Thermodynamics of baryon rich hadronic matter

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Hadron resonance gas (HRG) model can describe the QCD EoS as well as the 2nd order fluctuations of conserved charge reasonably well at zero net baryon density

How about higher order fluctuations and EoS at non-zero baryon density ?
How important are the repulsive baryon-baryon interactions ?

In this talk:

1) The virial expansion in the nucleon gas

2) HRG with with repulsive mean field

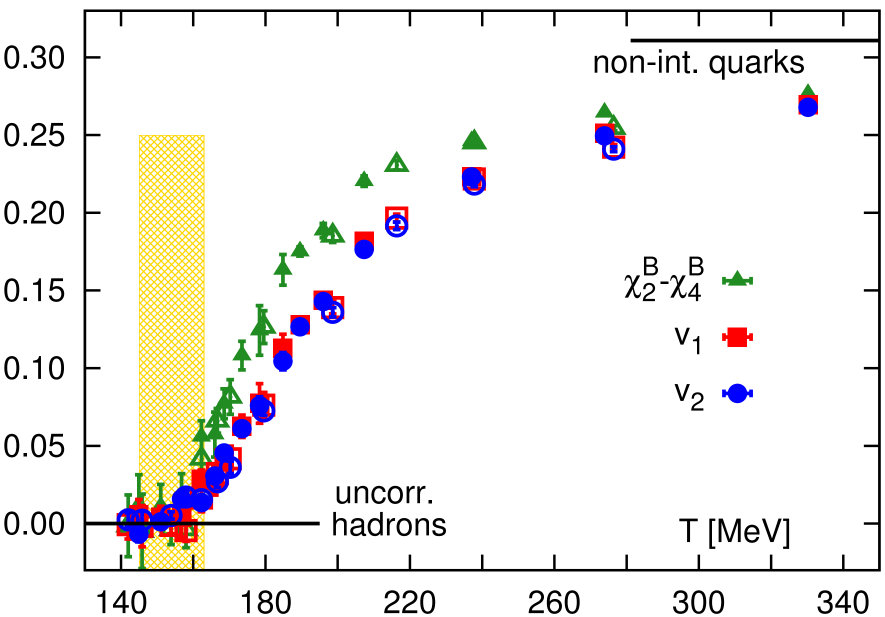
in collaboration with P. Huovinen, arXiv:1708.00879, work in progress

CPOD 2017, Stony Brook University, August , 7-11, 2017

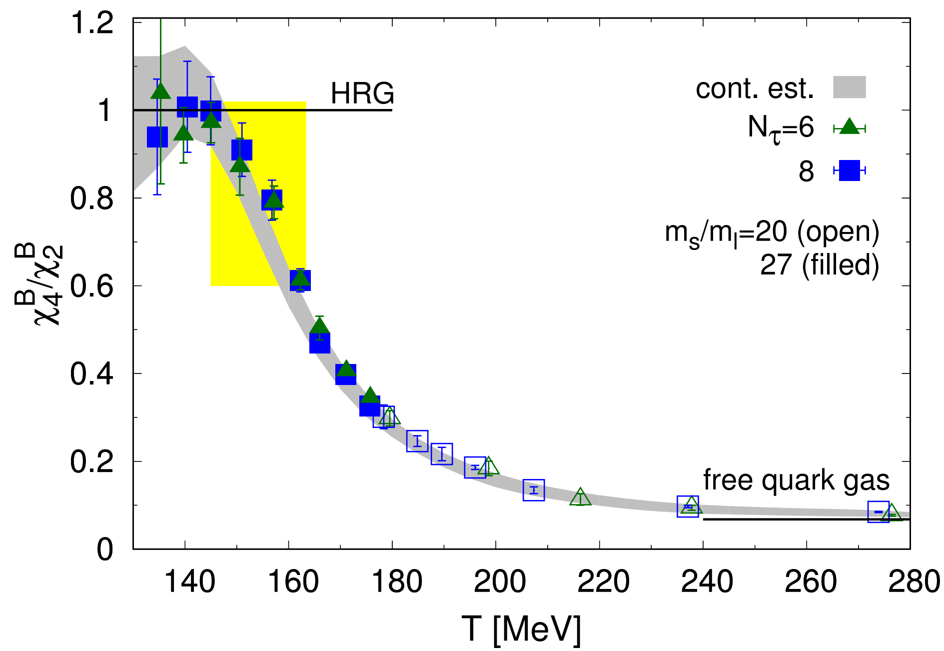
Higher order fluctuations of conserved charges in T>0 QCD

$$\chi_n^X = T^n \frac{\partial^n (p(T,\mu_X)/T^4)}{\partial \mu_X^n}\Bigg|_{\mu_X=0}, \quad \chi_{nm}^{XY} = T^{n+m} \frac{\partial^{n+m} (p(T,\mu_X,\mu_Y)/T^4)}{\partial \mu_X^n \partial \mu_Y^m}\Bigg|_{\mu_X=0,\mu_Y=0}.$$

