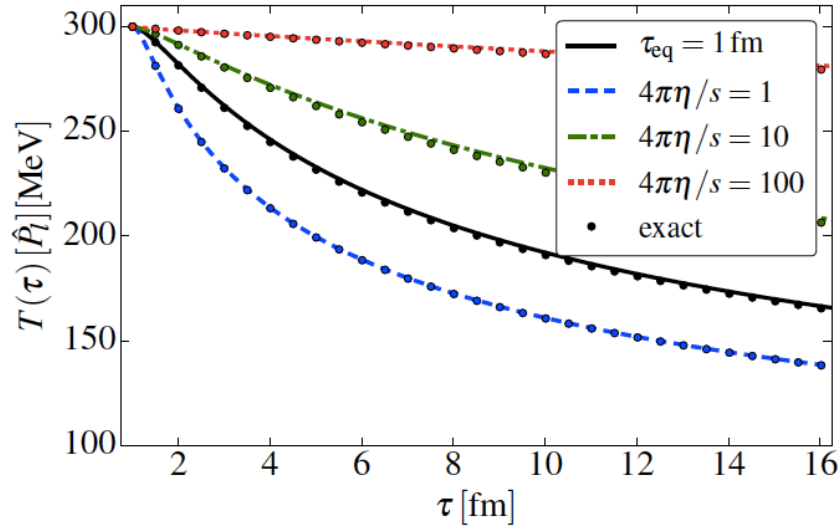
Example 2: 0+1d aHydro results in the conformal limit

E. Molnar, H. Niemi, and D. Rischke, arXiv: 1606.09019

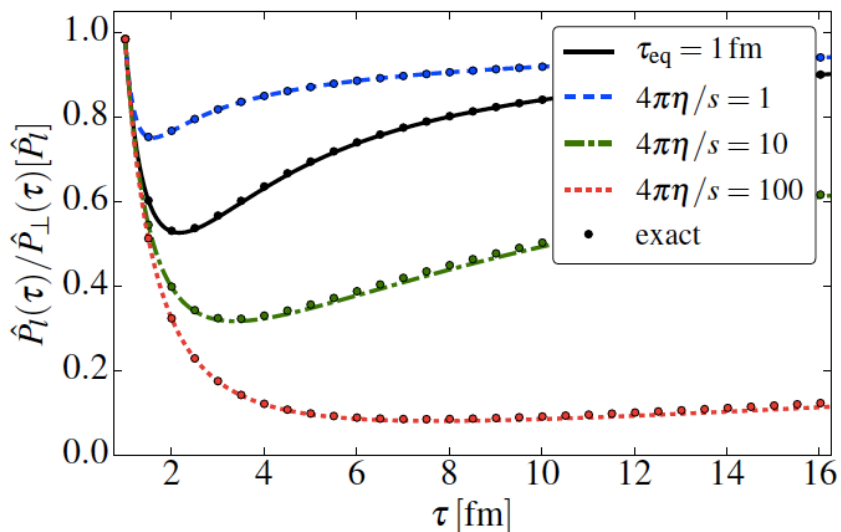
$\xi_0 = 0, T_0 = 300 \text{ MeV}, \lambda_0 = 1$



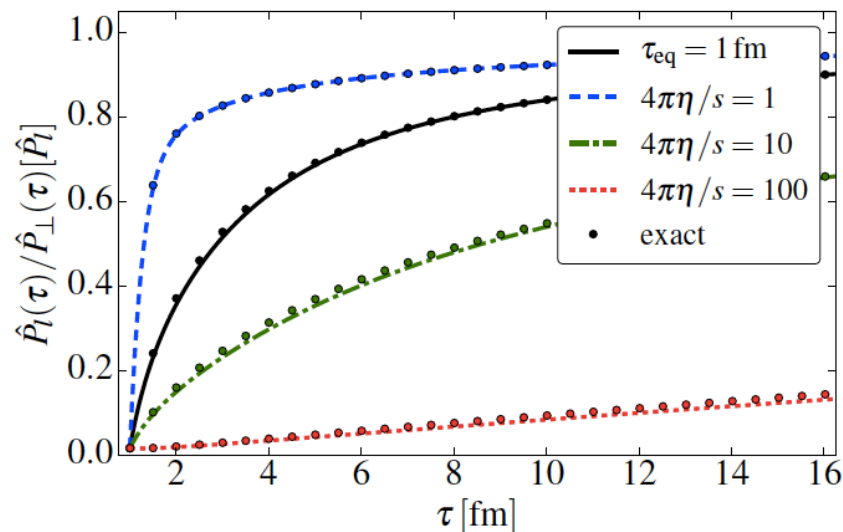
$\xi_0 = 100, T_0 = 300 \text{ MeV}, \lambda_0 = 1$



