Anatomy of Chiral Magnetic Effect In and Out-Of-Equilibrium

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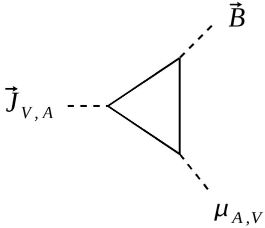
Reference: D. E. Kharzeev, M. A. Stephanov, H.-U. Yee, Phys.Rev. D95 (2017) no.5, 051901 (Rapid Communication)

Chiral Magnetic Effect

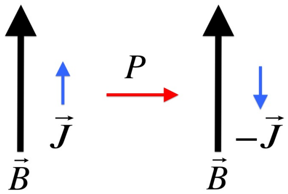
(Fukushima-Kharzeev-Warringa, Son-Zhitnitsky, Vilenkin)

$$\vec{J}_V = \frac{eN_c}{2\pi^2}\mu_A \vec{B}\,, \quad \vec{J}_A = \frac{eN_c}{2\pi^2}\mu_V \vec{B}$$

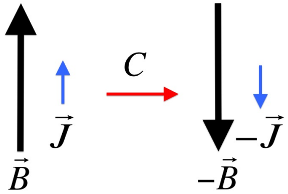


Note the $< AVV >$ structure

Discrete Symmetries



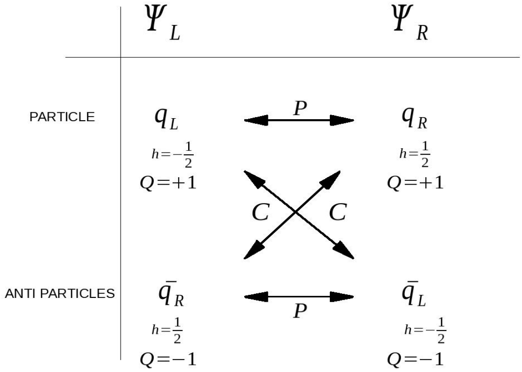
$$P = -1$$



$$C = +1$$

$\vec{J} = \sigma_\chi \vec{B}$ **is a P- and CP-odd phenomenon**

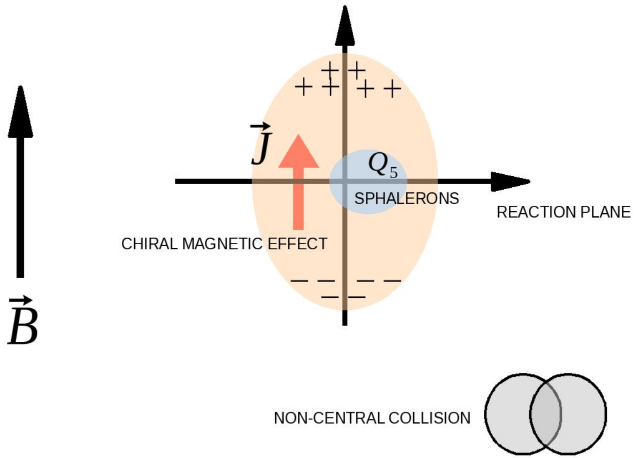
Axial Charge is P- and CP-odd



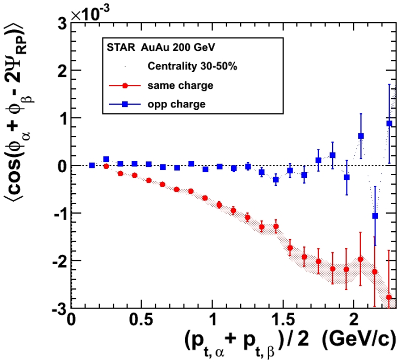
Axial Charge

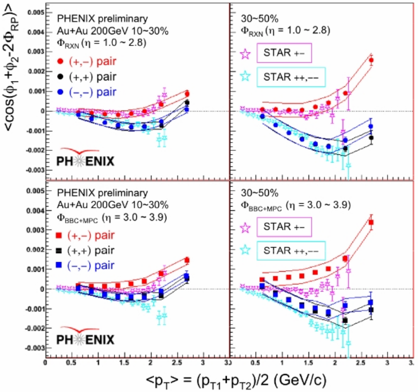
$$J^0_A = N(q_L) + N(\bar{q}_L) - N(q_R) - N(\bar{q}_R)$$

