Partial Thermalization of Correlations in Nuclear Collisions

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Fact: pp, pA exhibit ridge and anisotropy! Hydrodynamic flow?
Ask: How can we tell if these systems are near local thermal equilibrium?

I. Pre-equilibrium correlations and fluctuations
   a. Linearized Boltzmann equation plus Langevin noise
   b. Transport equation for the two-body correlation function

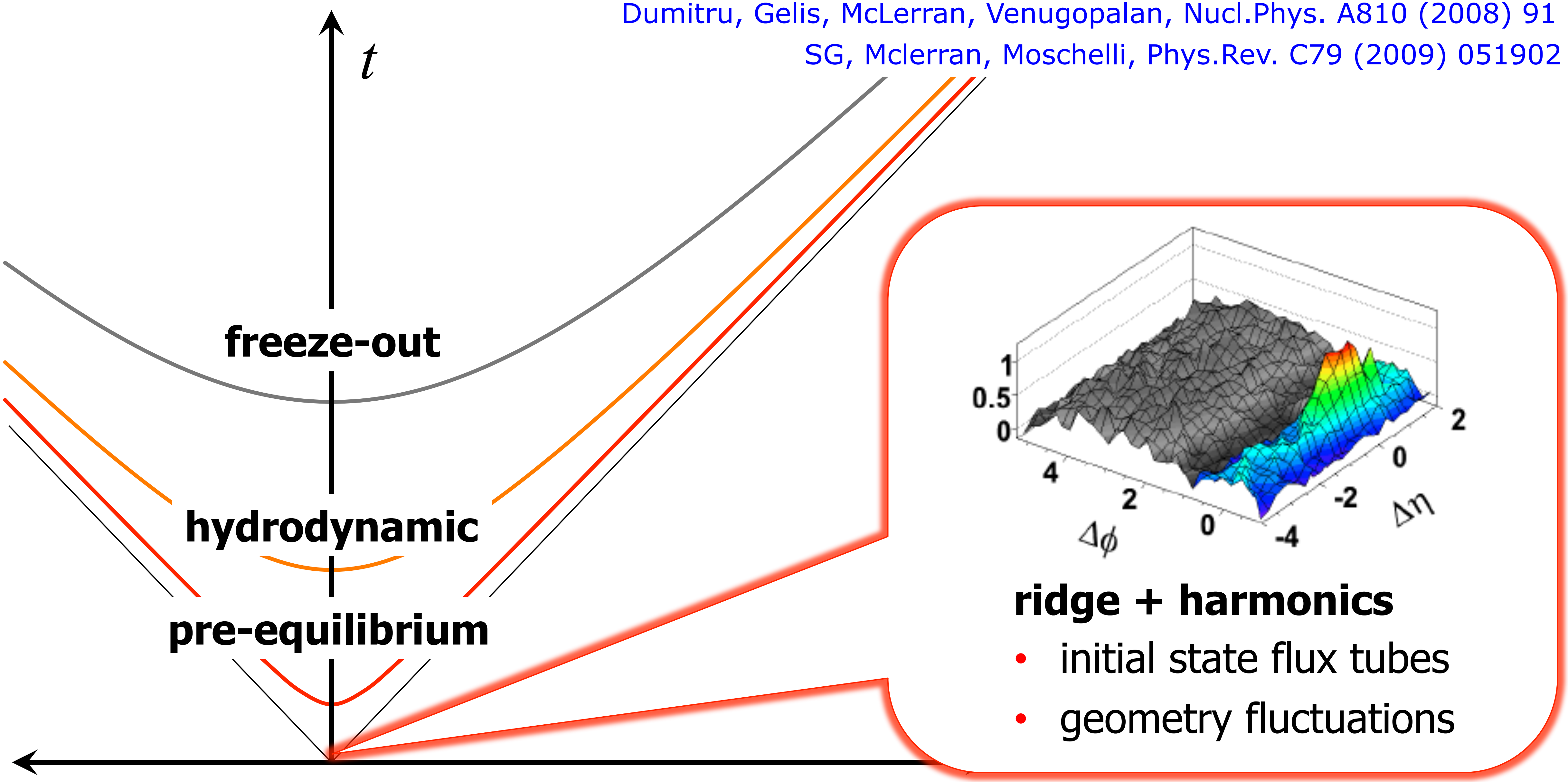
II. Measure transverse momentum fluctuations in pA and AA

SG, Moschelli & Zin, Phys Rev C95 (2017) 064901

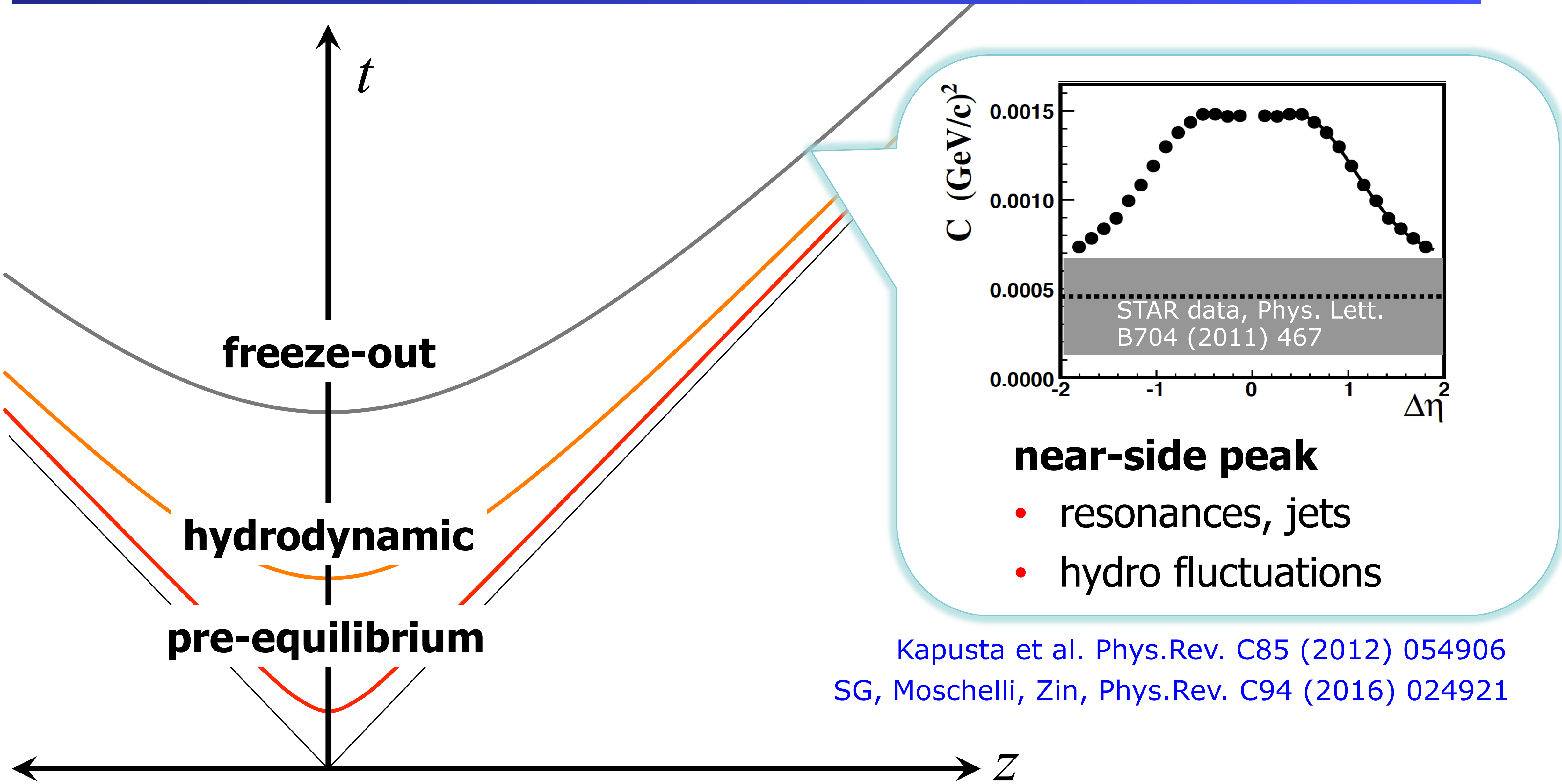
Long Range Rapidity Correlations → Early Stage

