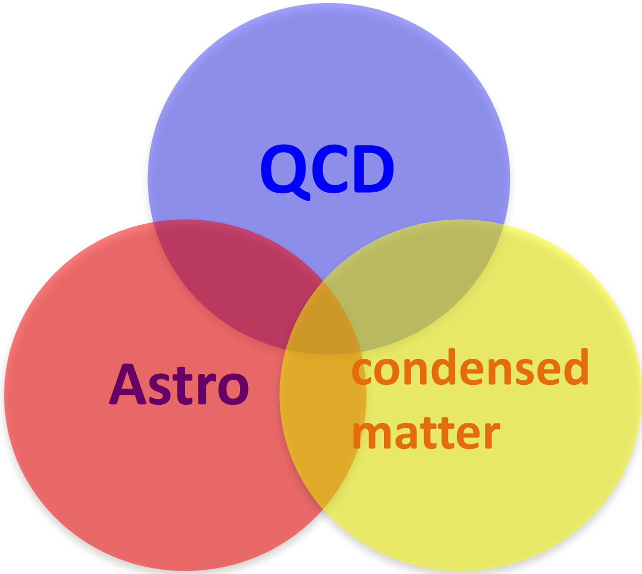
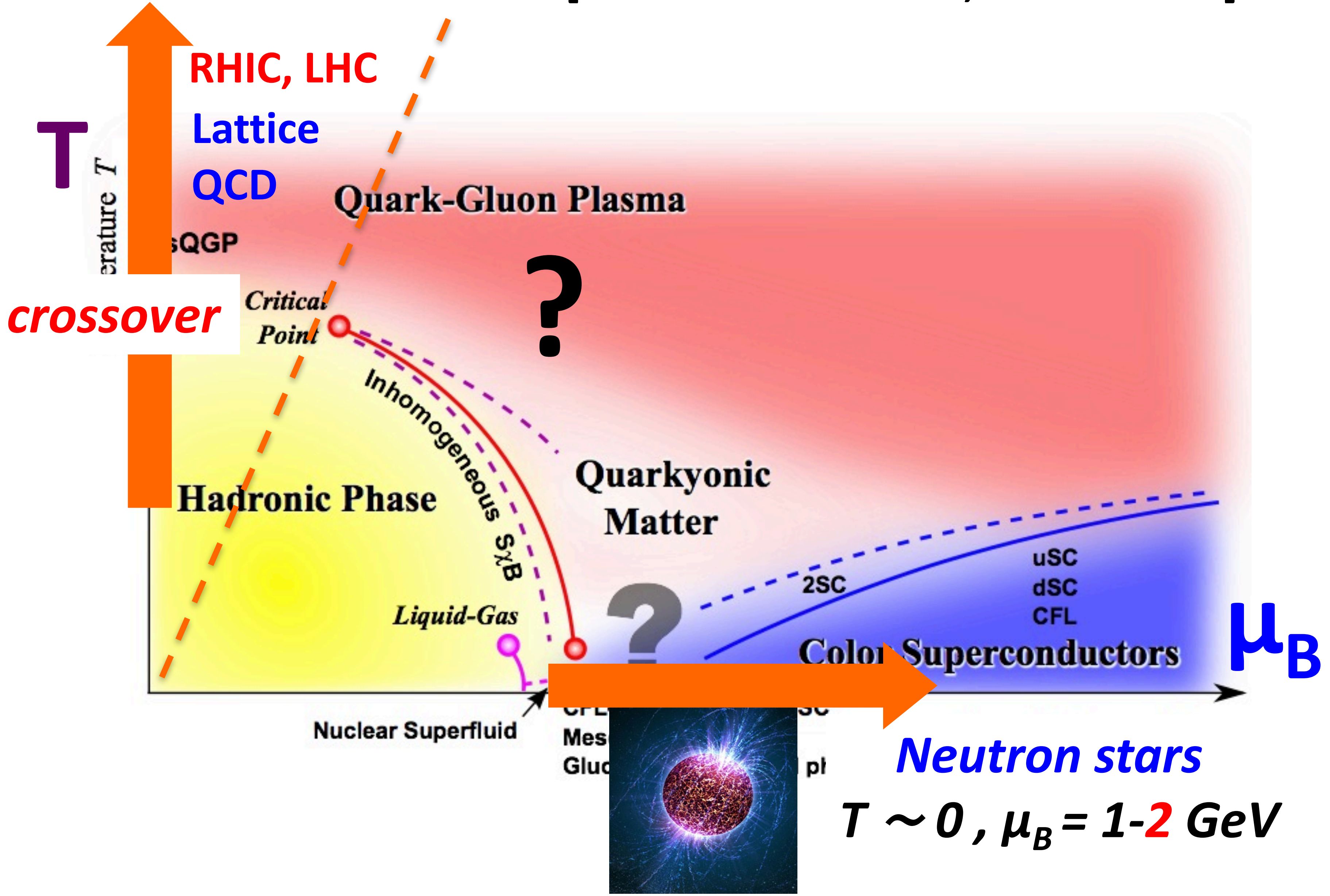
Phenomenological QCD equations of state for neutron star structure & mergers

Toru Kojo (CCNU)



- TK, P.D. Powell, Y. Song, G. Baym
  1412.1108 [hep-ph] , PRD91, 045043 (2015)
- TK, 1508.1108 [hep-ph], review in EPJA
- K. Fukushima & TK, 1509.1108, APJ817(2016)2
- TK, 1610.05486 [hep-ph], PLB769 (2017) 14
- Baym-Hatsuda-TK-Powell-Song-Takatsuka,
  (review) 1707.04966 [astro-ph]

[Fukushima-Hatsuda, review 2010]



T

RHIC, LHC

Lattice QCD

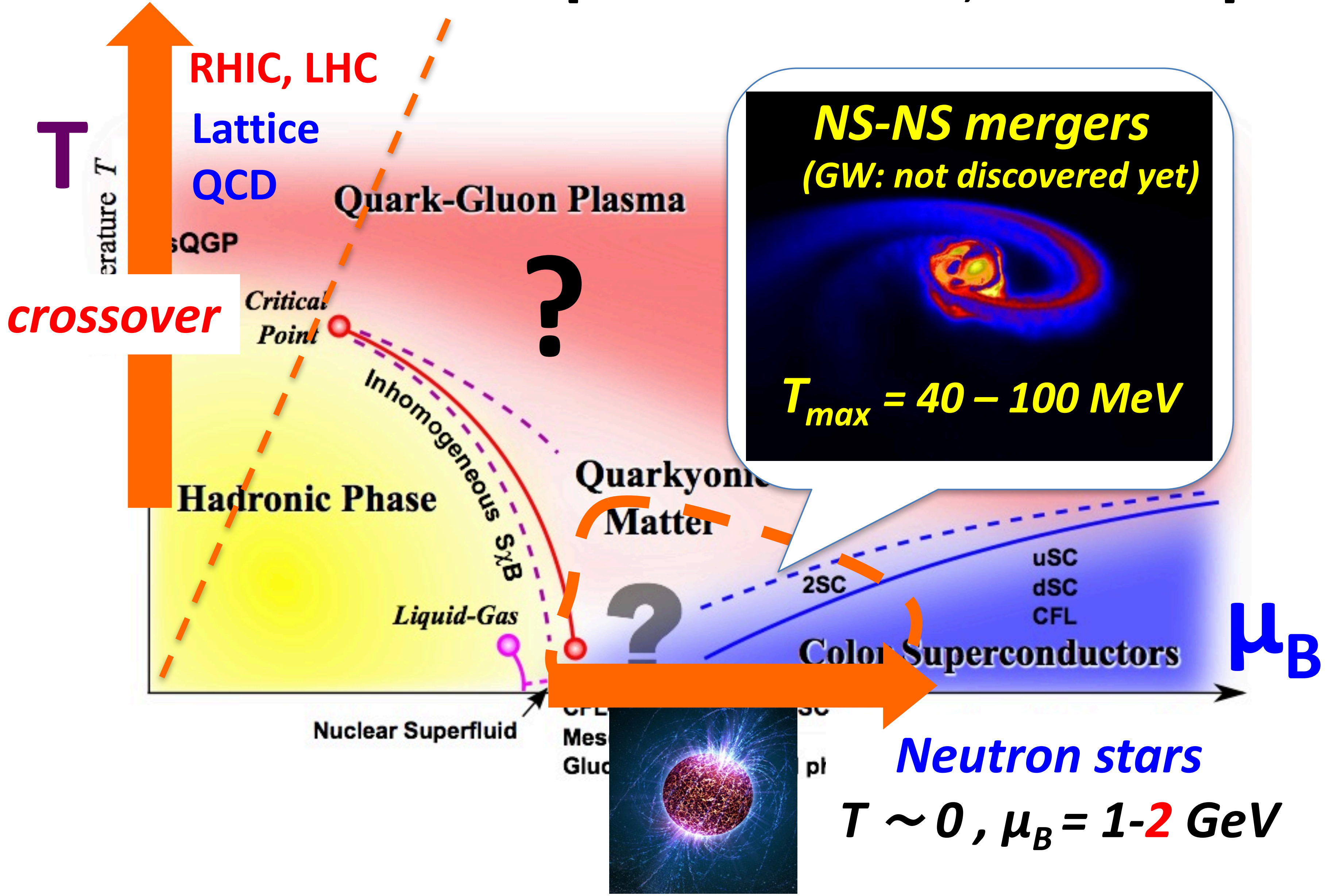
crossover

μ_B

Neutron stars

$T \sim 0$, $\mu_B = 1\text{-}2$ GeV

[Fukushima-Hatsuda, review 2010]



T

RHIC, LHC

Lattice QCD

crossover

NS-NS mergers

(GW: not discovered yet)

T_{max} = 40 – 100 MeV

μ_B

Neutron stars

$T \sim 0$, μ_B = 1-2 GeV

Strategy

Part I: NS structure, EoS at T=0

"observable"

EoS: $P(\mu)$ $\underset{\text{Lindblom (1992)}}{\overset{\text{GR : TOV eq.}}{\rightleftarrows}}$ ***M-R*** **relation**

$\hookrightarrow$ *a model for quark matter*

Part II: Perturbing T=0 EoS by (T, Y_e) *(lepton fraction)*

EoS: $P(\mu,T,Y_e)$ $\xrightarrow{\text{GR simulations}}$ *Gravitational waves*

&

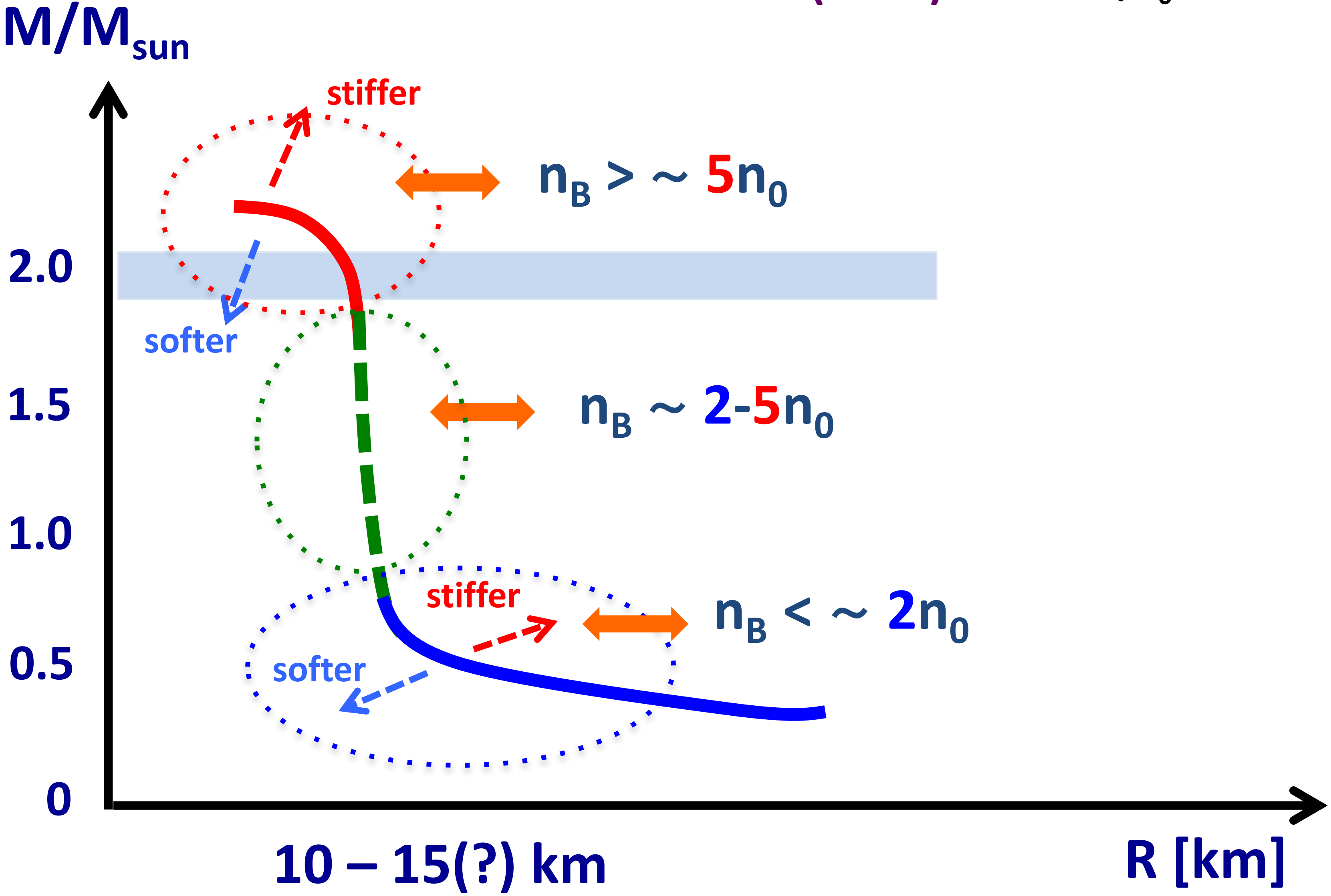
EW bursts

predictions

M-R relation & EoS

Lattimer & Prakash (2001)

($n_0 = 0.16\ \mathrm{fm}^{-3}$)



M-R relation & EoS

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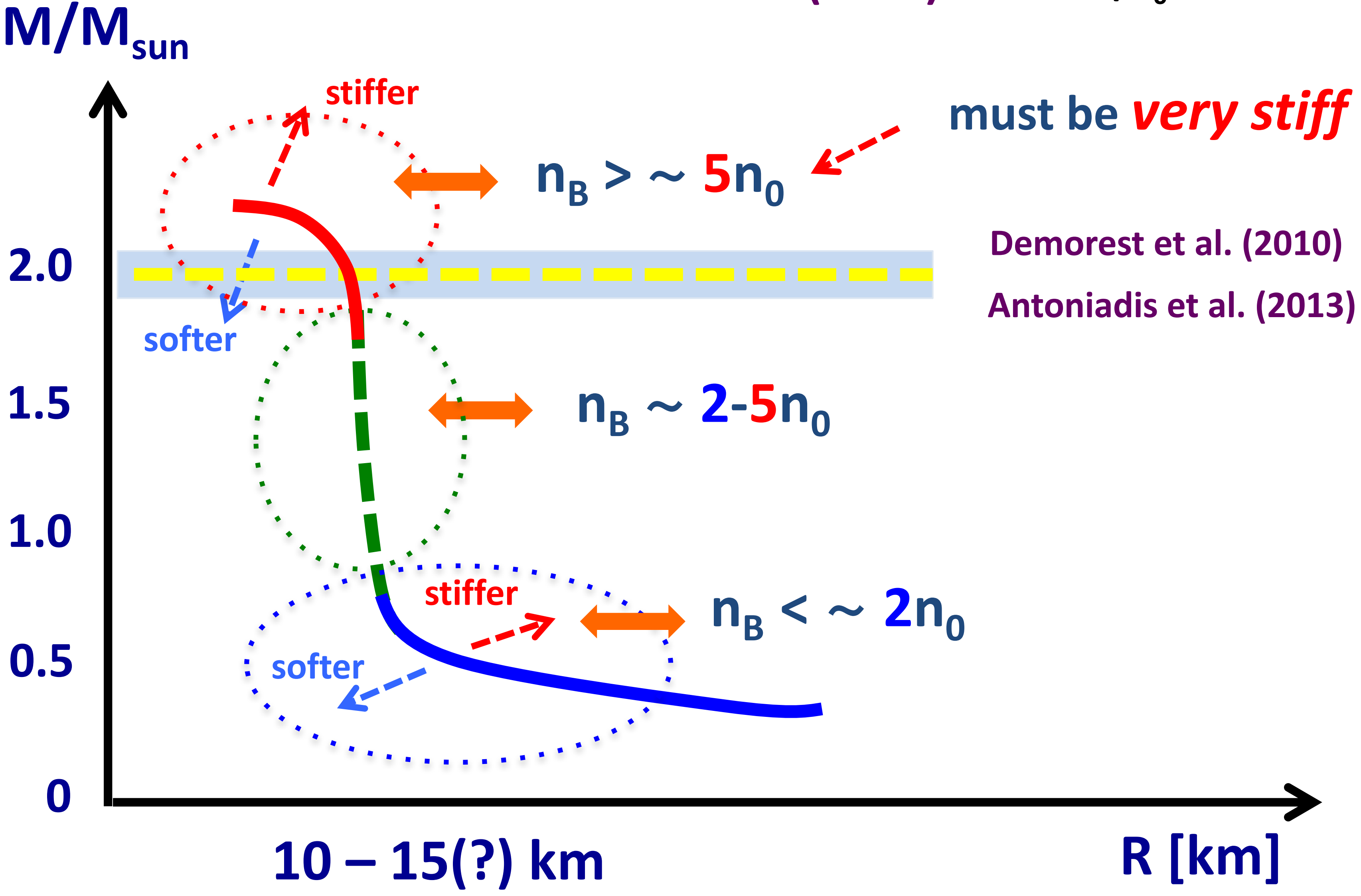


must be *very stiff*

$n_B > \sim 5n_0$

Demorest et al. (2010)

Antoniadis et al. (2013)

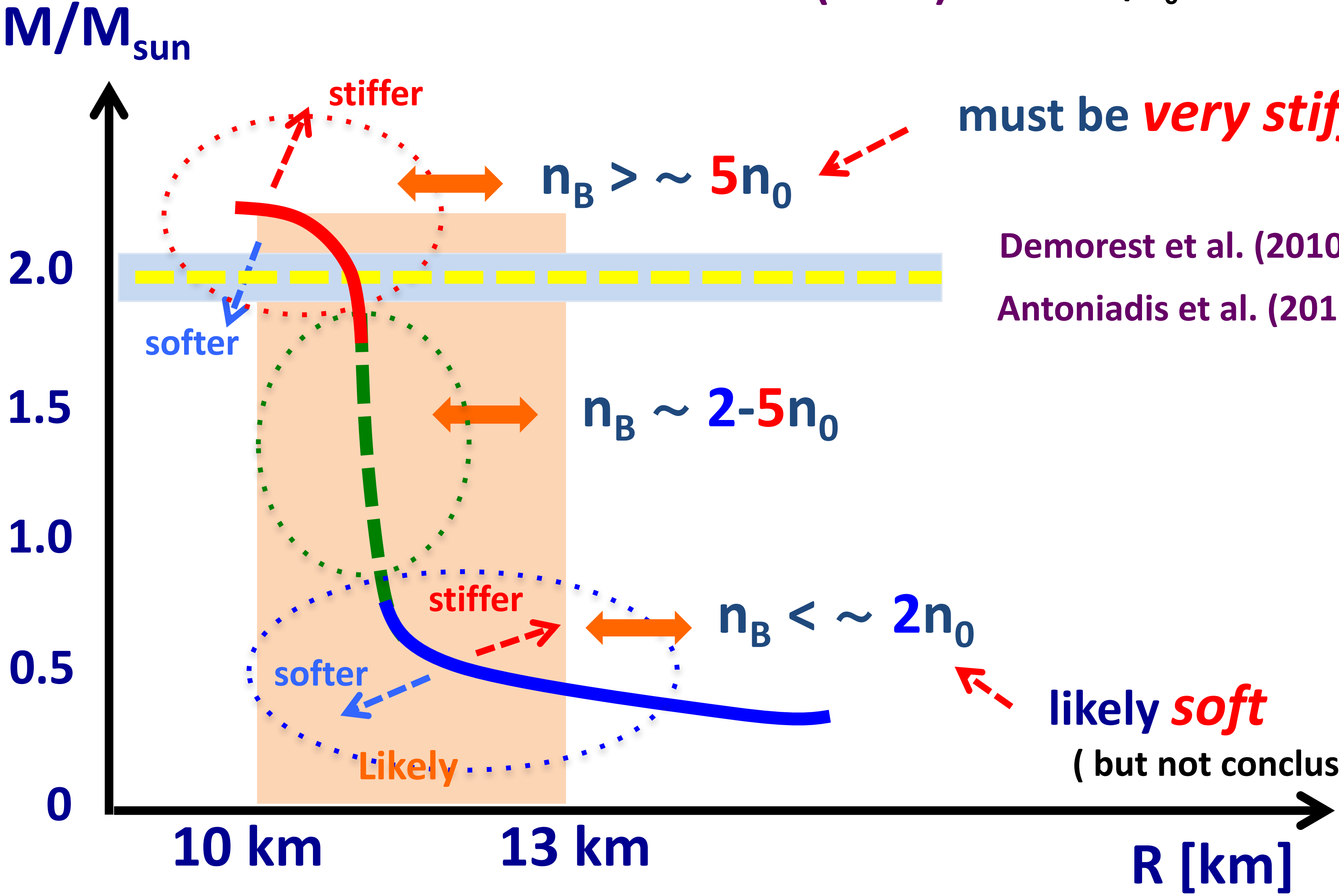
$n_B \sim 2\text{-}5n_0$

$n_B < \sim 2n_0$

M-R relation & *EoS*

Lattimer & Prakash (2001)

