Possible higher-order phase transition in large-*N* gauge theory

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<in prepration>

Outline

1. Introduction

2. Effective potential of the Polyakov loop

3. Phase structure

   3.1 General structure

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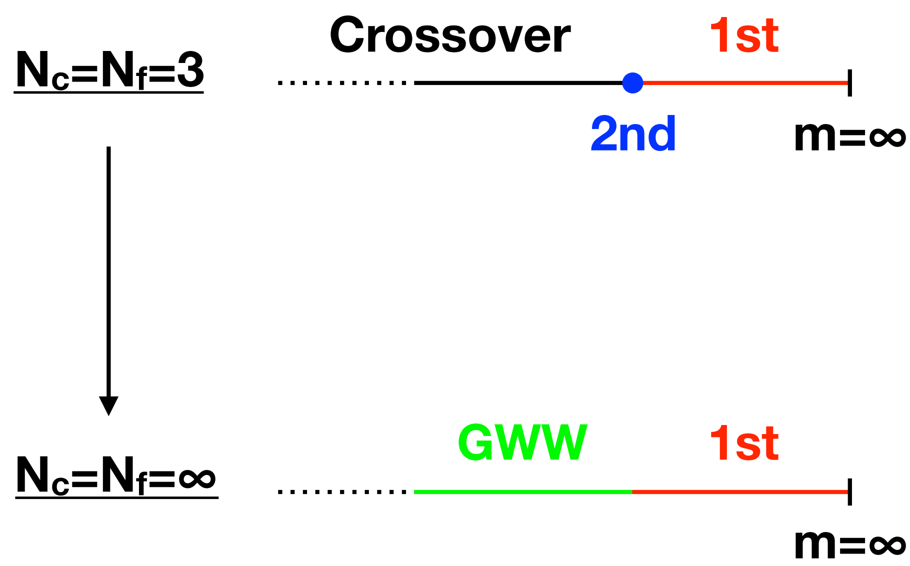


4. Conclusions

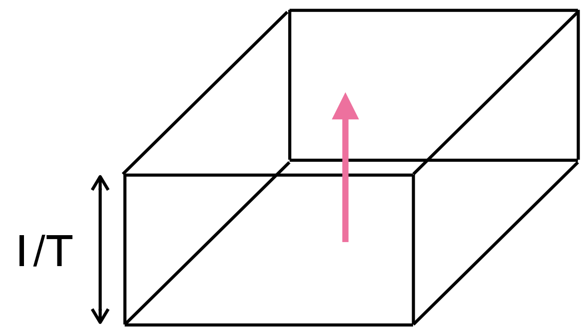
Introduction

Polyakov loop for SU(N)

- Order parameter

Static quark, $P(\vec{x})$

$$P(\vec{x}) = \mathcal{P} e^{i \int_0^{1/T} dx_4 A_4(x)}$$



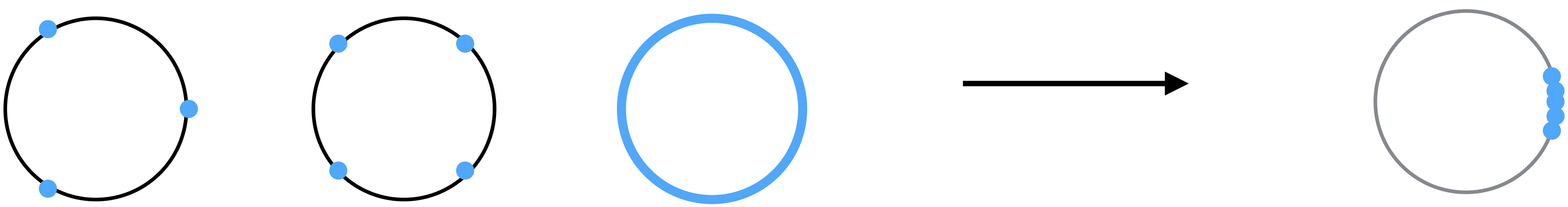
Center symmetry Z(N): $P \to zP$

- Eigenvalue distribution $P = \mathrm{diag}\left\{e^{i\theta_1}, e^{i\theta_2}, \cdots, e^{i\theta_N}\right\}$

Low T

High T

SU(3) SU(4) SU(∞)