Dumitru, Gelis, McLerran, Venugopalan, Nucl.Phys. A810 (2008) 91

SG, Mclerran, Moschelli, Phys.Rev. C79 (2009) 051902



Short Range Rapidity Correlations → Hydrodynamic Evolution



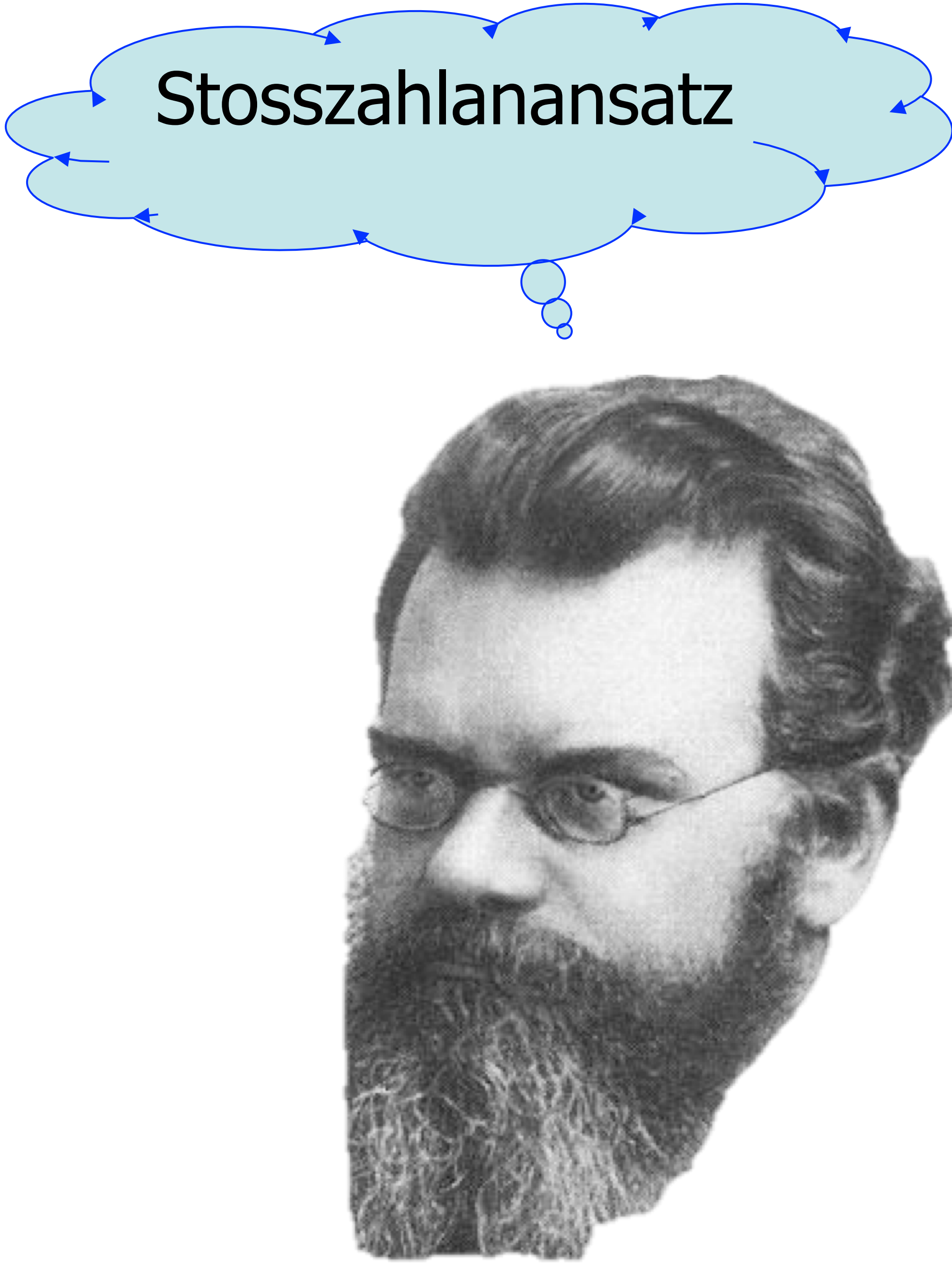
near-side peak

- resonances, jets
- hydro fluctuations

Kapusta et al. Phys.Rev. C85 (2012) 054906

SG, Moschelli, Zin, Phys.Rev. C94 (2016) 024921

Do Correlations Probe Equilibration?



Boltzmann equation: standard method to study approach to equilibrium

Problem: Boltzmann collision term neglects correlations

Boltzmann equation plus Langevin noise

- phase space "clumps"
- fluctuations $\Leftrightarrow$ correlations

Ask: what is the natural level of clumpiness?