Bazavov et al, PRL 111 (2013) 082301



Bazavov et al, PRD 95 (2017)054504



$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

see talks by Schmidt and Gunter

The above combinations should be 0 or 1 in HRG independent of details of hadron spectrum and HRG description breaks down close to the transition temperature (even below T_c)

Virial expansion in the nucleon gas

$$p = p^{ideal} + T \sum_{ij} b_2^{ij}(T) e^{\beta\mu_i} e^{\beta\mu_j}$$

b_2^{ij} can be related to the S-matrix of scattering of particles i and j

$\pi\pi$, KK, πN and NK scattering are dominated by resonances:

$$p \to p^{ideal}_{\pi,K,N} + p^{ideal}_{resonances}$$

No resonances in NN interactions

HRG model

Dashen, Ma, Berstein, PR 187 (1969) 345
Prakash, Venugopalan, NPA 546 (1992) 718

Gas of nucleons:

$$p(T,\mu) = p_0(T)\cosh(\beta\mu) + 2b_2(T) T \cosh(2\beta\mu)$$

$$p_0(T) = \frac{4M^2T^2}{\pi^2} K_2(\beta M)$$

$$b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE (\frac{ME}{2} + M^2) K_2\left(2\beta\sqrt{\frac{ME}{2} + M^2}\right) \frac{1}{4i} \mathrm{Tr}\left[S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S\right],$$

factorization in μ and T dependent part is broken

Virial expansion in the nucleon gas (cont'd)

Only the elastic part of the S-matrix is known

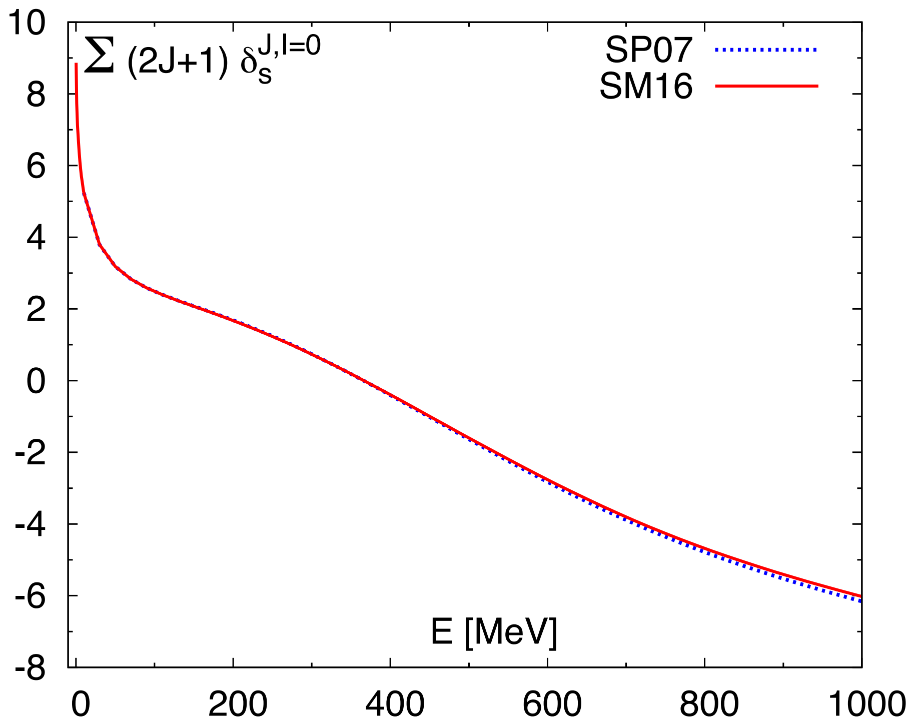
$$\frac{1}{4i}\mathrm{Tr}\left[S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE}S\right] \to \sum_{s=\pm}\sum_J (2J+1)\left(\frac{d\delta_s^{J,I=0}}{dE} + 3\frac{d\delta_s^{J,I=1}}{dE}\right)$$