($n_0 = 0.16\ \mathrm{fm}^{-3}$)



must be ***very stiff***

Demorest et al. (2010)

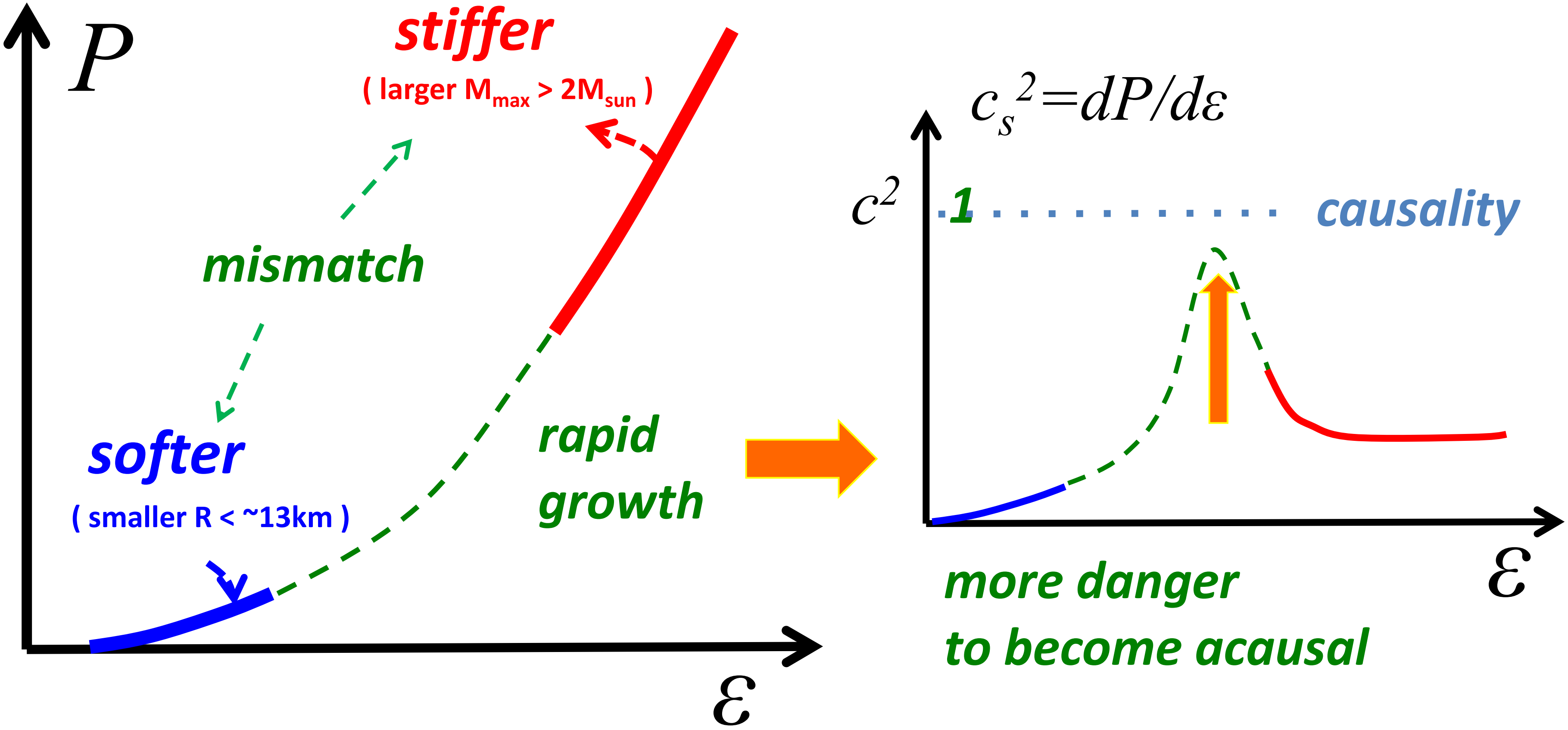
Antoniadis et al. (2013)

likely ***soft***

(but not conclusive)

Ozel et al. (2010), Steiner et al (2015) : X-ray analyses

Causality constraint on $2n_0$-$5n_0$ region



stiffer

(larger $M_{max} > 2M_{sun}$)

mismatch

softer

(smaller R < ~13km)

rapid growth

$c_s{}^2 = dP/d\varepsilon$

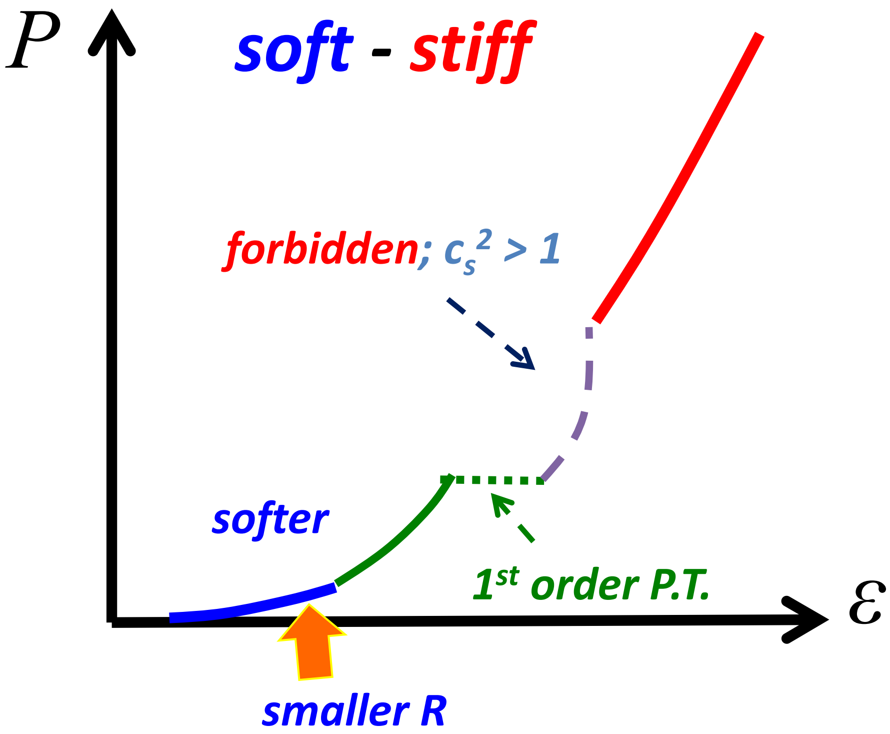
causality

more danger to become acausal

For *softer* - *stiffer EoS* ➡ ***less freedom*** **for $2n_0$- $5n_0$ region**

If we put 1st order H-Q transitions…

[more systematic analyses -> Han-Alford-Prakash 13]



If R is small (<~13km) ➡ disfavors strong 1st order P.T.

Ozel et al. (2010), Steiner et al (2015); target of NICER and GW detection

hadron-quark continuity ??

[Schaefer-Wilczek 98, Hatsuda et al. 07]

3-window modeling

(Masuda-Hatsuda-Takatsuka 12)

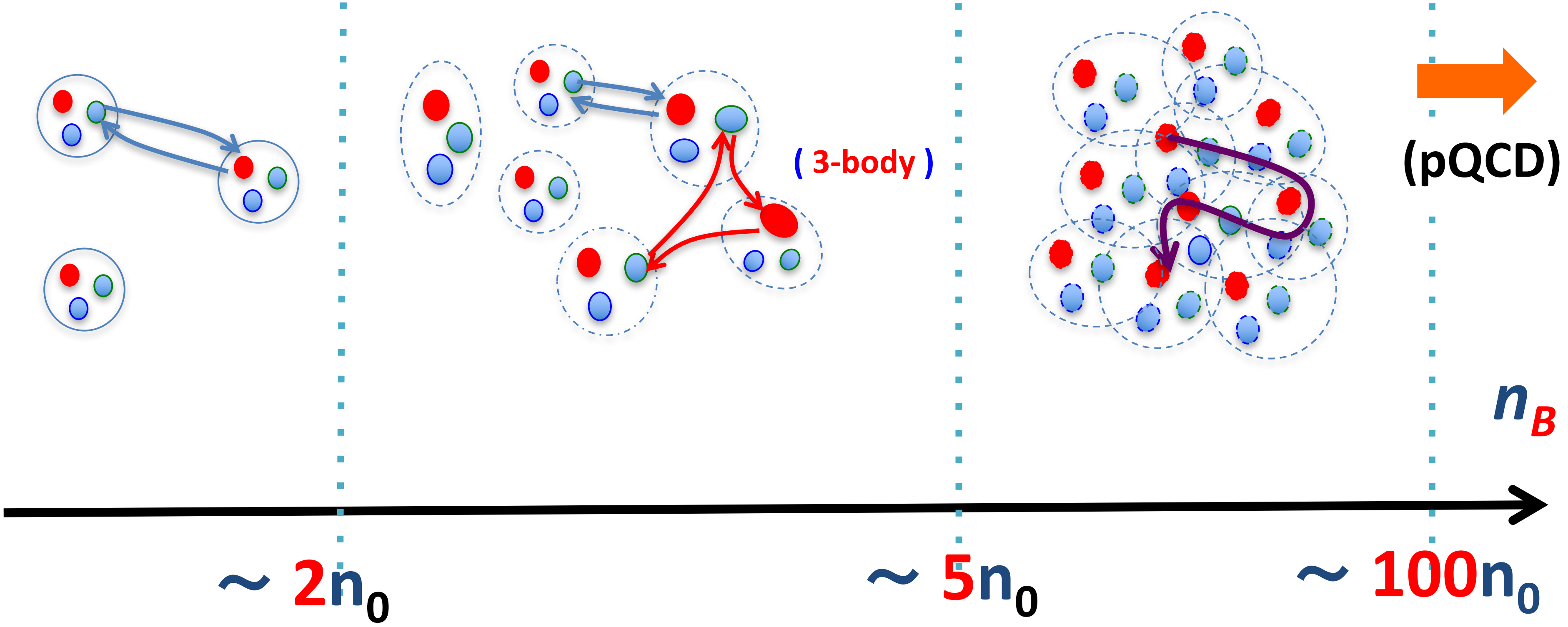
- **few meson exchange**
- **nucleons only**

- **many-meson exchange**

  (mobility --cf: Karsch-Satz ’80)
- **structural change of hadrons**

- **Baryons overlap**
- **Quark Fermi sea**

  $p_F \sim 400$ MeV



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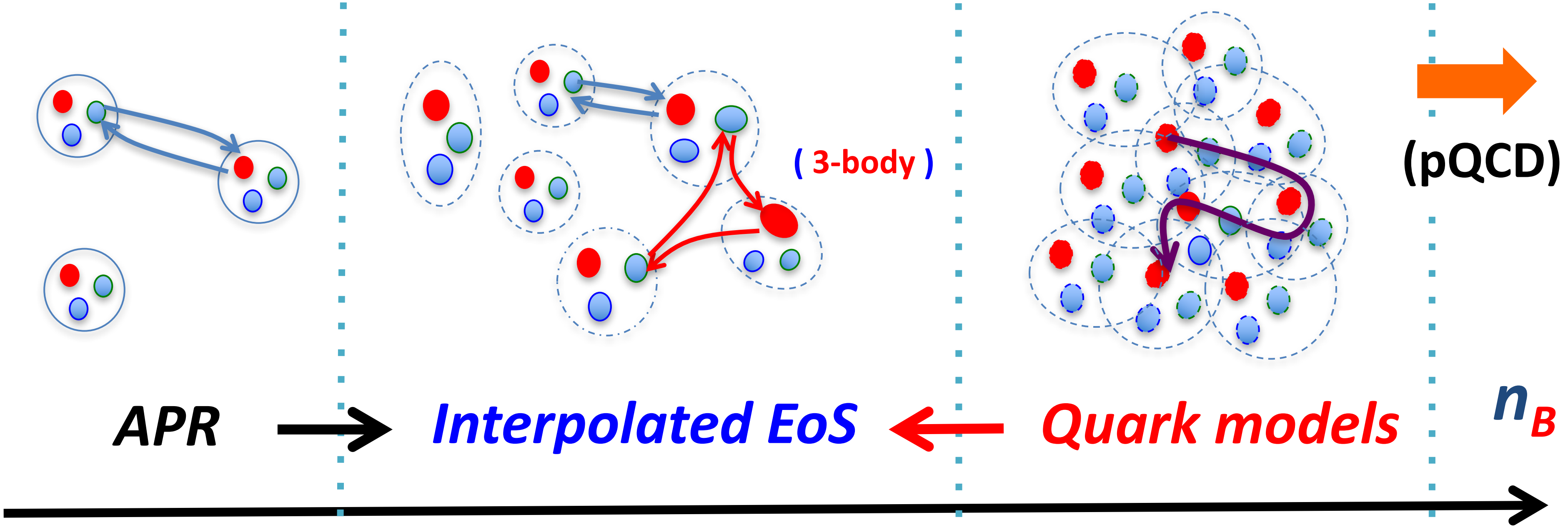


APR → ***Interpolated EoS*** ← ***Quark models*** n_B

$\sim 2n_0$ $\sim 5n_0$ $\sim 100n_0$

3-flavor quark MF model : template

Effective Hamiltonian (inspired by **hadron** & **nuclear** physics):

$$\mathcal{H}_{\rm eff} \sim \bar{\psi}\left[-\mathrm{i}\vec{\alpha}\cdot\vec{\partial}+m\right]\psi \;+\; \mathcal{H}_{\rm NJL}^{\rm 4Fermi+KMT}$$

→ structural change of *Dirac sea* & *quark bases*

$+ \; \mathcal{H}_{\rm conf}^{3q\rightarrow B}$ → will be *ignored* in the *percolated* domain

mag. part

$+ \; \mathcal{H}_{\rm OGE}$ → $- \, H \sum_{A,A'=2,5,7} \left(\bar{\psi}\mathrm{i}\gamma_5\lambda_A\tau_{A'}\psi_c\right)^2$

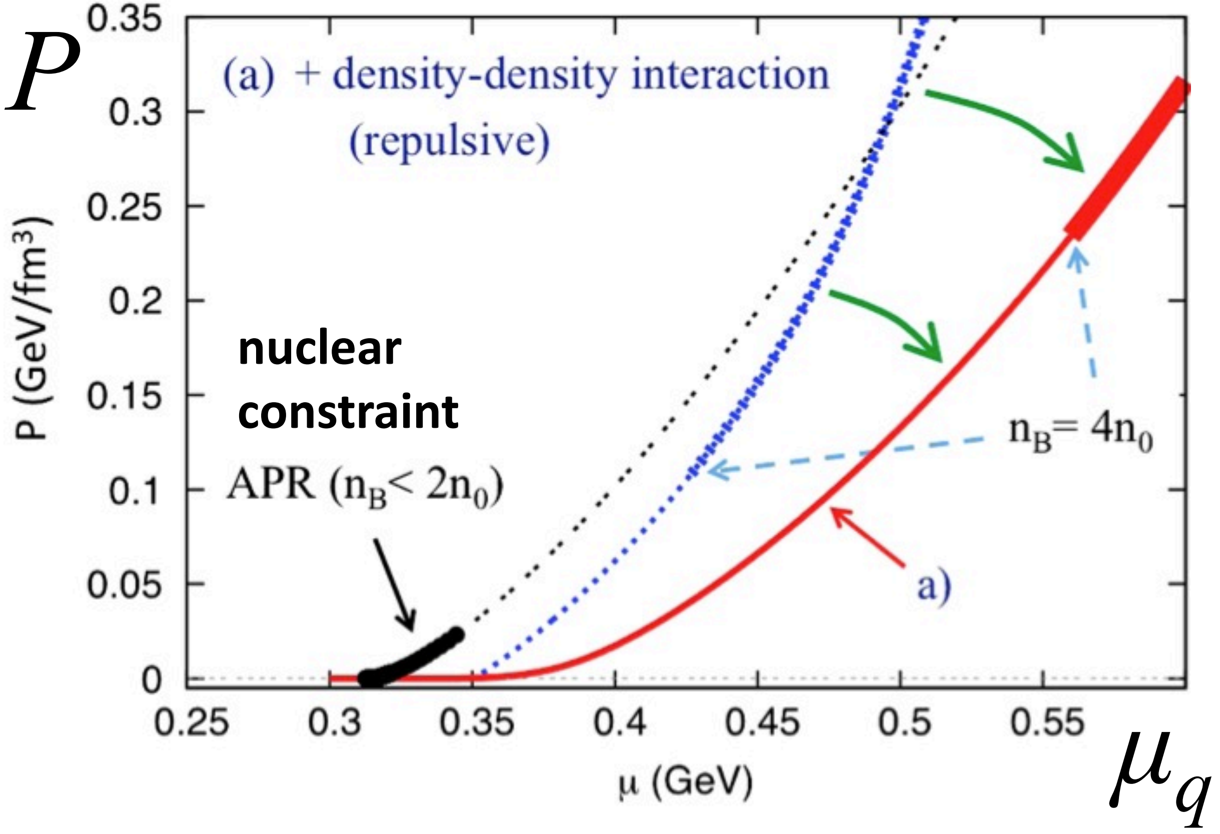
(cf: *N-Δ splitting*)

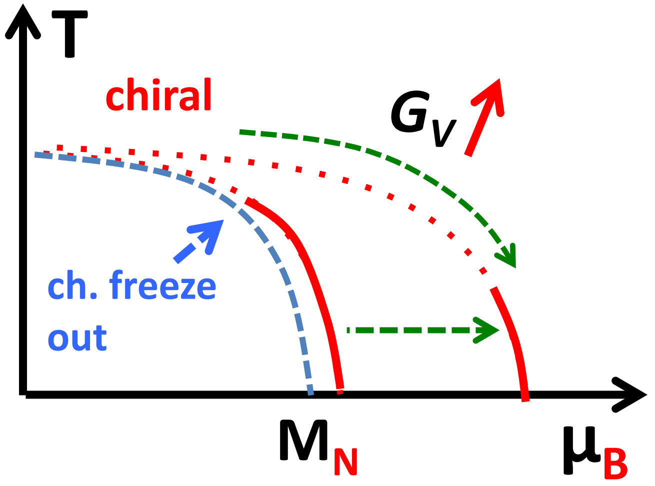
$+ \; \mathcal{H}_{\rm nucl}$ → $+ \, g_V\left(\bar{\psi}\gamma_0\psi\right)^2$ ~ *ω-exchange* (*repulsive*)

+ constraints (charge neutrality, β- equilibrium, color-neutrality)

