Transport coefficients of QGP in Magnetic fields

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Outline

Heavy Quark Diffusion in Strong Magnetic Fields at Weak Coupling and Implications for Elliptic Flow
K. Fukushima, K. hattori, H. U. Yee, Y. Yin, Phys.Rev. D93 (2016) no.7, 074028
Jet quenching parameter of the quark-gluon plasma in a strong magnetic field: Perturbative QCD and AdS/CFT correspondence
S. Li, K. A. Mamo, H. U. Yee, Phys.Rev. D94 (2016) no.8, 085016
Longitudinal Electric Conductivity in Strong Magnetic Field in Perturbative QCD.
K. Hattori, S. Li, D. Satow, H. U. Yee, Phys.Rev. D95 (2017) 076008.
K.Hattori, D. Satow, Phys.Rev. D94 (2016) no.11, 114032
Shear Viscosity of Quark-Gluon Plasma in Weak Magnetic Field in Perturbative QCD: Leading Log
S. Li, H. U. Yee, arXiv:1707.00795

1. Longitudinal Electric Conductivity in Strong Magnetic Field in Perturbative QCD

Introduction

Axial anomaly: $\partial_t n_A = \frac{e^2 N_c N_F}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B} - \frac{n_A}{\tau_R}$

The CME current: $\boldsymbol{J} = \frac{e^2 N_c N_F}{2\pi^2} \mu_A \boldsymbol{B} = \frac{e^2 N_c N_F}{2\pi^2 \chi} n_A \boldsymbol{B} = \frac{e^4 (N_c N_F)^2 B^2}{4\pi^4 \chi} tE$

For finite conductivity, we consider the relaxation dynamics of axial charge : finite quark mass or sphaleron dynamics.

Stationary solution: $n_A = \frac{e^2 N_c N_F}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B} \tau_R$

$$\boldsymbol{J} = \frac{e^4 (N_c N_F)^2 B^2}{4\pi^4 \chi} \tau_R \boldsymbol{E} \qquad \sigma_{zz} = \frac{e^4 (N_c N_F)^2 B^2}{4\pi^4 \chi} \tau_R$$

, where the inverse relaxation time goes as:

•Finite quark mass $\quad \frac{1}{\tau_{R,m}} \sim \frac{\alpha_s m_q^2}{T},$

$$\Downarrow \quad \alpha_s m_q^2 \gg \alpha_s^5 T^2$$

•Sphaleron dynamics: $\frac{1}{\tau_{R,s}} = \frac{(2N_F)^2 \Gamma_s}{2\chi T}, \qquad \Gamma_s \sim \alpha_s^5 log(\frac{1}{\alpha_s}) T^4$

Hierarchy of Energy Scale

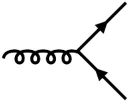
Assuming the hierarchy of energy scale $\alpha_s eB \ll (m_q^2, T^2) \ll eB$

A nice consistent Hard Thermal Loop (HTL) power counting scheme emerges: soft scale $m_D^2 \sim \alpha_s eB$ from HTL self-energy, hard scale $\sim T$, and UV cutoff $\Lambda_{UV}^2 \sim eB$

1.Focus on the LLL
2.Coupling with (1+1)-d quarks generates gluon Debye Screening mass.

Scattering process for finite conductivity

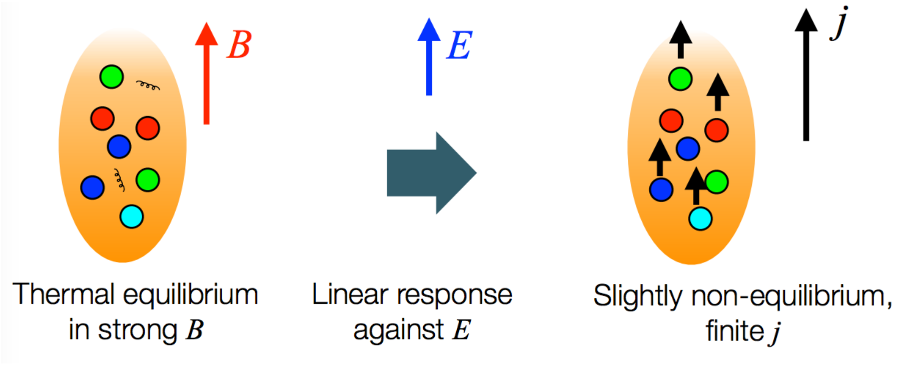
• $\mathbf{B} = \mathbf{0}$:
1 to 2 scattering is kinematically forbidden.