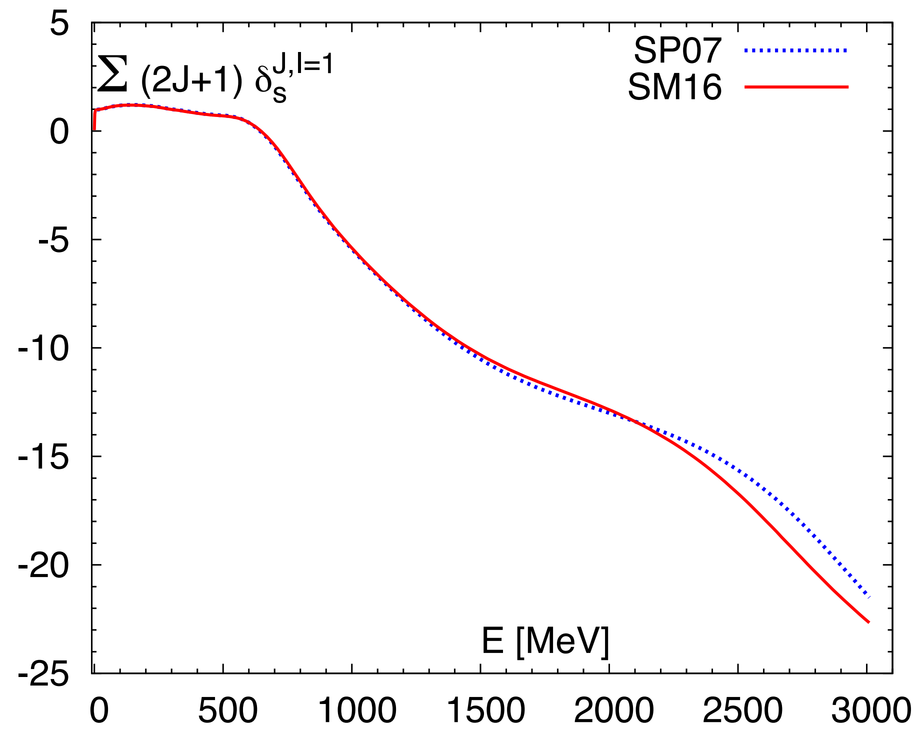
Use recent partial wave analysis results for NN scattering (SM16, SP07)

Use effective range expansion for $E < 1$ MeV

Workman et al, PRC 94 (2016) 065203

Arndt et al, PRC 76 (2007) 025209





$$\frac{d}{dE}\sum_{J,s}(2J+1)\delta_s^{J,I} \Rightarrow b_2(T) < 0$$

Repulsive mean field in the nucleon gas

Assume that the repulsive interactions reduce the single nucleon energies by $U = Kn_b$, where n_b is the single nucleon density

$$K \sim \int d^3 r V_{NN}(r) \Rightarrow K > 0$$

Nucleon and anti-nucleon densities

Olive, NPB 190 (1981) 483

$$n_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}, \quad \bar{n}_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p + \mu + \bar{U})}, \ E_p^2 = p^2 + M^2$$

$$\partial p / \partial \mu = n_b - \bar{n}_b \Rightarrow p(T,\mu) = T(n_b + \bar{n}_b) + \frac{K}{2}(n_b^2 + \bar{n}_b^2)$$

Small (zero) $\mu \Rightarrow \beta K n_b \ll 1$ and

$$n_b \simeq n_b^0(1 - \beta K n_b^0), \ \bar{n}_b \simeq \bar{n}_b^0(1 - \beta K \bar{n}_b^0) \Rightarrow$$

$$p(T,\mu) = T(n_b^0 + \bar{n}_b^0) - \frac{K}{2}\left(\left(n_b^0\right)^2 + \left(\bar{n}_b^0\right)^2\right)$$

or

$$p(T,\mu) = p_0(T)(\cosh(\beta\mu) - \frac{KM^2}{\pi^2} K_2(\beta M) \cosh(2\beta\mu))$$