Goal: *NS constraints* → $(\,G_s, H, g_V\,)_{@5\text{-}10n_0}$

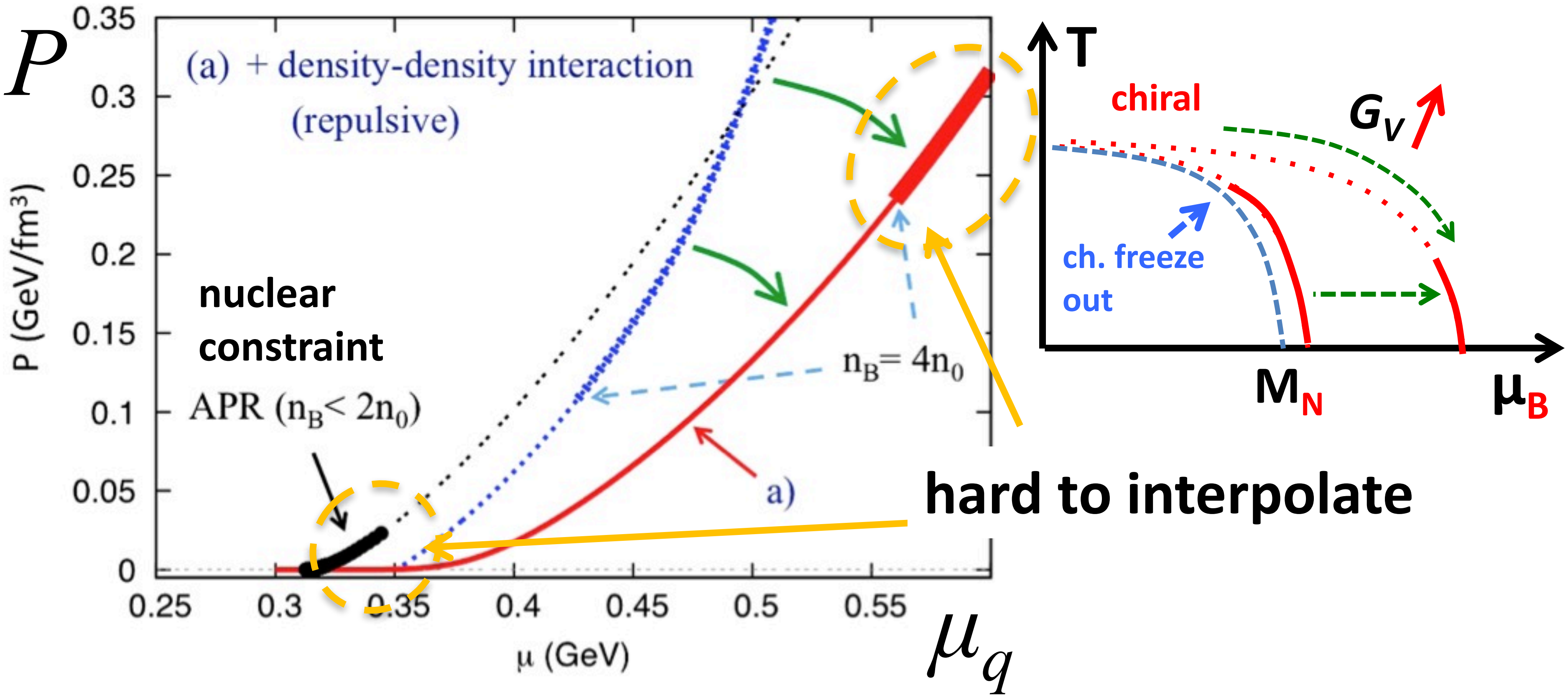
Standard + vector coupling





→ ***stiffen*** **EoS &** ***delay*** **the chiral restoration**

Standard + vector coupling



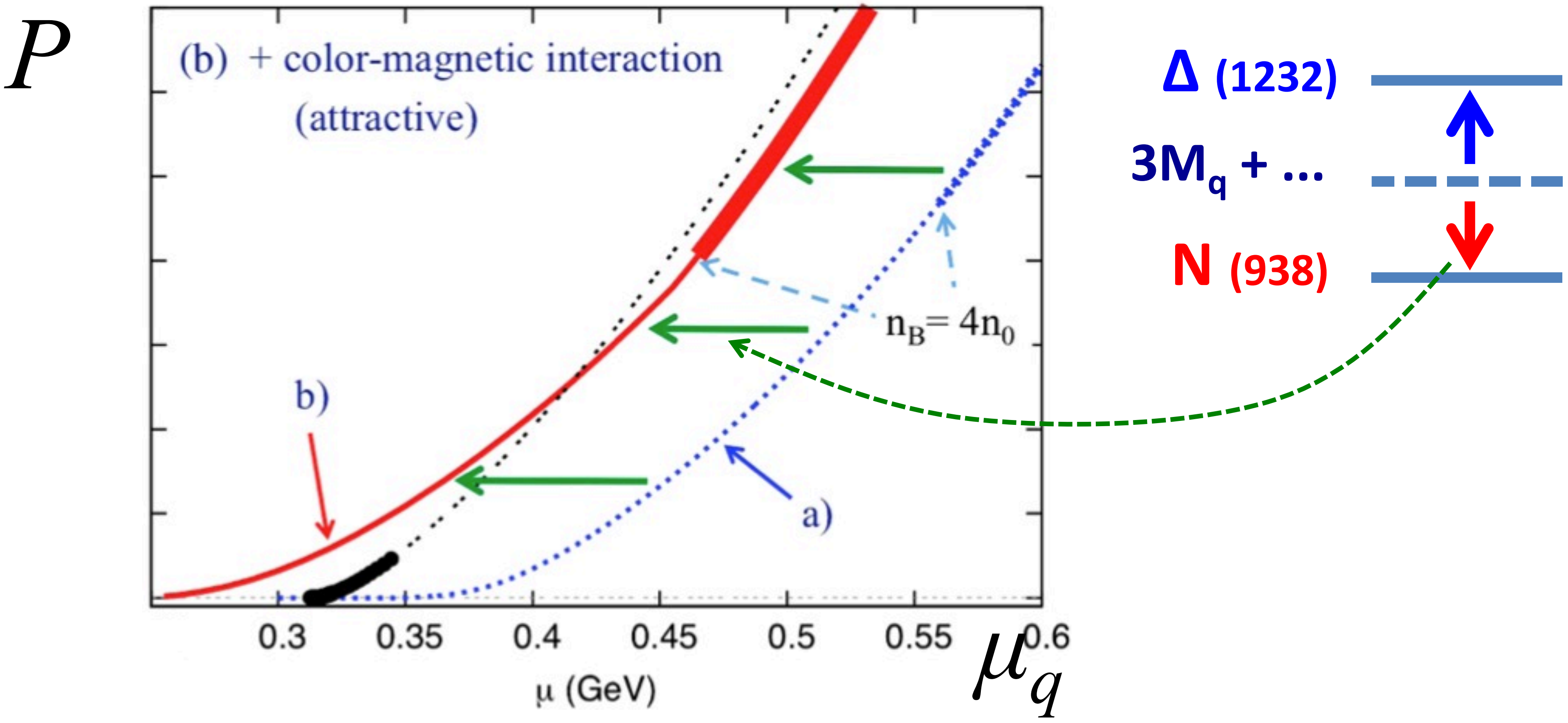
nuclear constraint

hard to interpolate

→ ***stiffen*** **EoS &** ***delay*** **the chiral restoration**

+ color magnetic interaction

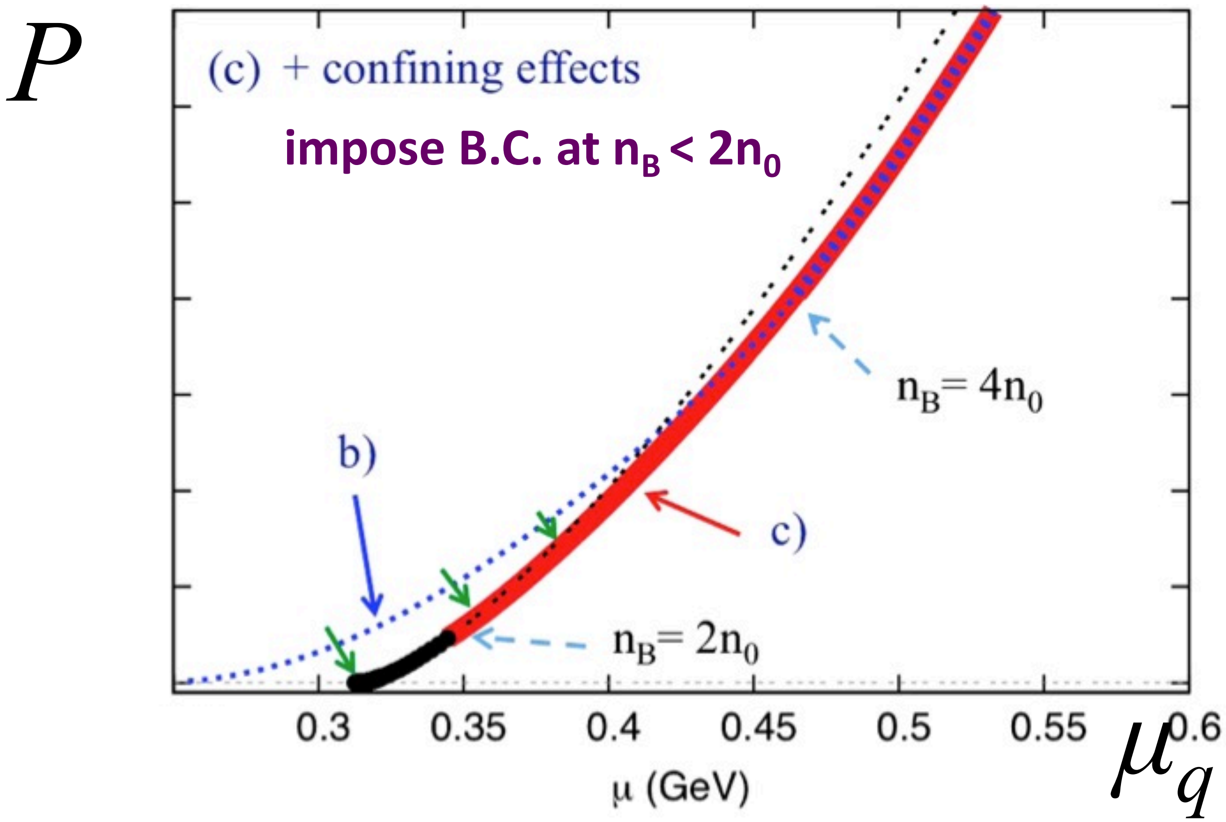
(in *MF*, effects appear as diquark condensate)

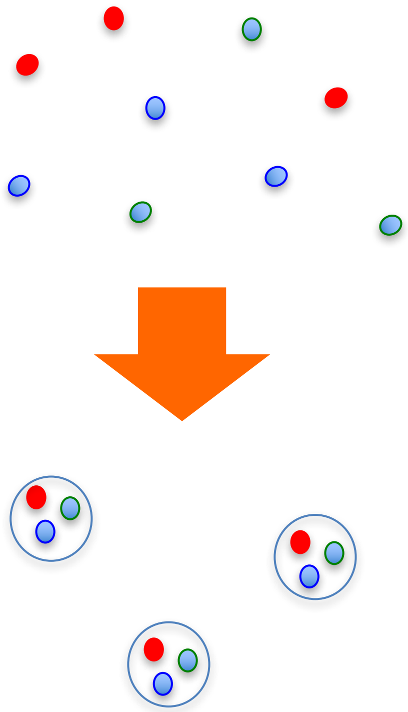


→ overall shift of P(μ) toward lower μ

+ APR constraint at low density

(*mimic confining effects*)

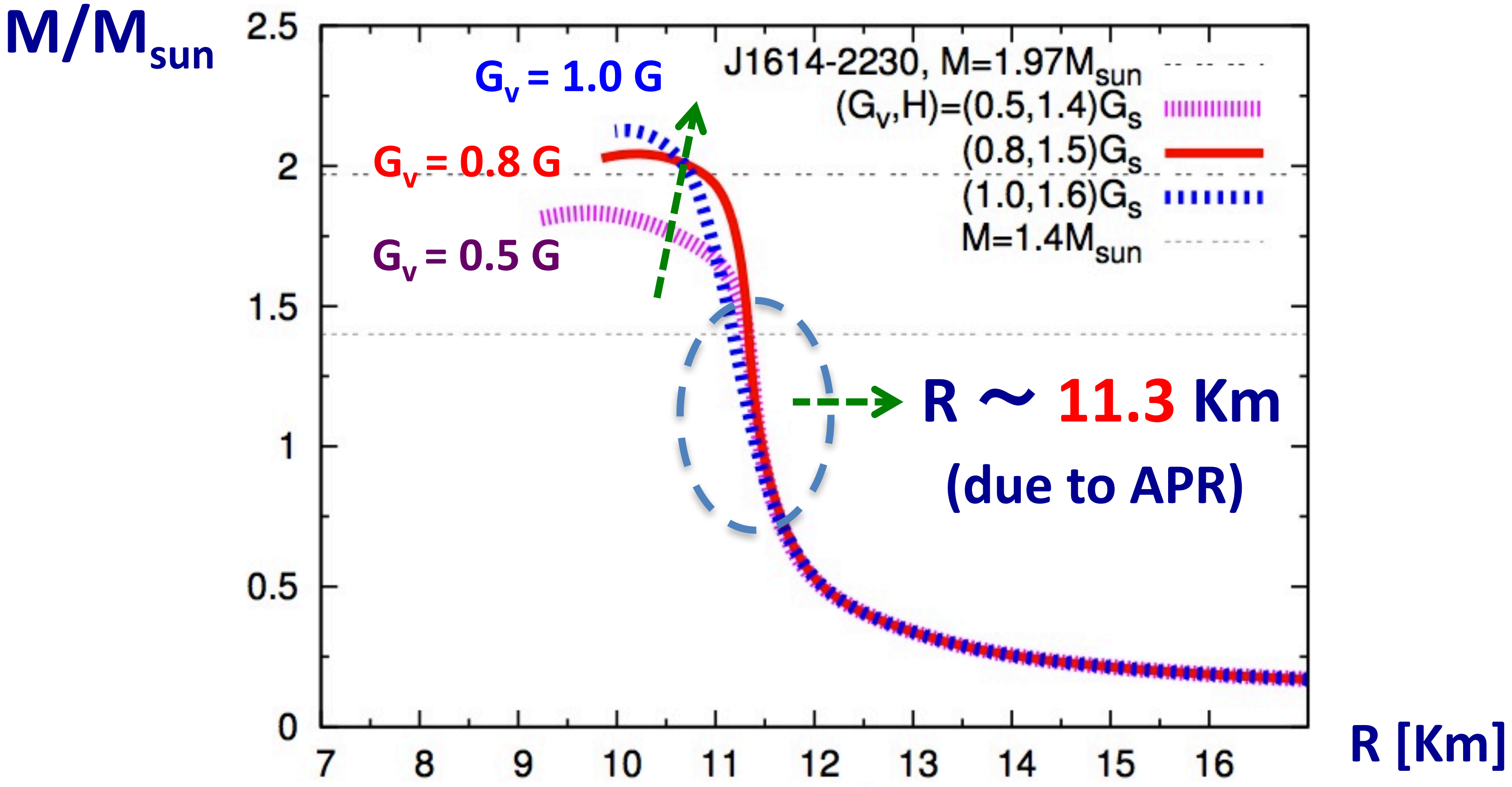




→ discard ***artificial*** excess of ***P*** at $n_B < \sim 5n_0$

(*like Polyakov loop effects in hot QCD*)

M-R curves



R ∼ 11.3 Km

(due to APR)

we need :

$$G_s \sim G_v \sim H \ \ @ \ \ n_B = 5\text{-}10\, n_0 \ \rightarrow \ O(G_s^{vac})$$

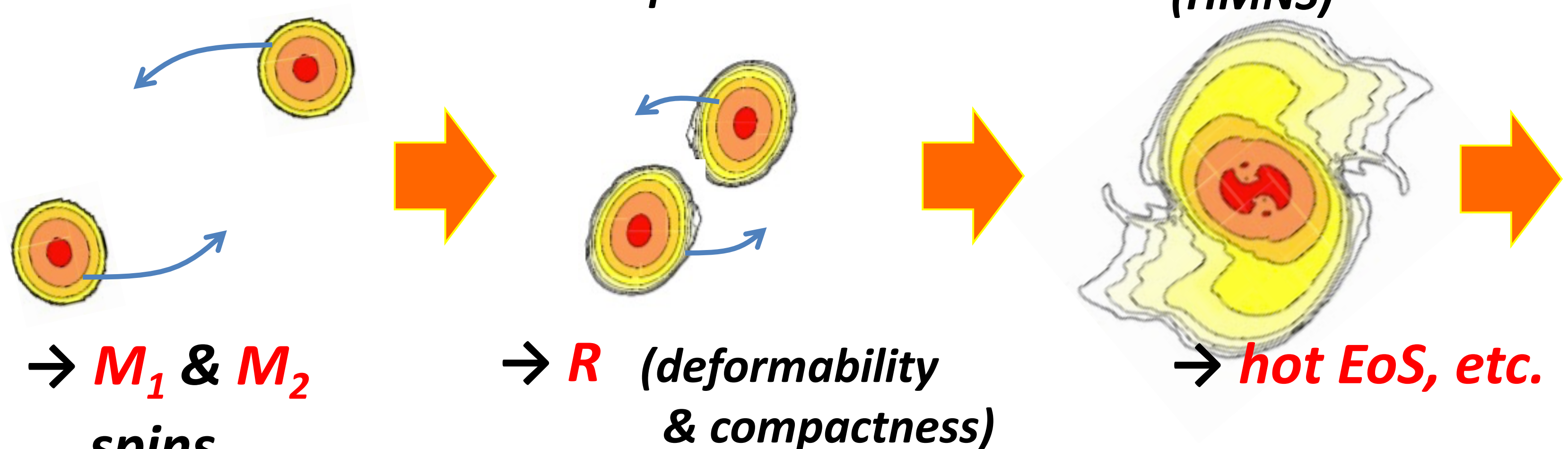
Perturbing T=0 quark EoS by (T, Y_e)

(for supernovae & NS-NS mergers)

[TK, in progress]

GW from NS-NS mergers

(0.1-10 (?) events / year)



Early inspiral

→ M_1 & M_2 *spins*

Tidally deformed phase

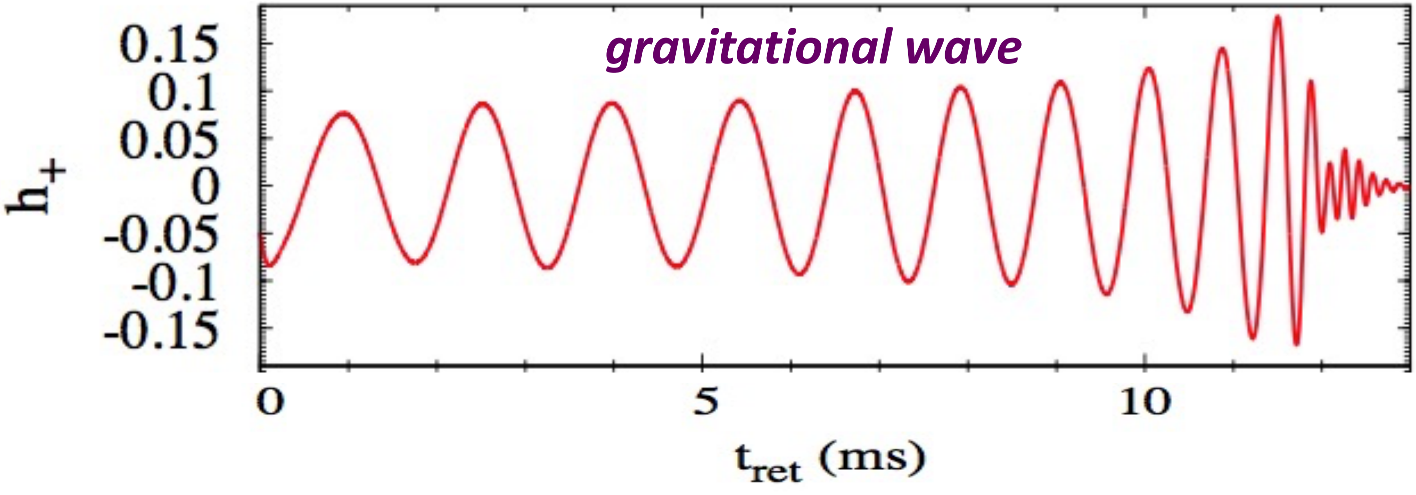
→ *R* *(deformability & compactness)*

Hyper Massive NS (HMNS)

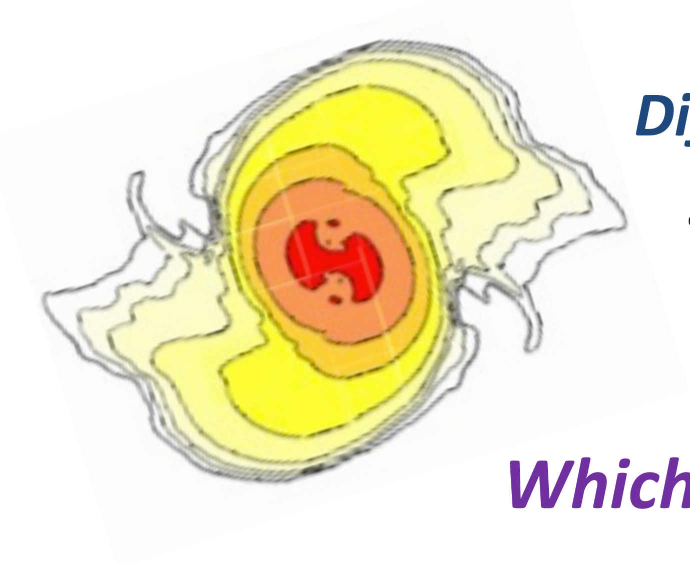
→ *hot EoS, etc.*

BH

(aLIGO, VIRGO, ...)