$\xi_0 = 0, T_0 = 300 \text{ MeV}, \lambda_0 = 1$



$\xi_0 = 100, T_0 = 300 \text{ MeV}, \lambda_0 = 1$



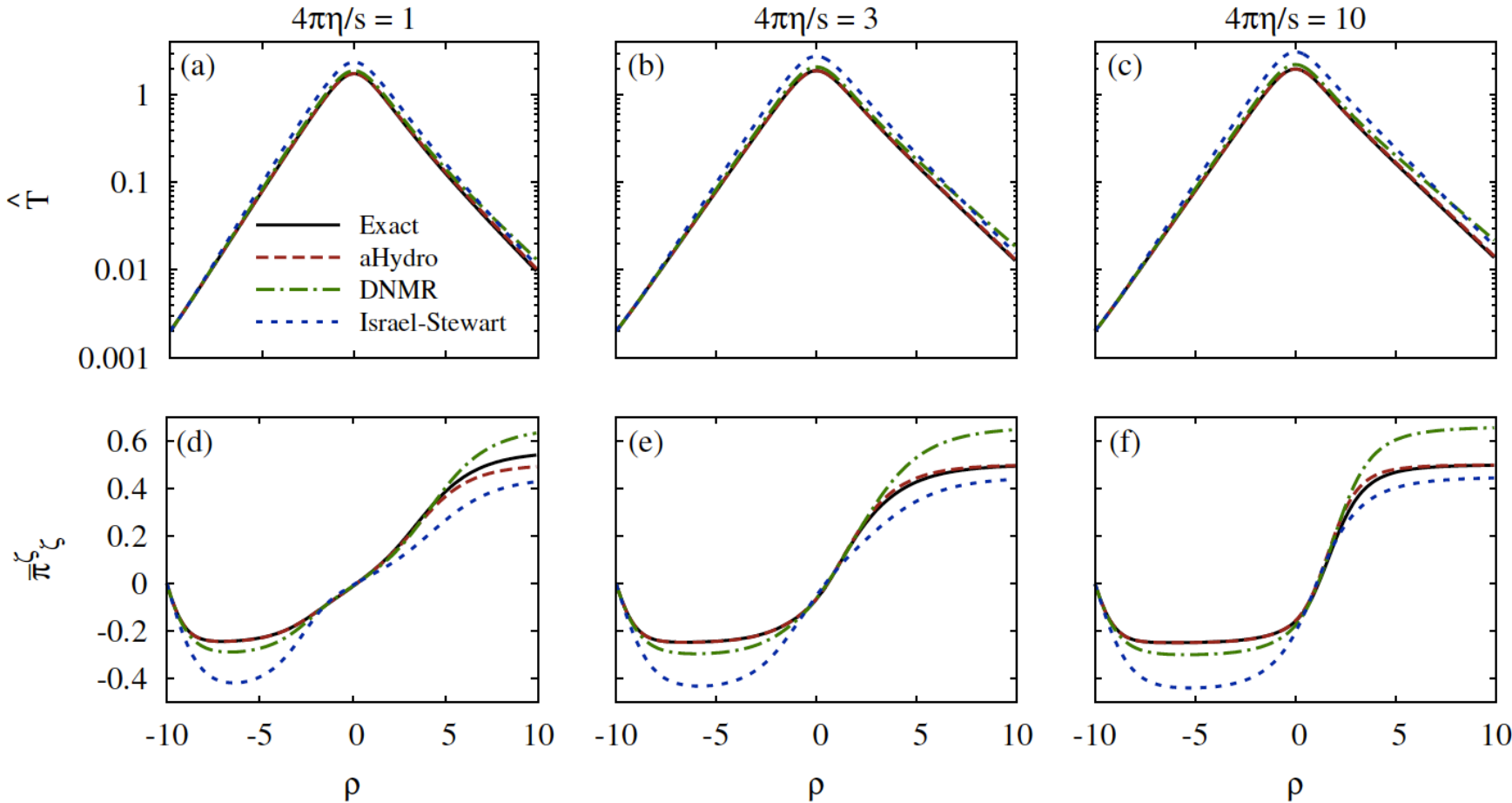
- In the conformal case

$$\tau_{\text{eq}} = 5 \frac{\eta}{T_s}$$

- $[\hat{P}_l]$ sign just represents one way of closing the dynamical equations, but the argument still valid that aHydro reproduces the exact solutions in this case even at very large η/s .
- λ_0 is the fugacity, when $\lambda_0 = 1$, the particle # is conserved.
- In the case of $\lambda_0 = 0$ (the particle # is not conserved), the agreement is the same between aHydro and exact solutions (not shown here).

Example 3: Gubser flow (1+1d) using aHydro

M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1410.6790



- ρ can be interpreted as de Sitter “time”.

$$\tau \rightarrow 0 \Leftrightarrow \rho \rightarrow -\infty$$

$$\tau \rightarrow \infty \Leftrightarrow \rho \rightarrow \infty$$

- \hat{T} the de Sitter effective temperature.
- The scaled shear is given by

$$\bar{\pi}_\zeta^\zeta = \hat{\pi}_\zeta^\zeta / (\hat{T} \hat{s})$$

The dynamical equations for a HydroQP

- The dynamical equations can be found by taking moments of the Boltzmann equation

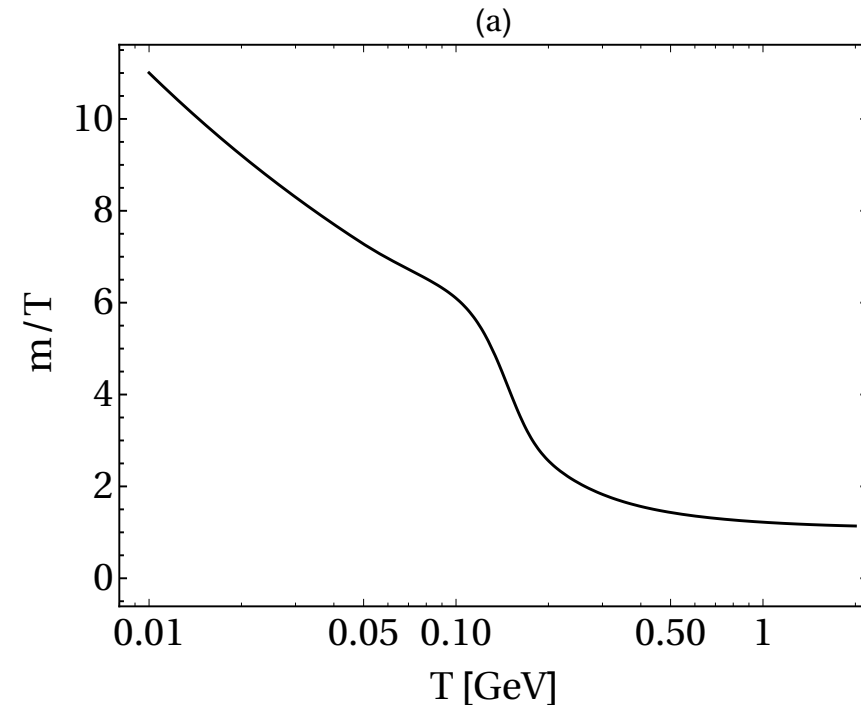
$$p^\mu \partial_\mu f + \frac{1}{2} \partial_i m^2 \partial_{(p)}^i f = -\mathcal{C}[f]$$

- Thermodynamic consistency is not guaranteed when including thermal mass $m(T)$, i.e., $S \neq \frac{\partial P}{\partial T}$. So, a background field is introduced

$$T^{\mu\nu} = T_{\text{kinetic}}^{\mu\nu} + g^{\mu\nu} B$$
$$\partial_\mu B = -\frac{1}{2} \partial_\mu m^2 \int dP f(x, p)$$

- From lattice results $m(T)$ can be obtained

$$\mathcal{S}_{\text{eq}} = 4\pi \tilde{N} m^3 K_3(m/T)$$



3+1d aHydroQP equations of motion

- In leading-order aHydro, we have eight degrees of freedom $\alpha_x, \alpha_y, \alpha_z, u_x, u_y, u_z, \lambda$, and T .
- We use 7 equations from the first and second moment and the matching condition to determine the effective temperature.

$$D_u \mathcal{E} + \mathcal{E} \theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu = 0$$

$$D_x \mathcal{P}_x + \mathcal{P}_x \theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu = 0$$

$$D_y \mathcal{P}_y + \mathcal{P}_y \theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu = 0$$

$$D_z \mathcal{P}_z + \mathcal{P}_z \theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu = 0$$

- First moment equations:

- Second moment equations:

$$D_u \mathcal{I}_x + \mathcal{I}_x (\theta_u + 2u_\mu D_x X^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_x]$$

$$D_u \mathcal{I}_y + \mathcal{I}_y (\theta_u + 2u_\mu D_y Y^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_y]$$

$$D_u \mathcal{I}_z + \mathcal{I}_z (\theta_u + 2u_\mu D_z Z^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_z]$$

- The matching condition:

$$\mathcal{E}(\boldsymbol{\alpha}, \lambda) = \mathcal{E}_{\text{eq}}(T)$$

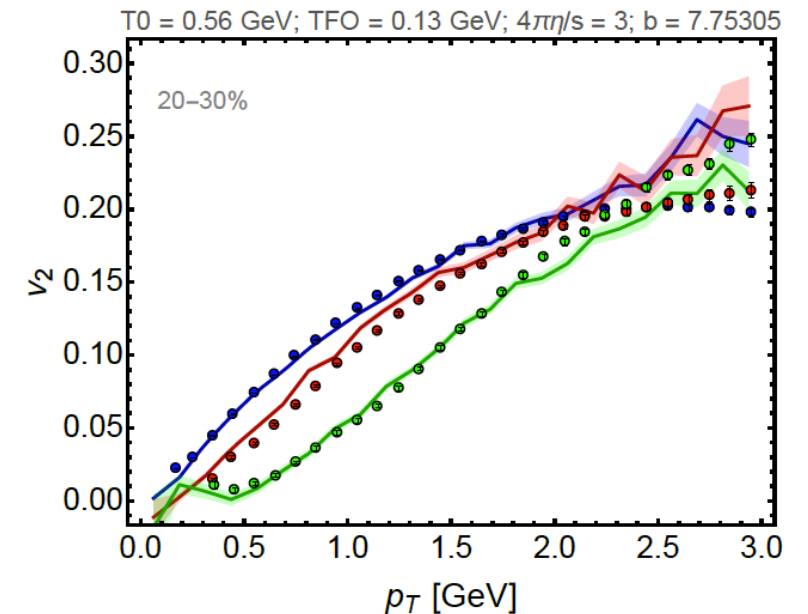
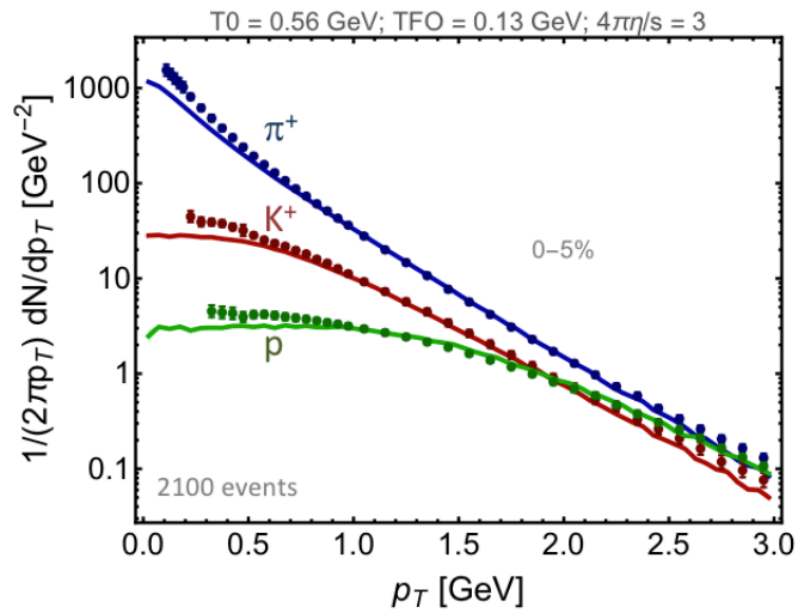
Comparisons with ALICE data using aHydro