• **Strong B** :
Gluon is effectively massive in
(1+1)-d $\quad E = \sqrt{k_z^2 + k_\perp^2 + M_D^2} \sim \sqrt{k_z^2 + k_\perp^2}$

$\Rightarrow$ Decay of Gluon into quark-antiquark pair becomes kinematically possible.
$\longrightarrow$ 1-to-2 scattering ($\sim \alpha_s$)is leading process than 2-to-2 process($\sim \alpha_s^2$).

Calculation of the longitudinal Conductivity



$$n^f(p_z, \beta, Z) = n^f_E(\epsilon_{p_z})$$
$$\epsilon_{p_z} = \sqrt{p_z^2 + m^2}$$

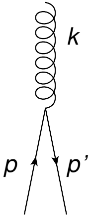
$$n^f(p_z, \beta, Z) = n^f_E(\epsilon_{p_z}) + \delta n^f(p_z, \beta, Z)$$

$$J_z = e_f(\frac{e_f B}{2\pi})2N_F \mathrm{dim}R \int \frac{dp_z}{(2\pi)} \mathbf{v_p} \delta n^f(p_z, \beta, Z). \quad \mathbf{v_p} = \frac{\mathbf{p_z}}{\epsilon_{p_z}}$$

Calculation of the Conductivity

The 1+1 Boltzman equation:

$$[\partial_t + \mathbf{v_p} \cdot \partial_\mathbf{x} + \mathbf{F_{ext}} \cdot \partial_\mathbf{p}] f(t, \mathbf{x}, \mathbf{p}) = C_{1->2}[f]$$



To first order, the Boltzman equation becomes:

$$[\partial_t + \mathbf{v_p} \cdot \partial_\mathbf{x} + \mathbf{F_{ext}} \cdot \partial_\mathbf{p}] f_0(t, \mathbf{x}, \mathbf{p}) = C_{1->2}[f_1](\mathbf{p}, \mathbf{x}, t)$$

Collision term in leading order:

$$C[f_+(p_z)] = \frac{1}{2E_p} \int |M|^2 [(1 - f_+(p_z))(1 - f_-(p_z'))f_g(k) - f_+(p_z)f_-(p_z')(1 + f_g(k)$$

$$|\mathcal{M}|^2_{colors} = 4C_2(f)g^2m^2$$

Vanishes at $\mathbf{m} = \mathbf{0}$ due to Chirality conservation!!!

Calculation of the finite conductivity

$[\partial_t + \mathbf{v_p} \cdot \partial_\mathbf{x} + \mathbf{F_{ext}} \cdot \partial_\mathbf{p}] f_0(t, \mathbf{x}, \mathbf{p}) = C_{1->2}[f_1](\mathbf{p}, \mathbf{x}, t)$

Linearizing :$f_\pm(p_z) = f_F^{eq}(E_p) + \beta f_F^{eq}(E_p)(1 - f_F^{eq}(E_p))\chi_\pm(p_z)$

Constant **E**: $\partial_t, \partial_z = 0$
Then we have:

$$-eE\frac{p_z}{E_p} f_F^{eq}(E_p)(1 - f_F^{eq}(E_p)) = \alpha_s C_2(R) m_q^2 \int dp'_z \frac{1}{E_p E_{p'}}$$

$$\times f_F^{eq}(E_p) f_F^{eq}(E_{p'})(1 + f_B^{eq}(E_p + E_{p'}))(\chi_+(p'_z) - \chi_+(p_z))$$

Quark Damping rate:
$\gamma_q = \frac{\alpha_s C_2(R) m_q^2}{E_p\left(1 - f_F^{eq}(E_p)\right)} \int dp'_z \frac{1}{E_{p'}} f_F^{eq}(E_{p'}) \left(1 + f_B^{eq}(E_p + E_{p'})\right)$

Solution:$\chi_+(p_z) = eE\frac{p_z}{E_p}\frac{1}{\gamma_q}$

Result

$$J_z = e(\frac{eB}{2\pi})2N_F \mathrm{dim}R \int \frac{dp_z}{(2\pi)}\, v_p\, \beta f_F^{eq}(E_p)(1-f_F^{eq}(E_p))eE\,\frac{p_z}{E_p}\frac{1}{\gamma_q}\,, \quad v_p = \frac{p_z}{E_p}\,.$$

The final expression for our longitudinal conductivity is then

$$\sigma_{zz} = \sum_f e_f^2 \frac{\mathrm{dim}R}{C_2(R)} \left(\frac{e_f B}{2\pi}\right) \frac{1}{\alpha_s T} \sigma_L(\bar{m}_f)$$