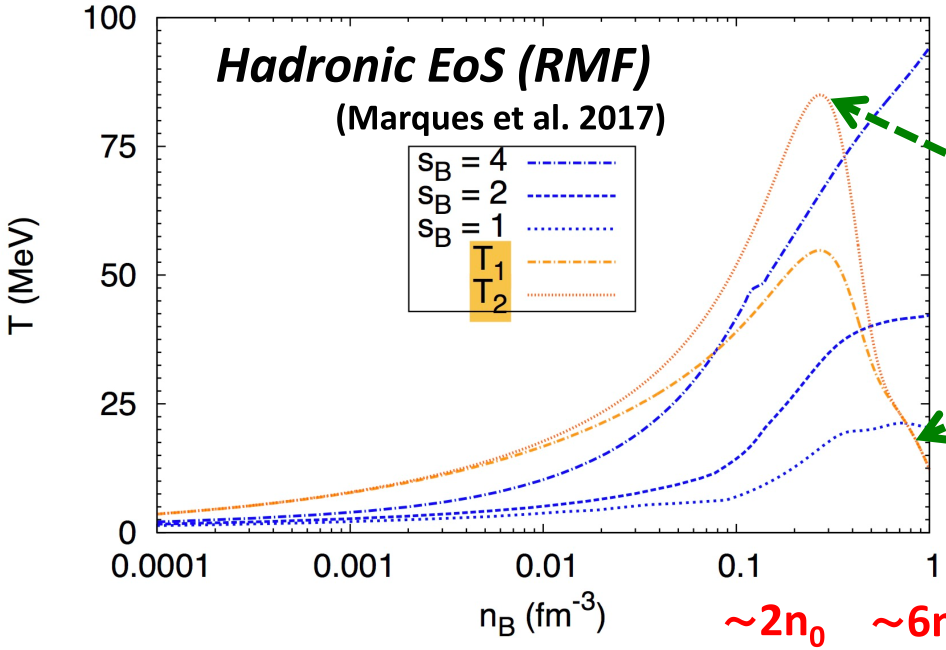
Hyper massive NS (HMNS)



Differential rotation* & *thermal pressure

→ **stars of *2-3M_{sun}* can survive for *~10ms***

Which density region is hot?



~$2n_0$ ~$6n_0$

For typical hadronic EoS coupled to GR simulations :

T = 30-100 MeV at n_B ~ $2n_0$

T = 10-20 MeV at n_B ~ $5n_0$

NOTE: profiles depends on *EoS*

Hot EoS for *post mergers*

- **Almost all GR simulations use hot *nuclear* EoS**

[Shen-EoS (Shen et al.), SLy EoS (Lattimer-Swetsy), ...]

- ***Hot quark* matter EoS (for $n_B > 5n_0$)**

***Normal* quark matter**

  - ***pQCD EoS*** **(gapless quarks & gapped gluons)** [Kurkela-Vuorinen ’16]
  - ***3-window EoS*** **(gapless quarks)** [Masuda-Hatsuda-Takatsuka ’15]
  - ***...***

gapless quarks → $\Delta P(T) \sim p_F^2 T^2 \quad (\gg T^4)$

***This work* → *Gapped* quark matter, *Color-Flavor-Locked* (CFL)**

For T < Δ ; $\Delta P(T) \sim T^4 + \ldots$

neutrinos, photons, NG modes

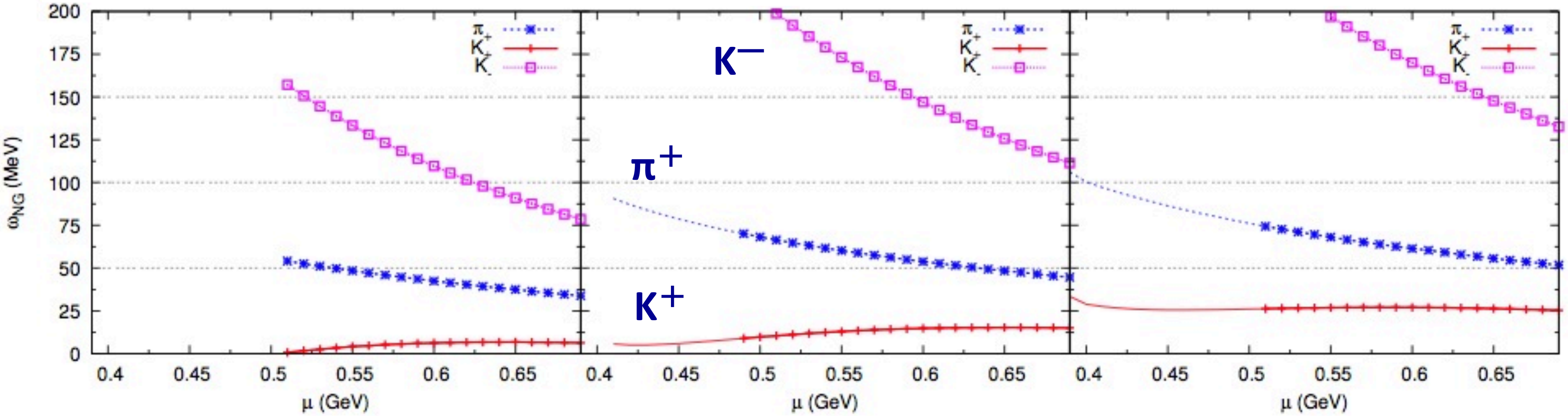
NG mode contributions (CFL color-super phase)

[Son-Stephanov 2000, Bedaque-Schafer 2002, ...]

- **setup consistent with T=0 NS descriptions**
  - explicit sym. breaking, mass & $U_A(1)$
  - neutrality conditions
  - coexistence of chiral and diquark condensates
  - keep “pa”, “pp”, “aa” contributions to be consistent with gap eq.

- **spectra in RPA (results consistent with EFT)** [Basler-Buballa ’10, TK16]

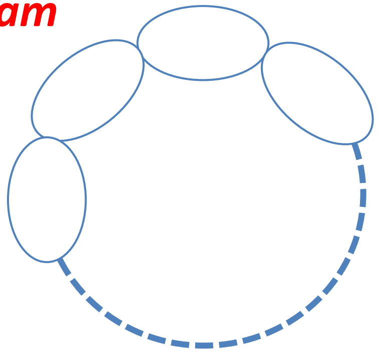
weak coupling ← ***setup for NS constraints*** **→ strong coupling**



most NG modes > 50 MeV; light K; more massive at stronger coupling

Thermodynamics (beyond low T regime)

Ring diagram



connected
Green's
function

NG bosons (bound states)

pre-formed pairs (p-a, p-p, a-a pairs)

decaying pairs (continuum)

very important to keep (see below)

The phase shift rep. of thermodynamic-potential :

[Beth-Uhlenbeck1939, Dashen-Ma-Bernstein 1969]

$$\Omega_X(T,\mu) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}\left[\underline{\omega} + \underline{T\ln\left(1-\mathrm{e}^{-\frac{\omega-\mu_X}{T}}\right)+T\ln\left(1-\mathrm{e}^{-\frac{\omega+\mu_X}{T}}\right)}\right]\boxed{\frac{\partial\delta_X(\omega,\vec{q})}{\partial\omega}}$$

$$\mathcal{G}/\mathcal{G}_0 = \left|\mathcal{G}/\mathcal{G}_0\right| \mathrm{e}^{\underline{\mathrm{i}\delta(\omega,\vec{q})}}$$

full/free
Green's function

phase shift

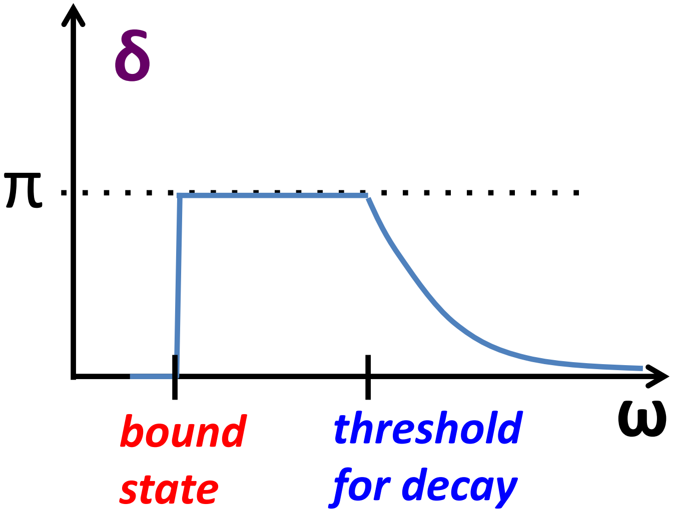
Constraint: Levinson's theorem

$$\mathcal{G}/\mathcal{G}_0 = |\mathcal{G}/\mathcal{G}_0|\,\mathrm{e}^{\underline{\mathrm{i}\delta(\omega,\vec{q})}}$$

Meaning: Total num. of states does not change by interactions

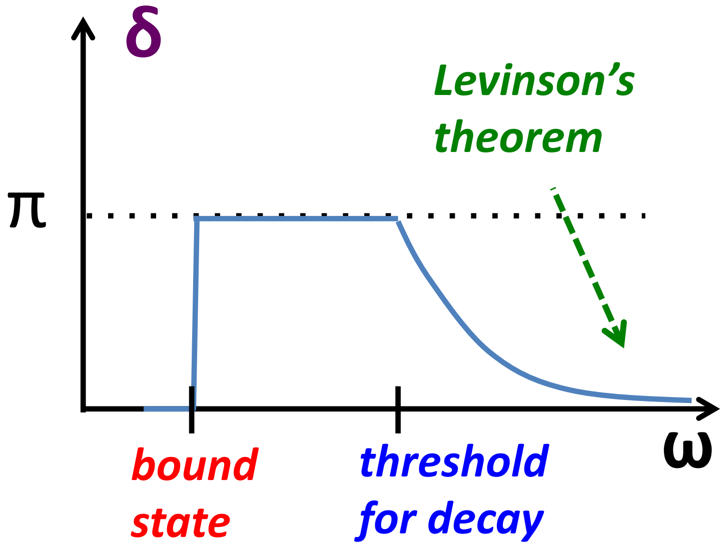
$$\begin{aligned} 0 &= \int_0^\infty \mathrm{d}E\,\mathrm{Tr}\left[\mathrm{Im}\mathcal{G} - \mathrm{Im}\mathcal{G}_0\right] \\ &= \int_0^\infty \mathrm{d}E\,\partial_E \mathrm{Tr}\left[\mathrm{Im}\ln \mathcal{G}^{-1}/\mathcal{G}_0^{-1}\right] \\ &= -\mathrm{Tr}\left[\delta(\infty) - \delta(0)\right] \end{aligned}$$

invariant

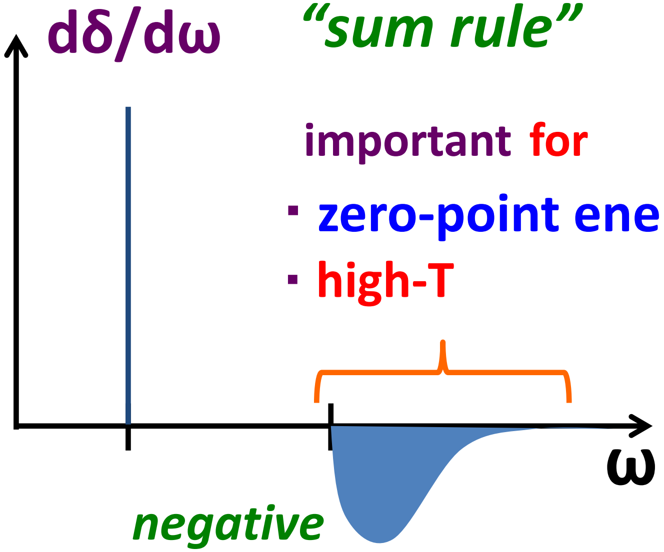


Phase shifts & Levinson's theorem

$$\Omega_X(T,\mu) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2\pi}\left[\underline{\omega} + \underline{T\ln\left(1-\mathrm{e}^{-\frac{\omega-\mu_X}{T}}\right) + T\ln\left(1-\mathrm{e}^{-\frac{\omega+\mu_X}{T}}\right)}\right]\boxed{\frac{\partial\delta_X(\omega,\vec{q})}{\partial\omega}}$$



d/dω



"sum rule"

important for

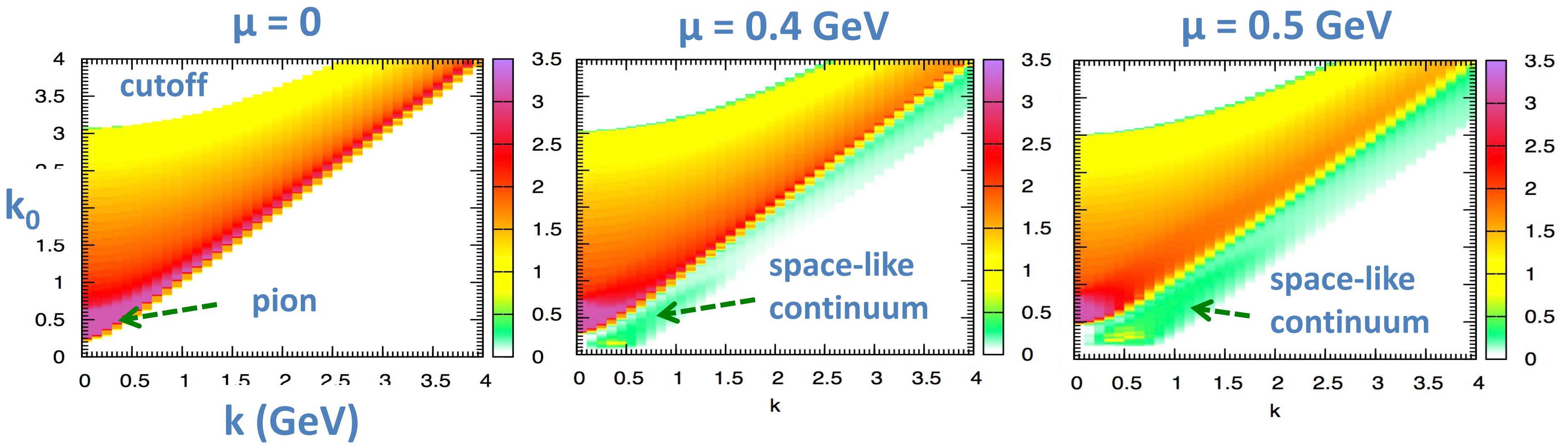
- zero-point energy
- high-T

origin: $\mathcal{G}^{\text{conn.}} = \mathcal{G}^{\text{full}} \underline{- \mathcal{G}_0}$

Pressure from low E and high E cancel one another; taming a meson (diquark) gas at high T

Phase shift $\delta(k_0, k)$: e.g. π-channel

particle-hole, particle-antiparticle



μ = 0

μ = 0.4 GeV

μ = 0.5 GeV

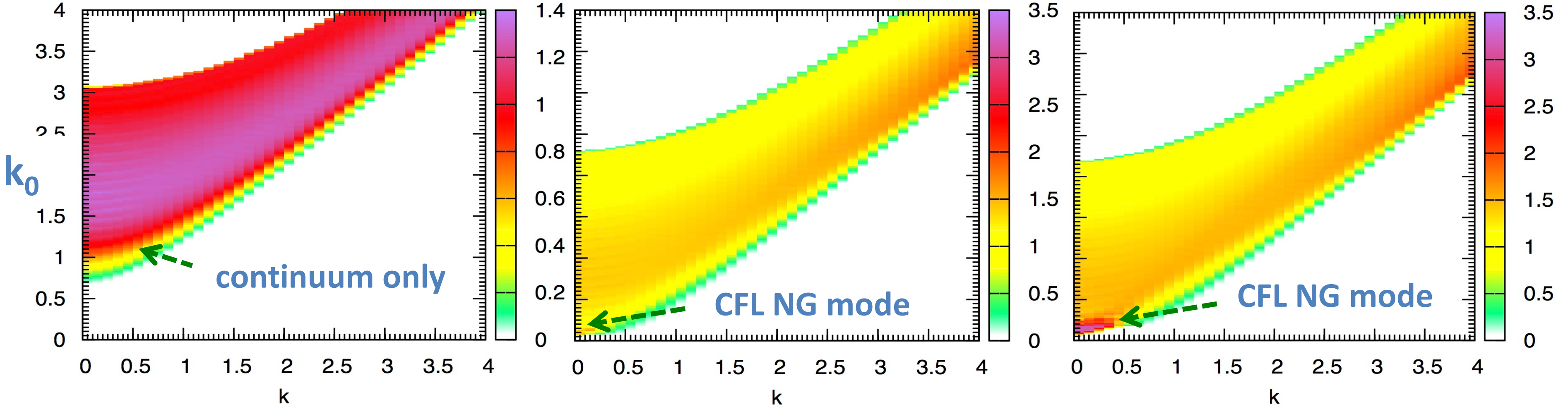
cutoff

pion

space-like continuum

space-like continuum

particle-particle, hole-hole



continuum only

CFL NG mode

CFL NG mode

Summary

- *Soft* EoS at *small n_B* & *stiff* EoS at *large n_B*

  → **crossover** or **weak 1st order** *from H to Q*

- [G_S, G_V, H] @ $5n_0$ ~ G_s^{vac}

  → gluons likely remain non-perturbative to n_B ~ 5-10 n_0

  (*Quarkyonic*)

- Quark matter EoS (MF + RPA correlation) for

  n_B = 5-10 n_0 & T = 10-100 MeV & Y_e = 0-0.5

  (*still under construction…*)

Back up

Collective modes in the CFL

- Symmetry breaking (in chiral limit):

$$U(1)_B \times U(1)_Q \times SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{C+L+R} \times U(1)_{Q'}$$

$$(1+1+8+8+8) - (8+1) \quad = \quad 8 \quad + \quad (\ 8 \quad + \quad 1\ )$$

Generators before and after the SSB

part of
massive gluons

NG bosons

- *+1 NG boson*: *effective* $U(1)_A$ → *light* η'

- In reality: *explicit* flavor sym. breaking in NSs

  mass, electric charges → 9 bosons are pseudo-NG modes

- *Effective chemical potentials* appear for *flavored* NG modes

  [high density EFT: Bedaque-Schafer 2002]

  → *small effects on* $\pi^{\pm}$, but significant effects on kaons

  (possibility of kaon condensations in the CFL)

NG modes in NSs

- **Most of the previous studies** [Son-Stephanov2000, Bedaque-Schafer 2002, ...]