Comparison of repulsive mean field and virial expansion

Repulsive mean field

$$p(T,\mu) = p_0(T) \times$$

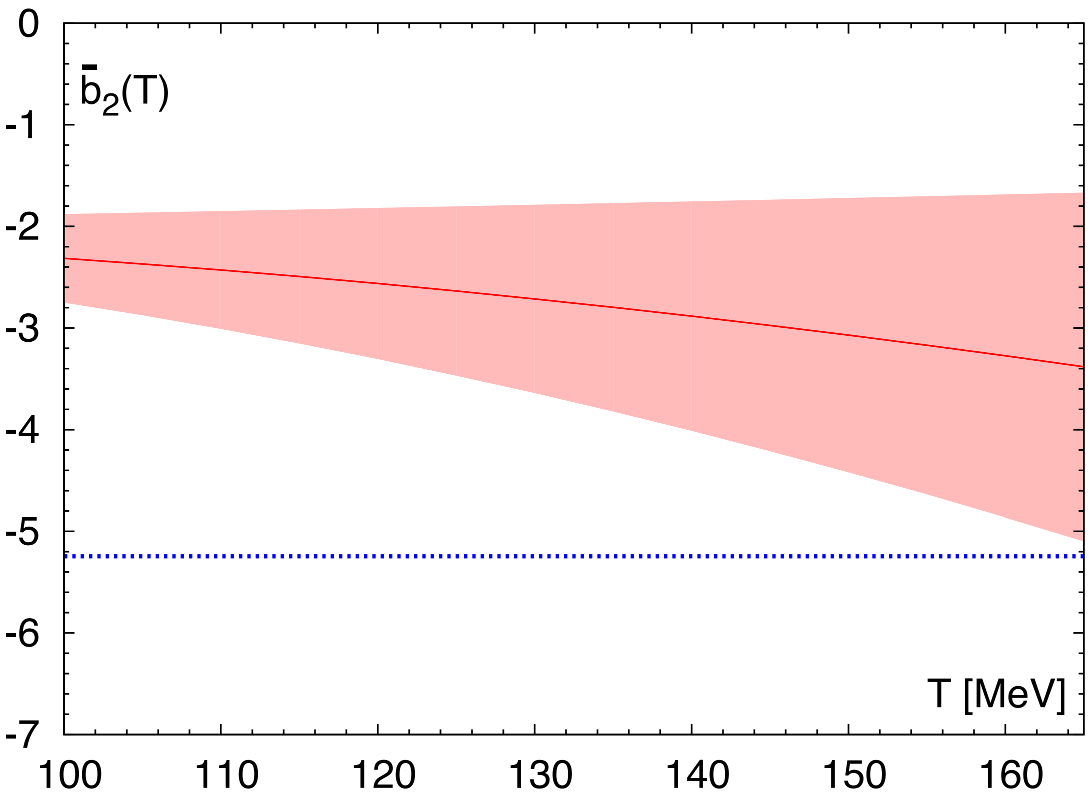
$$(\cosh(\beta\mu) - \frac{KM^2}{\pi^2} K_2(\beta M) \cosh(2\beta\mu))$$

2nd order virial expansion

$$p(T,\mu) = p_0(T) \times$$

$$(\cosh(\beta\mu) + \bar{b}_2(T) K_2(\beta M) \cosh(2\beta\mu))$$

$$\bar{b}_2(T) = \frac{2T b_2(T)}{p_0(T) K_2(\beta M)}$$



In-elastic interactions become important for $E > 400$ MeV
$\Rightarrow$ use σ_{el}/σ_{tot}
to estimate the uncertainties in $b_2(T)$ due to these effects

$$-\frac{KM^2}{\pi^2}$$

for typical phenomenological value $K = 450$ MeV fm^3

Sollfrank et al, PRC 55 (1997) 392

Hadron resonance gas with repulsive mean field

$$n_B(T,\mu_B,\mu_S,\mu_Q) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta \mu_{i,eff}}, \qquad \mu_{i,eff} = \sum_j q_i^j \mu_j - K n_B$$

$$\bar{n}_B(T,\mu_B,\mu_S,\mu_Q) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i) e^{\beta \bar{\mu}_{i,eff}}, \qquad \bar{\mu}_{i,eff} = -\sum_j q_i^j \mu_j - K \bar{n}_B$$

$$(q_i^1, q_i^2, q_i^3) = (B_i, S_i, Q_i)$$

strange and non-strange baryons interact the same way

$\partial p/\partial \mu_B = n_B - \bar{n}_B$ and leading order expansion in $\beta K n_B \Rightarrow$

$$p_B(T,\mu_B,\mu_S,\mu_Q) = T(n_B^0 + \bar{n}_B^0) - \frac{K}{2}\left(\left(n_B^0\right)^2 + \left(\bar{n}_B^0\right)^2\right)$$