- Here, the authors used the standard way for imposing the equation of state which is based on the factorization, the system is taken to be massless but the equation of state enters through the bulk variables.

$$\mathcal{E} = \mathcal{H}_3(\alpha)\mathcal{E}_{\text{iso}}(\lambda)$$

- We can replace the energy density then by the equilibrium one coming from realistic EoS.

- M. Nopoush, R. Ryblewski, and M. Strickland, 1610.10055
- M. Strickland, 1611.05056



- This model fits the data reasonably well when compared the spectra and differential v_2 .
- One troubling problem, the underestimation of the spectra at low p_T .

Comparisons with ALICE data using aHydroQP

- M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1703.05808
- M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1705.10191

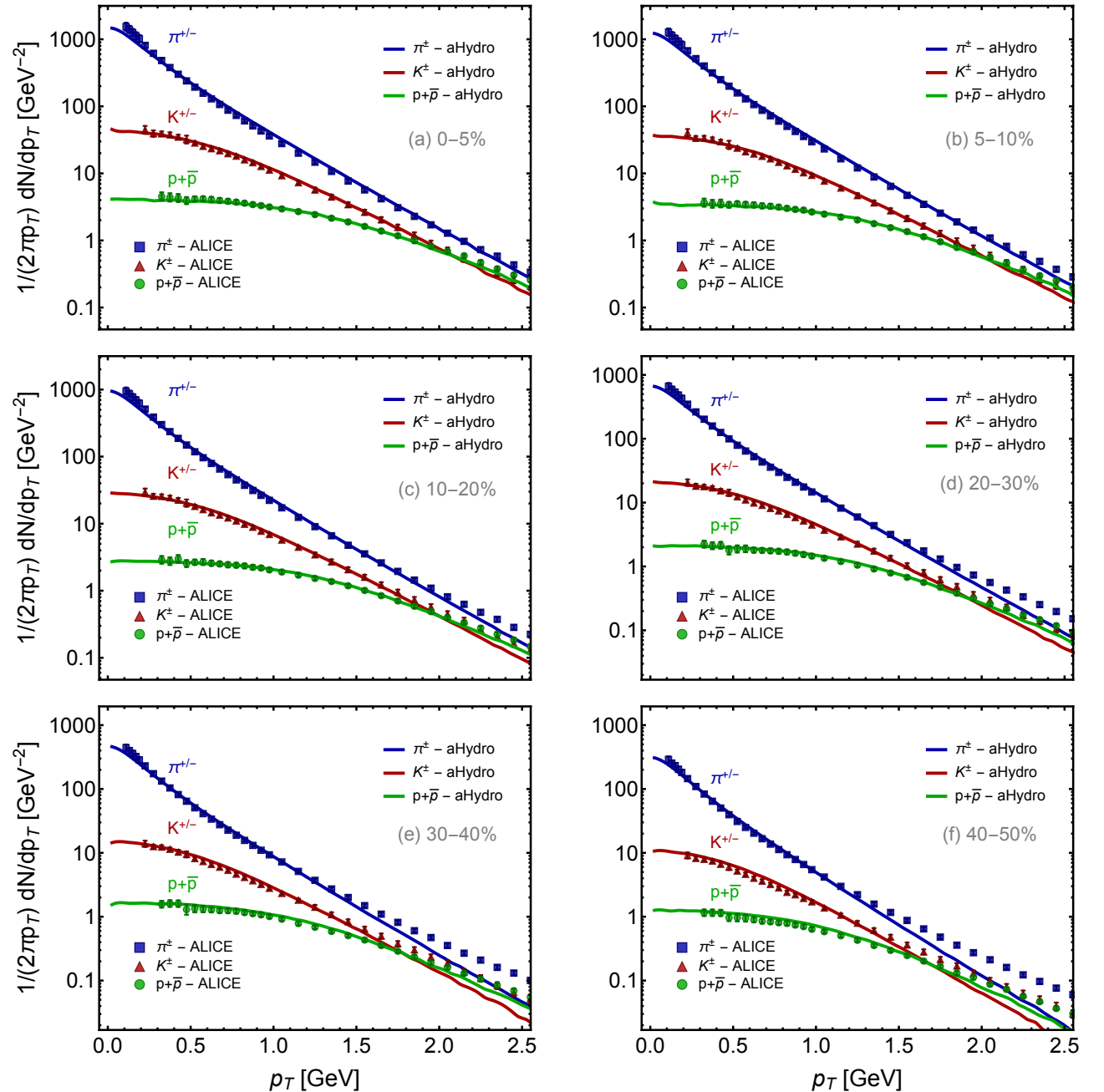
- We solve 3+1d quasiparticle anisotropic hydrodynamics.
- The system is assumed initially to be isotropic in momentum space.
- We used smooth Glauber initial conditions.
- We consider ALICE 2.76 TeV Pb-Pb collisions.
- We use anisotropic Cooper-Frye freeze-out to extract the freeze-out hypersurface.
- Then, we use THERMINATOR 2 to perform the hadronic production decays.
- The parameters we obtained from our fit are