  → for *high density* and/or *weak coupling limit*

  $(qq)(\bar{q}\bar{q})$ fluctuations with mass ∼ $O(m_q)$

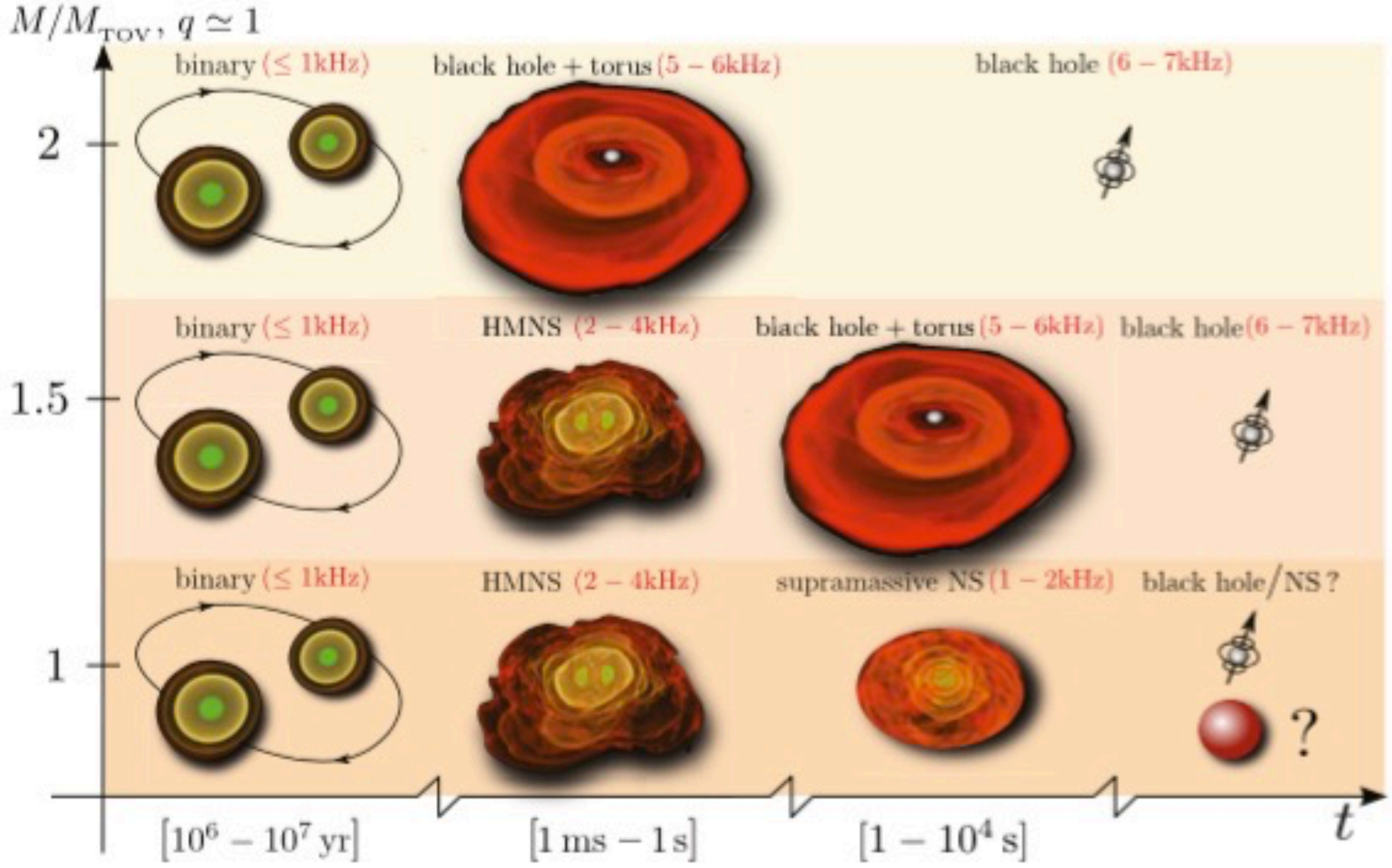
- **In *NSs*, the situation is not so clean...**
  - **matter is not weak coupling, and p_F ∼ 400-500 MeV**
  - **Chiral condensates likely remain**

    $(q\bar{q})$ fluctuations with mass ∼ $O(m_q^{1/2})$
  - **UA(1) breaking likely remains**

- **mixing with 2q-4q fluct. in *3-flavor limit*** [Yamamoto et al. 2007]

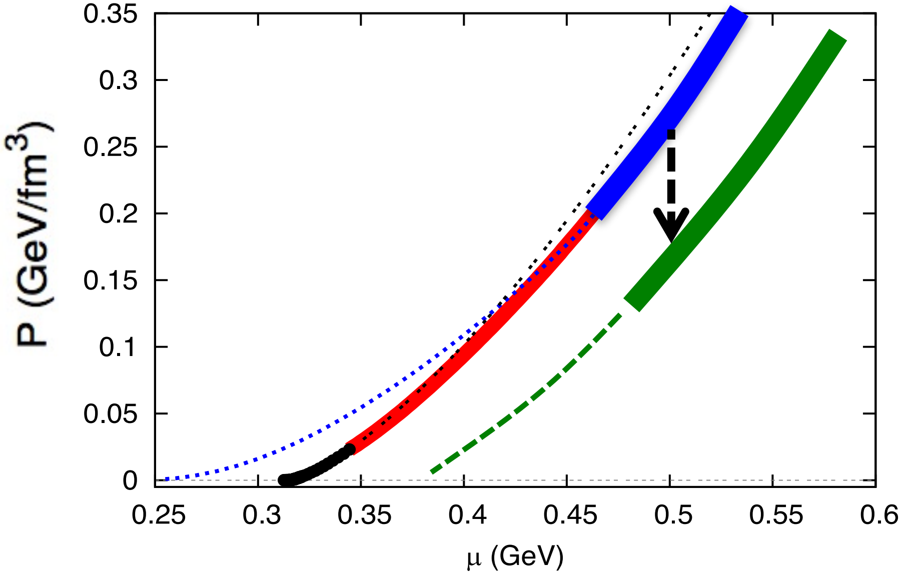
- **few model studies (but at that time NSs constraints are not available)**

  [Basler-Buballa '10, ...]


$M/M_{\rm TOV}$, $q \simeq 1$
2
1.5
1
binary ($\leq$ 1kHz)
black hole + torus (5 – 6kHz)
black hole (6 – 7kHz)
binary ($\leq$ 1kHz)
HMNS (2 – 4kHz)
black hole + torus (5 – 6kHz)
black hole (6 – 7kHz)
binary ($\leq$ 1kHz)
HMNS (2 – 4kHz)
supramassive NS (1 – 2kHz)
black hole/NS ?
?
[10^6 – 10^7 yr]
[1 ms – 1 s]
[1 – 10^4 s]
t

Discussion : Bag constant ?

P_{NJL} @ 5 n_0 → only 200 - 400 MeV fm^{-3}



If $B_g \sim \Lambda_{QCD}^4$ appears @ 5n_0

EoS → impossible to pass any constraints

Together with $G_V \sim H \sim G_s^{vac}$, *we claim :*

Gluons *should remain non-perturbative* to n_B ~ 5-10 n_0

Discussion : Bag constant ?

Def: $\mathcal{B} \equiv \epsilon^{vac}_{pert} - \epsilon^{vac}_{full} \sim \Lambda^4_{\rm QCD}$ **> 0**

- Energy *gain* by *non-pert. effects* ;

  e.g.) ChSB in Dirac sea, gluon condensation, ...

If μ is large enough :

(softening)

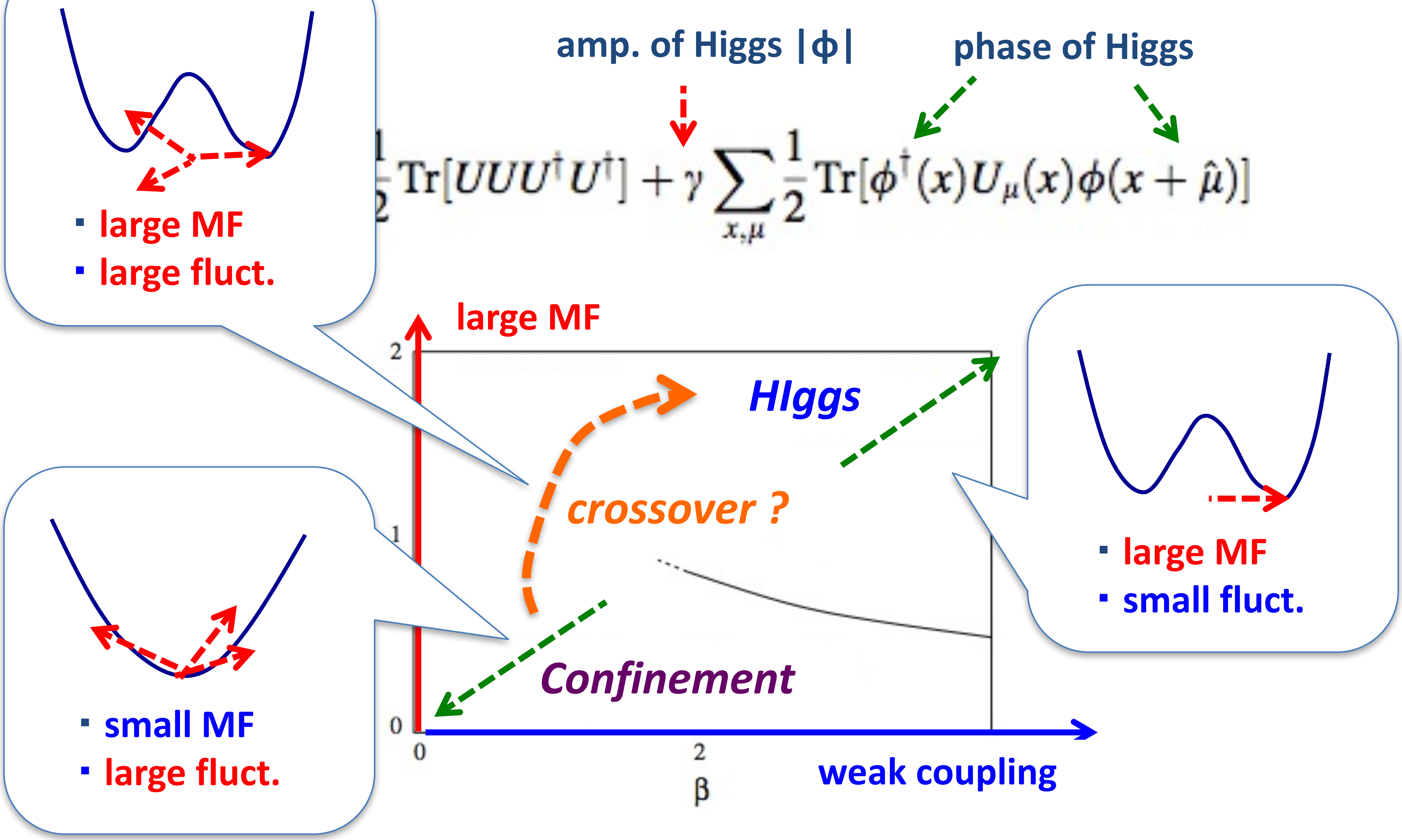
- Loss of *non-pert. effects* → $\left\{ \begin{array}{l} \epsilon_{\rm matter} \rightarrow \epsilon_{\rm matter} + \mathcal{B} \\ P_{\rm matter} \rightarrow P_{\rm matter} - \mathcal{B} \end{array} \right.$

- NJL takes into account the *vac. contributions* *only partially* ;

  it *misses* contributions from *gluonic* one, B_g

A question : Conf. vs Higgs ?

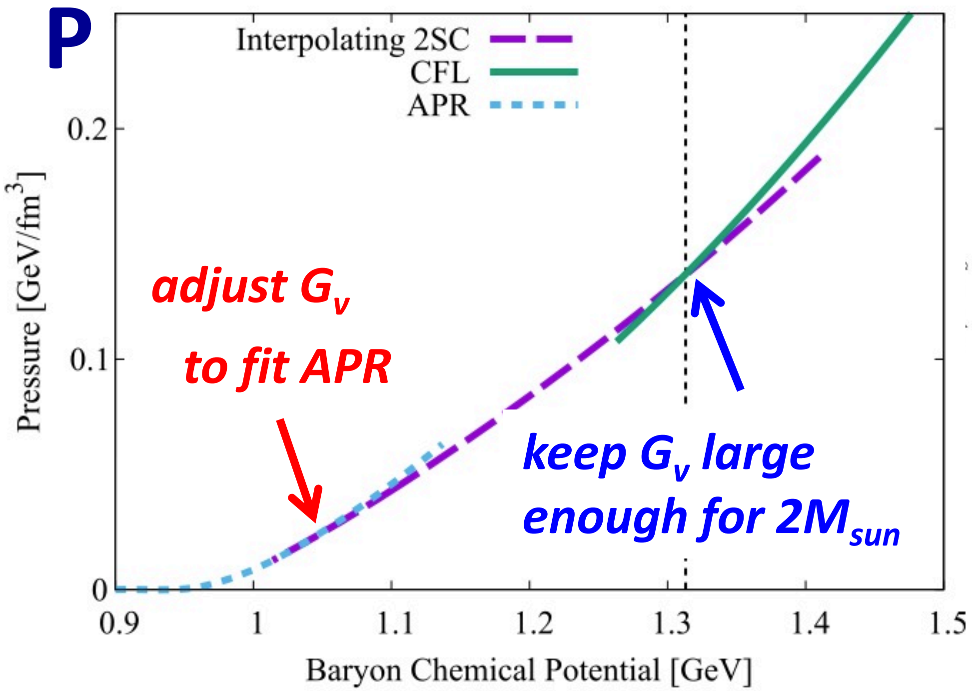
) Hi del *with const. amplitude* *[Fradkin-Shenkar 79]*



$$\frac{1}{2}\mathrm{Tr}[UUU^{\dagger}U^{\dagger}]+\gamma\sum_{x,\mu}\frac{1}{2}\mathrm{Tr}[\phi^{\dagger}(x)U_{\mu}(x)\phi(x+\hat{\mu})]$$

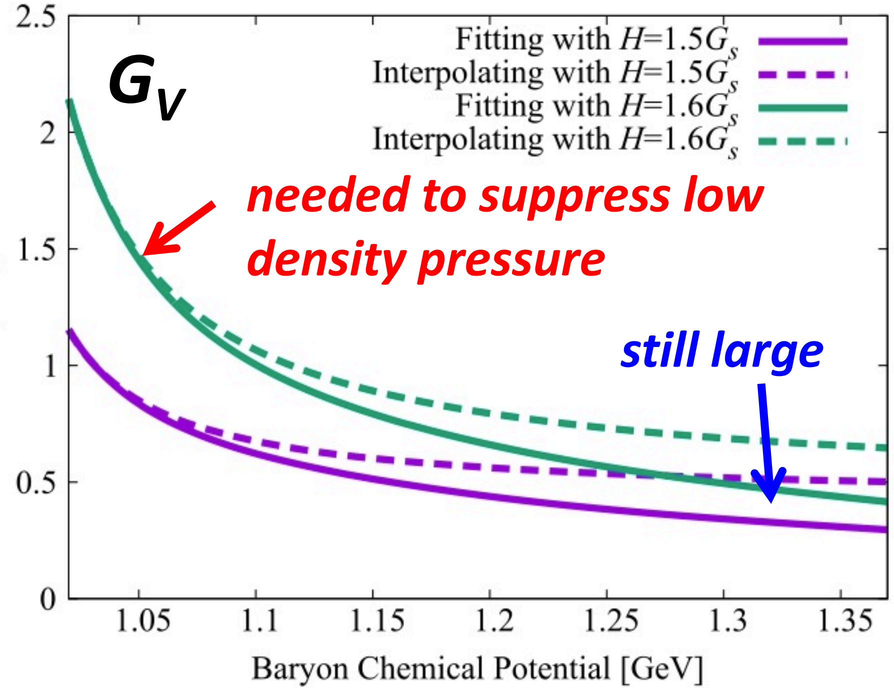
Discussion 2: value of G_V ?

***APR constrained NJL* with *running* $G_V(n_B)$** **[Fukushima-TK '15]**



adjust G_v to fit APR

keep G_v large enough for $2M_{sun}$



needed to suppress low density pressure

still large

would offer more concrete modeling for "unified" EoS

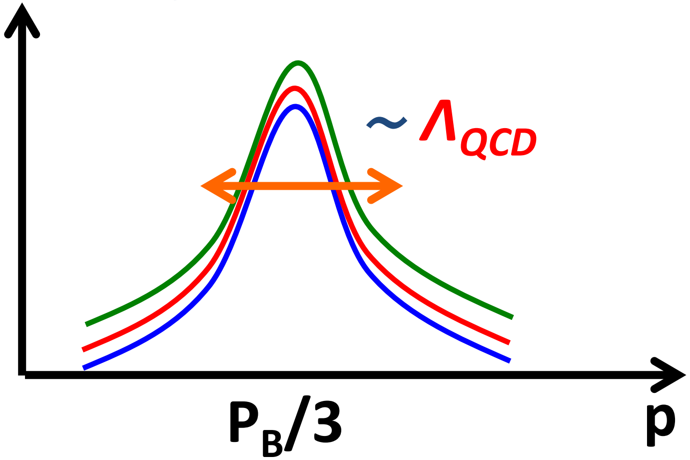
than 3-window descriptions

Discussion 3: Hyperon problems ?

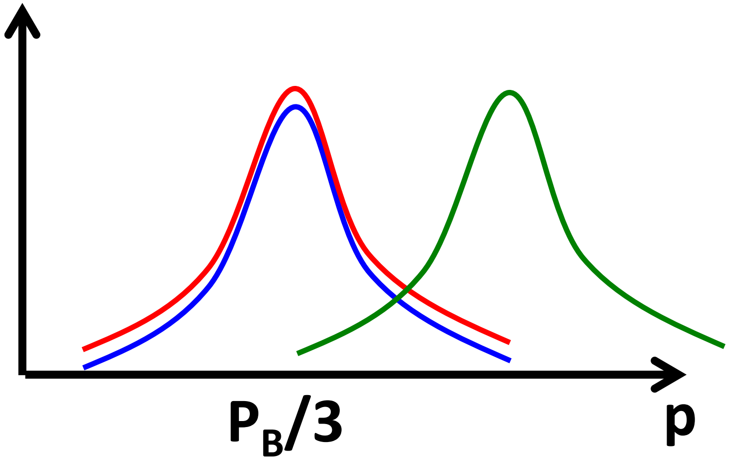
How did we avoid hyperon softening ?

- μ_B^{th} for **strangeness** :
  - $\mu_B \sim 3M_s \sim$ **1.5 GeV** (**quark** picture)
  - $\mu_B \sim \mu_\Lambda , \mu_\Sigma \sim$ **1.1-1.2 GeV** (**hadron** picture)
    (uds, uus,...)

- ***A quark w.f. for a baryon*** (e.g. Isgur-Kahl)



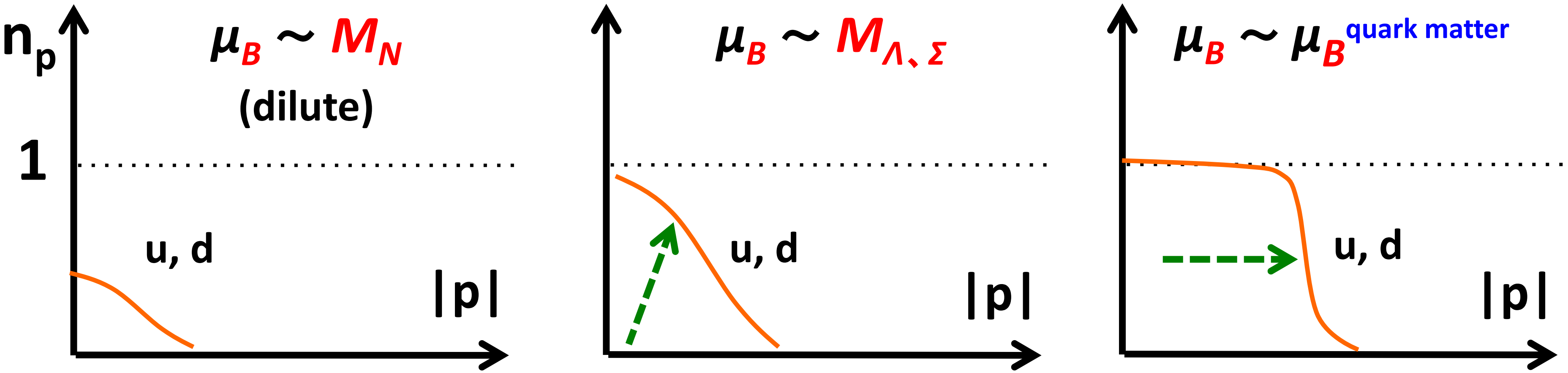
ground state



excited state

Discussion 3: Hyperon problems ?

- *Quark descriptions of hadronic matter* :



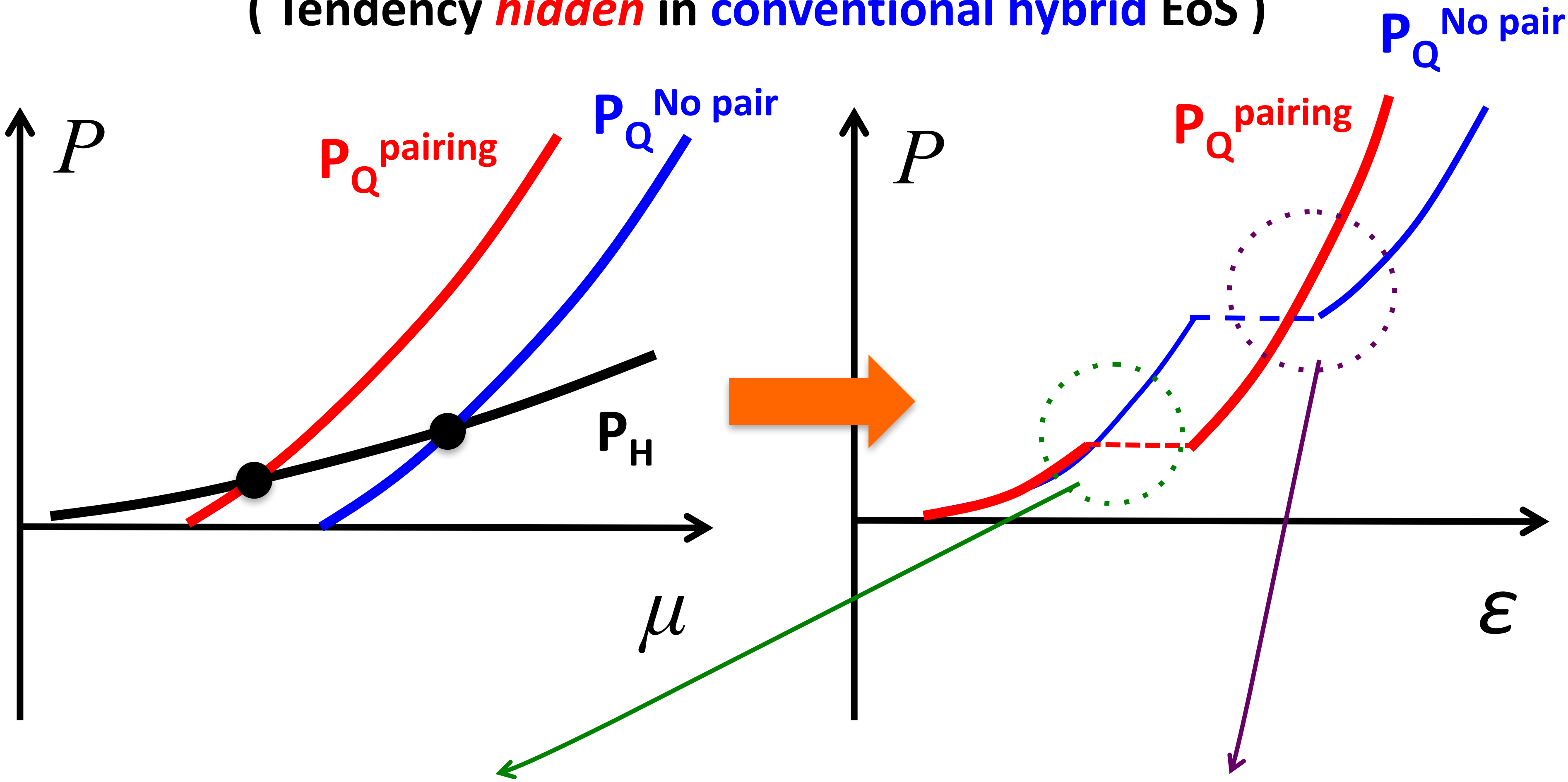
How to put hyperons ??

