Characterizing hydrodynamic fluctuations in heavy-ion collisions from effective field theory approach

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(Collaboration with Hong Liu and Yi Yin)

Effective action of hydrodynamics - Motivations

- Hydrodynamics formulated mainly in terms of conservation equations
  - Conservation of energy $\nabla_\mu T^{\mu\nu} = 0$
  - Conservation of currents $\nabla_\mu J^\mu = 0$
- But noises have to be incorporated phenomenologically
- Nonlinear interactions and non-equilibrium systems can be treated systematically in this framework
- One can also apply field theory technique (loop corrections etc) if an action is available

P. Kovtun, G. Moore, P. Romatschke JHEP 07 (2014) 123

Alternative approach to hydrodynamics - Effective action

- Hydrodynamics captures the low energy behaviour of a many body system

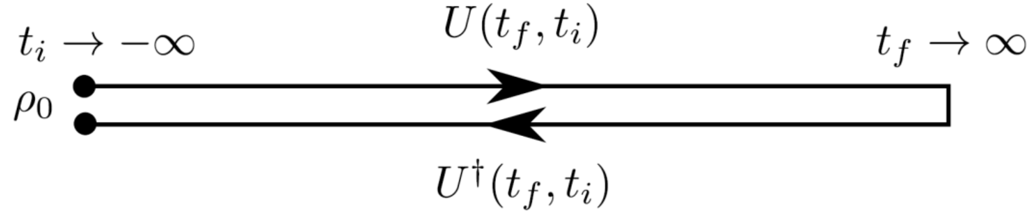
- An effective theory for a system in a macroscopic state

$$Tr(\rho_0 \cdots\cdots) = \int D\psi_1\, D\psi_2\, (\cdots\cdots)\, e^{iS[\psi_1]-iS[\psi_2]}$$

- Starting point : A microscopic action S_{micro}
- Real time evolution of the density matrix and expectation values of the physical quantities : Schwinger-Keldysh formalism (Closed Time Path, CTP)

M.Crossley, P. Glorioso, H. Liu arXiv:1511.03646
arXiv:1701.07817

Quick review of CTP



- In a non-equilibrium system : the final state is not known.
- Use this CTP contour to avoid this problem
- The degrees of freedom are doubled $\psi \to (\psi_1, \psi_2)$
- Insert operators along the contour to compute correlation functions
  - A generating functional $e^{W[\phi_1,\phi_2]} = Tr(\rho_0 \mathcal{P}\, e^{i\int dt\, (\mathcal{O}_1(t)\phi_1(t) - \mathcal{O}_2(t)\phi_2(t))})$

J. S. Schwinger J. Math. Phys. 2 (1961) 407.
L. V. Keldysh Sov. Phys. JETP 20 (1965) 1018

Properties of CTP

$$e^{W[\phi_1,\phi_2]} = Tr(\rho_0 \mathcal{P}\, e^{i\int dt\, (\mathcal{O}_1(t)\phi_1(t) - \mathcal{O}_2(t)\phi_2(t))})$$

- $W = 0$ when $\phi_1 = \phi_2$
- Another convenient basis :
  - $\phi_r = \frac{1}{2}(\phi_1 + \phi_2)$ average field and $\phi_a = (\phi_1 - \phi_2)$ is the noise.
- In this basis, the two point functions are

$$e^{W[\phi_r,\phi_a]} = Tr(\rho_0 \mathcal{P}\, e^{i\int dt\, (\mathcal{O}_a(t)\phi_r(t) + \mathcal{O}_r(t)\phi_a(t))})$$

$$G_{ra}(t_1,t_2) = G_R(t_1,t_2) \qquad G_{ar}(t_1,t_2) = G_A(t_1,t_2) \qquad G_{rr}(t_1,t_2) = G_S(t_1,t_2)$$

- Given that $\rho_0 = \frac{1}{Z_0} e^{-\beta_0(H-\mu_0 Q)}$, W satisfies the Kubo-Martin-Schwinger condition. This leads to the Fluctuation-dissipation theorem

$$G_S = \frac{i}{2}\coth\frac{\beta_0\omega}{2}(G_A - G_R)$$

Effective action

$$Tr(\rho_0 \cdots\cdots) = \int D\psi_1 \, D\psi_2 \, (\cdots\cdots) \, e^{iS[\psi_1]-iS[\psi_2]}$$

- $\psi_{1,2}$ contains
  - long-lived gapless (hydrodynamical modes)
  - short-lived gapless modes and gapped modes (Integrate out)
- The effective action can then be written as

$$Tr(\rho_0 \cdots\cdots) = \int D\chi_1 \, D\chi_2 \, (\cdots\cdots) \, e^{iS_{hydro}[\chi_1,\chi_2,\rho_0]}$$

- This cannot be done explicitly -> write down the effective action as a derivative expansion based on the symmetry expected in hydrodynamics.