$$\Downarrow$$

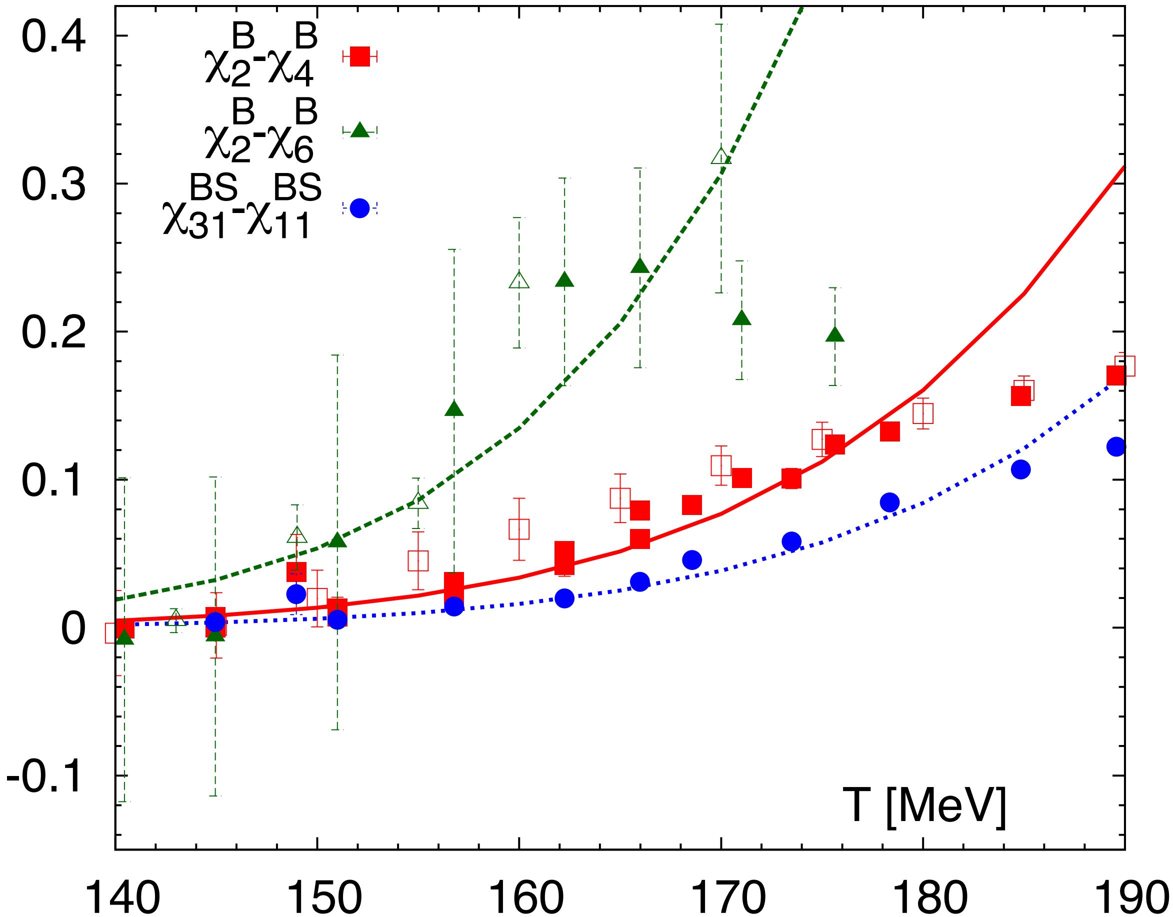
$$\chi_n^B = \chi_n^{B(0)} - 2^n \beta^4 K \left(N_B^0\right)^2, \qquad (n \text{ even})$$

$$\chi_{n1}^{BS} = \chi_n^{BS(0)} + 2^{n+1}\beta^5 K N_B^0 (p_B^{S1} + 2p_B^{S2} + 3p_B^{S3}) \qquad (n \text{ odd})$$

$$N_B^0(T) = \frac{T}{2\pi^2}\sum_i g_i M_i^2 K_2(\beta M_i)$$

Comparison with lattice QCD results

Assume that only ground state baryons (octet + decuplet) contribute to n_B
higher resonances are treated as free particles



Filled symbols: HISQ

Bazavov et al,
PRL 111 (2013) 082301,
PRD 95 (2017)054504

Open symbols: stout

4th order

Bellwied et al
PRD 92 (2015) 114505

6th order

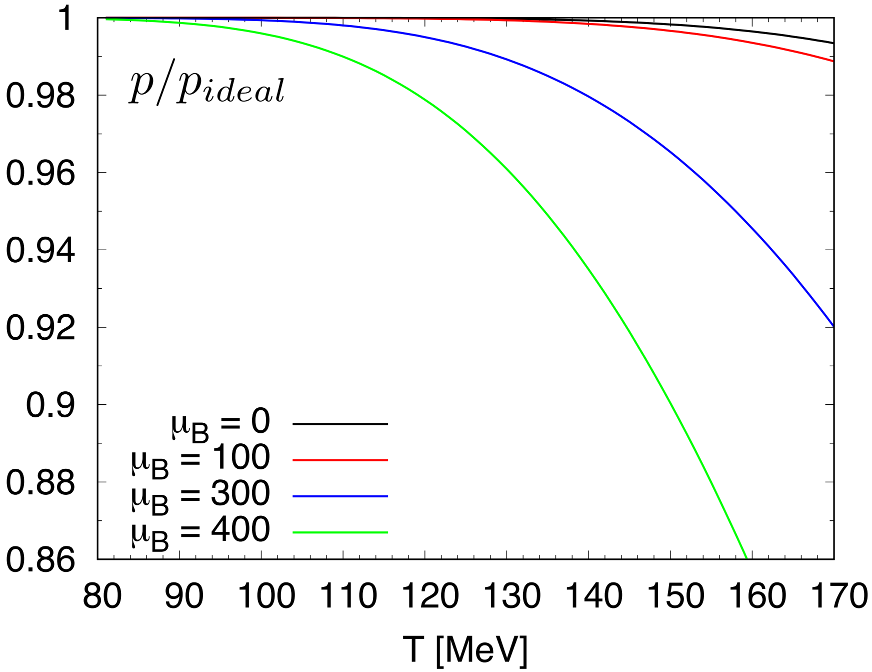
D'Elia et al,
PRD 95 (2017) 094503

Repulsive mean field calculations can explain the differences between certain higher order fluctuations and correlations; v_2 is not described by this simple model