- $M_{\Lambda、\Sigma}$ at *low P* is *rejected* by quark Pauli blocking on (u,d)
- $M_{\Lambda、\Sigma}$ at *high P* avoid the blocking, but *is energetic*

[Note: this argument becomes *more powerful* at *higher n_B*]

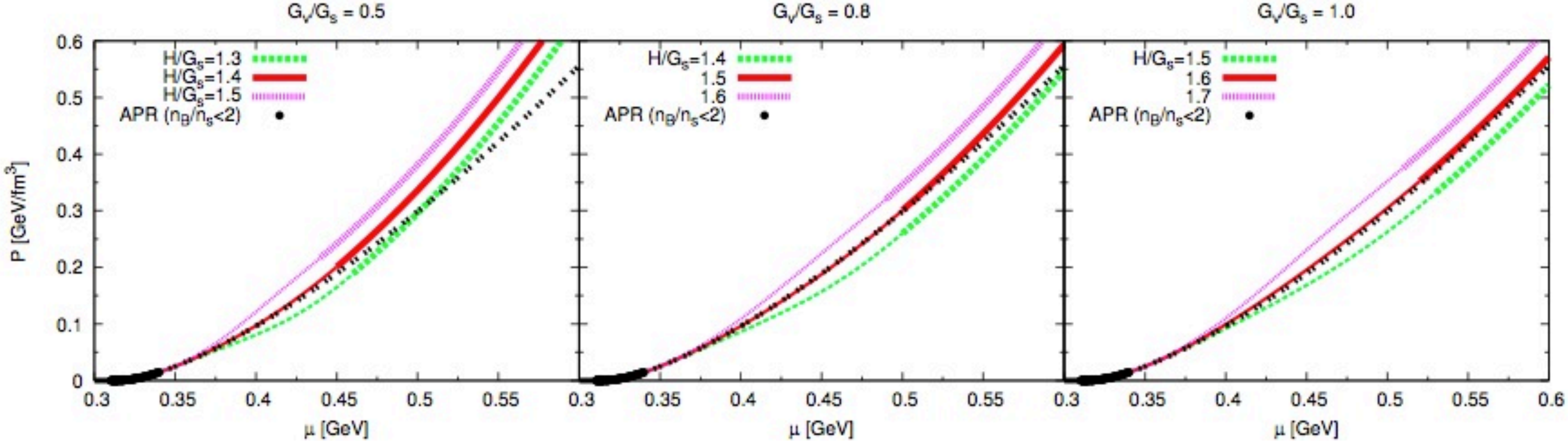
“Pairing” *can stiffen EoS*

(Tendency *hidden* in conventional hybrid EoS)

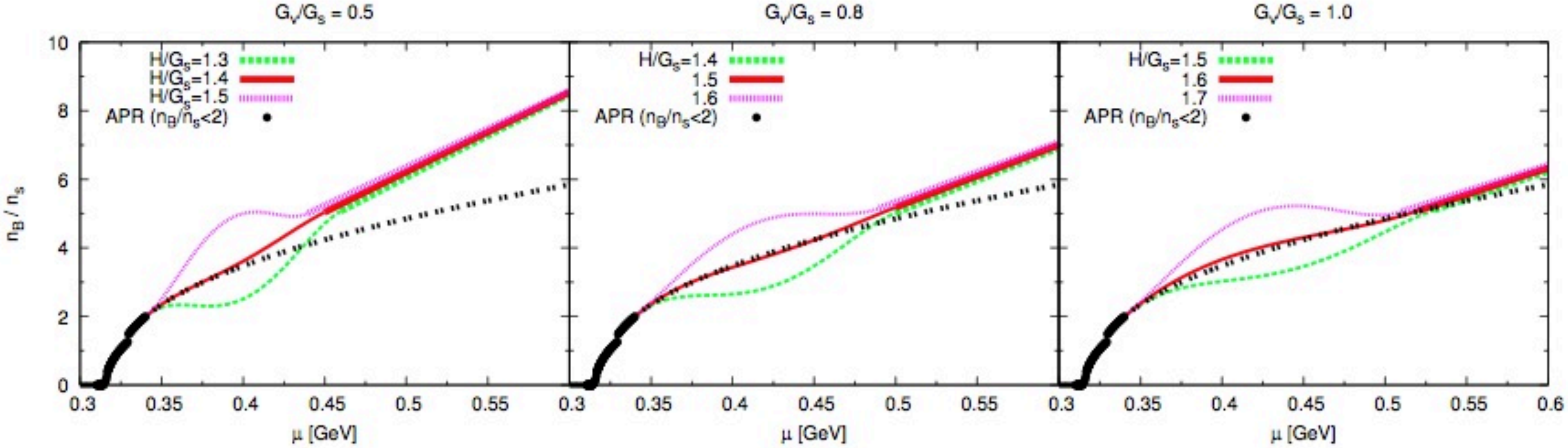


→ ***Softening*** **at *low* n_B & *stiffening* at *high* n_B**

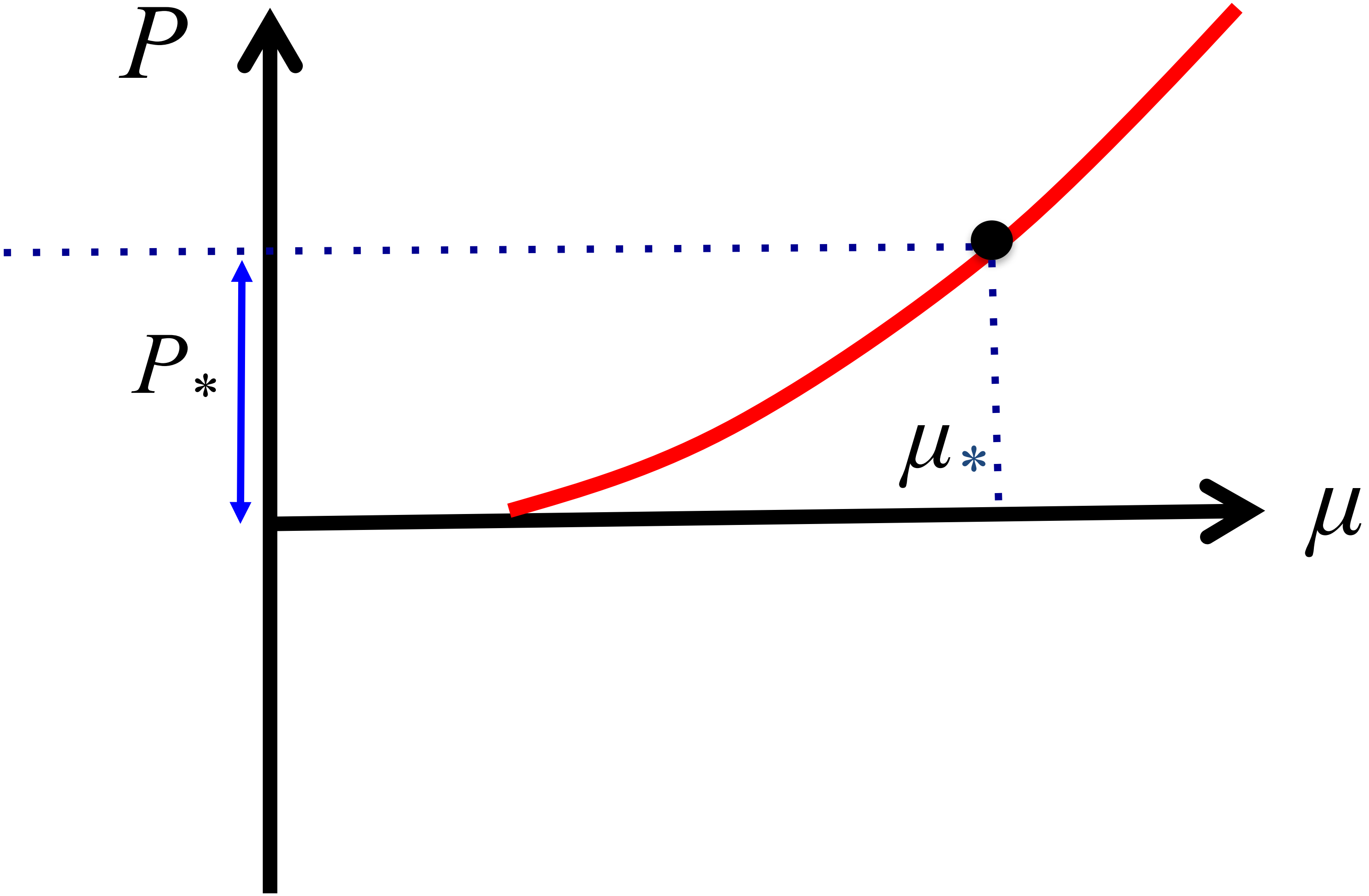
P v.s. μ



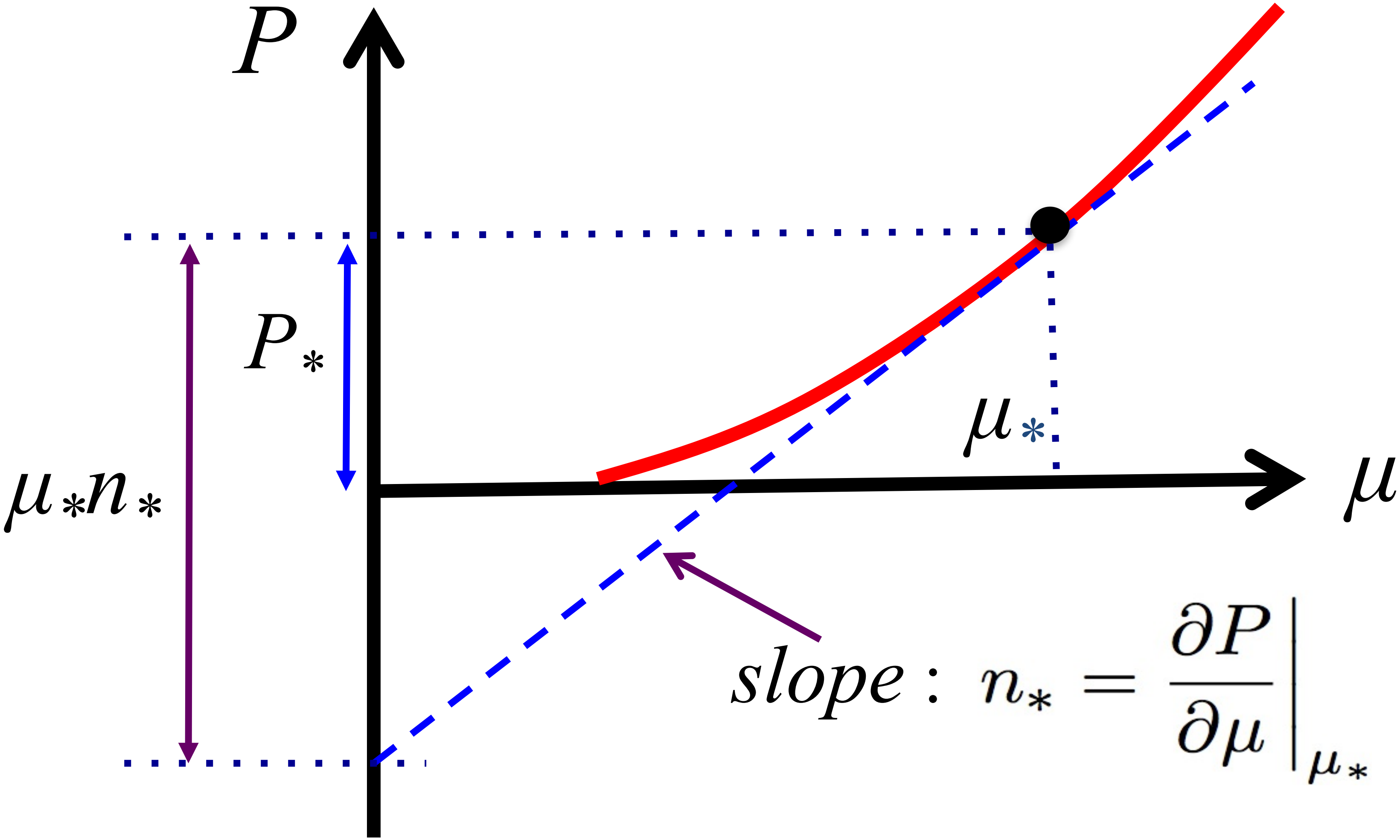
n_B/n_0 v.s. μ



How stiff EoS looks like in P(μ) curves

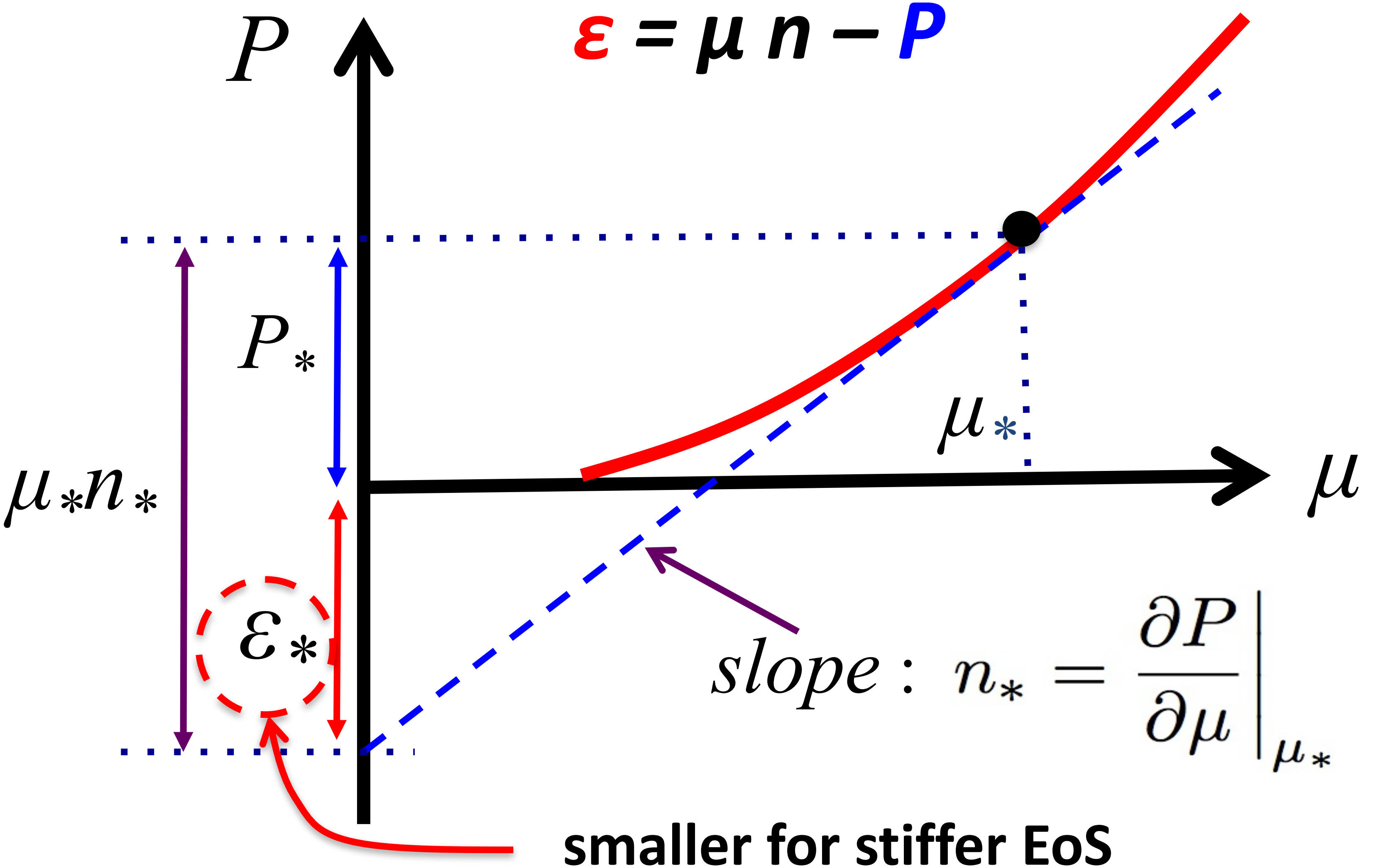


How stiff EoS looks like in P(μ) curves



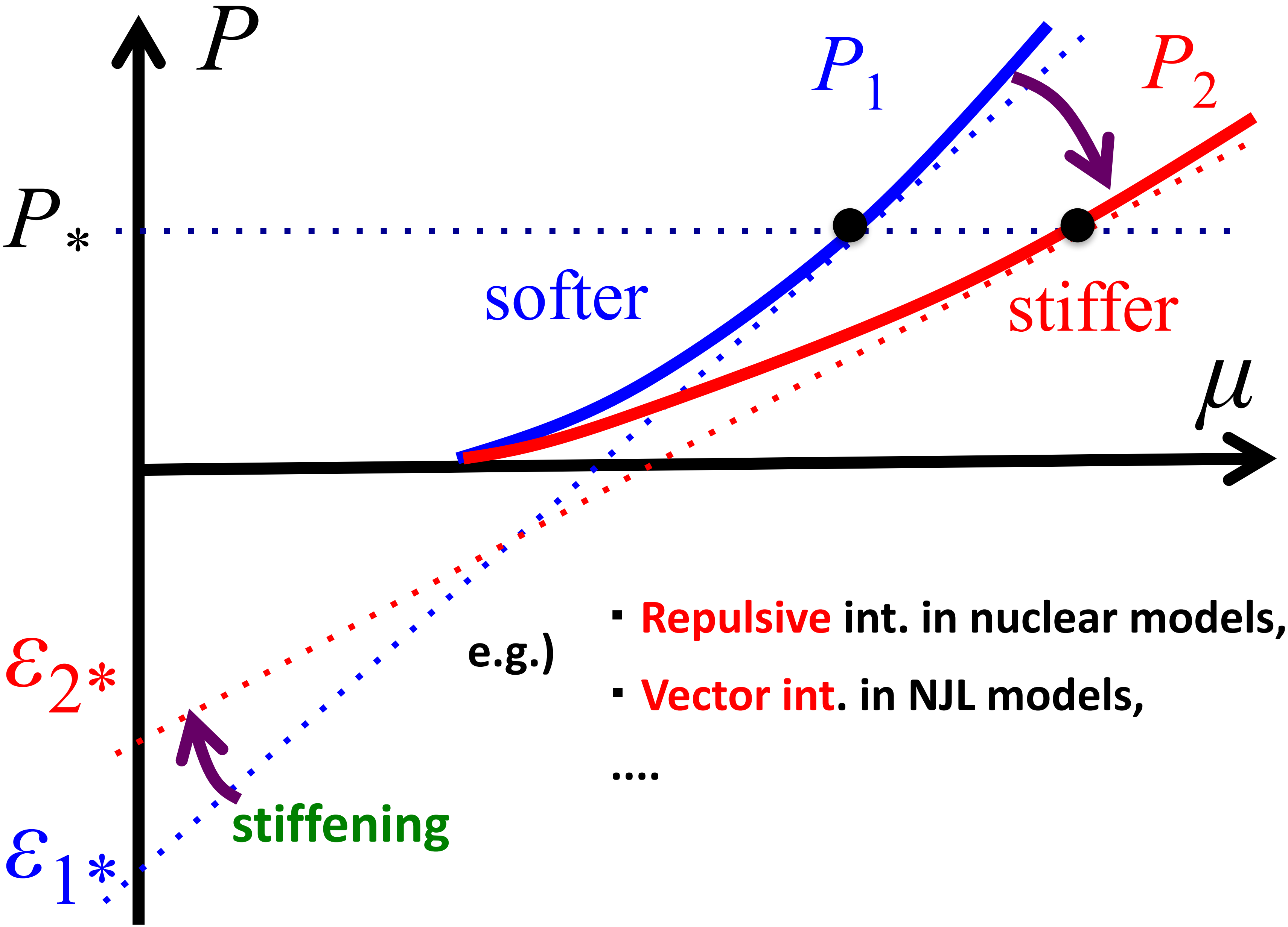
$$\text{slope}: \; n_* = \left.\frac{\partial P}{\partial \mu}\right|_{\mu_*}$$