$\langle \mathrm{tr} P(\vec{x}) \rangle = 0$

$\langle \mathrm{tr} P(\vec{x}) \rangle \neq 0$

- Simplification at N=∞

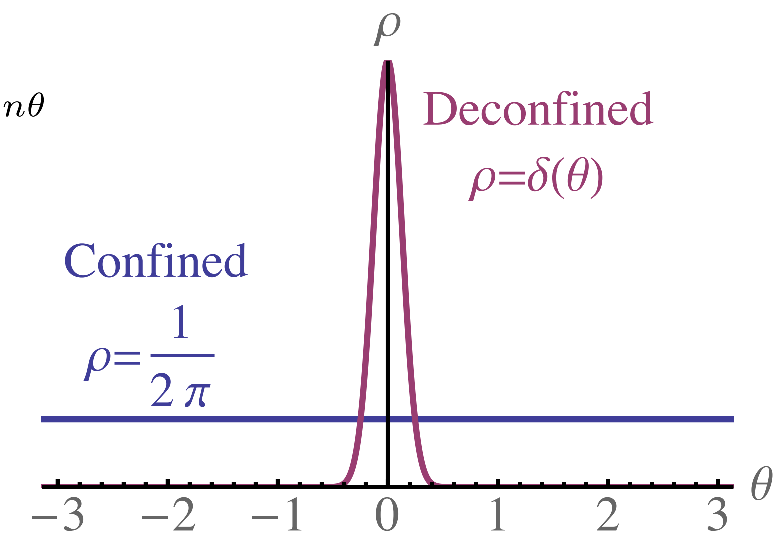
$$\rho_n = \frac{1}{N}\mathrm{tr}_F P^n = \frac{1}{N}\sum_{i=1}^{N} e^{in\theta_i} \xrightarrow{N\to\infty} \int d\theta \rho(\theta) e^{in\theta}$$

<E. Brezin, C. Itzykson, G. Parisi, and J. Zuber, 1978>

Two constraints:

1. Normalization: $\int_{-\pi}^{\pi} d\theta \rho(\theta) = 1$
2. Nonnegative: $\rho(\theta) \geq 0$

Polyakov loop for SU(∞)



- Third-order phase transition

<u>- 2-D Wilson action</u>

<D. Gross and E. Witten, 1980>

<S. R. Wadia, 1980> etc..

<u>- Strong-coupling with heavy quarks</u>

<F. Green and F. Karsch, 1984>

<P.H. Damgaard and A. Paktos, 1986> etc..

<u>- S^1 X S^3</u>

<O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, 2004>

<S. Hands, T. Hollowood and J. Myers, 2010> etc..

<u>- Matrix models</u>

<A. Dumitru, J. Lenaghan and R. Pisarski, 2005>

<R. Pisarski and V. Skokov, 2012> etc..

<u>- Lattice simulations</u>

<F. Bursa and M. Teper, 2006> etc..

See Review: <B. Lucini and M. Panero, 2013>

Effective potential

SU(N)

- Effective potential of the Polyakov loop

Constant background normalized Polyakov loop: $\rho_n = \frac{1}{N}\mathrm{tr}_F P^n$

$$\exp\left[-\mathcal{V}T^{d-1}N^2 V_{\text{eff}}(\rho_n)\right] = \int DA_\mu e^{-S_{YM}(A_\mu)} \prod_{m=1}^{N-1} \delta(\rho_m - \frac{1}{\mathcal{V}N}\int d\mathbf{x}\,\mathrm{tr}_F P^m)$$

<A. Dumitru, Y. Guo and C. P. Korthals Altes, 2014>

- Z(N) symmetric potential

Center symmetry Z(N): $\rho_n \to z^n \rho_n$

$$V_{\text{eff}}(\rho_n) = \sum_n \tilde{a}_n \rho_n \rho_{-n} + \sum_{m,n} \tilde{a}_{m,n} \rho_m \rho_n \rho_{-m-n} + \ldots$$

SU(∞)

- Perturbation theory up to 2 loops

$$V_{\text{eff}}(\rho_n) = -(c_1 + c_2\lambda) \sum_{n=1}^{\infty} \frac{1}{n^4} |\rho_n|^2 + \mathcal{O}(3 \text{ loops})$$

<A. Dumitru, Y. Guo and C. P. Korthals Altes, 2014>

- Strong coupling

  - Leading order: only the double-trace terms. <L. Del Debbio and A. Patella, 2009>

  - Beyond leading order… <Work in progress>

- ρ_1 drives the phase transition.

<u>In this talk</u>

$$V_{\text{eff}}(\rho_n) = \sum_{n=1}^{\infty} a_n |\rho_n|^2 + b_1 |\rho_1|^4 - h(\rho_1 + \rho_1^*)$$