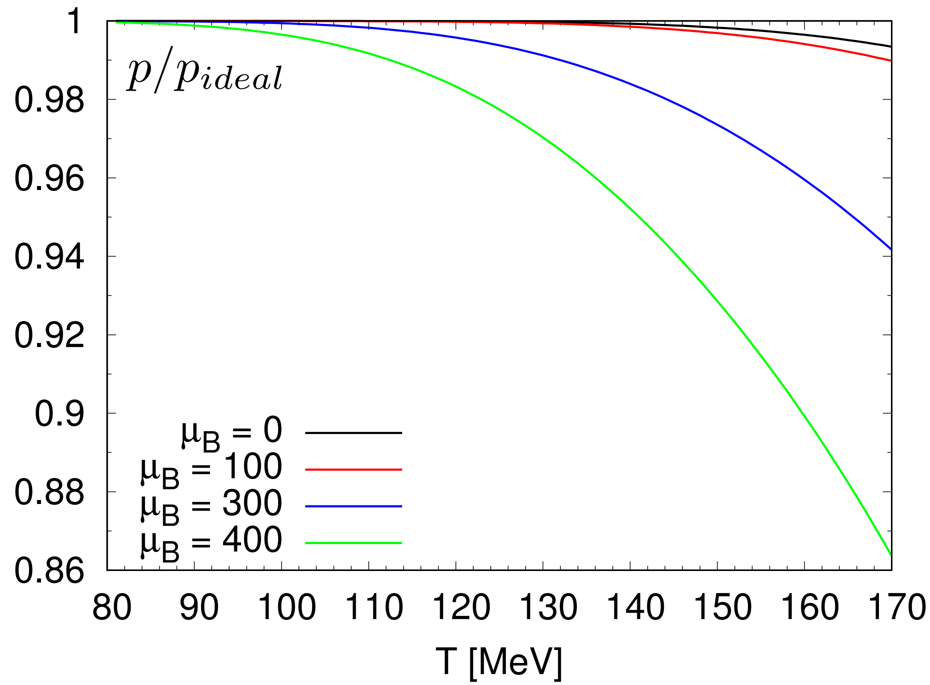
Pressure with repulsive mean field

Use expanded expressions (in $K\, n_B$) to calculate the pressure

$\mu_S = 0$



$n_S = 0$

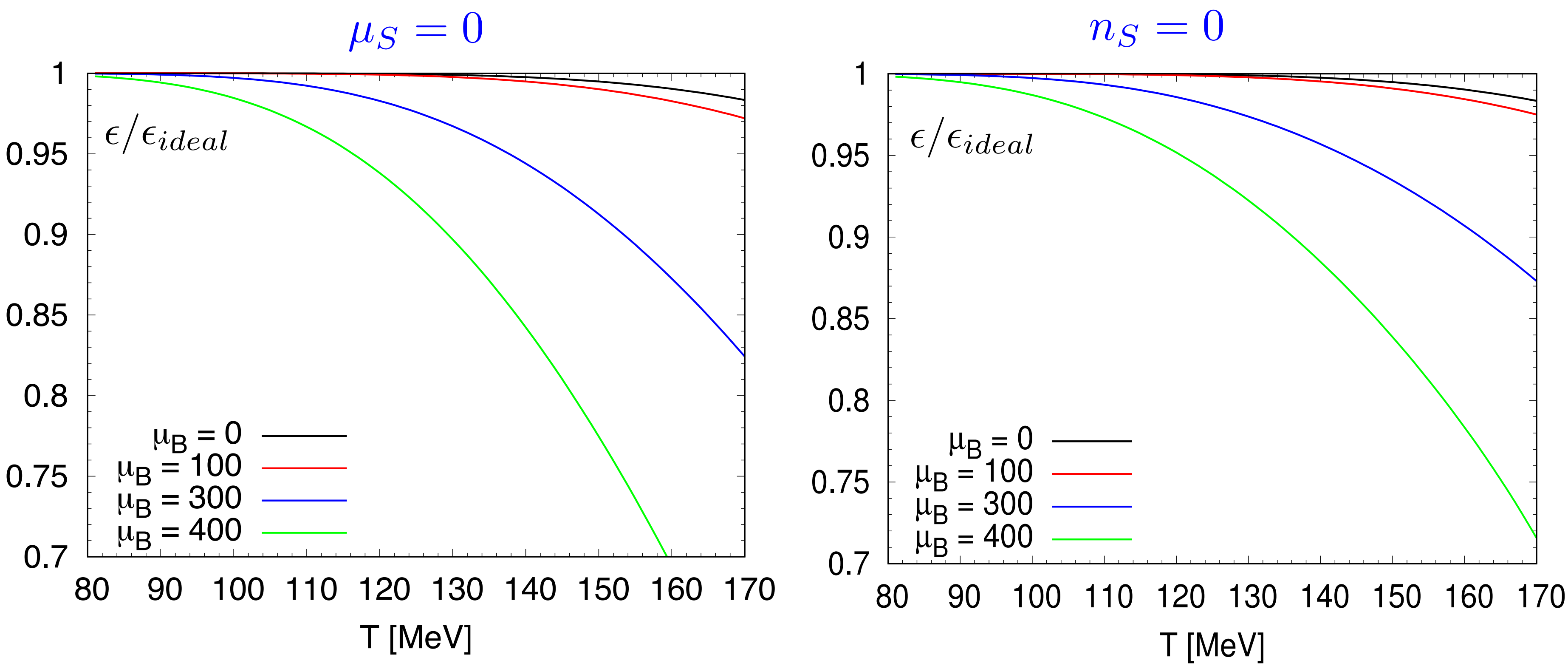


Virial expansion works only for baryon chemical potential < *400* MeV

The repulsive mean field reduces the pressure up 24%