How stiff EoS looks like in P(μ) curves



smaller for stiffer EoS

Example of stiffening 1

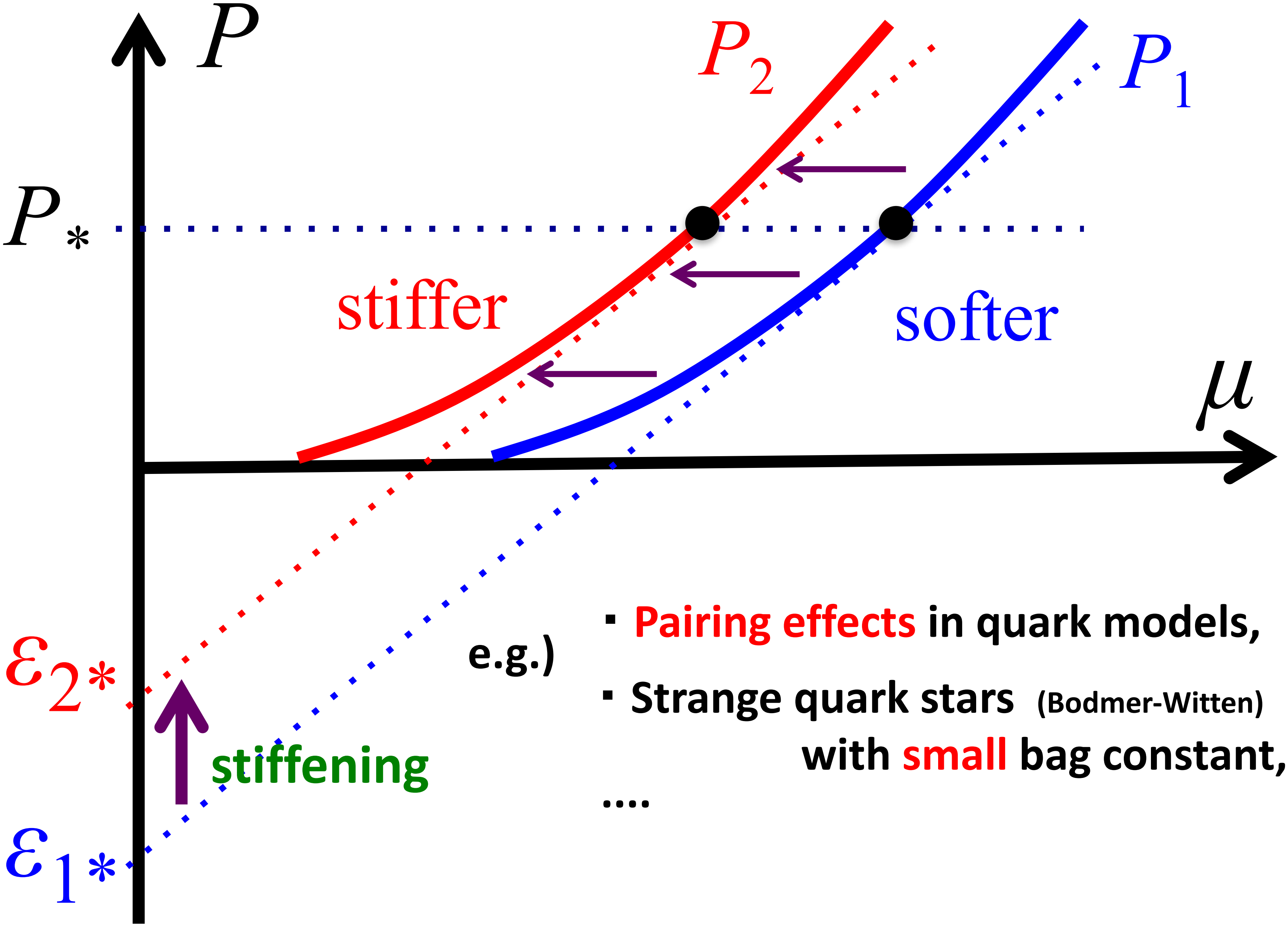


e.g.)

- **Repulsive int. in nuclear models,**
- **Vector int. in NJL models,**

….

Example of stiffening 2



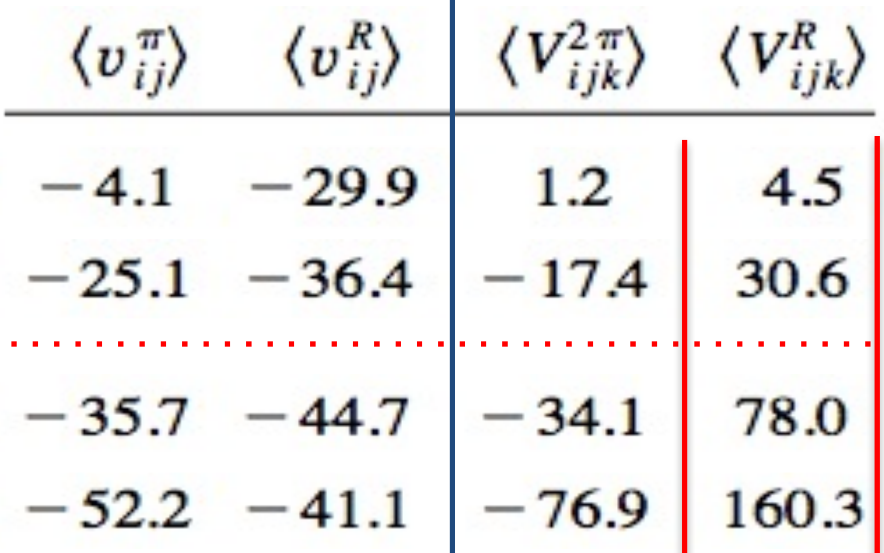
e.g.)

- **Pairing effects in quark models,**
- **Strange quark stars (Bodmer-Witten) with small bag constant,**

....

Nuclear EoS : convergence ?

Many-body interaction (APR-A18+UIX case)



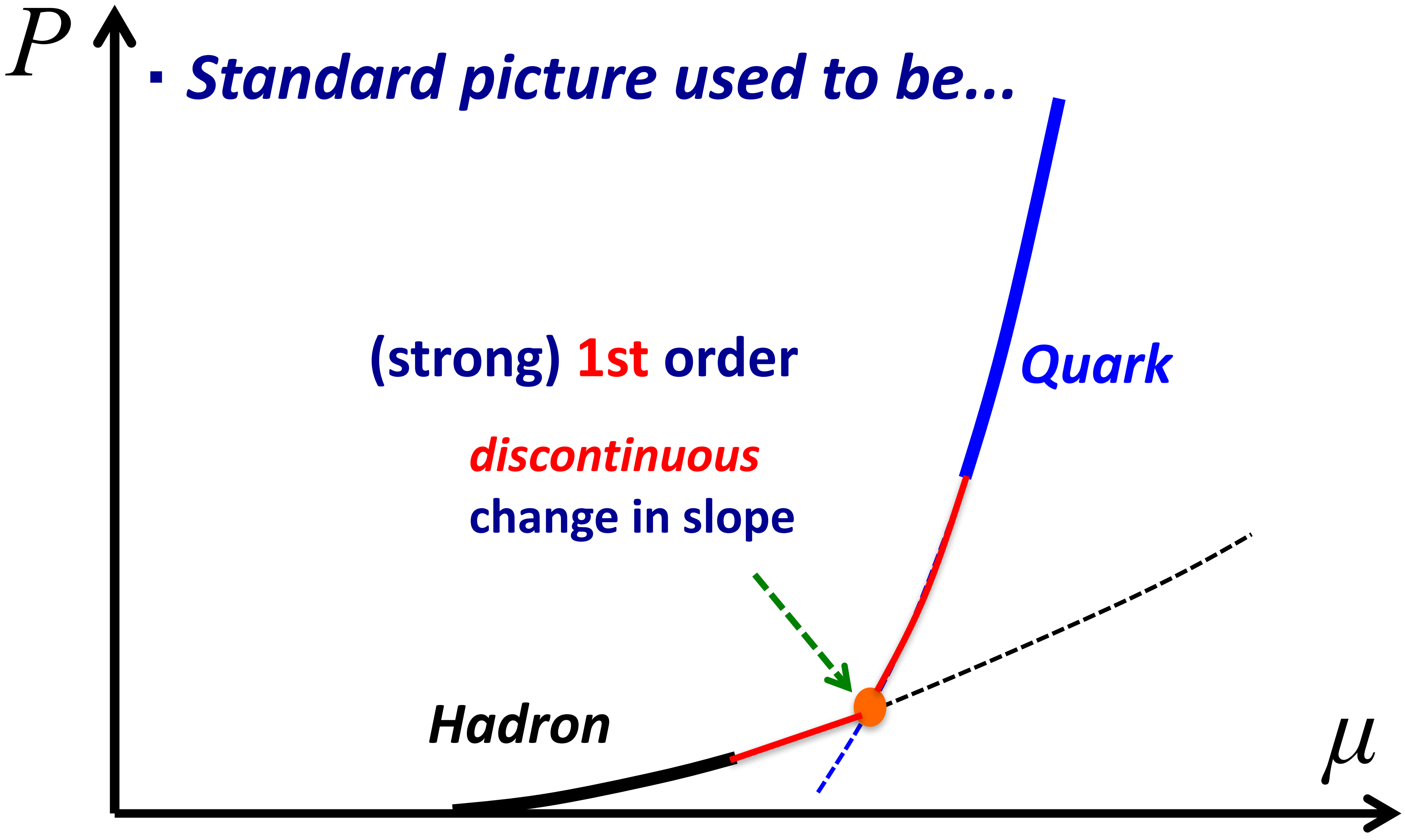
n_B	2 –body int. $\langle v^{\pi}_{ij} \rangle$	2 –body int. $\langle v^{R}_{ij} \rangle$	3 –body int. $\langle V^{2\pi}_{ijk} \rangle$	3 –body int. $\langle V^{R}_{ijk} \rangle$	4 –body int. (our guess)
n_0	−4.1	−29.9	1.2	4.5	small
2 n_0	−25.1	−36.4	−17.4	30.6	marginal
3 n_0	−35.7	−44.7	−34.1	78.0	large
4 n_0	−52.2	−41.1	−76.9	160.3	

grow rapidly !!

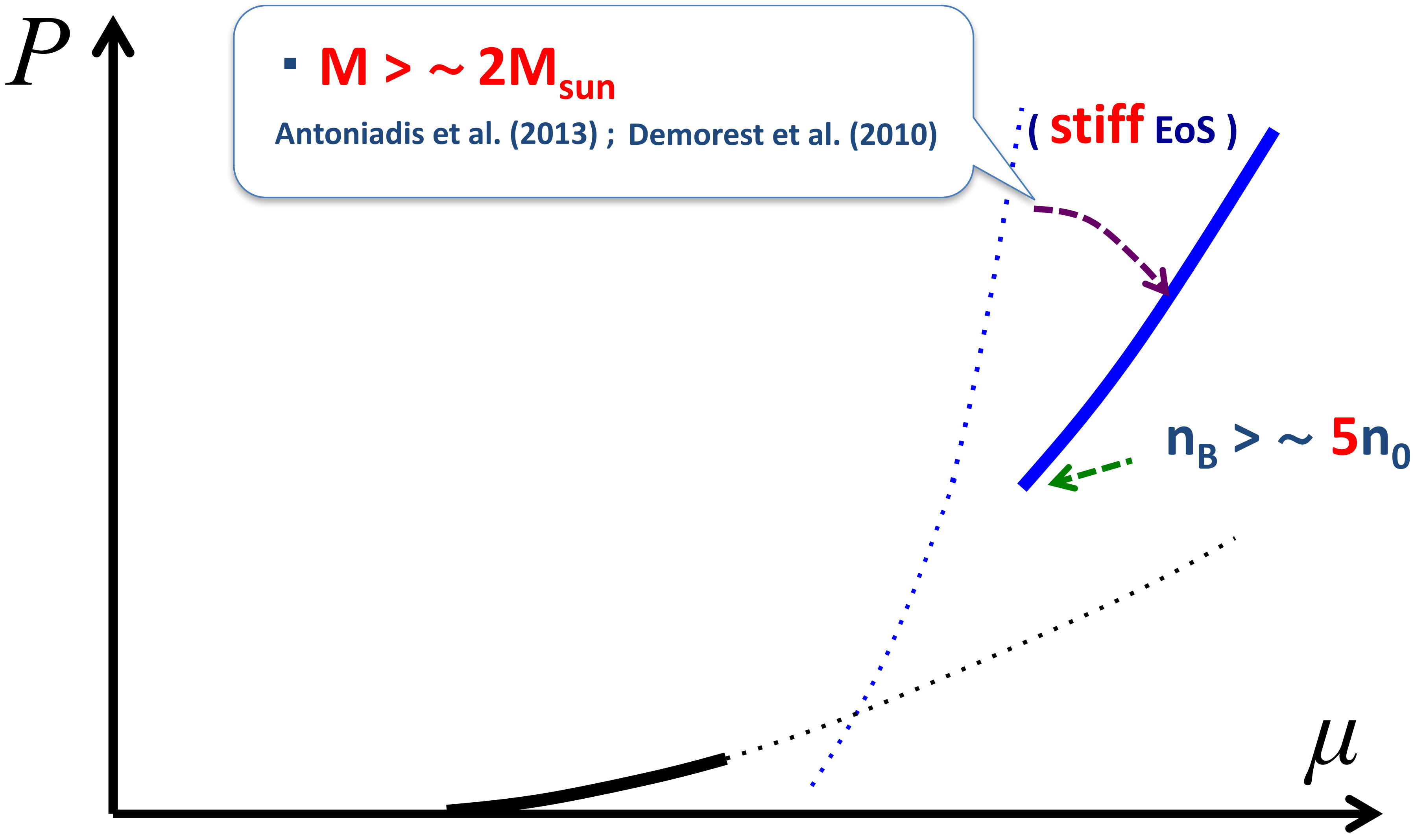
$$\boxed{\langle V_{N\text{-body}} \rangle \sim c_N \, (n_B / n_0)^N}$$

Observational constraints on $P(\mu)$

- *Standard picture used to be...*



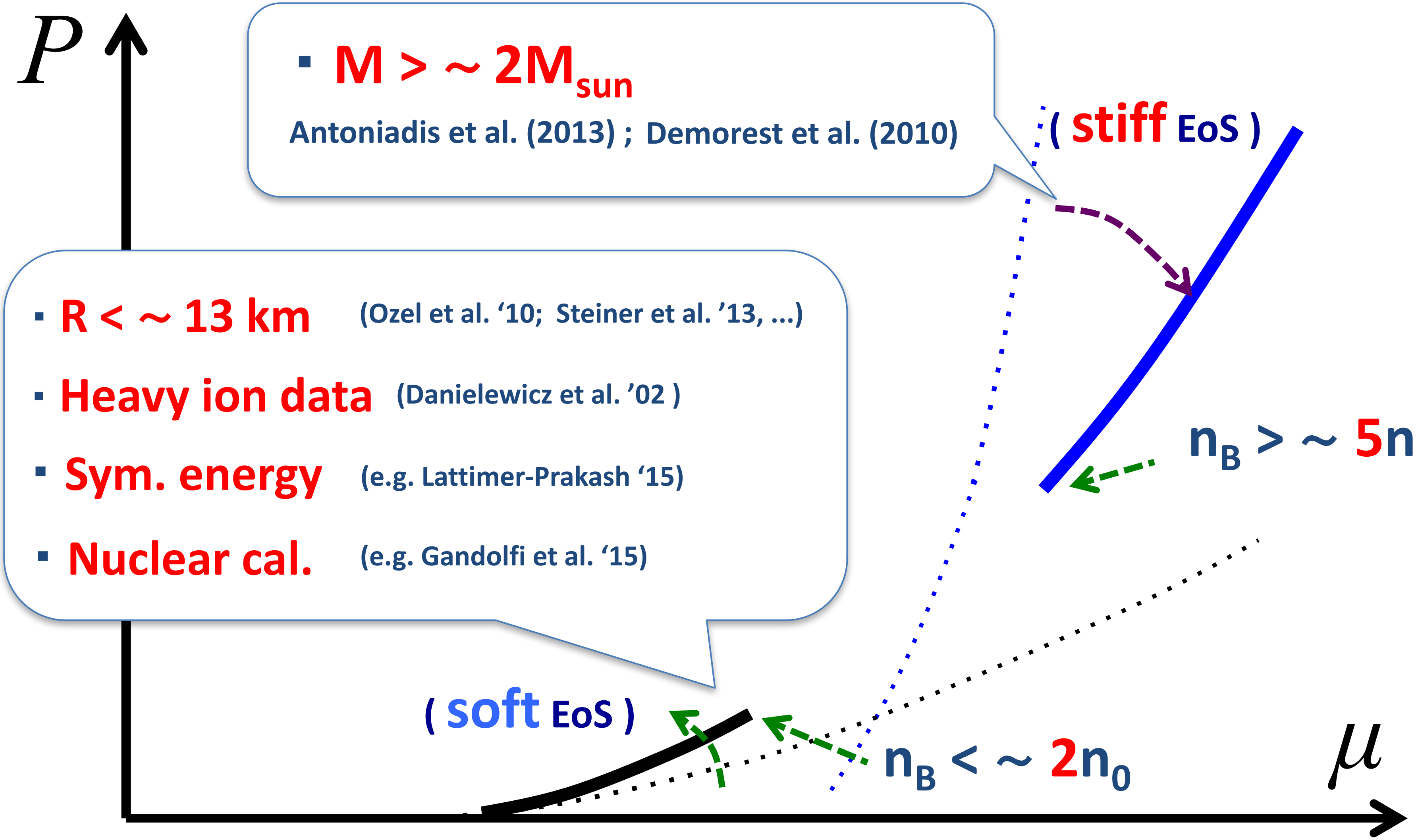
Observational constraints on $P(\mu)$



- $M > \sim 2M_{sun}$

Antoniadis et al. (2013) ; Demorest et al. (2010)

Observational constraints on P(μ)