$$\tau_0 = 0.25 \text{ fm}/c$$

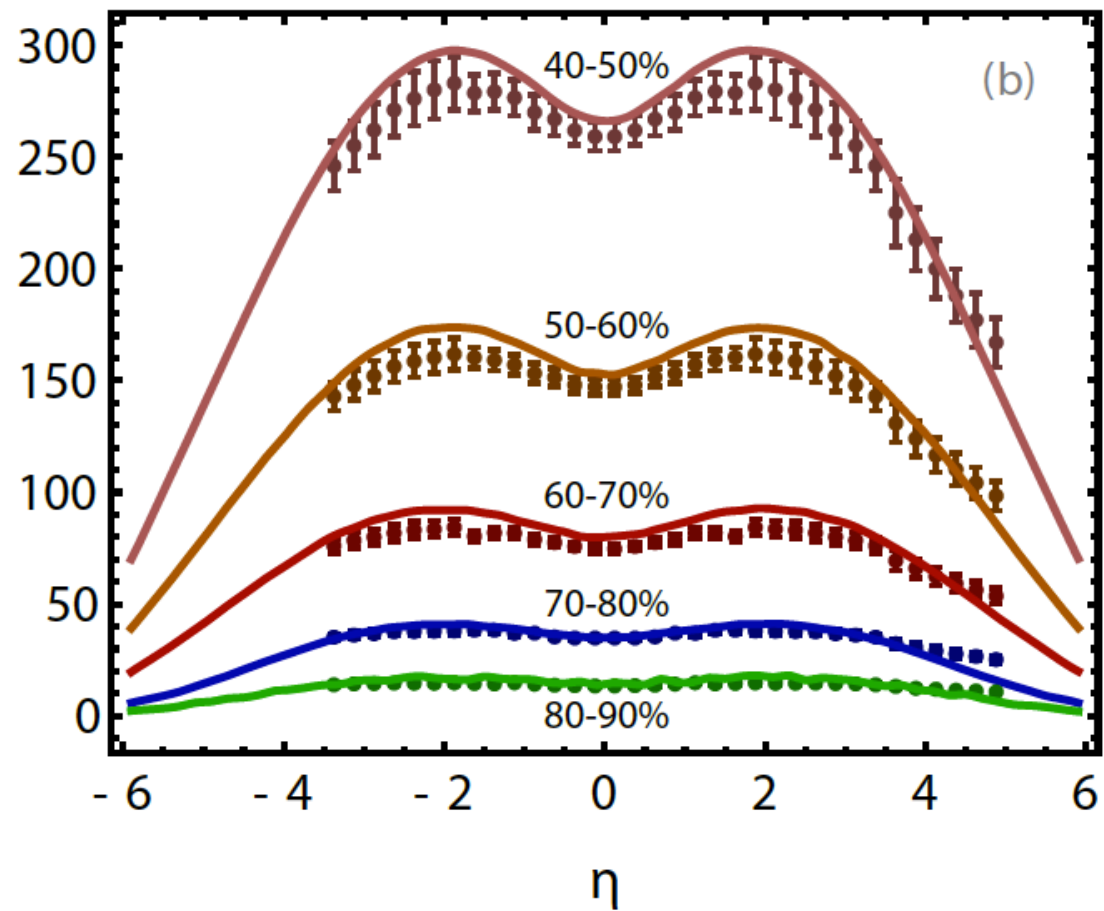
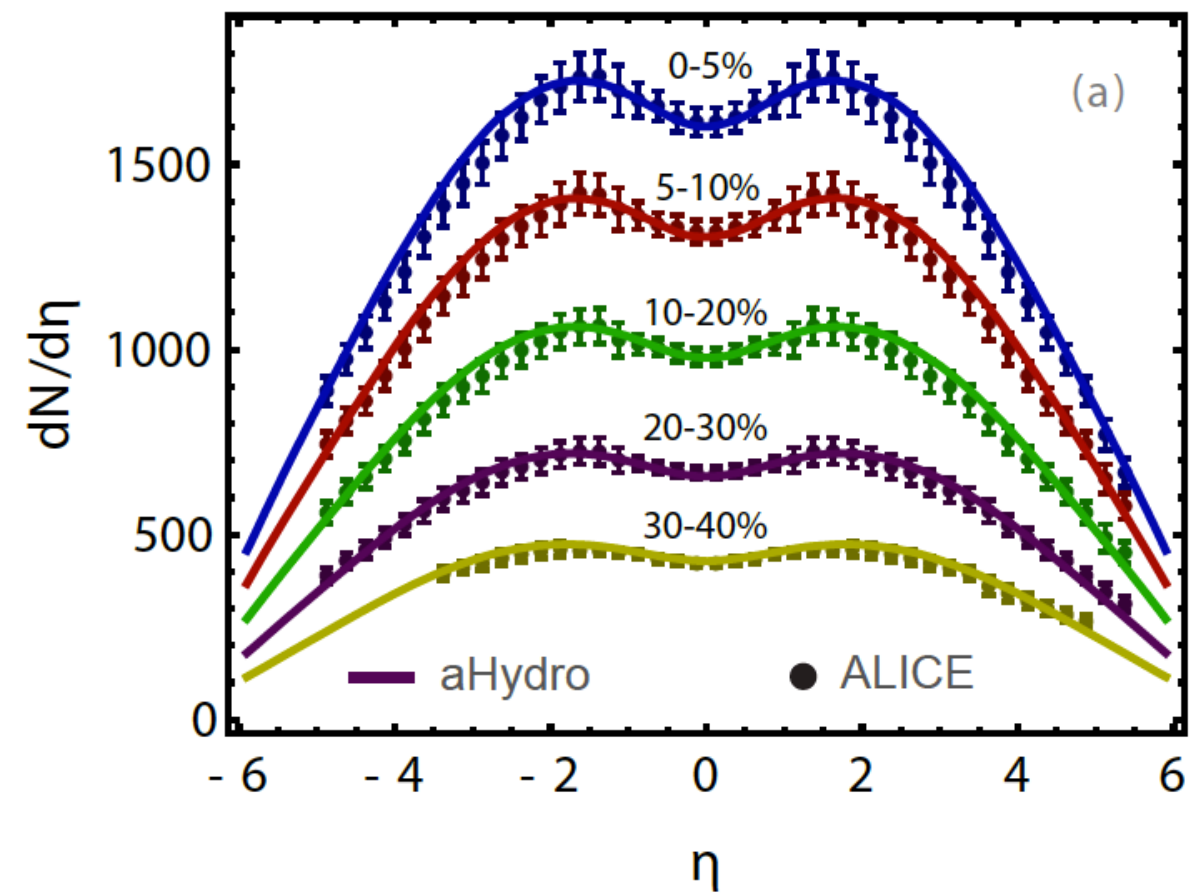
$$\begin{aligned} T_0 &= 600 \text{ MeV} \\ \frac{\eta}{s} &= 0.159 \\ T_{\text{FO}} &= 130 \text{ MeV} \end{aligned}$$

Pions, kaons, and protons spectra

- We fix the underestimation of the spectra predicted by the standard way for imposing EoS.
- Our fits to the data show very good agreement including the mass splitting between different hadrons (pion, kaon, protons).
- The largest differences appear at relatively high centrality classes, e.g, our model shows good agreement only up to $p_T \approx 1.5$ GeV in 40-50%, where in 0-5% up to $p_T \approx 2.5$ GeV.