Boltzmann Equation in the Relaxation Time Approximation

$$\left(\frac{\partial}{\partial t}+\vec{v}_{\vec{p}}\cdot\vec{\nabla}\right)f=-\nu\left(f-f^{e}\right)$$

phase space $f(\vec{p},\vec{x})$
relaxation time ν^{-1}
drift velocity $\vec{v}_{\vec{p}}=\vec{p}/E$

local equilibrium distribution:

$$f^{e}=\exp\left\{-\gamma(E-\vec{p}\cdot\vec{u}-\mu)/T\right\}$$

temperature T
velocity $\vec{u}$
chemical potential μ

conservation laws require that we choose $T,\vec{u},\mu$ so that f^{e} gives the same energy, momentum and particle density as f

Conservation Laws

demand f^{e} and f give same **energy density**: $e = \int d^3p\, f\, E = \int d^3p\, f^e E$

momentum density: $\vec{g} = \int d^3p\, f\, \vec{p} = \int d^3p\, f^e \vec{p}$

energy conservation: multiply Boltzmann equation by E and integrate

$$\int E\left(\partial_t + \vec{v}_{\vec{p}} \cdot \vec{\nabla}\right) f = -\nu \int E\left(f - f^e\right) \equiv 0$$

$$\partial_t \int Ef + \vec{\nabla} \cdot \int E\, \vec{v}_{\vec{p}} f = 0 \longrightarrow \partial_t e + \vec{\nabla} \cdot \vec{g} = 0$$

multiply Boltzmann equation by p_i or 1 give momentum or number conservation

Linearized Relaxation Time Approximation

$$\left(\frac{\partial}{\partial t}+\vec{v}_{\vec{p}}\cdot\vec{\nabla}\right)f=-\nu\left(1-P\right)f, \qquad f=f^{e}+\delta f,\quad \delta f\ll f^{e}$$

conservation laws enforced by operator P

- constructed so that $\int p^{\mu}(1-P)f=0=\int(1-P)f$
- integrating the Boltzmann eq'n times $p^{\mu},1\Rightarrow$ conservation eq'ns

anisotropic pressure depends on initial distribution $f_{0}(\vec{p},\vec{x})$

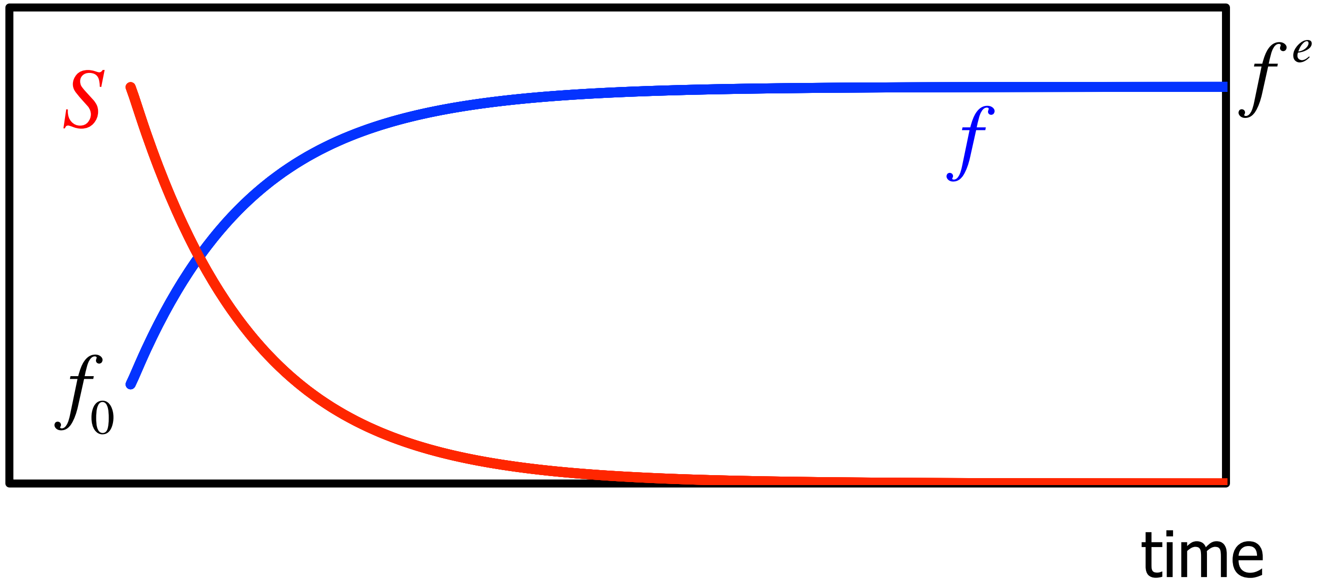
BONUS: projection operator $Pf=f^{e}$

Linearized Boltzmann Solution

$$f(\vec{p},\vec{x},t) = f_0(\vec{p},\vec{x}-\vec{v}_p t)\,S + f^e(\vec{p},\vec{x}-\vec{v}_p t,t)\,(1-S)$$

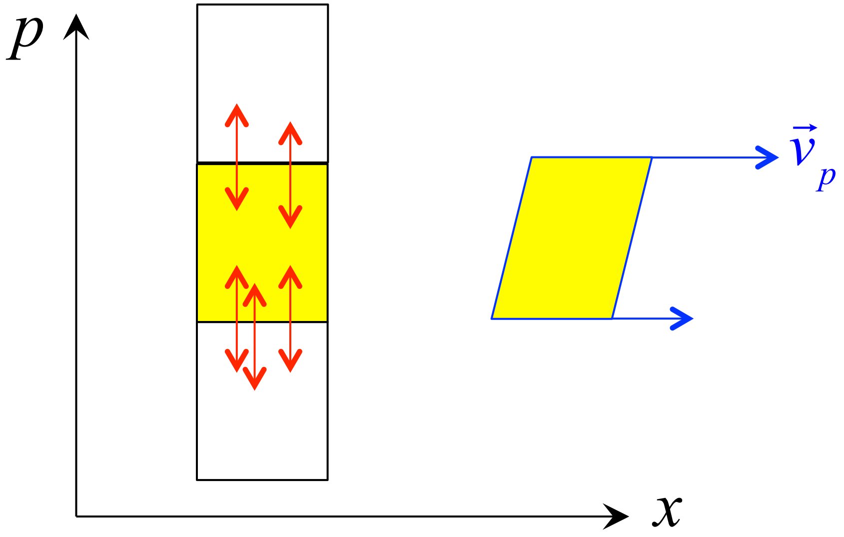
survival probability: parton doesn't scatter before time t