Possible experimental consequence of chiral magnetic effect

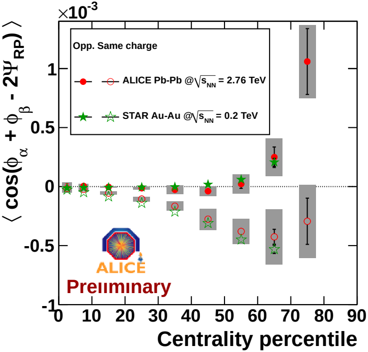


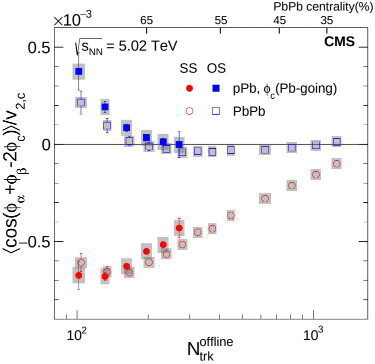
Experiments at RHIC





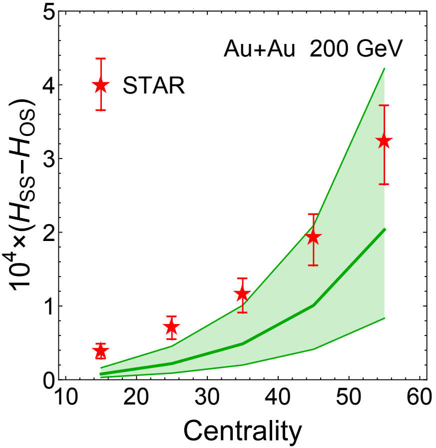
Experiments at LHC





Cutting-Edge Simulations of Hydrodynamics with Chiral Magnetic Effect

Yin Jiang, Shuzhe Shi, Yi Yin, Jinfeng Liao
(arXiv:1611.04586 [nucl-th])



In the chiral basis $J_{R,L} = \frac{1}{2}(J_V \pm J_A)$

Chiral Magnetic Effect

(Fukushima-Kharzeev-Warringa, Son-Zhitnitsky, Vilenkin)

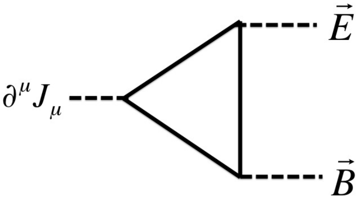
$$\vec{J}_{R,L} = \pm \frac{1}{4\pi^2} \mu_{R,L} \vec{B}$$

Chiral Vortical Effect

(Erdmenger.*et al*, Banerjee.*et al*, Vilenkin)

$$\vec{J}_{R,L} = \pm \frac{1}{4\pi^2} \left(\mu_{R,L}^2 + \frac{\pi^2}{3} T^2 \right) \vec{\omega} \,, \quad \vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{v}$$

They are robust and protected by Chiral Anomaly



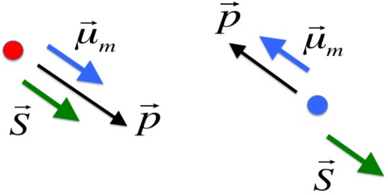
They have been checked

- **at strong coupling**
  (**HUY, Rebhan-Schmitt-Stricker, Gynther.***et al*)
- **in hydrodynamics** (**Son-Surowka**)
- **on lattices**
  (**Buividovich.***et al*, **Abramczyk.***et al*, **Yamamoto, Bali.***et al*)

We will see that the weak coupling picture is a bit more subtle

Quasi-particle picture of CME (Kharzeev-Warringa)

Quantized Weyl particles (p) and anti-particles ($\bar{p}$)

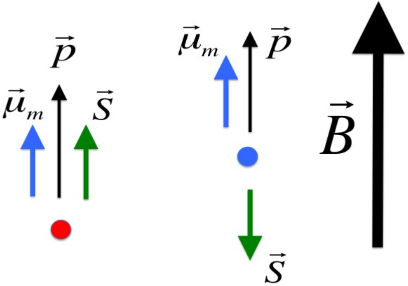


$Q = +1$ $\qquad$ $Q = -1$

$h = +\frac{1}{2}$ $\qquad$ $h = -\frac{1}{2}$

$$\vec{S} = \pm\frac{1}{2}\frac{\vec{p}}{|\vec{p}|}\,, \qquad \vec{\mu}_M = \pm\frac{\vec{S}}{|\vec{p}|} = \frac{1}{2}\frac{\vec{p}}{|\vec{p}|^2}$$