For the strangeness neutral case the effects of the repulsive interactions are smaller.

Energy density with repulsive mean field

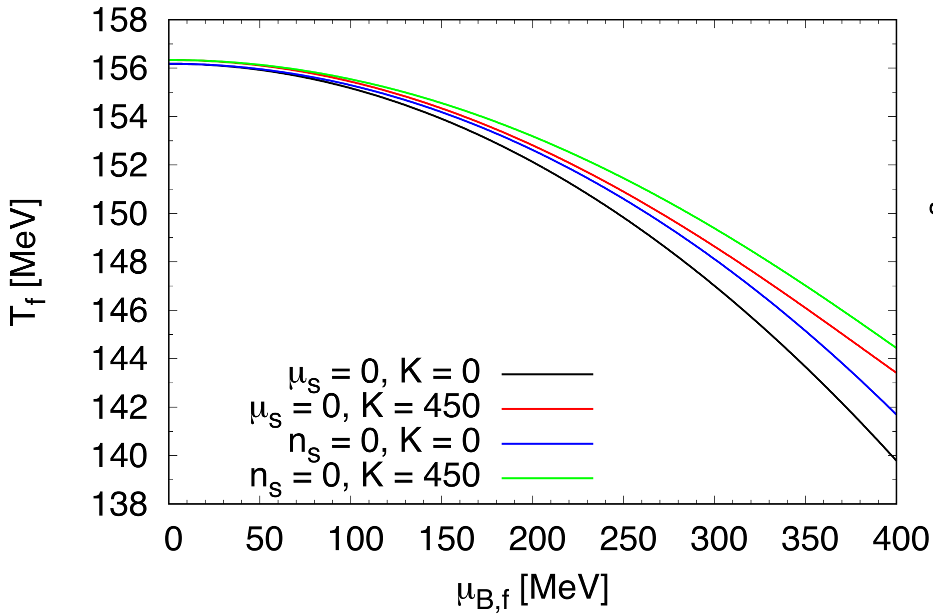


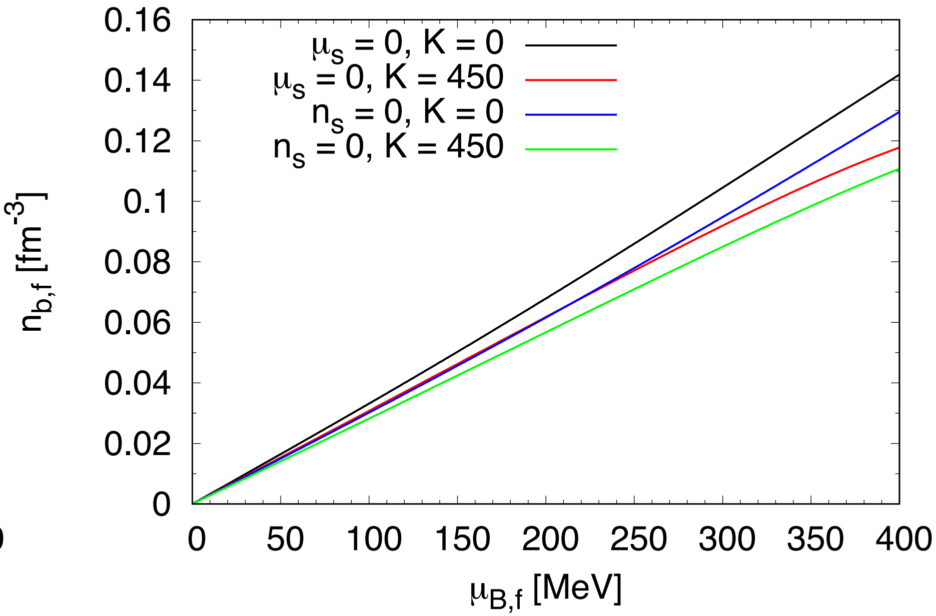
Repulsive mean field reduces the energy density up to 30%

For the strangeness neutral case the effects of the repulsive interactions are smaller.