$$S = \exp\left\{-\int_{t_0}^{t} \nu(t')\,dt'\right\}$$



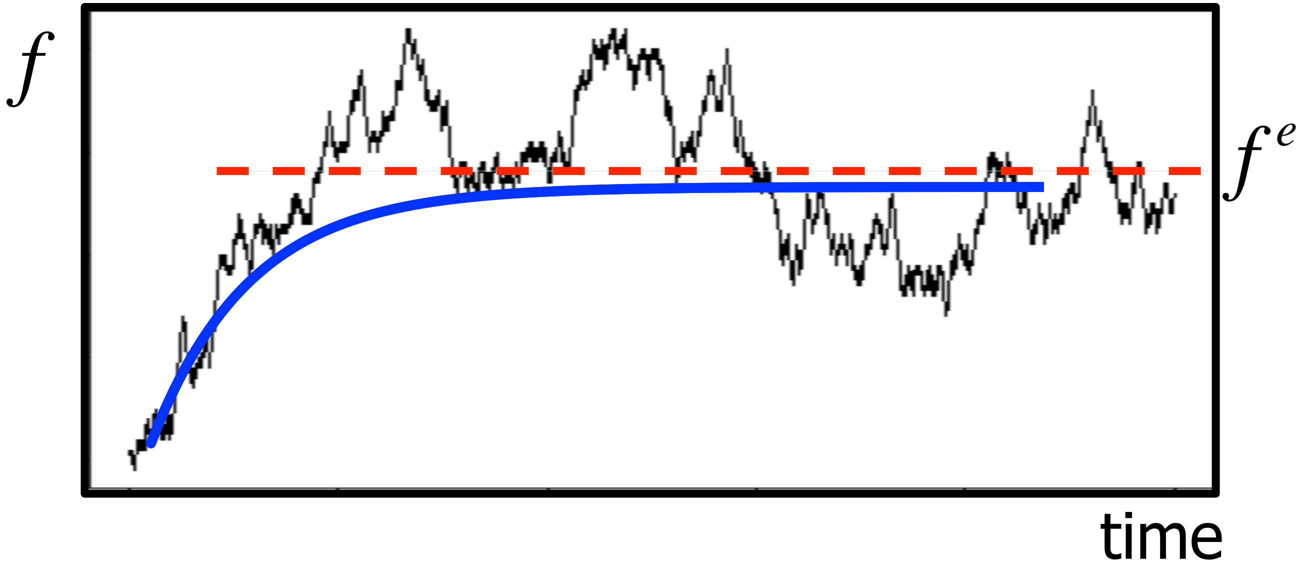
Noise in the Boltzmann Equation

- scattering shuffles particles in phase space
  $\Longrightarrow$ fluctuations and dissipation
- drift is deterministic



approach to equilibrium

- dissipation drives $f \longrightarrow f^e$
- random walk due to fluctuations
- equilibrium state fluctuates



Noisy Thermalization

- events with same initial and equilibrium state – only thermal noise
- illustrative special case: infinite spatially-uniform system

Boltzmann-Langevin $$\frac{\partial}{\partial t} f = -\nu\left(f - f^e\right) + \text{noise}$$

linearize $$Pf = f^e$$

difference equation $$\Delta f = -\nu(1-P) f \Delta t + \Delta W$$

noise $$\langle \Delta W \rangle = 0 \qquad \langle \Delta W(p_1, x_1) \Delta W(p_2, x_2) \rangle = \Gamma_{12} \Delta t$$

Evolution of Correlations: Uniform Infinite System

noisy two-body equation $$\partial_t \langle f_1 f_2 \rangle = -\nu \left(2 - P_1 - P_2\right) \langle f_1 f_2 \rangle + \Gamma_{12}$$

noisy equilibrium state $$\nu \left(2 - P_1 - P_2\right) \langle f_1 f_2 \rangle^e = \Gamma_{12}$$

find: relaxation equation $$\partial_t G_{12} = -\nu \left(2 - P_1 - P_2\right) G_{12}$$

correlation function $$G_{12} = \langle f_1 f_2 \rangle - \langle f_1 f_2 \rangle^e$$

Noise is Necessary for Equilibrium Fluctuations

noisy two-body equation

$$\partial_t \langle f_1 f_2 \rangle = -\nu(2 - P_1 - P_2)\langle f_1 f_2 \rangle + \Gamma_{12}$$

noisy equilibrium state

$$2\nu\underbrace{\left(\langle f_1\, f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle\right)^e}_{\text{nonzero for infinite system}} = \Gamma_{12}$$

find: relaxation equation

$$\partial_t G_{12} = -\nu(2 - P_1 - P_2) G_{12}$$

correlation function

$$G_{12} = \langle f_1\, f_2 \rangle - \langle f_1\, f_2 \rangle^e$$

Noise is Necessary for Equilibrium Fluctuations

noisy two-body equation

$$\partial_t \langle f_1 f_2 \rangle = -\nu\left(2 - P_1 - P_2\right)\langle f_1 f_2 \rangle + \Gamma_{12}$$

noisy equilibrium state

$$\nu\left(2 - P_1 - P_2\right)\langle f_1\, f_2 \rangle^e = \Gamma_{12}$$

find: relaxation equation

$$\partial_t G_{12} = -\nu\left(2 - P_1 - P_2\right) G_{12}$$

correlation function

$$G_{12} = \langle f_1\, f_2 \rangle - \underbrace{\langle f_1 \rangle \langle f_2 \rangle - \langle f_1 \rangle \delta(1-2)}_{}$$

equilibrium: uncorrelated distinct pairs

Expanding System

- expansion eventually leads to freeze out
- equilibrium can be **approached** but never achieved

$$\left(\frac{\partial}{\partial t}+\vec{v}_{\vec{p}_1}\cdot\vec{\nabla}_1+\vec{v}_{\vec{p}_2}\cdot\vec{\nabla}_2\right)G_{12}=$$

$$=-\nu\left(2-P_1-P_2\right)G_{12}+\nu P_1 P_2\left(\langle f_1\rangle-f^e\right)\delta(1-2)$$

$$G_{12}=\langle f_1 f_2\rangle-\langle f_1\rangle\langle f_2\rangle-\langle f_1\rangle\delta(1-2)$$

correlation source: expansion pulls one-body distribution away from equilibrium

Expanding System

- expansion eventually leads to freeze out
- equilibrium can be **approached** but never achieved

$$\underbrace{\left(\frac{\partial}{\partial t}+\vec{v}_{\vec{p}_1}\cdot\vec{\nabla}_1+\vec{v}_{\vec{p}_2}\cdot\vec{\nabla}_2\right)}_{\text{drift}}G_{12}=$$

$$=-\nu\left(2-P_1-P_2\right)G_{12}+\nu P_1P_2\left(\left\langle f_1\right\rangle-f^e\right)\delta(1-2)$$

$$G_{12}=\left\langle f_1\,f_2\right\rangle-\left\langle f_1\right\rangle\left\langle f_2\right\rangle-\left\langle f_1\right\rangle\delta(1-2)$$