Phase Structure

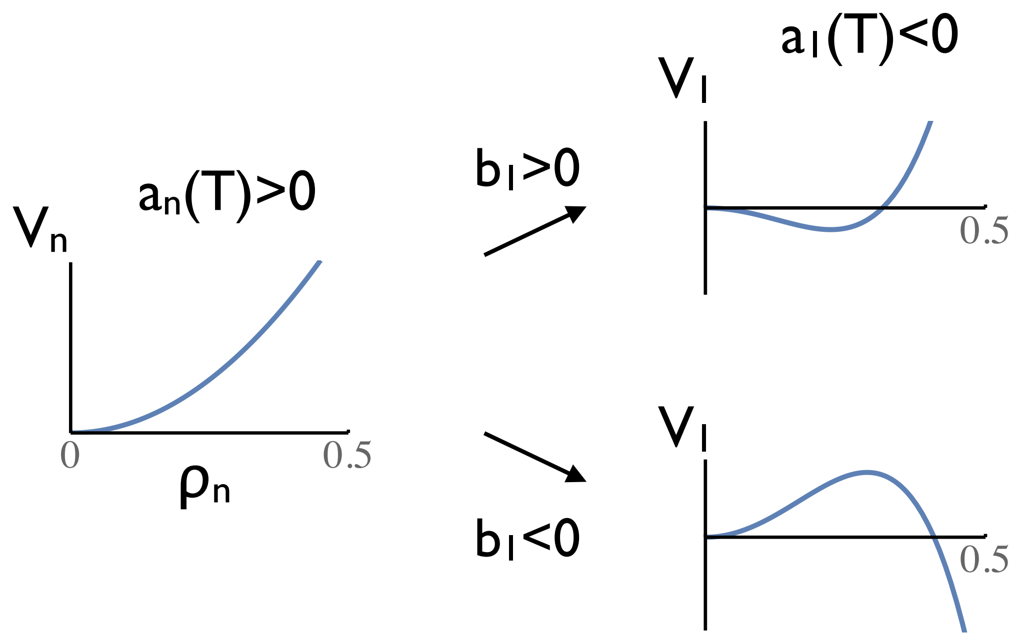
General structure

- Landau energy for each loop

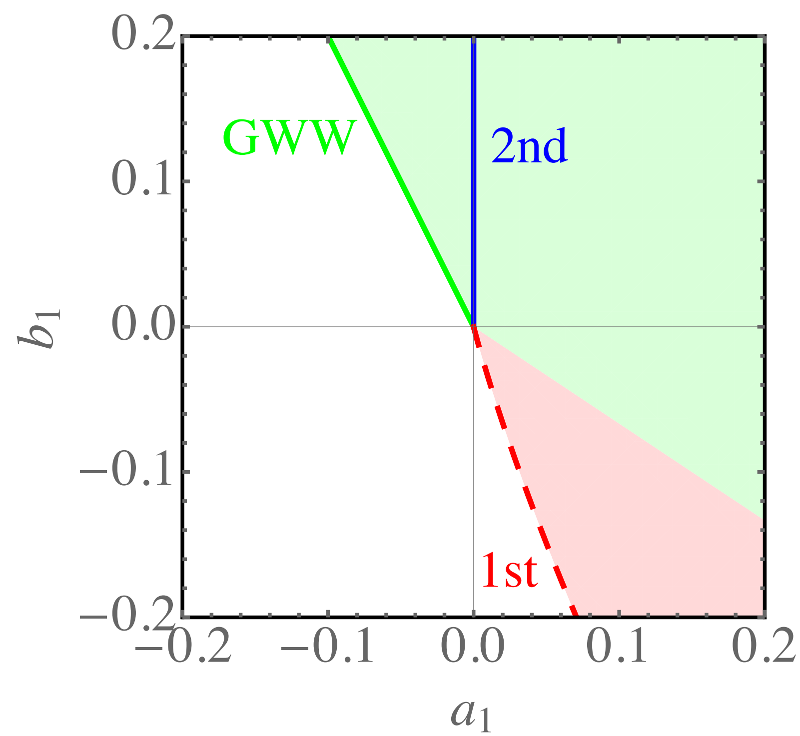
$$V_{eff} = \sum_{n=1}^{\infty} V_n(\rho_n) \qquad \text{where} \qquad V_1 = a_1 \left|\rho_1\right|^2 + b_1 \left|\rho_1\right|^4 - h(\rho_1 + \rho_1^*)$$

$$V_{n>1} = a_n \left|\rho_n\right|^2$$

- Zero external field, h=0.