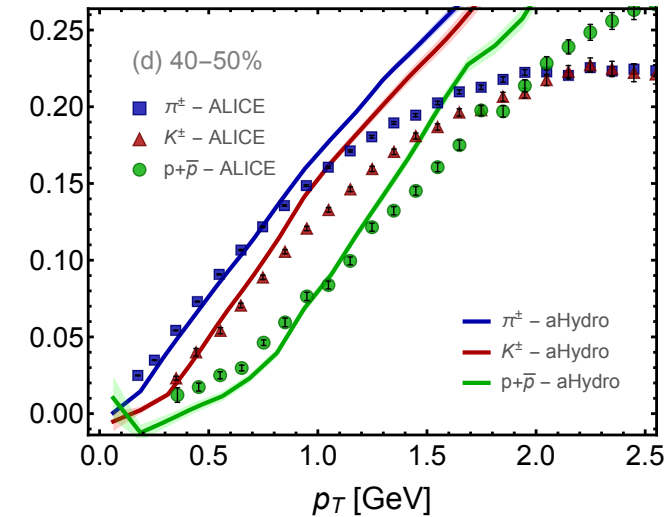
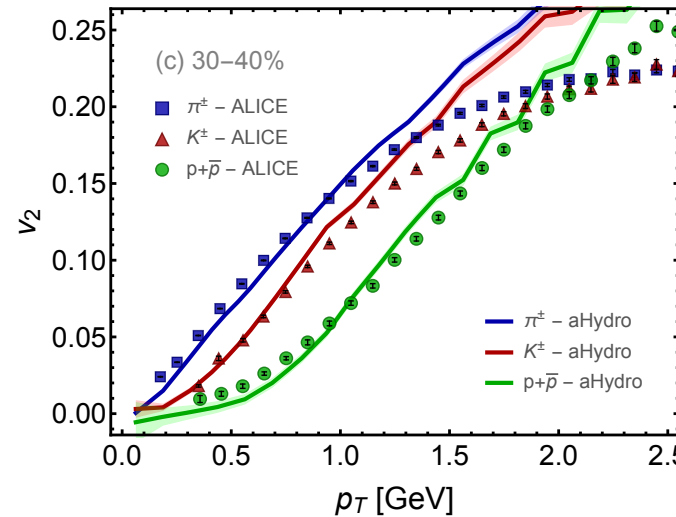
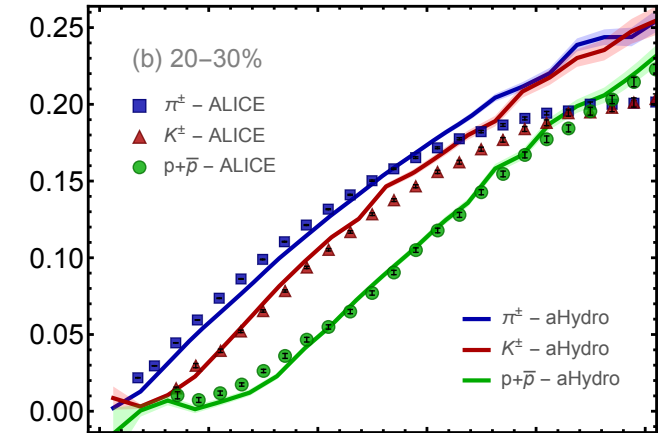
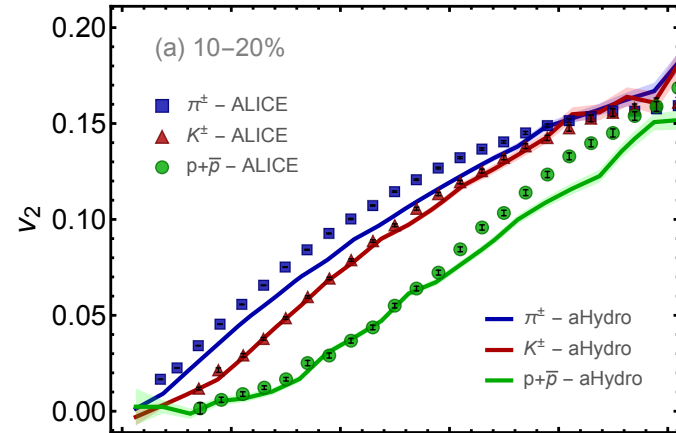


Charged-hadron multiplicity



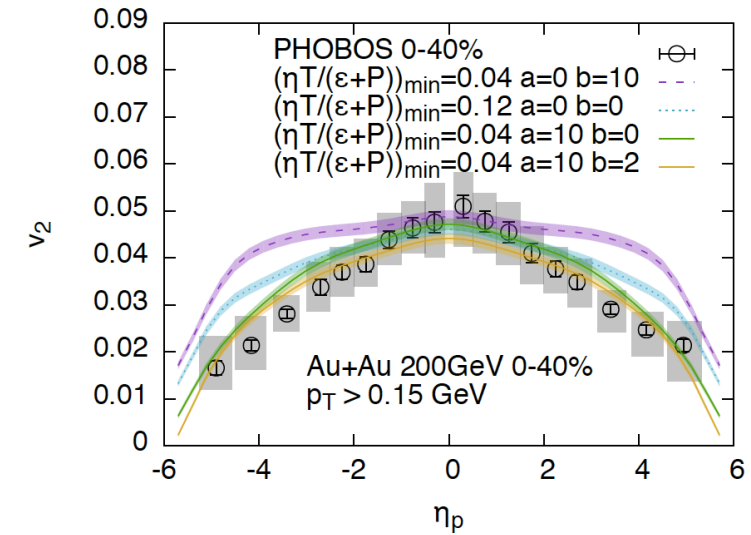
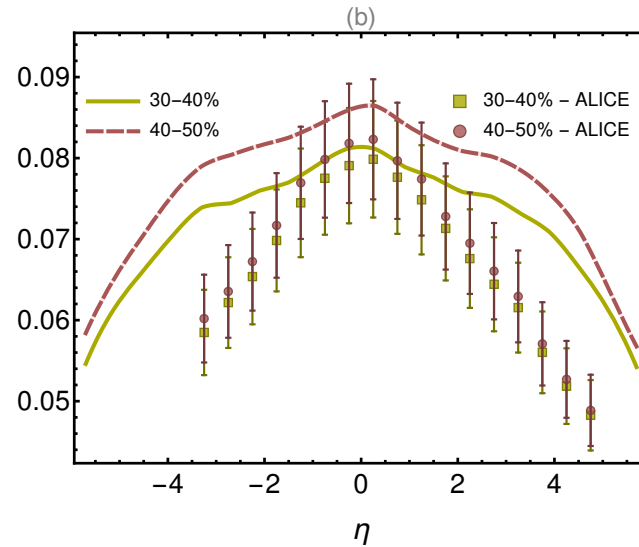
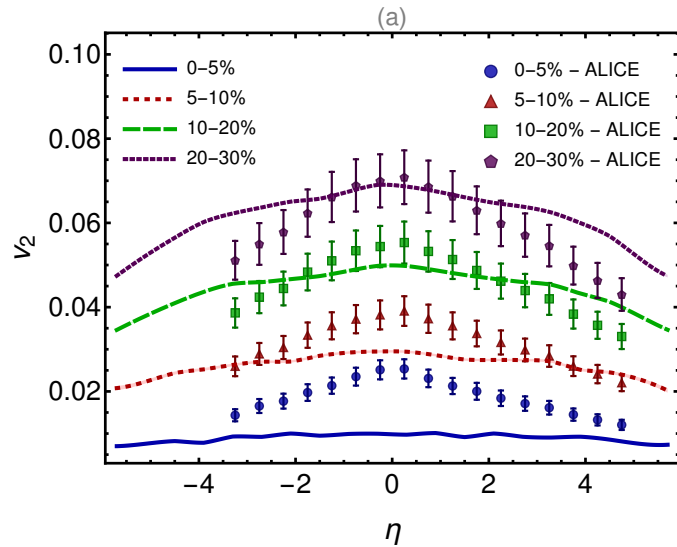
V_2 for identified hadrons as a function of p_T

- Overall, our model shows a quite reasonable description of the data.
- In panels (b) and (c), the agreement is quite good, our model reproduces the data for pion, kaon, and proton out to $p_T \sim 1.5$, 1.5, and 2.5 GeV.
- In panel (a) and (d), the agreement is less.
- This difference can be related to using smooth Glauber initial conditions and/or constant η/s .



The pseudorapidity dependence of v_2

G. Denicol, A. Monnai, and B. Schenke arXiv:1512.01538



- We show the pseudorapidity dependence of v_2 for different centrality classes.
- At large pseudorapidity, we note that our model results do not fall fast enough compared with experimental data.
- This can be related to using a constant η/s , using a temperature-dependent $\eta/s(T)$ can improve the agreement (work in progress).