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$$\left(\frac{\partial}{\partial t}+\vec{v}_{\vec{p}_1}\cdot\vec{\nabla}_1+\vec{v}_{\vec{p}_2}\cdot\vec{\nabla}_2\right)G_{12}=$$

$$=\underbrace{-\nu\left(2-P_1-P_2\right)G_{12}}_{\text{relaxation}}+\nu P_1 P_2\left(\langle f_1\rangle-f^e\right)\delta(1-2)$$

$$G_{12}=\langle f_1 f_2\rangle-\langle f_1\rangle\langle f_2\rangle-\langle f_1\rangle\delta(1-2)$$

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Expanding System

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$$=-\nu\left(2-P_1-P_2\right)G_{12}+\underbrace{\nu P_1 P_2\left(\langle f_1\rangle-f^e\right)\delta(1-2)}_{\text{source}}$$

$$G_{12}=\langle f_1 f_2\rangle-\langle f_1\rangle\langle f_2\rangle-\langle f_1\rangle\delta(1-2)$$

correlation source: expansion pulls one-body distribution away from equilibrium

Phase Space Correlation Function

formal solution using the method of characteristics:

$$G(\vec{p}_1, \vec{x}_1, \vec{p}_2, \vec{x}_2, t) = G^e_{12} + A_{12}\, S + B_{12} S^2 \qquad S = \exp\left\{ -\int_{t_0}^{t} \nu(t')\, dt' \right\}$$

- local equilibrium G^e_{12} describes asymptotic state of expanding system
- initial conditions determine A_{12}, B_{12}
- drift $\Rightarrow$ these depend on positions $\vec{x}_i - \vec{v}_{pi} t$, where t is measured in a frame co-moving with the matter
- solution must also be averaged over initial conditions

Transverse Momentum Fluctuations

SG, Phys.Rev.Lett. 92 (2004) 162301

SG & Moschelli, Phys.Rev. C85 (2012) 014905

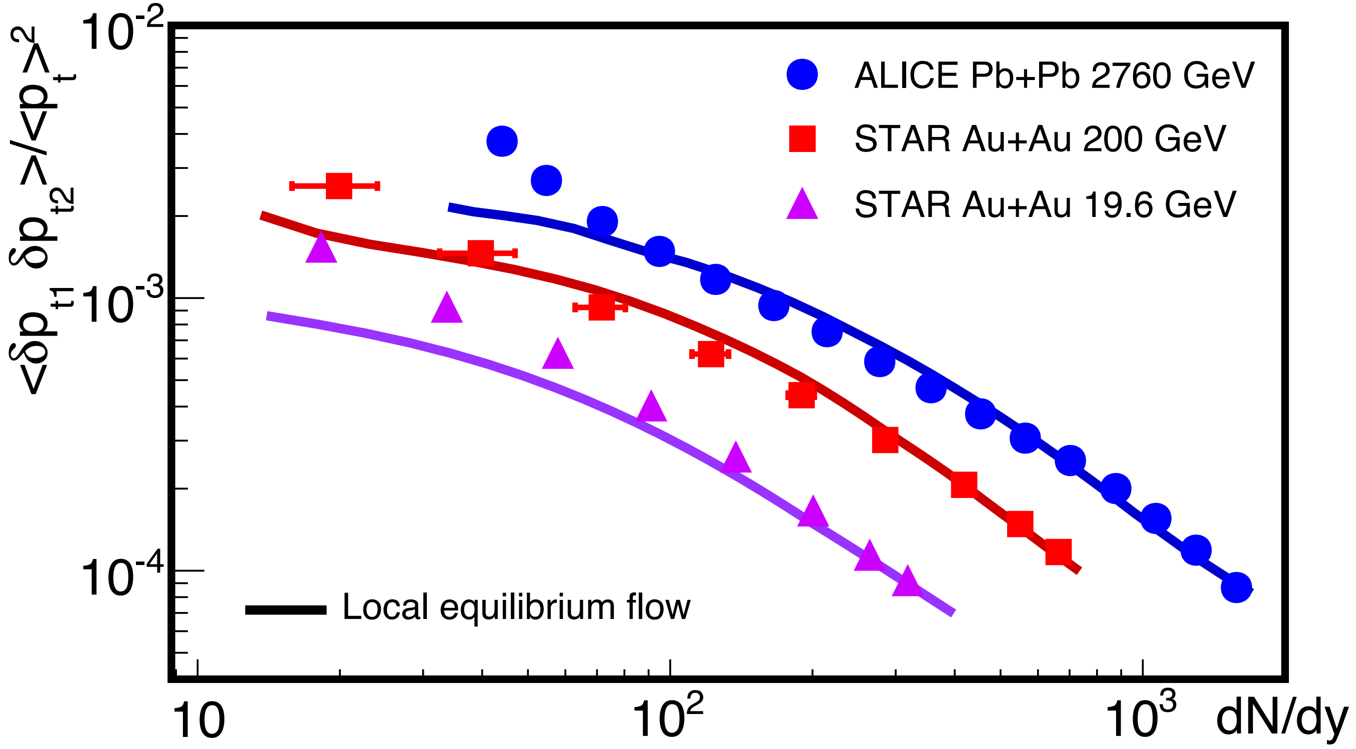
p_t covariance – probe of thermalization

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

$$\delta p_t \equiv p_t - \langle p_t \rangle$$

transverse flow important

- blast wave/hydro describes central ion collisions
- systematic deviation in peripheral?