Below T_c Above T_c



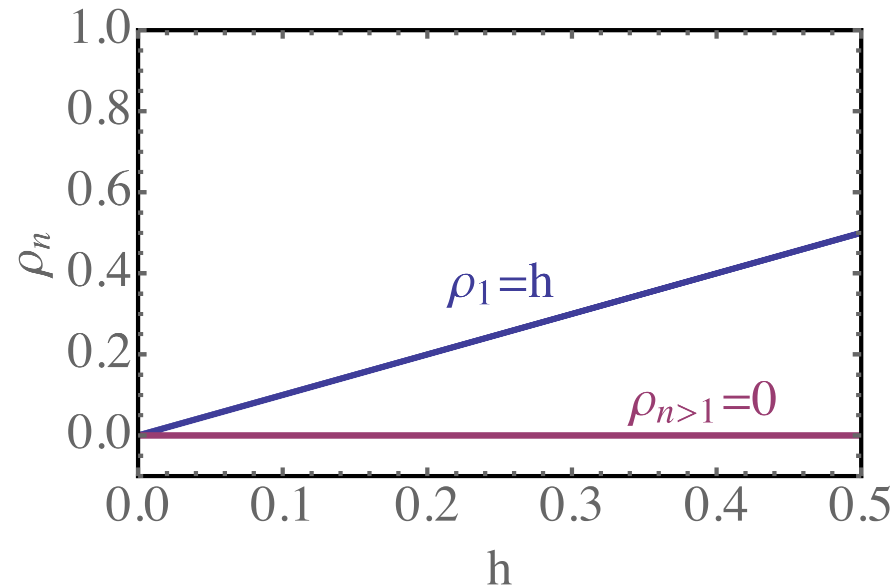
General structure

- Zero quartic coupling

$$V_1 = a_1 \left|\rho_1\right|^2 - 2h\rho_1$$

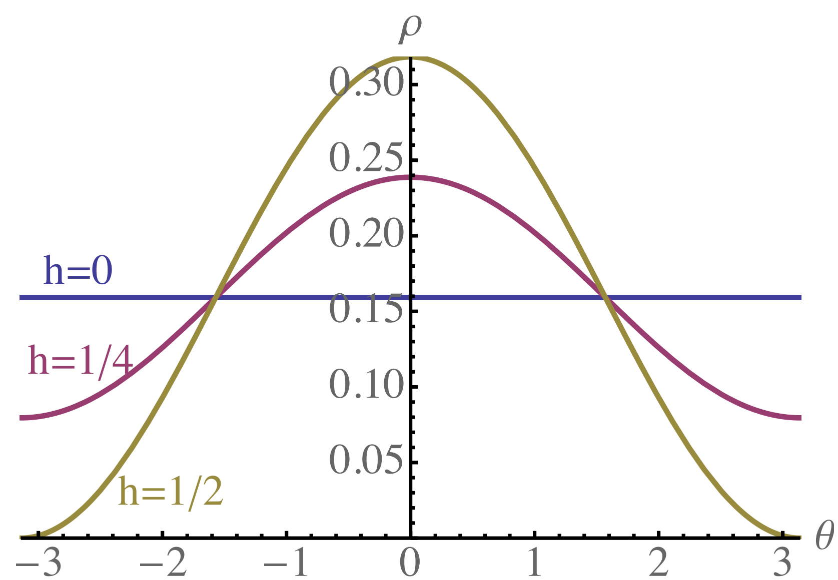
$$V_{n>1} = a_n \left|\rho_n\right|^2$$

$$\longrightarrow \qquad \rho = \frac{1}{2\pi} + \frac{\rho_1}{\pi}\cos\theta$$



Simple Landau picture breaks down at $\rho_1 = 1/2$:

"Gross-Witten-Wadia (GWW) point"