Quasi-particle picture of CME (Kharzeev-Warringa)



Energy shift in a magnetic field: $\Delta E = -\vec{\mu}_M \cdot \vec{B} = -\frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}$

It gives rise to a tendency to align the momentum along the magnetic field direction

Let's try to be more quantitative

The energy shift $\Delta E = -\frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}$ will modify the equilibrium distribution of particles ($f_+^{\rm eq}$) and anti-particles ($f_-^{\rm eq}$)

from

$$f_\pm^{(0)} \equiv \left(\exp[\beta(|\vec{p}| \mp \mu)] + 1\right)^{-1}$$

to

$$\begin{aligned} f_\pm^{\rm eq} &= \left(\exp[\beta(|\vec{p}| - \frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2} \mp \mu)] + 1\right)^{-1} \\ &\approx f_\pm^{(0)} + \beta f_\pm^{(0)}(1 - f_\pm^{(0)})\frac{\vec{p}\cdot\vec{B}}{2|\vec{p}|^2} + \mathcal{O}(B^2) \end{aligned}$$

The net current is

$$
\begin{aligned}
\vec{J} &= \int \frac{d^3\vec{p}}{(2\pi)^3}\,\dot{\vec{x}}\,\left(f_+^{\rm eq} - f_-^{\rm eq}\right) = \int \frac{d^3\vec{p}}{(2\pi)^3}\,\frac{\vec{p}}{|\vec{p}|}\,\left(f_+^{\rm eq} - f_-^{\rm eq}\right) \\
&= \frac{\beta}{2}\int \frac{d^3\vec{p}}{(2\pi)^3}\,\frac{\vec{p}}{|\vec{p}|}\,\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) \\
&= \frac{1}{3}\cdot\frac{1}{4\pi^2}\vec{B}\times\beta\int_0^\infty dp\; p\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) \\
&= \frac{1}{3}\cdot\frac{\mu}{4\pi^2}\vec{B}
\end{aligned}
$$

where

$$
\beta\int_0^\infty dp\; p\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) = \mu
$$

independent of temperature

This contribution from the energy shift explains only $\frac{1}{3}$ of the full result

Identifying the remaining $\frac{2}{3}$ contribution to the CME needs a complete picture of microscopic motions of fermions under a magnetic field

Berry Phase in Momentum Space

(Son-Yamamoto, Stephanov-Yin, Qun Wang *et al*)
(QFT worldline approach Mueller-Venugopalan)

The motion of Weyl particle is described by the action

$$S_+ = \int dt \; \left(\vec{p} \cdot \dot{\vec{x}} + \vec{A} \cdot \dot{\vec{x}} + A_0 - \mathcal{E} - \vec{\mathcal{A}}_p \cdot \dot{\vec{p}} \right)$$

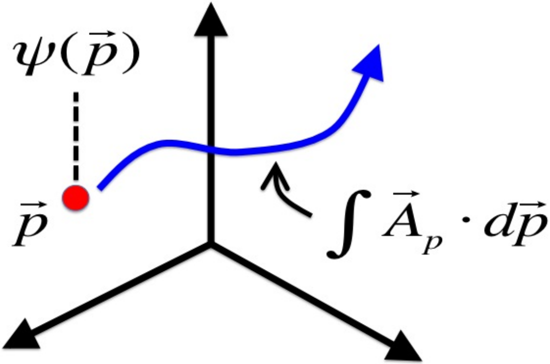
where the last term is the Berry's connection coming from the chiral spinor wave-function

$$\vec{\mathcal{A}}_p = i\psi^\dagger(\vec{p})\vec{\nabla}_p\psi(\vec{p}) \,, \quad (\vec{\sigma}\cdot\vec{p})\psi(\vec{p}) = |\vec{p}|\psi(\vec{p})$$