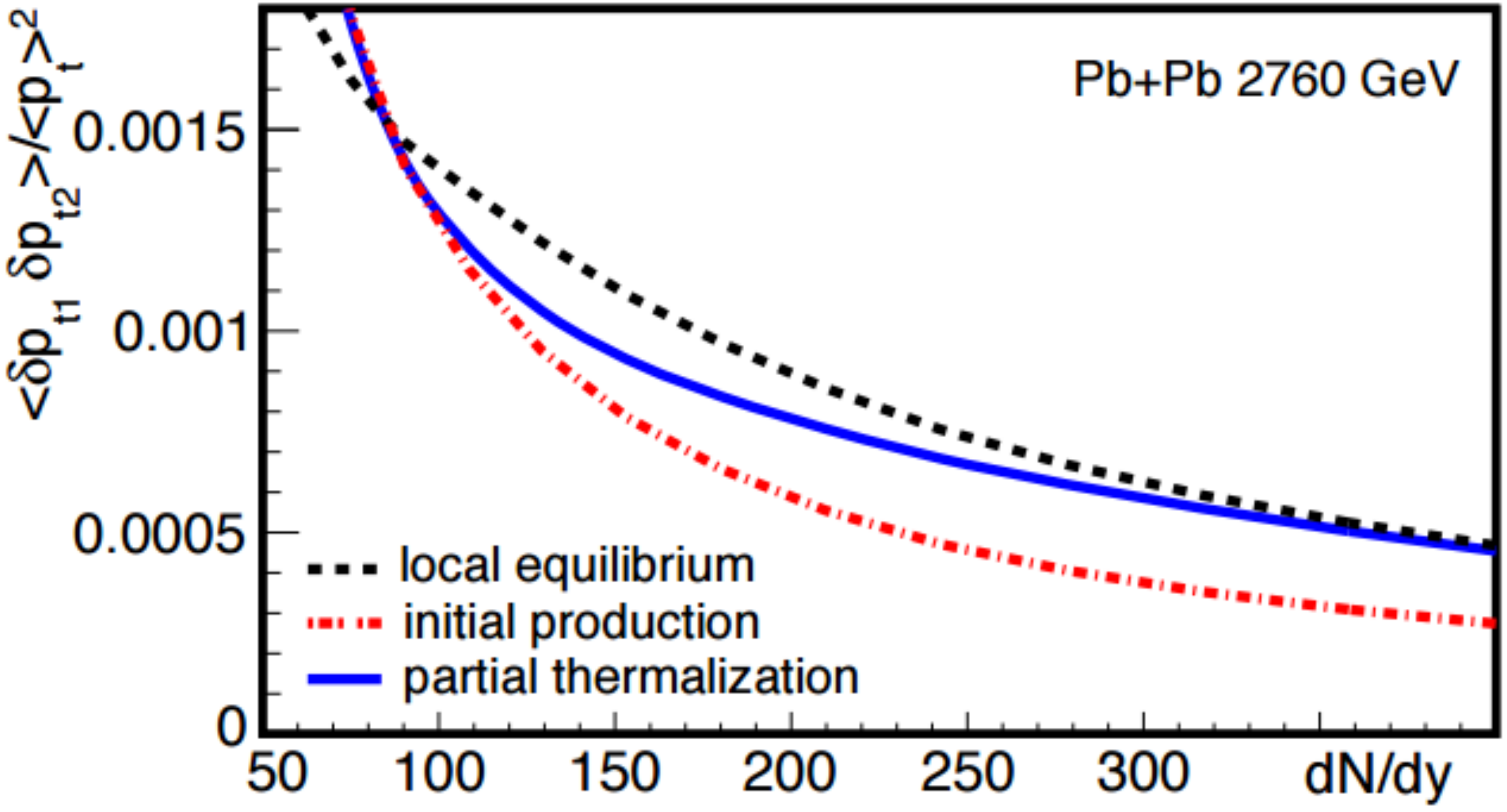
ASK: is the peripheral deviation due **incomplete thermalization**?

Compute Heavy Ion Fluctuations

$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \left\langle \int \delta p_{t1}\, \delta p_{t2} G_{12} \right\rangle$$

$$G_{12} = G_{12}^{e} + A_{12}\, S + B_{12} S^2$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \langle \delta p_{t1} \delta p_{t2} \rangle_0 S^2 + \langle \delta p_{t1} \delta p_{t2} \rangle^e \left(1 - S^2\right)$$



Partial thermalization

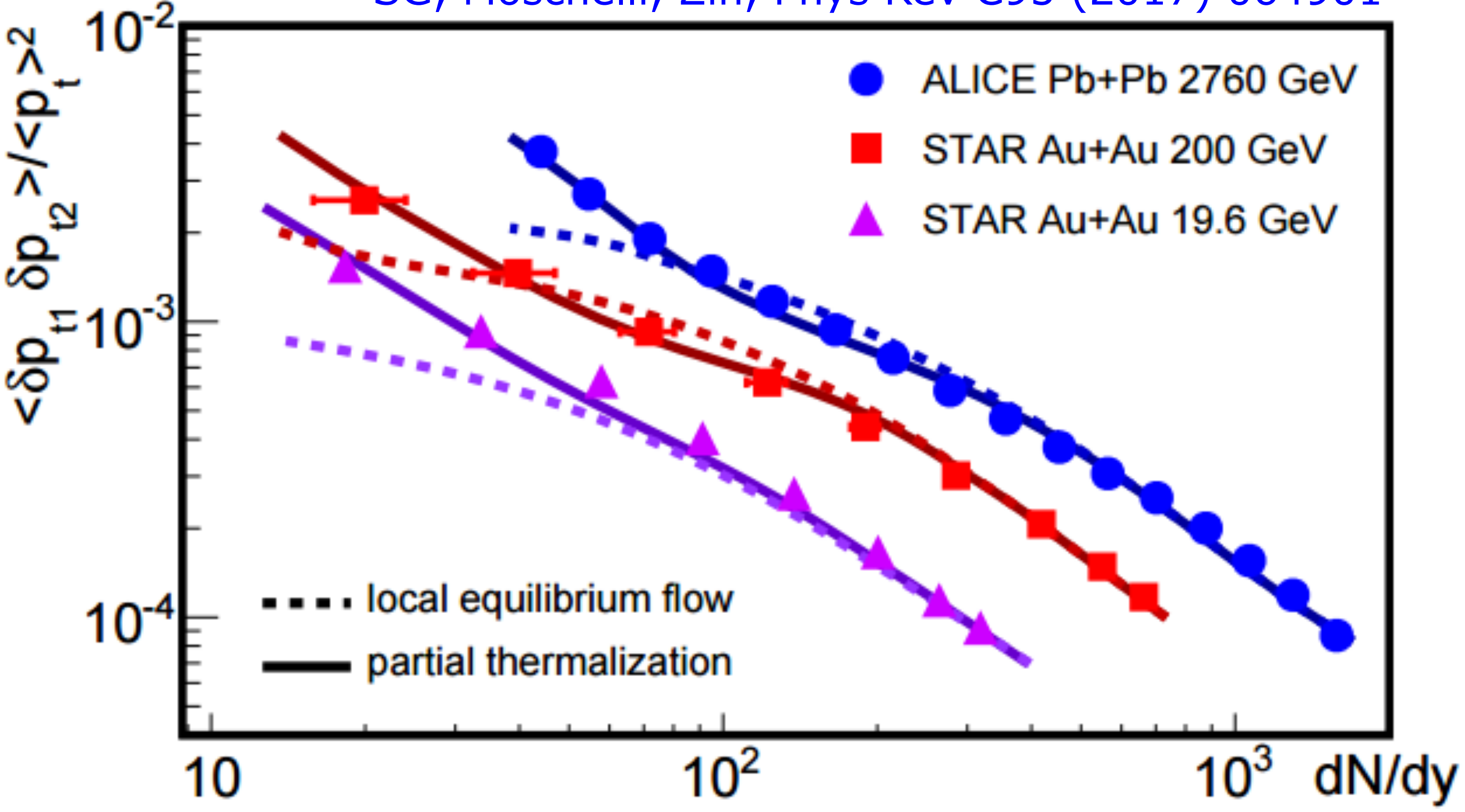
- long-lived central systems equilibrate
- short-lived peripheral systems don't