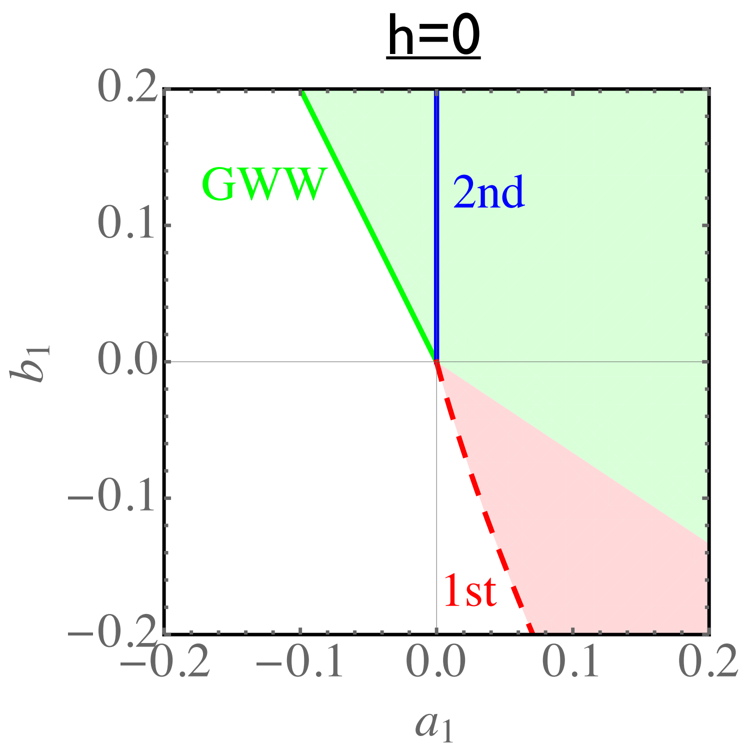


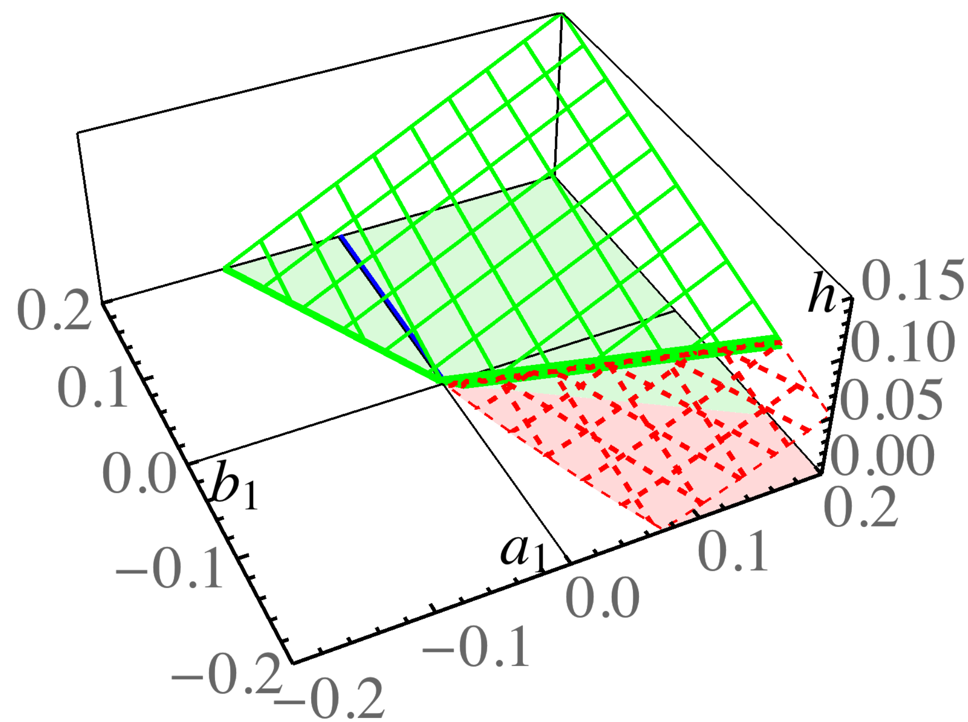
General structure

- GWW surface

$$V_1 = \frac{1}{16}\left(4a_1 + b_1 - 16h\right) + \underline{\left(a_1 + \frac{b_1}{2} - 2h\right)}\,\delta\rho_1 + \underline{\left(a_1 + \frac{3b_1}{2}\right)}\,\delta\rho_1^2 + 2b_1\delta\rho_1^3 + b_1\delta\rho_1^4$$