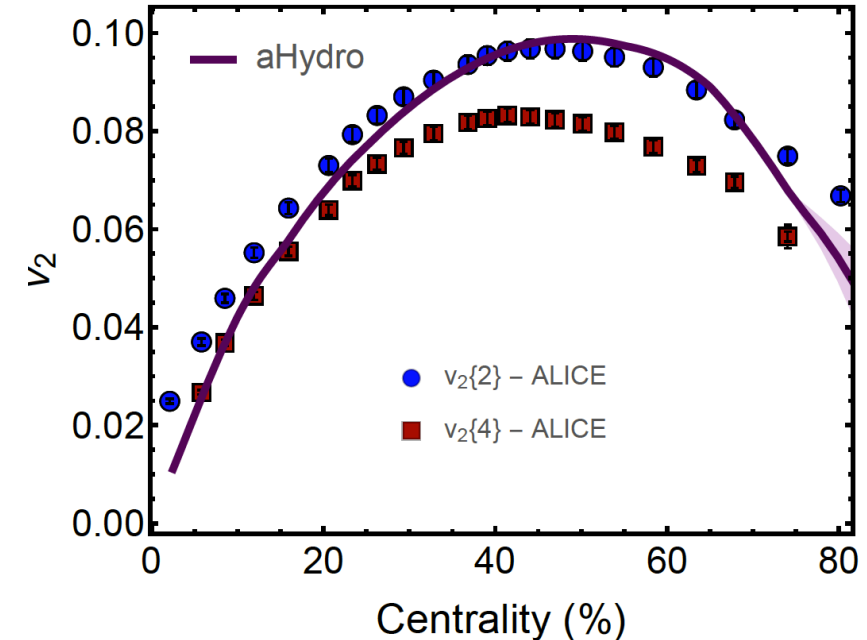
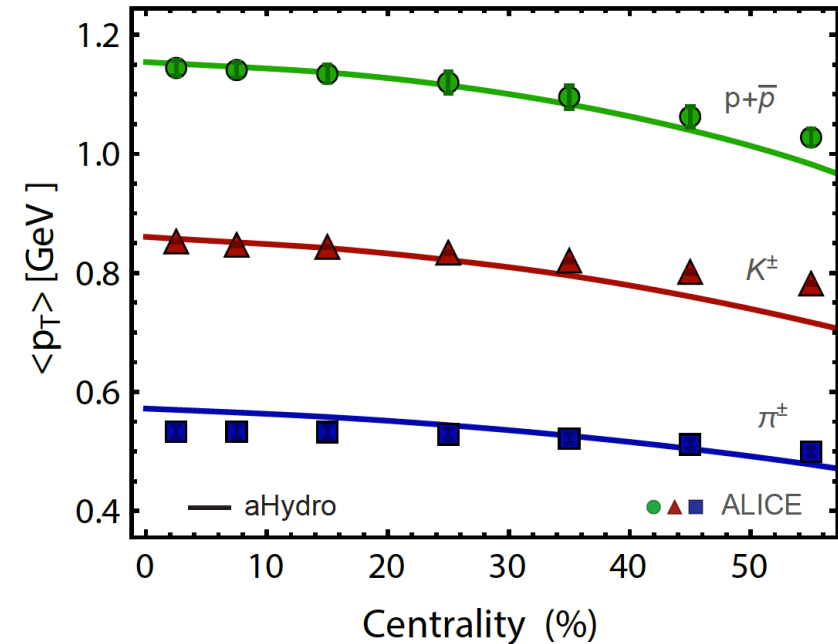
The average transverse momentum

- Our model reproduces the data quite well for the mean transverse momentum as a function of centrality.
- The agreement that our model provides is similar to what other models find.

See: S. Ryu, J. F. Paquet, etc , arXiv: 1502.01675

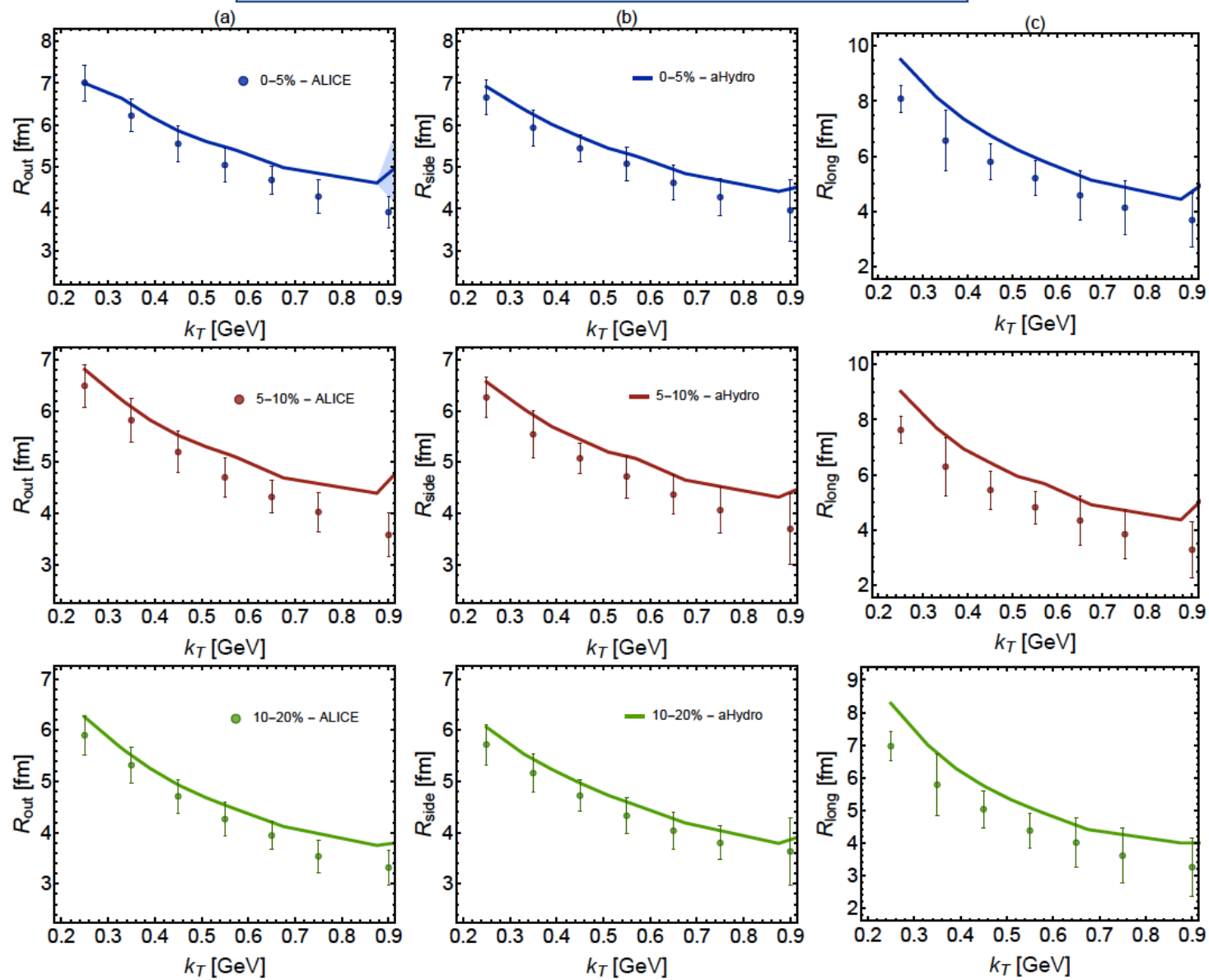
V_2 as a function of centrality

- We compared also the integrated elliptic flow for charged hadrons as a function of centrality where our model shows a quite good agreement.
- Our model underestimates the v_2 at very central collisions, this is due to using smooth initial conditions.