Effective action from generating functions

- Conserved quantities -> generating functions -> correlation functions

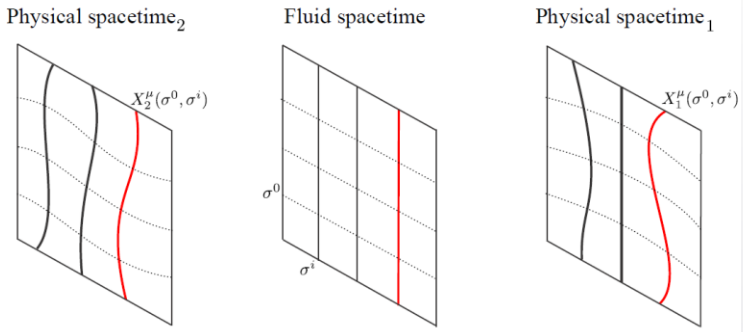
- Generating function:
  - integrating out the dynamical field of the field theory

$$e^{W[g_{1\mu\nu},g_{2\mu\nu}]} = Tr\left(\rho_0 \mathcal{P}\, e^{i\int d^d x\,(T_1^{\mu\nu} g_{1\mu\nu} - T_2^{\mu\nu} g_{2\mu\nu})}\right)$$

- In this formalism, the corresponding dynamical field which we have integrated out is X^μ

$$e^{W[g_{1\mu\nu},g_{2\mu\nu}]} \Rightarrow \int DX_{1\mu} DX_{2\mu} e^{iI[h_{1ab};h_{2ab}]} \qquad h_{iab} = \frac{\partial X_i^\mu}{\partial \sigma^a} g_{i\mu\nu} \frac{\partial X_i^\nu}{\partial \sigma^b}$$

Effective action - interpretation



- The dynamical fields $X_i^\mu(\sigma^a)$ = physical spacetime
- σ^a = fluid spacetime
- The σ^a coordinates : label the fluid elements and their individual time
- $X^\mu \Rightarrow$ coordinates of the corresponding fluid element labelled by σ^a.
- Expressing the action in terms of $X^\mu(\sigma^a)$ = Lagrange description
- $\sigma^a(X^\mu)$ corresponds to the Euler description.

Connection to hydrodynamics variables

- With the interpretation of $X_i^\mu(\sigma)$, we can then identify the standard hydrodynamics variables

$$u_i^\mu = \frac{1}{b_i}\frac{\partial X_i^\mu}{\partial \sigma^0}$$

- And in order to include temperature into the system, we introduce an extra scalar field $\phi(\sigma)$. The temperature is then

$$T(\sigma) = T_0 e^{-\phi(\sigma)}$$

Symmetries of the fluid spacetime

- With the fluid spacetime coordinates σ^a in mind, we expect they satisfy the following symmetries.
- Time-independent reparameterisations (relabelling fluid elements)

$$\sigma^i \to \sigma'^i(\sigma^i), \qquad \sigma^0 \to \sigma^0$$

- Time-diffeomorphisms (each fluid element has its own internal clock)

$$\sigma^0 \to \sigma'^0(\sigma^0, \sigma^i), \qquad \sigma^i \to \sigma^i$$

Effective action as an expansion

- Symmetries -> write down action order by order in the noise (X_a) and derivative $\left(\partial_{\sigma^0}, \partial_{\sigma^i}\right)$

$$S = \int d^d\sigma \left|\det \frac{\partial X}{\partial \sigma}\right| \sqrt{-g} \left(\mathcal{L}^{(1,0)}_{ideal} + \left(\mathcal{L}^{(1,1)} + \mathcal{L}^{(2,0)}\right) + \cdots\cdots \right)$$

- (n, m) represents the number of of a-field and number of derivative in the corresponding part of the lagrangian

- The terms are related by KMS condition

Bjorken flow

- In heavy ion collisions, the flow of QGP can be described by the Bjorken flow
- It is convenient to consider the flow in a Boost invariant coordinate given by (τ, η, x, y) where the proper time $\tau = \sqrt{t^2 - z^2}$ and rapidity $\eta = \arctan\left(\frac{z}{t}\right)$
- Then the velocity field is $u^\mu = (1,0,0,0)$ with $u^\mu u_\mu = -1$ and the metric is $g_{\mu\nu} = \mathrm{diag}\,(-1, \tau^2, 1,1)$

Effective action for Bjorken flow up to second order expansions

- The Lagrangian in Landau frame is

$$\mathcal{L} = \frac{1}{2}\left(\frac{\epsilon}{b^2}\dot{X}_r{}^{\mu}\dot{X}_r{}^{\nu} + p\Delta^{\mu\nu} - \eta_{vis}\beta^{-1}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} - \frac{\Delta^{\mu\nu}\Delta^{\alpha\beta}}{3}\right)\left(\nabla_\alpha\beta_\beta + \nabla_\beta\beta_\alpha\right)\right)\mathrm{G}_{\mathrm{a}\mu\nu}$$

$$+\frac{i\eta_{vis}}{2}\beta^{-1}\Delta^{\mu\alpha}\Delta^{\nu\beta}\mathrm{G}_{a\mu\nu}G_{a\alpha\beta} + O(G_a^3)$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + \frac{1}{b^2}\dot{X}^{\mu}\dot{X}^{\nu} \qquad \mathrm{G}_{a\mu\nu} = g_{a\mu\nu} + \nabla_\mu X_{a\nu} + \nabla_\nu X_{a\mu} \qquad \beta_\mu = \beta u_\mu$$