Heavy Ion Fluctuations

- initial conditions from PYTHIA plus Glauber
- event-averaged $\langle \delta p_{t1} \delta p_{t2} \rangle^e$ from blast wave
- energy independent S fit to data

ALICE, Eur. Phys. J. C74, 3077 (2014)
STAR, Phys. Rev. C72, 044902 (2005)
STAR, J. Novak, MSU PhD thesis (2013)

SG, Moschelli, Zin, Phys Rev C95 (2017) 064901

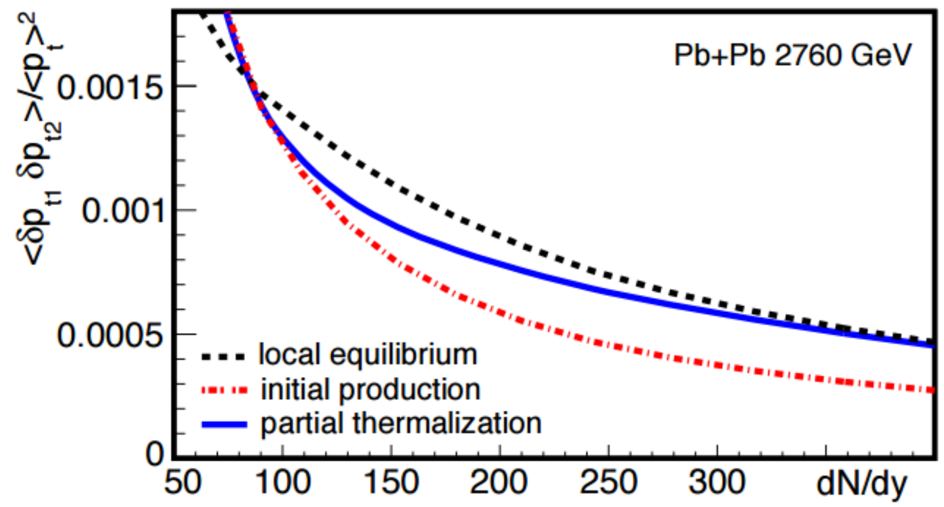


S from data suggests **relaxation time** is

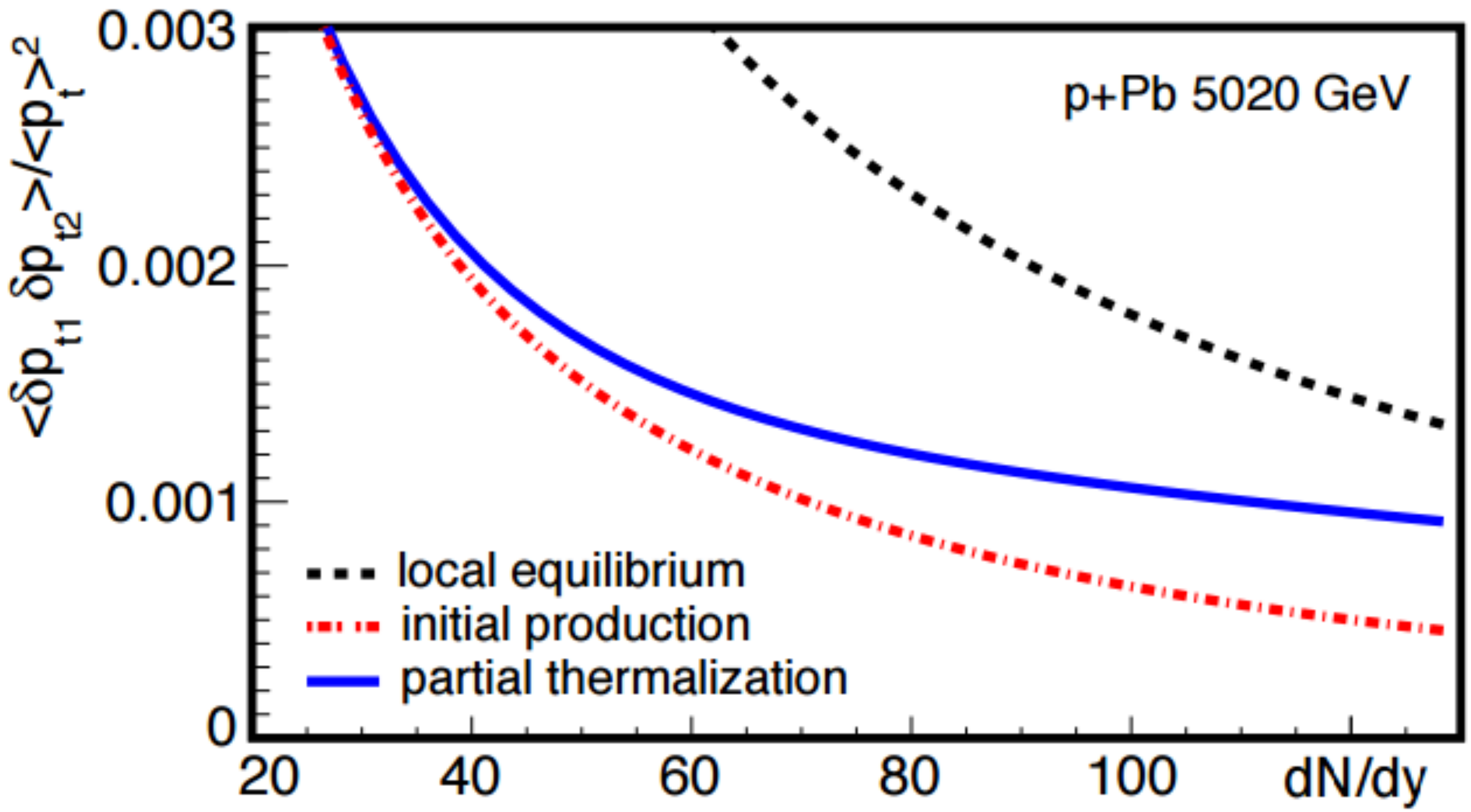
$\nu^{-1} \sim 0.3$ fm

at $\tau_0 \sim 0.6$ fm.