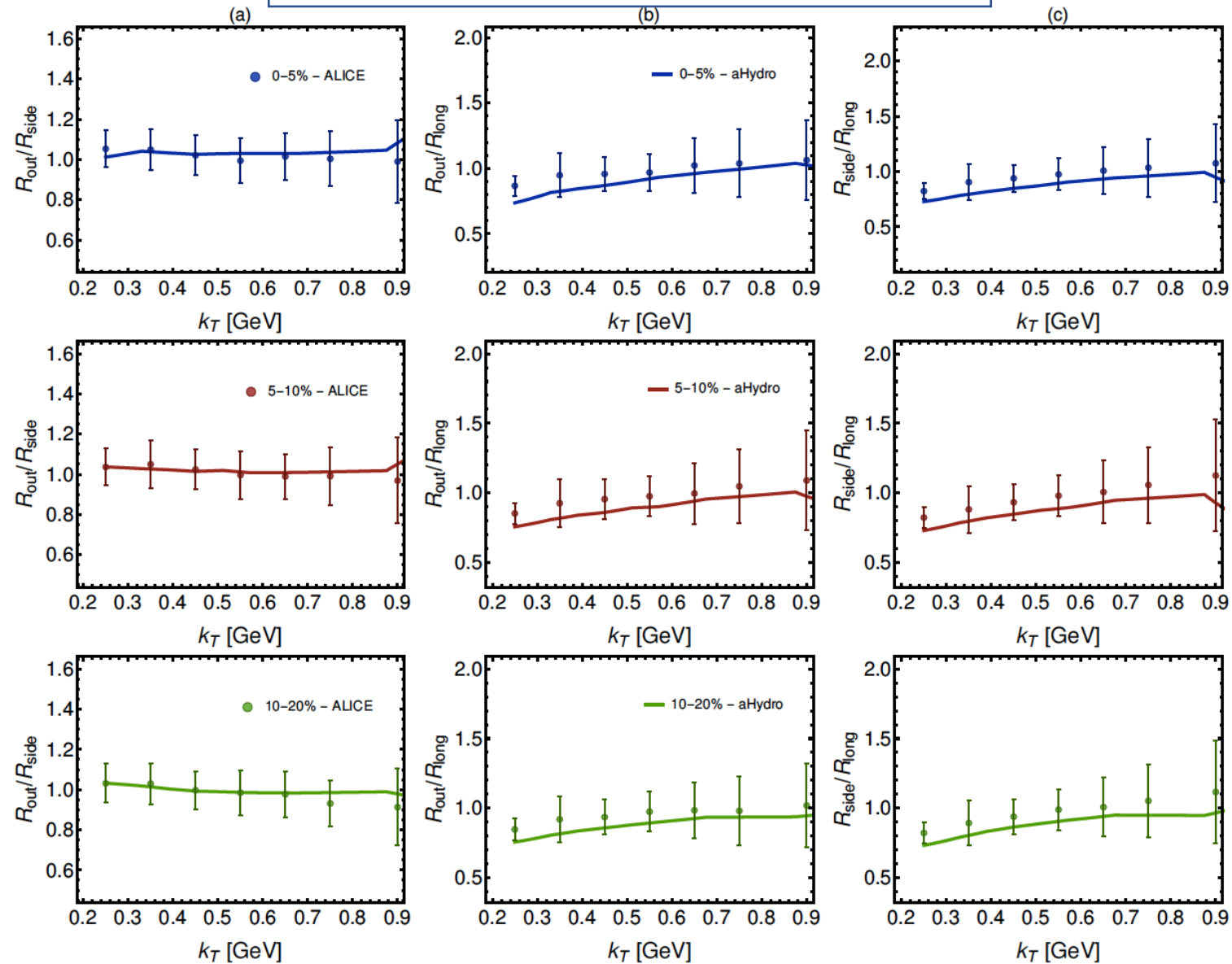
HBT radii

M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1705.10191

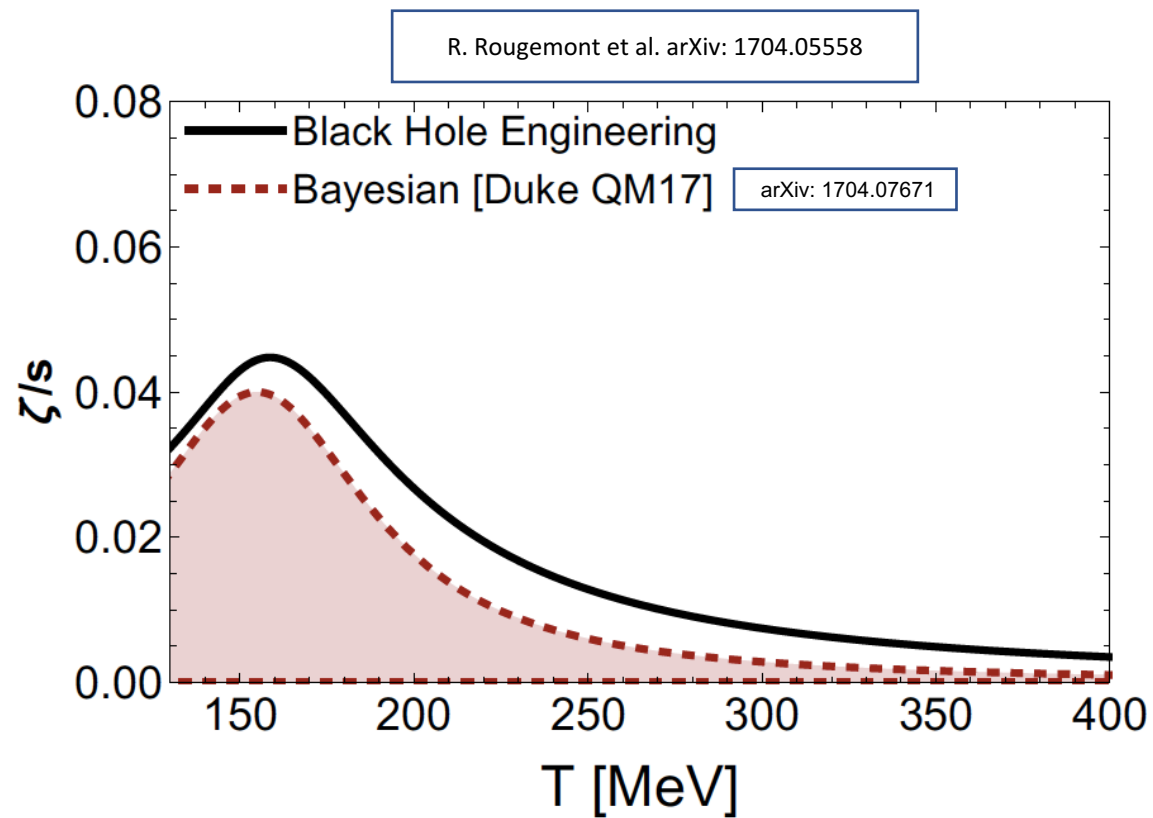
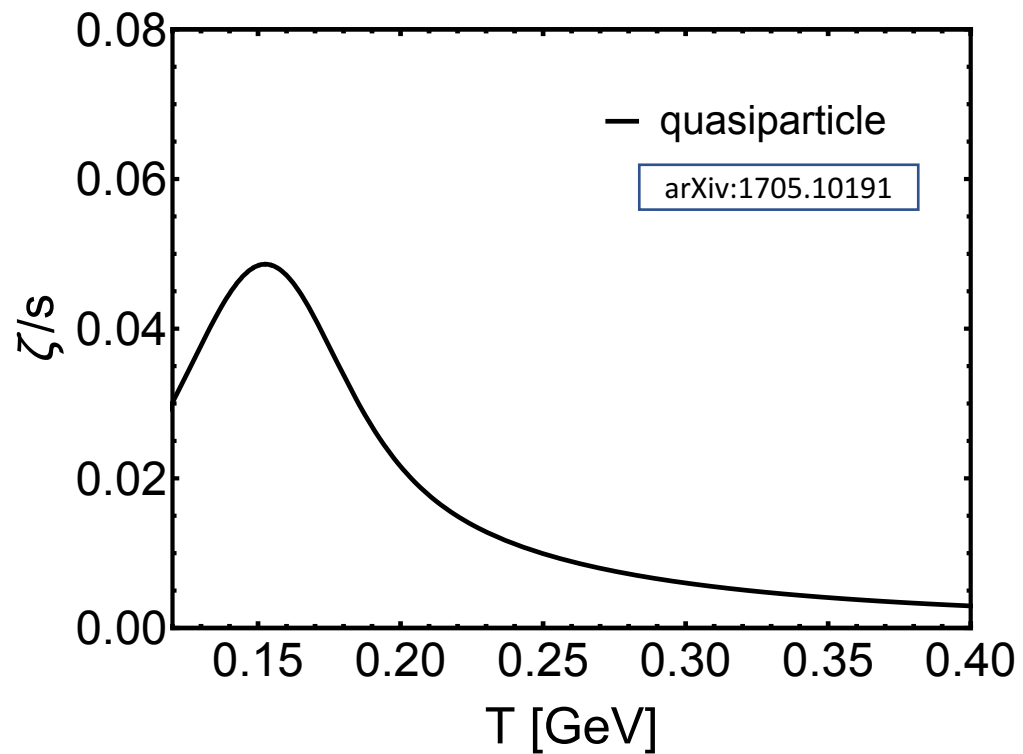