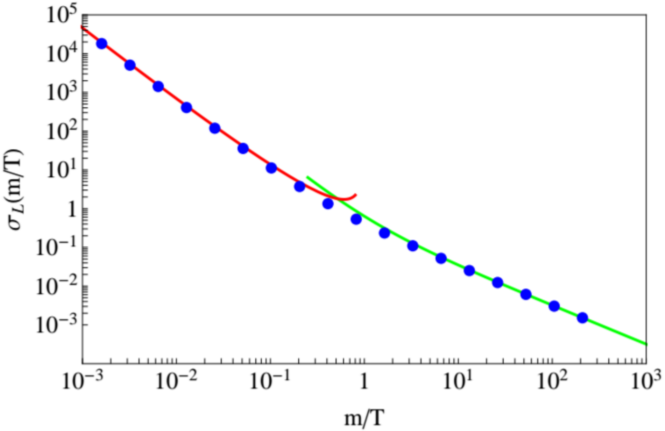
where:

$$\sigma_L(\bar{m}) = \frac{2}{\bar{m}^2} \int_0^\infty \frac{d\bar{p}}{(2\pi)} \frac{\bar{p}^2}{\epsilon_{\bar{p}}} \frac{n_F(\epsilon_{\bar{p}})\,(1-n_F(\epsilon_{\bar{p}}))^2}{\int_0^\infty \frac{d\bar{p}'}{\epsilon_{\bar{p}'}} n_F(\epsilon_{\bar{p}'})(1+n_B(\epsilon_{\bar{p}}+\epsilon_{\bar{p}'}))}$$

$\bar{m} = \frac{m}{T}$ and $\bar{p} = \frac{p}{T}$

- $\sigma_L(\bar{m}) \to \frac{1}{\pi \bar{m}^2 \log(1/\bar{m})}$, In the small $\bar{m} \to 0$ limit
- $\sigma_L(\bar{m}) \to \frac{1}{\pi \bar{m}}$ in the opposite limit ($\bar{m} \to \infty$)

Result



10^5
10^4
10^3
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
$\sigma_L(m/T)$
10^{-3}
10^{-2}
10^{-1}
1
10^1
10^2
10^3
m/T

2. Shear Viscosity of Quark-Gluon Plasma in Weak Magnetic Field in Perturbative QCD: Leading Log

Shear Viscosity in the external magnetic fields

•**Hydrodynamic degrees of freedom:** $T(x)$, $u^\mu(x)$.

•**Hydrodynamic variables in the presence of B:** $T(x)$, $u_{\|}^{\mu}(x)$.

The spatial component of the current is:$\boldsymbol{j} = \sigma \boldsymbol{v}_\perp \times \boldsymbol{B}$

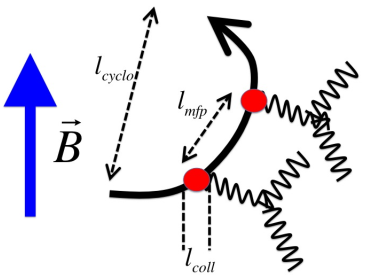
$\partial_t \boldsymbol{v}_\perp = \frac{1}{(\epsilon+p)} \boldsymbol{j} \times \boldsymbol{B} = -\frac{\sigma B^2}{(\epsilon+p)} \boldsymbol{v}_\perp \equiv -\frac{1}{\tau_R} \boldsymbol{v}_\perp$ **(quasi-normal modes)**

•**The energy-momentum tensor up to first order in gradient generally takes a form:**

$T^{\mu\nu} = (\epsilon + p_{\|}) u_{\|}^{\mu} u_{\|}^{\nu} + p_{\|} \eta_{\|}^{\mu\nu} + p_\perp \delta_\perp^{\mu\nu} - \eta(\partial_\perp^\mu u_{\|}^\nu + \partial_\perp^\nu u_{\|}^\mu) - (\zeta(u_{\|}^\mu u_{\|}^\nu + \eta_{\|}^{\mu\nu}) + \zeta' \delta_\perp^{\mu\nu})(\partial_{\|} \cdot u_{\|})$

Length scales

Assume the energy scales: $eB \sim g^4 \log(1/g) T^2 \ll T^2$



l_{coll}, satisfies $T^{-1} \ll l_{coll} \ll (gT)^{-1}$

$l_{cyclo} \sim p/(eB) \sim (g^4 \log(1/g) T)^{-1}$

$l_{mfp} \sim \mathcal{C}^{-1} \sim (g^4 \log(1/g) T)^{-1}$

$\frac{\partial f}{\partial t} + \hat{\boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = \mathcal{C}[f]$