Partial thermalization moves to **central** p+Pb collisions!



SG, Moschelli, Zin, Phys Rev C95 (2017) 064901



Summary: Boltzmann-Langevin approach to pre-equilibrium correlations

New transport equation for non-equilibrium correlations

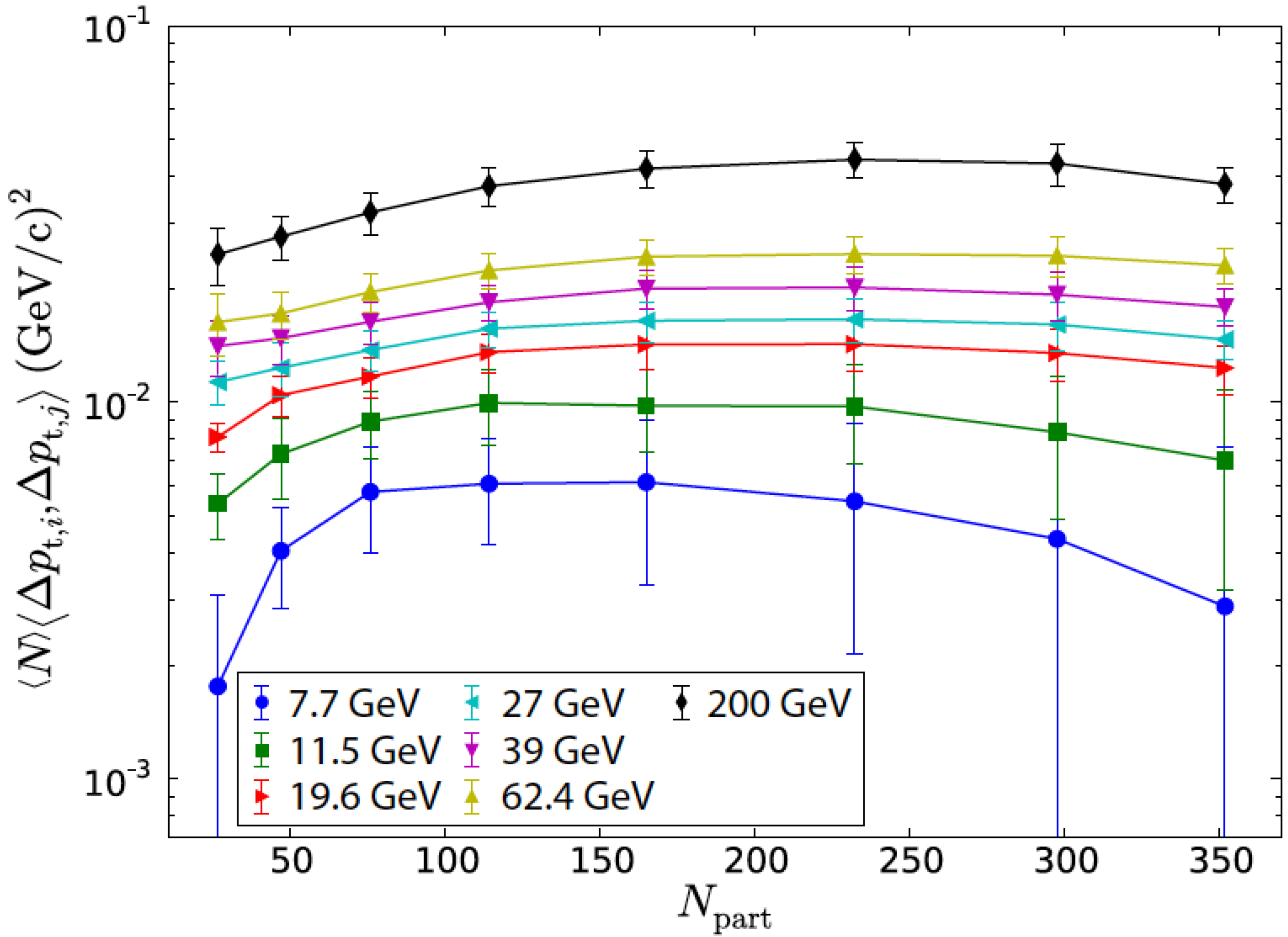
- New tool for understanding the general features of correlations
- Consistent description of collisional noise off equilibrium
- Test Monte Carlo transport codes

First steps in describing pre-equilibrium correlations.

- Color fields add a momentum gradient term to drift
  - Ryblewski & Florkowski Phys.Rev. D88 (2013) 034028
- Bose statistics, condensation of overpopulated modes
  - Blaizot, Rept. Prog. Phys. 80 (2017) no.3, 032301
- Comparison to experiment
  - Pratt & Young, Phys.Rev. C95 (2017) no.5, 054901

Beam Energy Scan Data

STAR, J. Novak, MSU PhD thesis (2013)



Boltzmann Equation in the Relaxation Time Approximation

$$\left(\frac{\partial}{\partial t} + \vec{v}_{\vec{p}} \cdot \vec{\nabla}\right) f = -\nu\left(f - f^{e}\right)$$

relaxation time ν^{-1}

local equilibrium distribution $f^{e} = \exp\left\{-\gamma(E - \vec{p}\cdot\vec{u} - \mu)/T\right\}$

define off-equilibrium temperature T, velocity $\vec{u}$ and chemical potential μ

$$\int d^3p\, f \begin{Bmatrix} 1 \\ \vec{p} \\ E \end{Bmatrix} = \int d^3p\, f^{e} \begin{Bmatrix} 1 \\ \vec{p} \\ E \end{Bmatrix}$$

Conservation Laws

choose $T, \vec{u}, \mu$ so that f^{e} and f give same energy density e, momentum density $\vec{g}$ and particle density n

$$\int d^3p\, f \begin{Bmatrix} 1 \\ \vec{p} \\ E \end{Bmatrix} = \int d^3p\, f^e \begin{Bmatrix} 1 \\ \vec{p} \\ E \end{Bmatrix}$$

multiply Boltzmann equation by p^{μ} or 1 and integrating implies conservation eq'ns

energy conservation:

$$\int E\left(\partial_t + \vec{v}_{\vec{p}} \cdot \vec{\nabla}\right) f = -\nu \int E\left(f - f^e\right) \equiv 0$$

$$\partial_t \int Ef + \vec{\nabla} \cdot \int E\, \vec{v}_{\vec{p}} f = 0$$

$$\vec{v}_{\vec{p}} = \vec{p} / E$$

$$\partial_t e + \vec{\nabla} \cdot \vec{g} = 0$$