where $\rho_1 = \frac{1}{2} + \delta\rho_1$ with $\delta\rho_1 < 0$

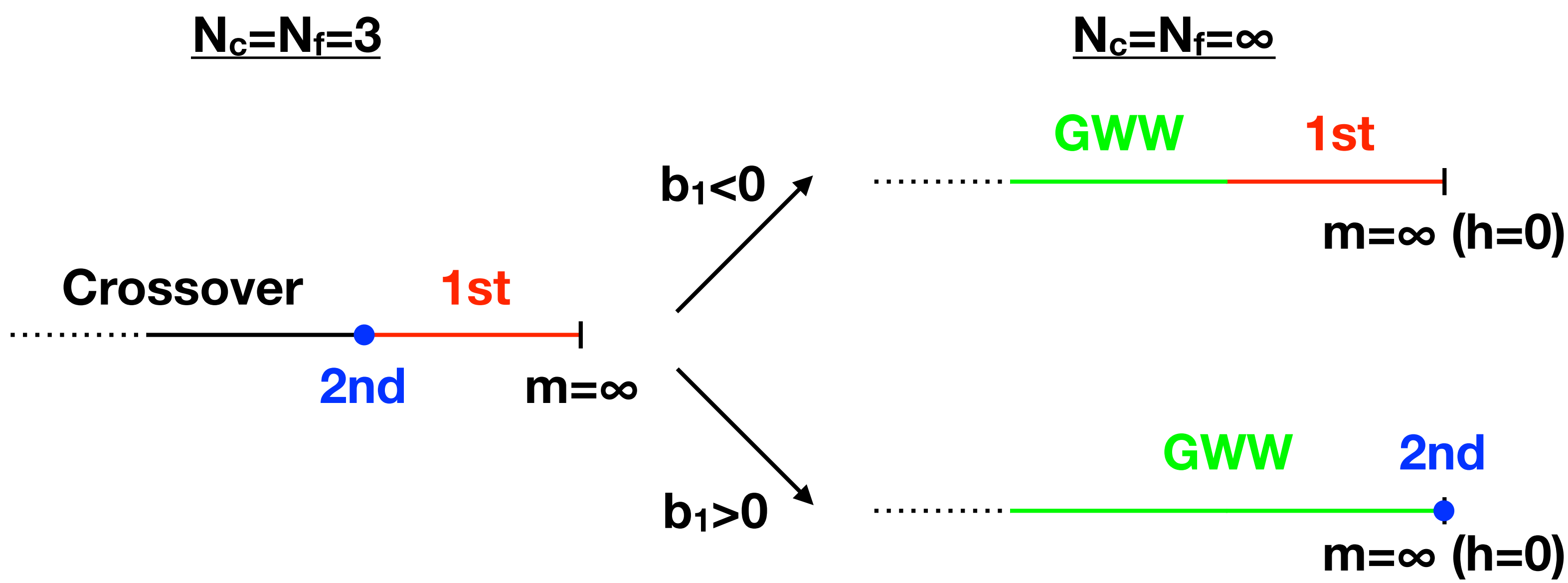




- Higher-order phase transition at GWW

General structure

- Heavy quarks: $h \sim \exp(-m/T)$



Crossover becomes a higher-order phase transition at $\rho_1 = 1/2$.

Models

$$V_{\text{eff}} = V_{\text{pert}} + V_{\text{nonpert}}$$

- Two-loop gluonic effective potential in *d* dimensions, V_{pert}

$$V_{\text{pert}} = -\sum_{n=1}^{\infty} \frac{p_1}{n^d} |\rho_n|^2$$

Z(N) maximally broken

- Nonperturbative confining potential, V_{nonpert}

$$V_{\text{nonpert}} = \sum_{n=1}^{\infty} \frac{c_1}{n^s} |\rho_n|^2 \quad \text{with} \quad s \leq d \quad \text{and} \quad c_1(T/T_c) > p_1 \text{ below } T_c$$

- s=1 and 2 are the Haar measure and mass deformation types, respectively.

<P. Meisinger, T. Miller, and M. Ogilvie, 2002>

- We consider N=∞ and s=1,2,3,4.

Models

- ρ_1 driving the phase transition.

$$a_n = \frac{c_1}{n^s} - \frac{p_1}{n^d}$$

$$V_{\text{eff}} = -p_1 \left|\rho_1\right|^2 + b_1 \left|\rho_1\right|^4 \boxed{- h\left(\rho_1 + \rho_1^*\right) + \sum_{n=1}^{\infty} c_1 \frac{\left|\rho_n\right|^2}{n^s}}$$

- Equation of motion:

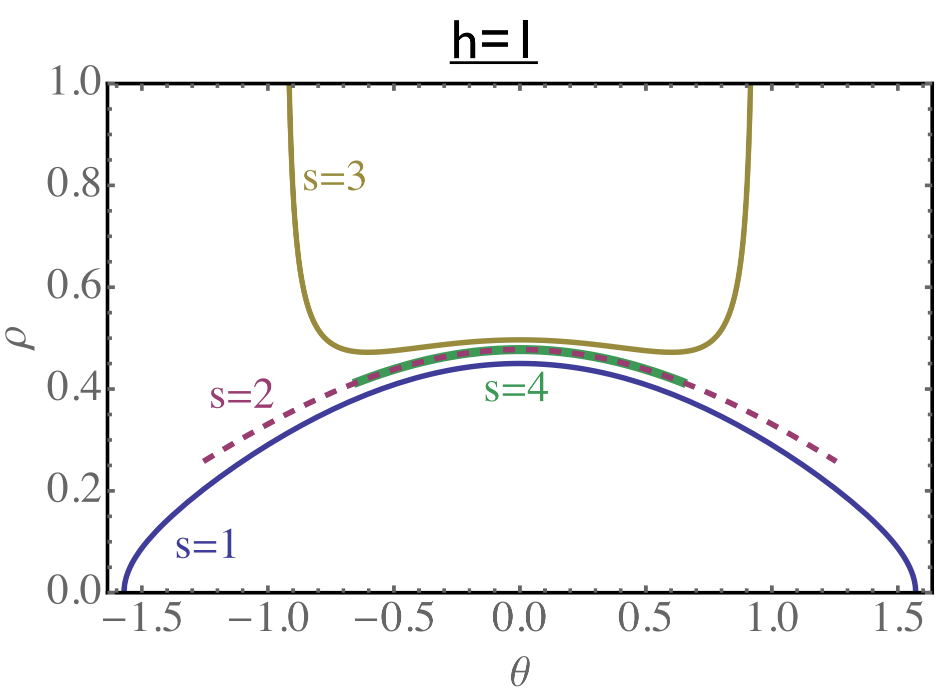
$$h \sin\theta = \int_{-\pi}^{\pi} d\theta' \rho(\theta') \sum_{n=1}^{\infty} \frac{\sin(n\theta - n\theta')}{n^{s-1}}$$