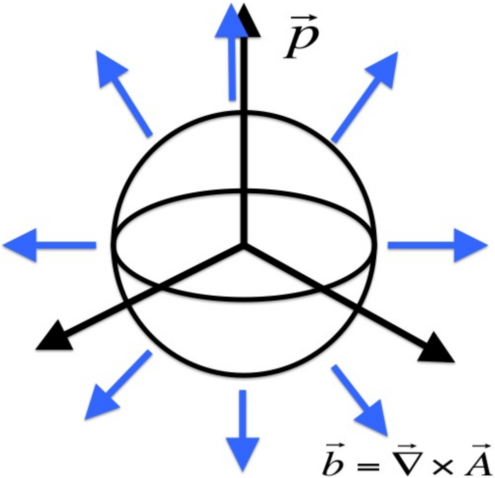
whose curvature is of the monopole form

$$\vec{b} \equiv \vec{\nabla} \times \vec{\mathcal{A}}_p = \frac{\vec{p}}{2|\vec{p}|^3}$$

Picture of Berry's Phase in Momentum Space



Picture of Berry's Phase in Momentum Space



$$\vec{b} = \vec{\nabla} \times \vec{A}_p = \frac{\vec{p}}{2|\vec{p}|^3}$$

Recall the Lorentz force

$$\dot{\vec{p}} = \dot{\vec{x}} \times \vec{B}$$

Having a magnetic field $\vec{b}$ in momentum space would imply

$$\dot{\vec{x}} = \hat{p} + \dot{\vec{p}} \times \vec{b}$$

where the first term is the original velocity

It is easy to see that the combined effects of both magnetic fields is to give a net velocity along $\vec{B}$ direction.

The result is (Stephanov-Yin)

$$\sqrt{G}\;\dot{\vec{x}} \;=\; \frac{\partial\mathcal{E}}{\partial\vec{p}} + \vec{B}\left(\frac{\partial\mathcal{E}}{\partial\vec{p}}\cdot\vec{b}\right) = \frac{\vec{p}}{|\vec{p}|} + \frac{\vec{p}(\vec{p}\cdot\vec{B})}{|\vec{p}|^4} + \mathcal{O}(B^2)$$

The second term is the new velocity from triangle anomaly

The current from this new velocity is

$$\begin{aligned}\vec{J} &= \int \frac{d^3\vec{p}}{(2\pi)^3}\,\sqrt{G}\,\dot{\vec{x}}\left(f_+^{(0)} - f_-^{(0)}\right)\\ &= \int \frac{d^3\vec{p}}{(2\pi)^3}\,\frac{\vec{p}(\vec{p}\cdot\vec{B})}{|\vec{p}|^4}\left(f_+^{(0)} - f_-^{(0)}\right)\\ &= \frac{2}{3}\cdot\frac{1}{4\pi^2}\vec{B}\times\int_0^\infty dp\,\left(f_+^{(0)} - f_-^{(0)}\right)\\ &= \frac{2}{3}\cdot\frac{\mu}{4\pi^2}\vec{B}\end{aligned}$$

where

$$\int_0^\infty dp\,\left(f_+^{(0)} - f_-^{(0)}\right) = \mu$$

independent of temperature

Breaking Up The Equilibrium CME Value

$\frac{1}{3}$ comes from the modification of equilibrium distribution due to energy shift

Let's call it "energetic" contribution

$\frac{2}{3}$ comes from a new component of velocity due to anomaly, which is more kinematic

Let's call it "kinematic" contribution

The point: In out-of-equilibrium conditions, the "energetic" contribution is expected to be lost, while the "kinematic" contribution always exists

Let's consider "shaking" the magnetic field with some frequency ω

If the shaking is slower than the relaxation time scale of achieving equilibrium τ_R, the system will be able to adjust itself to equilibrium at each moment of time, and we expect the full value of CME

If $\omega \gg \tau_R^{-1}$, the distribution will not be able to follow equilibrium distribution, so $\frac{1}{3}$ of CME will be lost while $\frac{2}{3}$ should remain

Chiral Magnetic Conductivity at Finite ω (Kharzeev-Warringa)

Chiral Magnetic Effect is a response of the current to an external magnetic field

$$\vec{J}(\omega) = \sigma_\chi(\omega)\vec{B}(\omega)$$

The "chiral magnetic conductivity" $\sigma_\chi(\omega)$ is computed from the P-odd retarded function