HBT radii ratios

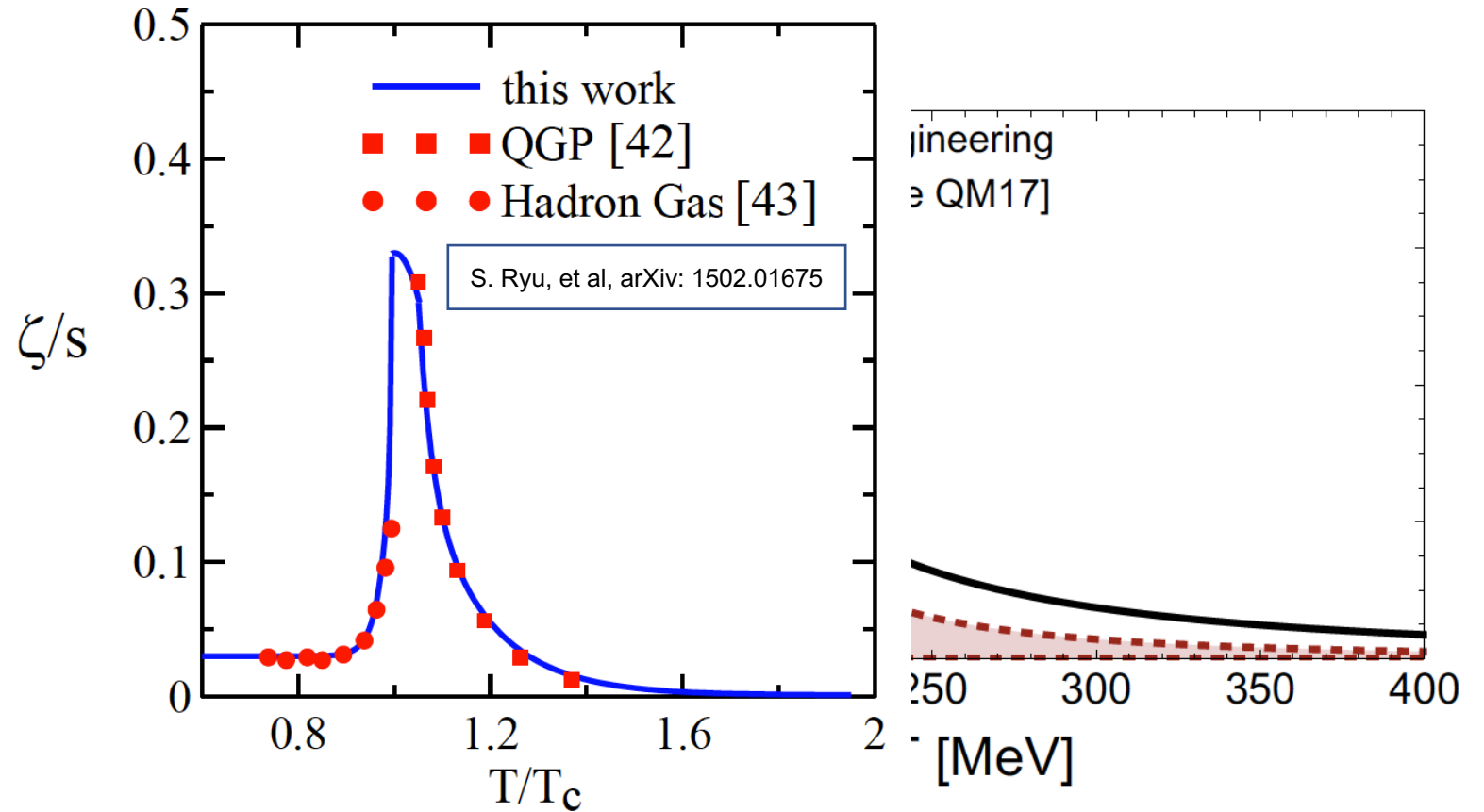
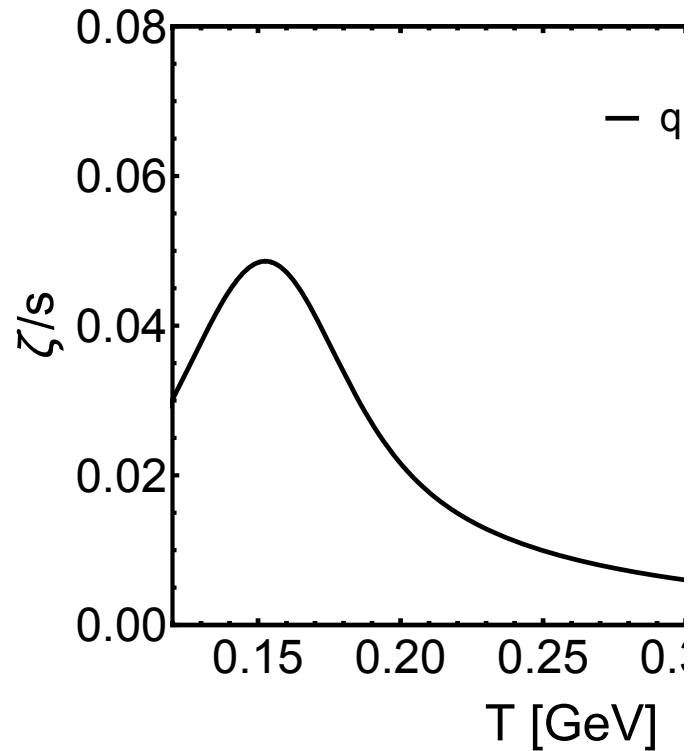
M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1705.10191



The bulk viscosity



The bulk viscosity



- The peak value of the bulk viscosity predicted by our model (quasiparticle) is much lower than other models predictions, e.g, the peak value predicted in (arXiv: 1502.01675) was $\zeta/s \sim 0.3$.
- Note that we use $\eta/s = 0.159$, however they use $\eta/s = 0.09$.

Conclusions and outlook

- Anisotropic hydrodynamics takes into account momentum-space anisotropies of the QGP from the beginning.
- Quasiparticle anisotropic hydrodynamics more self-consistently treats the non-conformality of the QGP than prior approaches.
- Using aHydroQP, we were able to fit the ALICE 2.76 TeV Pb-Pb collisions quite well.
- Our model predicts $\frac{\eta}{s} = 0.159$, $T_0 = 600$ MeV at $\tau_0 = 0.25$ fm/c, $T_{FO} = 130$ MeV, and the peak value of $\zeta/s \sim 0.05$ which is smaller than what other viscous studies found ~ 0.3 .

Looking to future

- We are working on improving our code in many ways
 - Including fluctuating initial conditions.
 - Including temperature dependence of $\frac{\eta}{s}$
 - Including elastic hadronic collisions using some available codes on the market like URQMD or SMASH.
 - Realistic collisional kernels instead of relaxation time approximation (RTA).
- We are working also on using our code at RHIC energies.

THANK YOU