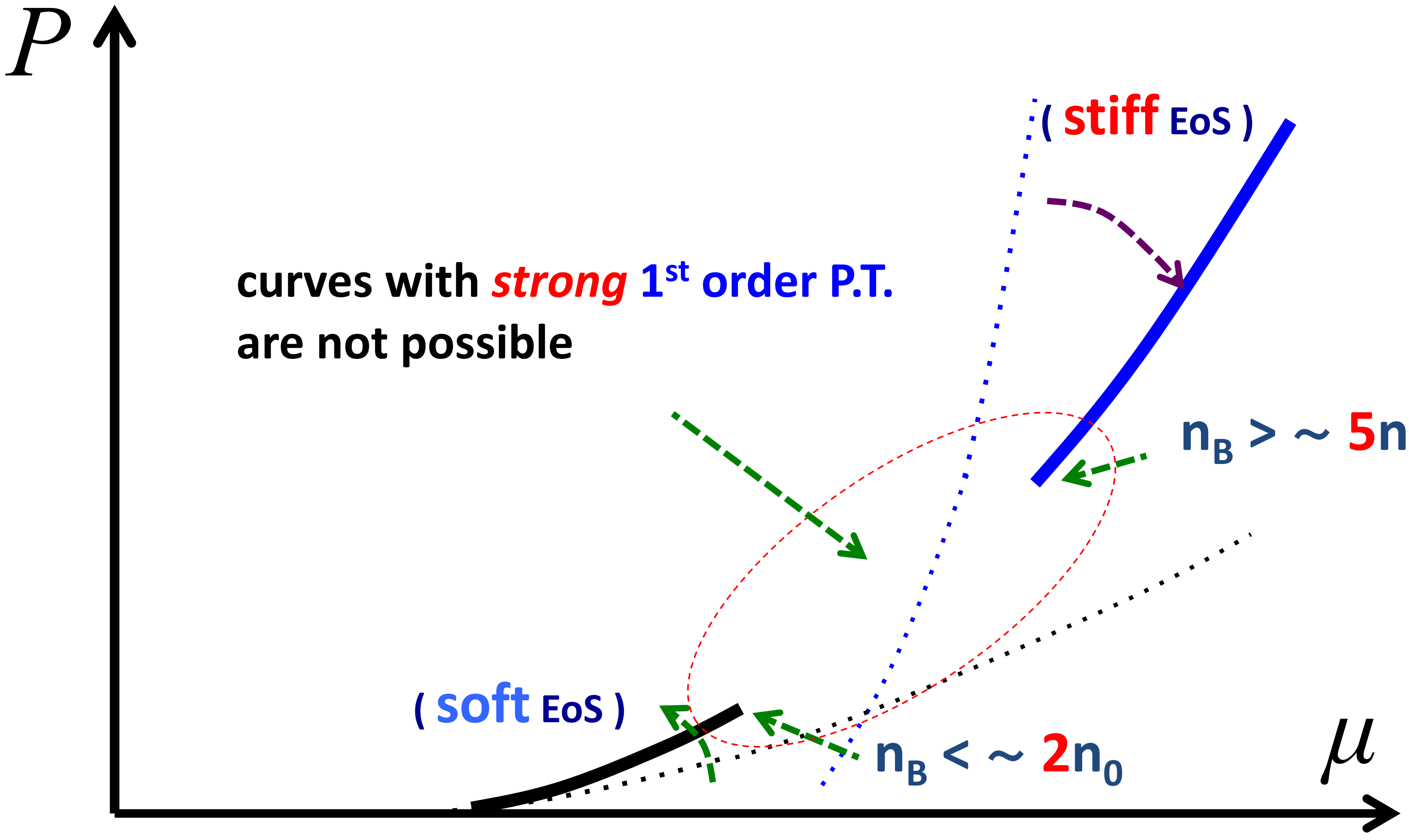


- **$M > \sim 2M_{sun}$**

Antoniadis et al. (2013) ; Demorest et al. (2010)

- **$R < \sim 13$ km** (Ozel et al. '10; Steiner et al. '13, ...)
- **Heavy ion data** (Danielewicz et al. '02)
- **Sym. energy** (e.g. Lattimer-Prakash '15)
- **Nuclear cal.** (e.g. Gandolfi et al. '15)

Observational *constraints* on *P(μ)*



curves with *strong* 1st order P.T. are not possible

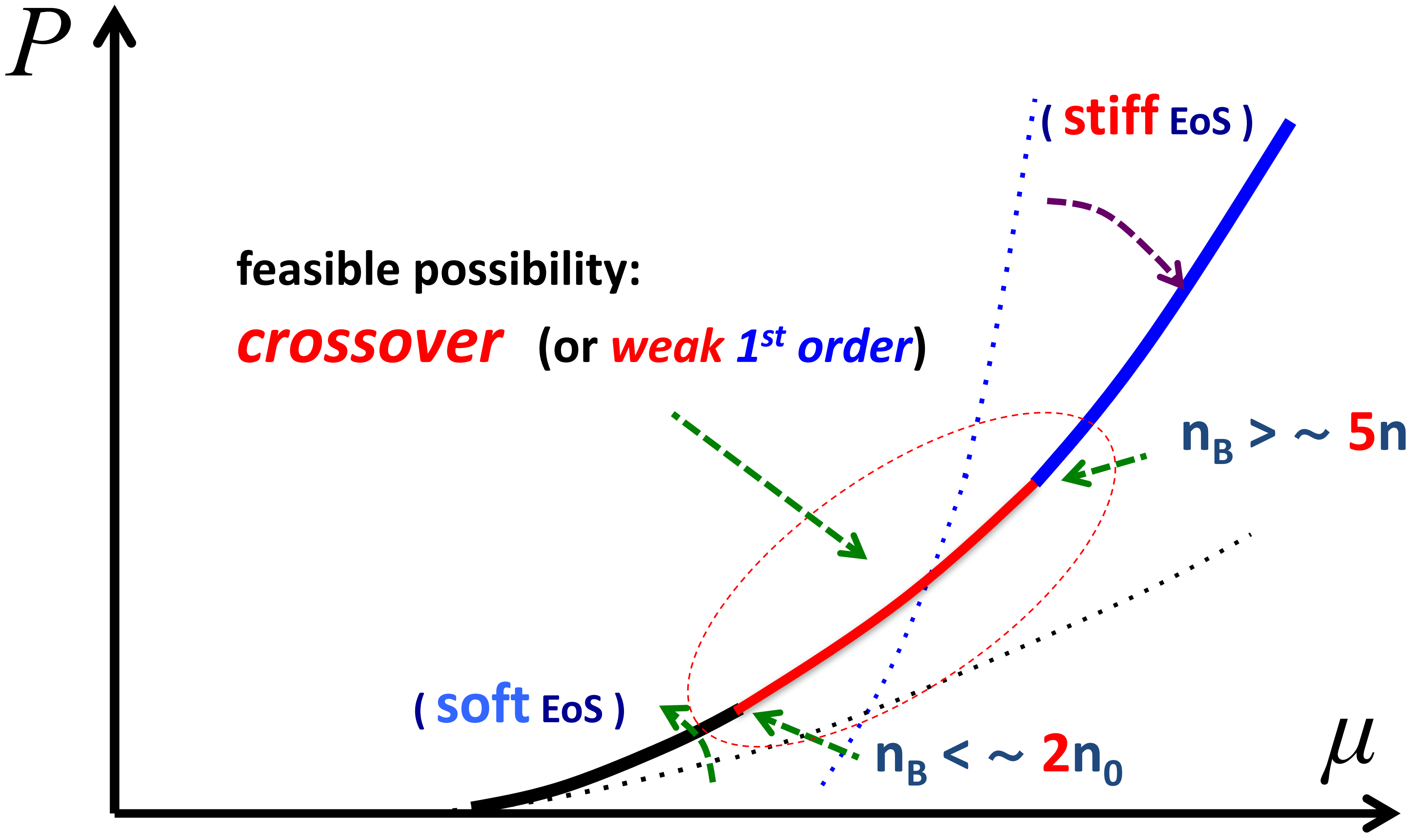
(**stiff** EoS)

$n_B > \sim 5n_0$

(**soft** EoS)

$n_B < \sim 2n_0$

Observational constraints on P(μ)



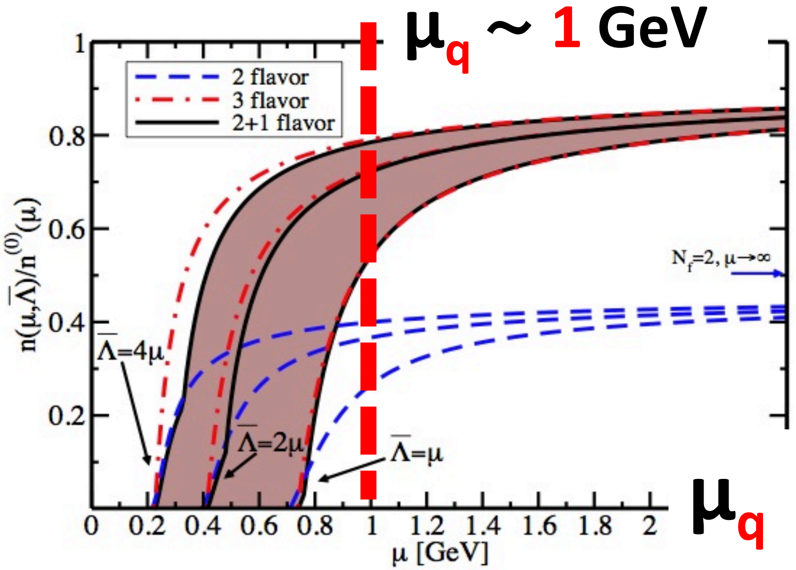
Theoretical guides at N_c=3

- 3-loop *pQCD at large μ_q*

  [Freedman-McLerran 78; Baluni 78
  Kurkela-Romatschke-Vuorinen 09, ...]

  - large α_s corrections at μ_q < 1 GeV

    → soft gluons important at n_B < 100 n_0



- *Nuclear calculations* (ChEFT+many-body) *at small μ_q*

  - reliable at $n_B \sim n_0$

    [Akmal et al. (APR) 98; Gandolfi et al. 12, ...]

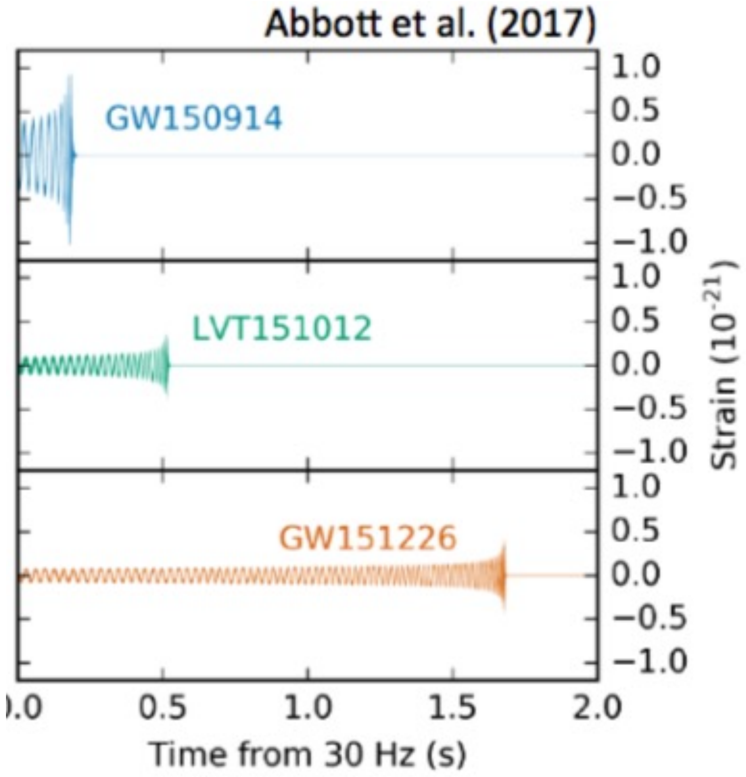
  - At n_B > 2n_0
    - *convergence* problems : < $V_{2\text{-body}}$ > ~ < $V_{3\text{-body}}$ > ~ ...
    - *hyperon softening*, unless introducing ad hoc repulsion
    - *changes in hadron w.f.* & *Dirac sea negligible?*

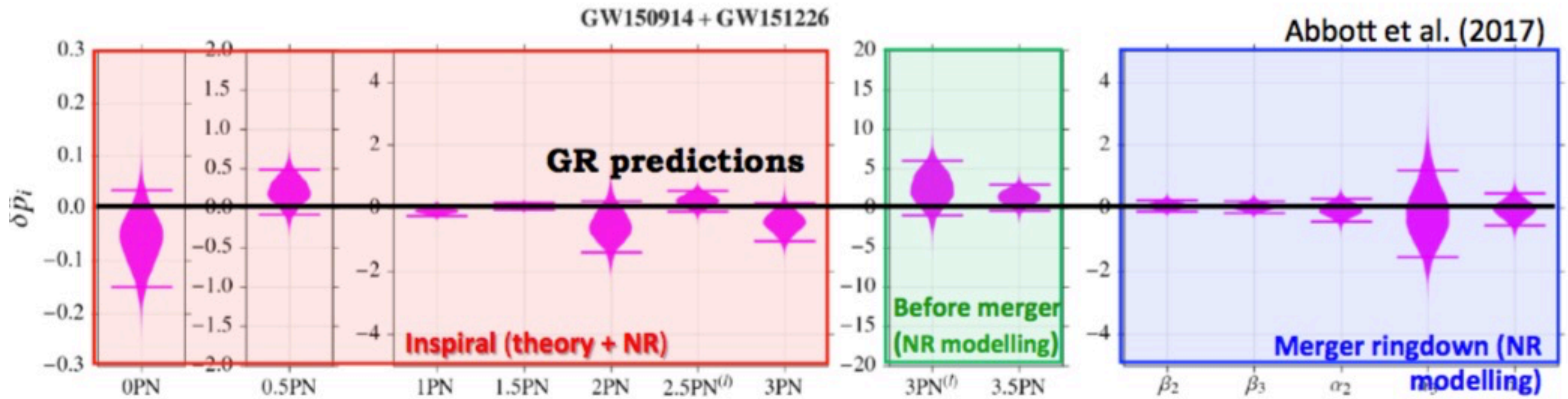
GW159014 : the discovery of GWs

BH-BH mergers

→ larger amplitudes

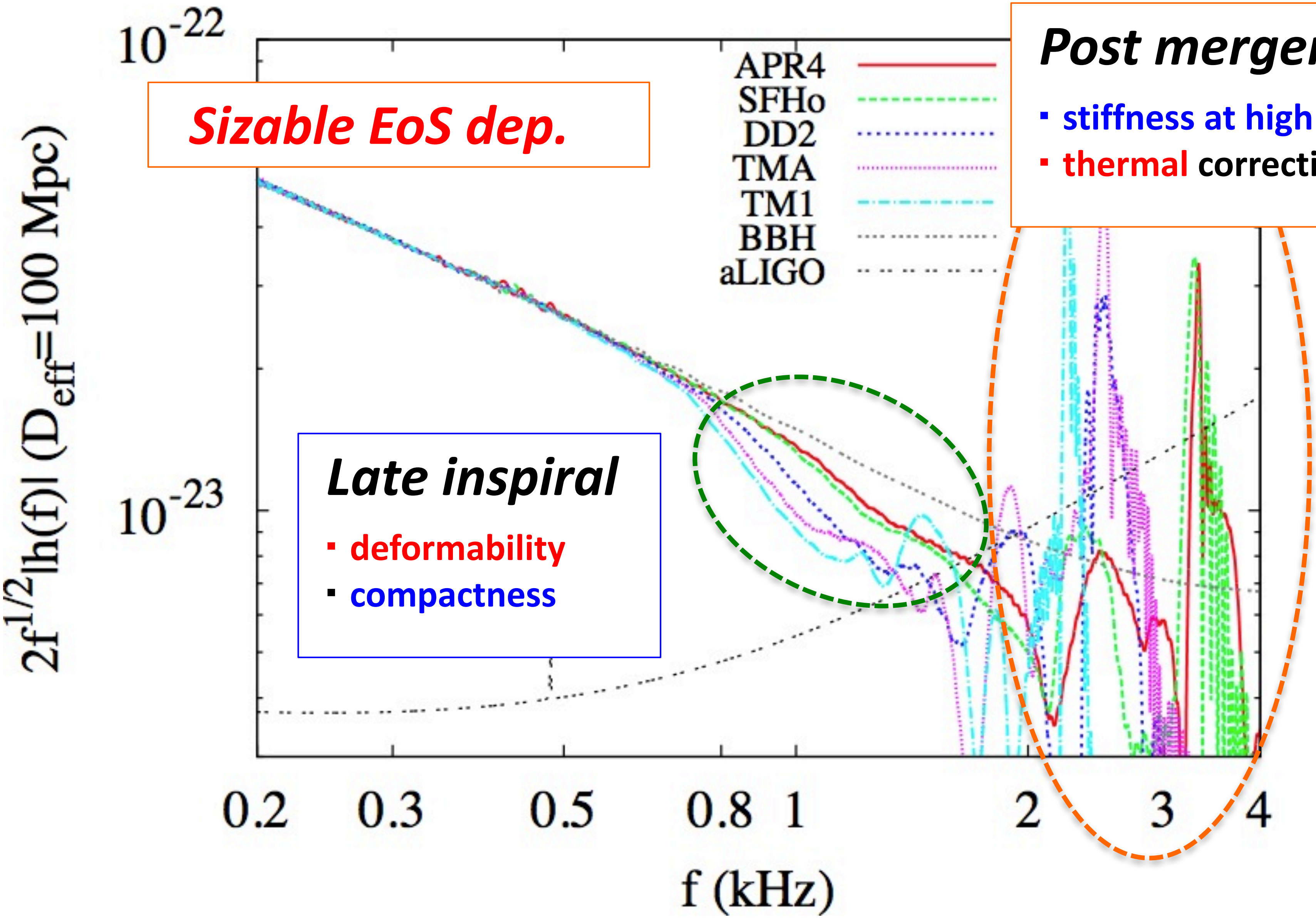
The wave patterns:
consistent with
the general relativity
in strong field regimes





Frequency spectrum

GR simulations, **Hotokezaka et al. 2016**



Sizable EoS dep.

Late inspiral

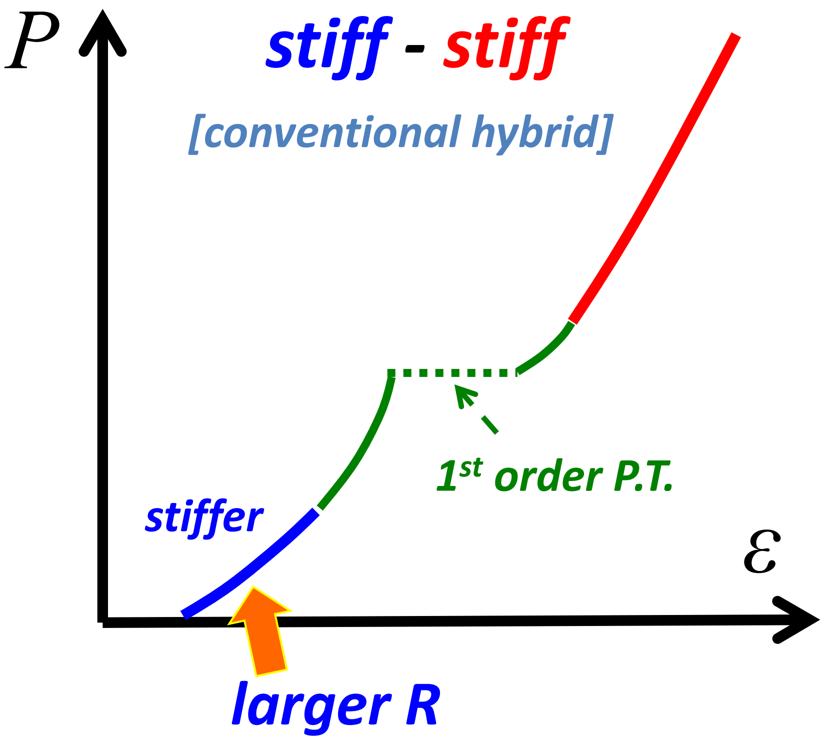
- deformability
- compactness

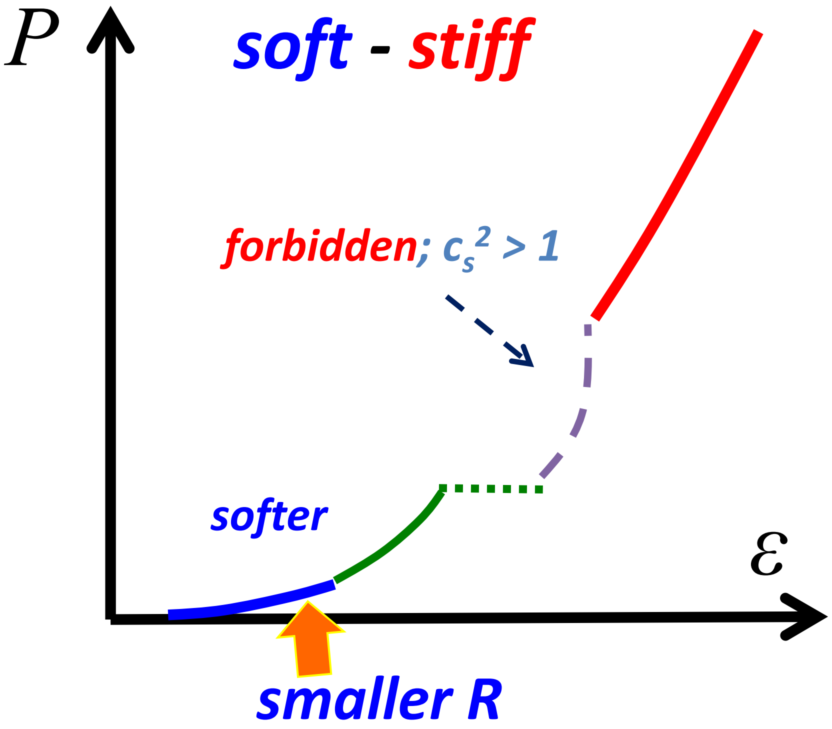
Post merger

- stiffness at high density
- thermal corrections

If we put 1st order H-Q transitions…

[more systematic analyses -> Han-Alford-Prakash 13]





If R is small (<~13km) ➡ ***disfavors strong 1st order P.T.***

Ozel et al. (2010), Steiner et al (2015); target of NICER and GW detection

hadron-quark continuity ?? *[Schaefer-Wilczek 98, Hatsuda et al. 07]*