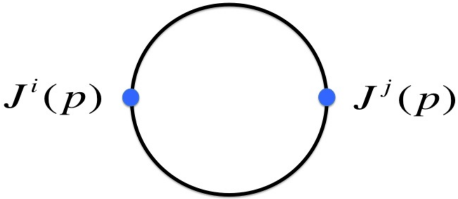
$$G_R^{ij} = -i\theta(t-t')\langle\left[J^i(t), J^j(t')\right]\rangle \sim i\sigma_\chi(\omega, k)\epsilon^{ijl}k_l$$

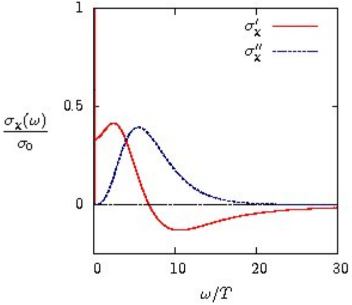
Expectation from the above analysis

- **For small frequency $\omega \ll \tau_R^{-1}$, we recover full equilibrium value, $\sigma_\chi(\omega) \to \frac{\mu}{4\pi^2}$**
- **For $\omega \gg \tau_R^{-1}$, we should get only $\frac{2}{3}$ of the full result, $\sigma_\chi(\omega) \to \frac{2}{3} \cdot \frac{\mu}{4\pi^2}$**

A Puzzle in Free Fermion Computations
(Kharzeev-Warringa)

One loop computation with free fermion





(Kharzeev-Warringa)

There is a sudden drop in the real part σ'_χ to the value $\frac{1}{3}$ of the full CME for any $\omega > 0$. This behavior is characterized by a function

$$\frac{\omega}{\omega + i\epsilon}$$

which is 0 at $\omega = 0$, but is 1 for any $\omega > 0$.

This means that we have missed an **additional out-of-equilibrium contribution** to CME that behaves

- For $\omega \to 0$, this additional contribution should vanish, that is, it should not change the equilibrium value
- For $\omega \to \infty$, this contribution should become $-\frac{1}{3}$ of the full result

Where do we find this additional contribution ?

Recall that we used the current

$$\vec{J} = \int \frac{d^3\vec{p}}{(2\pi)^3}\ \dot{\vec{x}}\ \left(f_+^{\rm eq} - f_-^{\rm eq}\right)$$

There is a gradient correction to it, arising from the magnetization current

$$\vec{J}^{(1)} = \vec{\nabla} \times \vec{M}\,, \quad \vec{M} = \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}}{2|\vec{p}|^2} \left(f_+ + f_-\right) \right)$$

This correction also contributes to CME at finite frequency $\omega \neq 0$!

(Kharzeev-Stephanov-HUY, Phys.Rev. D95 (2017) no.5, 051901)

The key is the Bianchi identity

$$\frac{d\vec{B}}{dt} + \vec{\nabla} \times \vec{E} = 0\,, \quad -i\omega\vec{B} + \vec{\nabla} \times \vec{E} = 0$$

Recall the equation of motion $\dot{\vec{P}} = \pm\vec{E}$, and the kinetic equation

$$\frac{\partial f_\pm}{\partial t} + \dot{\vec{p}} \cdot \frac{\partial f_\pm}{\partial \vec{p}} = -i\omega f_\pm \pm \vec{E} \cdot \frac{\partial f_\pm}{\partial \vec{p}} = 0$$

whose solution is

$$f_\pm = f^{(0)} + f^{(1)}\,, \quad f^{(1)} = \mp i \frac{\vec{E} \cdot \hat{\boldsymbol{p}}}{(\omega + i\epsilon)} \frac{\partial f^{(0)}}{\partial |\boldsymbol{p}|}$$