Freezout at constant energy density

Assume that freeze-out happens at energy density of 330 MeV/fm^3

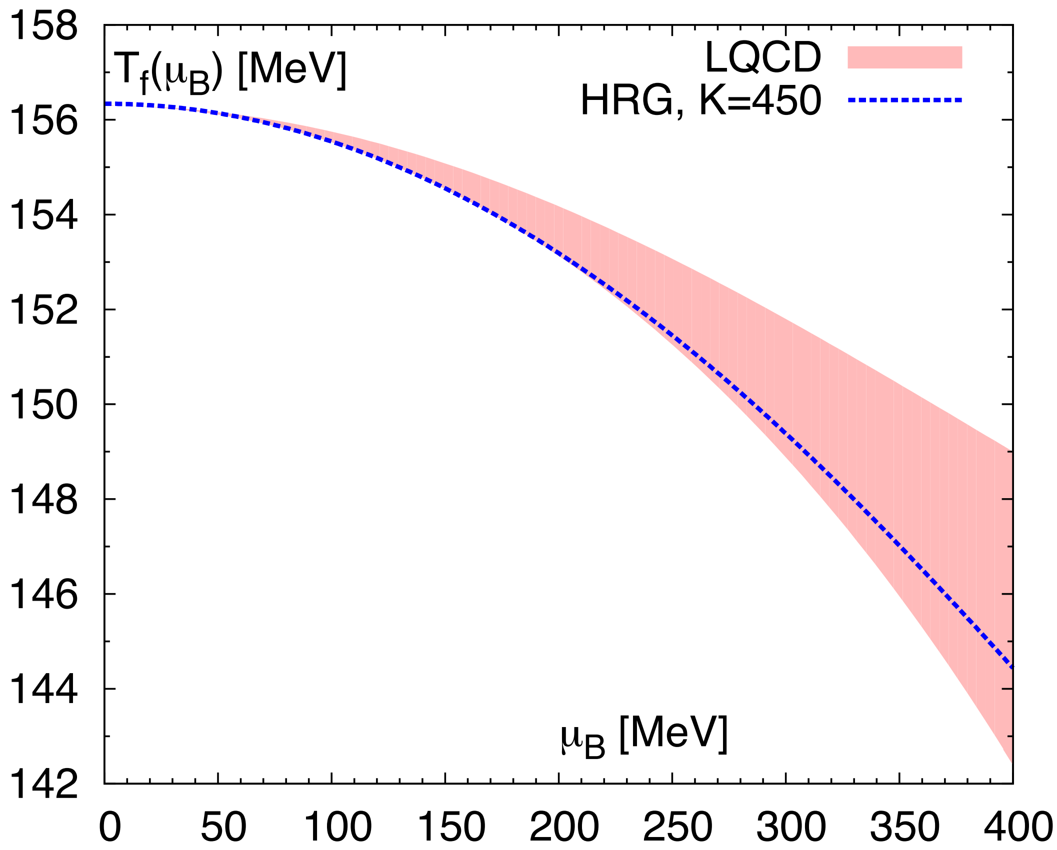




Strangeness neutrality and repulsive interaction reduce the curvature of the freeze-out temperature

Strangeness neutrality and repulsive interactions reduce the net baryon density at the freeze-out

Freeze-out line in HRG vs. lattice



The curvature of the freeze-out line corresponding to constant energy density ~330 MeV/fm^3 calculated in HRG model with repulsive interactions agrees with lattice result of Bazavov et al, PRD 95 (2017)054504

Summary

- Repulsive baryon-baryon interactions are important for higher order baryon number fluctuations and baryon strangeness correlations and for EoS at non-zero baryon density (already seen in the studies using excluded volume approach, see the talk by Vovchenko)

- Mean field approach is very similar to the virial expansion in the low density regime => constraints on the mean field values

- The simplest mean field approach can describe the differences between second and higher order baryon number fluctuations as well as baryon strangeness correlations, but certain baryon-strangeness correlations cannot be described by this simple model

- The virial expansion is applicable for baryon chemical potential < *400* MeV

- Future: virial expansion for strange baryons, missing baryons, going beyond the leading order density expansion