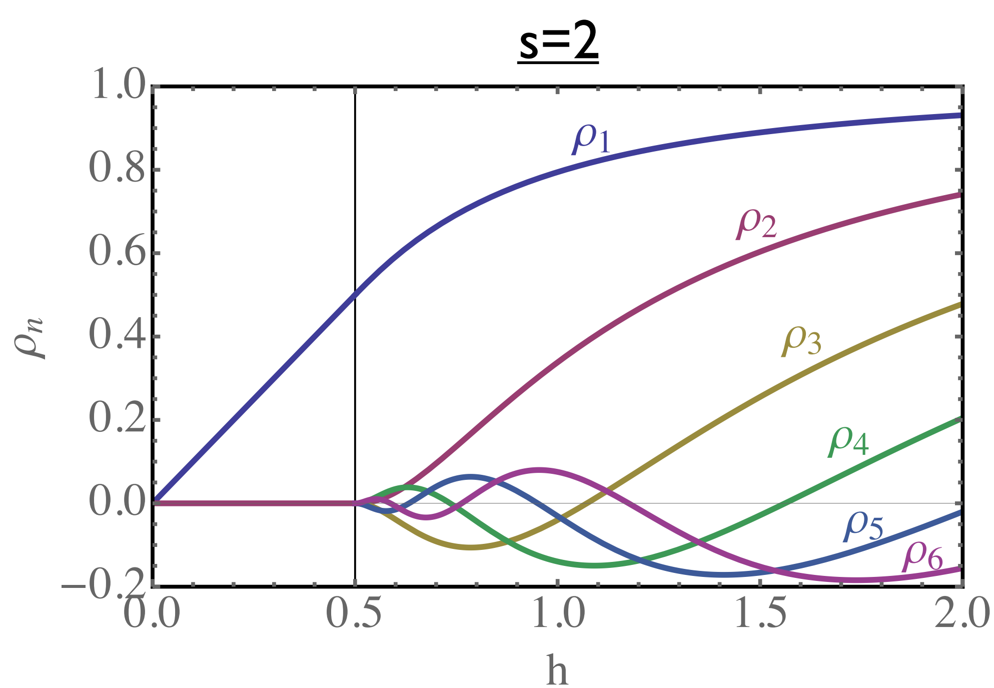
Models

- **Eigenvalue distributions:** $-\theta_0(h) \le \theta \le \theta_0(h)$

s=1,3: $$\rho = C_1(h)\cos\frac{\theta}{2}\left(\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta}{2}\right)^{1/2} + C_2(h)\cos^3\frac{\theta}{2}\left(\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta}{2}\right)^{-1/2}$$

s=2,4: $$\rho = \frac{1}{2\pi}(1+2h\cos\theta) + (1-x_0(h))\,\frac{\delta(\theta-\theta_0)+\delta(\theta+\theta_0)}{2}$$





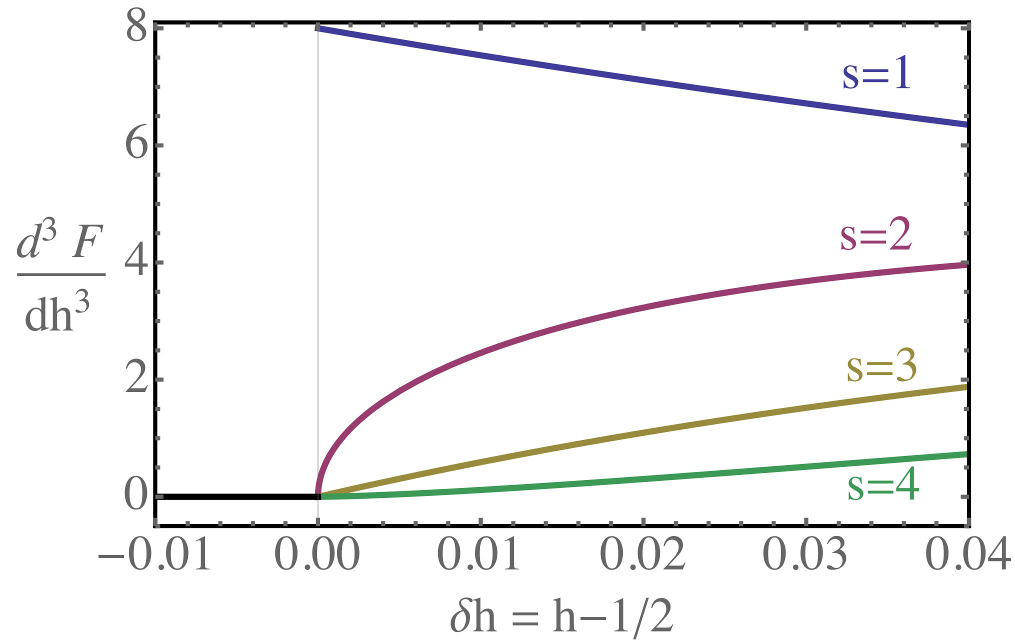
Models

- Free energy

Below GWW

$$F = -\frac{1}{4} - \delta h - \delta h^2$$

Just above GWW



$$F = -\frac{1}{4} - \delta h - \delta h^2 + v_s \delta h^{(5+s)/2} + \mathcal{O}(\delta h^{(7+s)/2})$$ for s = 1, 2, 3, 4

Higher-order phase transition: 3rd, 4th, and 5th.