- The standard equation of motion can be recovered by varying the action w.r.t the a-variables $X_{a\mu}$ and for Bjorken flow

$$\partial_\tau\epsilon + \frac{\epsilon + p}{\tau} = \frac{4\eta_{vis}}{3\tau}$$

Correction from fluctuation

- With the effective action, we can include the effect of fluctuation by expanding the dynamical fields about the classical solution

$$X^{\mu}_{r/a} = X^{\mu cl}_{r/a} + \delta X^{\mu}_{r/a}, \qquad \phi = \phi^{cl} + \delta\phi$$

- The two-point correlation functions of the fluctuation can then be read off from the action expanded to second order in fluctuation

$$\hat{D}\left(X^{cl}_{r}, \partial_{\mu}\right) G_{\delta X_{r/a} \delta X_{r/a}}(x-y) = \delta(x-y)$$

Compare to other approach

- Assuming $\frac{\omega \eta_{vis}}{(\epsilon+p)c_s^2} \ll 1$ and taking $\gamma_\eta = \frac{\eta_{vis}}{\epsilon+p}$ to be a constant, we reproduce the same equations for the perturbation of the stress-tensor

$$0 = \left(\frac{\partial}{\partial\tau} + \frac{1+c_s^2}{\tau}\right)\delta e + i\vec{k}_\perp \cdot \vec{g}_\perp + i\kappa g^\eta + \xi^\tau, \quad (56a)$$

$$\vec{0}_\perp = \left(\frac{\partial}{\partial\tau} + \frac{1}{\tau}\right)\vec{g}_\perp + c_s^2 i\vec{k}_\perp \delta e + \gamma_\eta \left(k_\perp^2 + \frac{\kappa^2}{\tau^2}\right)\vec{g}_\perp + \frac{1}{3}\gamma_\eta \vec{k}_\perp \left(\vec{k}_\perp \cdot \vec{g}_\perp + \kappa g^\eta\right) + \vec{\xi}_\perp, \quad (56b)$$

$$0 = \left(\frac{\partial}{\partial\tau} + \frac{3}{\tau}\right) g^\eta + \frac{c_s^2 i\kappa}{\tau^2}\delta e + \gamma_\eta \left(k_\perp^2 + \frac{\kappa^2}{\tau^2}\right) g^\eta + \frac{1}{3\tau^2}\gamma_\eta \kappa \left(\vec{k}_\perp \cdot \vec{g}_\perp + \kappa g^\eta\right) + \xi^\eta. \quad (56c)$$

Y. Akamatsu, A. Mazeliauskas, D. Teaney Phys.Rev. C95 (2017) no.1, 014909

Background field method

- Technique in QFT -> 1PI effective action
- Instead of expanding about the classical solution, we consider an expansion about an arbitrary background X_0 and construct a 1PI effect action by integrating out the action of second order fluctuation
- At second order -> a Gaussian integral

$$e^{\Gamma[\mathrm{X}_0]} = e^{i\left(S[X_0]+\frac{1}{2}\mathrm{Tr}\left(\ln\left[\frac{\delta^2 \mathrm{S}[X_0]}{\delta\left(X(x)\right)\delta\left(X(y)\right)}\right]\right)\right)}$$

- "Noise-corrected" correlation function can be obtained by differentiating w.r.t. the background fields.

Background field method for Bjorken flow

- The fluctuations of the action at second order in $(\delta X_r, \delta X_a)$ take the following form in the CTP formalism

$$\begin{pmatrix}\delta X_r & \delta X_a\end{pmatrix}\begin{pmatrix}0 & D \\ D^\dagger & C\end{pmatrix}\begin{pmatrix}\delta X_r \\ \delta X_a\end{pmatrix}$$

- The Green functions are then given by the inverse of the matrix

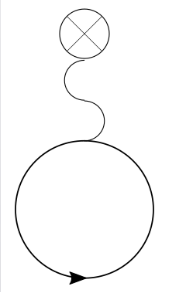
$$\begin{pmatrix}-\frac{C}{D^\dagger D} & \frac{1}{D^\dagger} \\ \frac{1}{D} & 0\end{pmatrix}$$

Bjorken flow

- The stress tensor receives loop correction in the following form

$$\partial_\tau \epsilon + \frac{\epsilon+p}{\tau} - \frac{4\eta_{vis}}{3\tau} = \text{loops}$$

$\delta T^{\mu\nu}_{1-loop}$=



Conclusion

- CTP formalism provides a good framework for effective field theory of hydrodynamics
- This formalism reproduces the conventional perturbation treatment of hydrodynamics
- With the effective action, we can apply field theory technique to capture noise corrections systematically
- We applied the background field method to include the full quantum correction to the simple Bjorken flow