Inserting this solution into the gradient correction $\vec{J}^{(1)}$

$$\vec{J}^{(1)} = \frac{i}{\omega + i\epsilon}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left(\frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}\times\vec{\nabla}(\vec{E}\cdot\hat{\boldsymbol{p}})\left(\frac{\partial f_+^{(0)}}{\partial|\boldsymbol{p}|}-\frac{\partial f_-^{(0)}}{\partial|\boldsymbol{p}|}\right)\right)$$

$$= -\frac{i}{6(\omega+i\epsilon)}(\vec{\nabla}\times\vec{E})\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{|\boldsymbol{p}|}\left(\frac{\partial f_+^{(0)}}{\partial|\boldsymbol{p}|}-\frac{\partial f_-^{(0)}}{\partial|\boldsymbol{p}|}\right)$$

$$= \frac{\omega}{6(\omega+i\epsilon)}\vec{B}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{|\boldsymbol{p}|}\left(\frac{\partial f_+^{(0)}}{\partial|\boldsymbol{p}|}-\frac{\partial f_-^{(0)}}{\partial|\boldsymbol{p}|}\right)$$

$$= \frac{\mu}{4\pi^2}\left(-\frac{1}{3}\frac{\omega}{\omega+i\epsilon}\right)\vec{B}$$

This behaves precisely as what we need:

It goes to 0 in $\omega \to 0$ and $-\frac{1}{3}$ in $\omega \to \infty$

Summary of CME in Equilibrium and Out-of-Equilibrium at Weak Coupling

Three contributions:

1) Equilibrium Distribution
2) Anomalous Velocity
3) Magnetization Current

	$\omega \ll \tau_R^{-1}$	$\omega \gg \tau_R^{-1}$
1)	$\frac{1}{3}$	0
2)	$\frac{2}{3}$	$\frac{2}{3}$
3)	0	$-\frac{1}{3}$
Total	1	$\frac{1}{3}$

Note that this applies to only the kinetic regime $\omega \ll T$ where T is the typical momentum of dominant charge carriers

Towards a complete picture in perturbative QCD

With a finite relaxation time τ_R, we have a good approximate formula in the kinetic regime $\omega \ll T$

$$\sigma_\chi(\omega) = \sigma_0 \left(1 - \frac{2}{3}\frac{\omega}{\omega + i\tau_R^{-1}}\right)$$

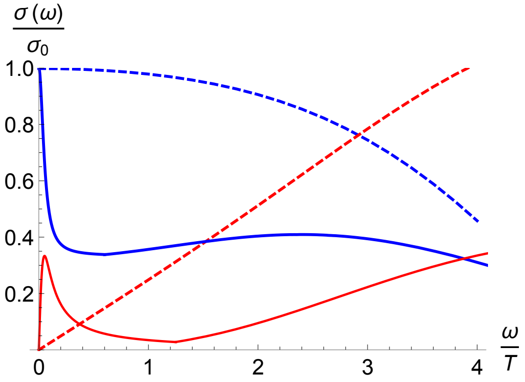
Two-flavor perturbative QCD in leading log gives

(Jimenez-Alba, HUY)

$$\tau_R^{-1} \sim 1.3\alpha_s^2 \log(1/\alpha_s) T$$

In the quantum regime $\omega \gg T$, a free 1-loop diagram suffices in leading order

A complete picture of $\sigma_\chi(\omega)$ in leading order of perturbative QCD



$\alpha_s = 0.2$, μ_A/T**=0.1**

Blue (Red) curves are real (imaginary) part of $\sigma_\chi(\omega)$
Dotted curves are from the AdS/CFT

Dirac/Weyl semi-metal systems

The g-factor can be significantly different from the relativistic value $g = 2$: Bi_2Se_3 has $g = 20 - 30$

The energetic contribution and the magnetization current should be proportional to the magnetic moment

$$\boldsymbol{J}^E = \frac{g}{6}\left(1 - \frac{\omega}{\omega + i\tau_R^{-1}}\right)\sigma_0 \boldsymbol{B}\,, \quad \boldsymbol{J}^M = -\frac{g}{6}\frac{\omega}{\omega + i\tau_R^{-1}}\sigma_0 \boldsymbol{B}$$

The kinematic contribution is independent of ω and is fixed to reproduce the total equilibrium value σ_0

$$\boldsymbol{J}^{KM} = \left(1 - \frac{g}{6}\right)\sigma_0 \boldsymbol{B}$$

The total current is

$$\boldsymbol{J} = \left(1 - \frac{g}{3}\frac{\omega}{\omega + i\tau_R^{-1}}\right)\sigma_0 \boldsymbol{B}$$

Thank you!