Models

- ✦ Connection to lattice simulations at large N

  - The quartic coupling is nonpositive: $b_1 \leq 0$

  - ρ_1 is near 1/2 at T_c.

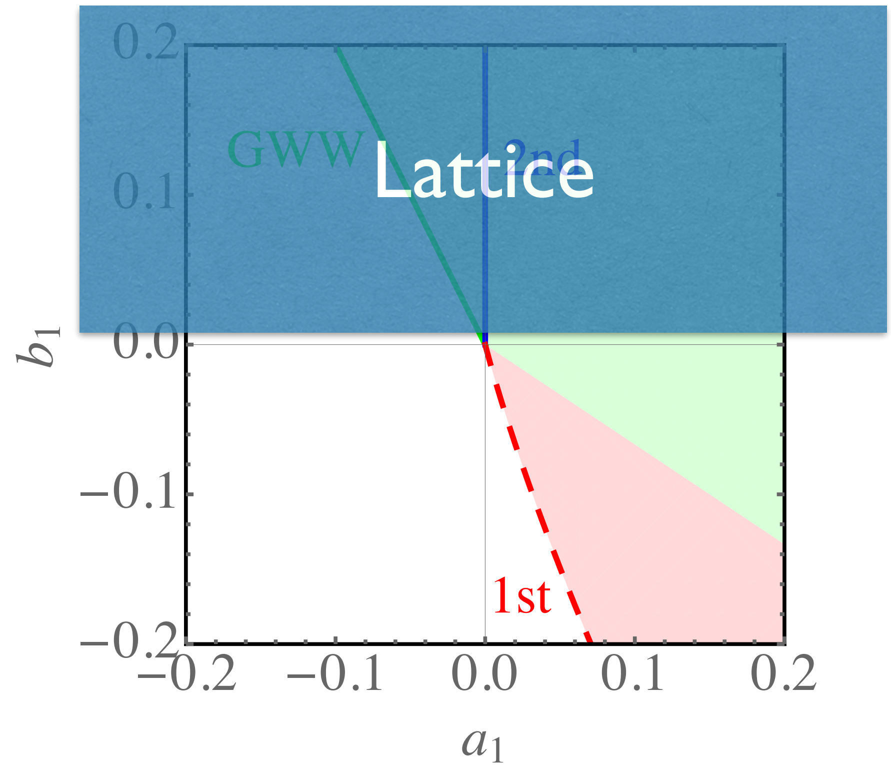
    <A. Mykkanen, M. Panero, and K. Rummukainen, 2011>

    ⟶ $|b_1|$ should be small.

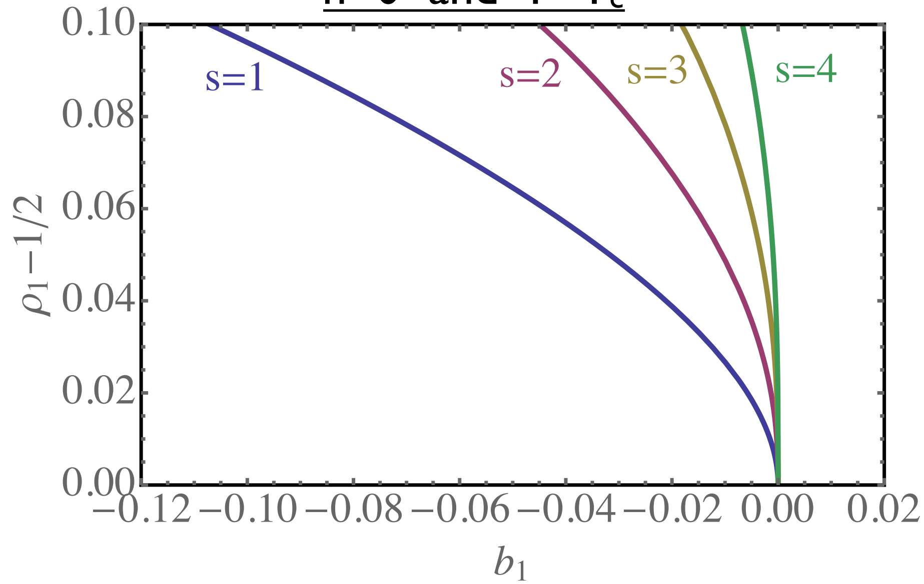
  - Could observe a GWW point.

    <F. Bursa and M. Teper, 2006>

Lattice could differentiate the type of confining potential.



h=0 and T=T_c



Conclusions

- General structure of the phase diagram using the simple Landau theory in SU(∞) is discussed.

- Depending on the type of confining potential, we have a higher-order phase transition at the GWW point. This can be tested in lattice simulations at large N.