$\dot{\boldsymbol{p}} = \pm q_F \hat{\boldsymbol{p}} \times (e\boldsymbol{B})$

$\bar{B} \equiv \frac{eB}{g^4 \log(1/g) T^2} \sim l_{mfp}/l_{cyclo}$

Computation of Shear Viscosity in B

Boltzmann equations:

$$\partial_t f^q + \hat{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}} f^q + q_F e(\hat{\boldsymbol{p}} \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{p}} f^q = \mathcal{C}^q$$

$$\partial_t f^{\bar{q}} + \hat{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}} f^{\bar{q}} - q_F e(\hat{\boldsymbol{p}} \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{p}} f^{\bar{q}} = \mathcal{C}^{\bar{q}}$$

$$\partial_t f^g + \hat{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}} f^g = \mathcal{C}^g$$

where $f = f_{eq}(\boldsymbol{p}, u_{\parallel}^{\mu}(x)) + \delta f(\boldsymbol{p})$,

$f_{eq}^{q,\bar{q},g}(\boldsymbol{p}, u_{\parallel}^{\mu}) = 1/(e^{-\frac{1}{T} p_\mu u_{\parallel}^{\mu}} \pm 1)\,, \quad p^\mu = (|\boldsymbol{p}|, \boldsymbol{p})$

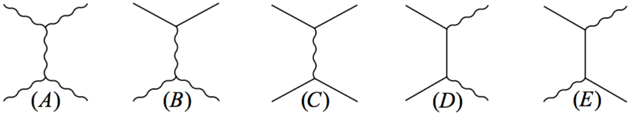
δf **satisfies the linearized Boltzmann equations:**

$$(\hat{\boldsymbol{p}}_\perp \cdot \partial_\perp v_z)\beta p_z n_F(\boldsymbol{p})(1 - n_F(\boldsymbol{p})) + q_F e(\hat{\boldsymbol{p}}_\perp \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{p}} \delta f^q = \mathcal{C}^q[\delta f^q, \delta f^{\bar{q}}, \delta f^g]$$

$$(\hat{\boldsymbol{p}}_\perp \cdot \partial_\perp v_z)\beta p_z n_F(\boldsymbol{p})(1 - n_F(\boldsymbol{p})) - q_F e(\hat{\boldsymbol{p}}_\perp \times \boldsymbol{B}) \cdot \partial_{\boldsymbol{p}} \delta f^{\bar{q}} = \mathcal{C}^{\bar{q}}[\delta f^q, \delta f^{\bar{q}}, \delta f^g]$$

$$(\hat{\boldsymbol{p}}_\perp \cdot \partial_\perp v_z)\beta p_z n_B(\boldsymbol{p})(1 + n_B(\boldsymbol{p})) = \mathcal{C}^g[\delta f^q, \delta f^{\bar{q}}, \delta f^g]$$

Collision terms



•$A - C$ **t-channel soft gluon exchanges process which give rise to momentum diffusion.**

•$D - E$ **t-channel soft fermion exchanges process which convert fermions into gluons and vice versa.**

•**All other diagrams do not contribute to the transport coefficients in the leading log.**

Solution

Writing δf **as**: $\delta f^a = n_{F/B}(1 \mp n_{F/B})\chi^a\,, \quad a = q, \bar{q}, g$

•**Ansatz:**

$$\chi^q(\boldsymbol{p}) = (\boldsymbol{p}_\perp \cdot \partial_\perp v_z)\, p_z\, \chi_+(|\boldsymbol{p}|) + (\boldsymbol{p}_\perp \times \partial_\perp v_z)\, p_z\, \chi_-(|\boldsymbol{p}|)$$

$$\chi^{\bar{q}}(\boldsymbol{p}) = (\boldsymbol{p}_\perp \cdot \partial_\perp v_z)\, p_z\, \chi_+(|\boldsymbol{p}|) - (\boldsymbol{p}_\perp \times \partial_\perp v_z)\, p_z\, \chi_-(|\boldsymbol{p}|)$$

$$\chi^g(\boldsymbol{p}) = (\boldsymbol{p}_\perp \cdot \partial_\perp v_z)\, p_z\, \chi_G(|\boldsymbol{p}|)$$

•**The transverse energy-momentum tensor of our interest is:**

$T^{\perp z} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\boldsymbol{p}_\perp p_z}{E_{\boldsymbol{p}}} \left(\nu_q \sum_F (\delta f^q + \delta f^{\bar{q}}) + \nu_g \delta f^g\right)$

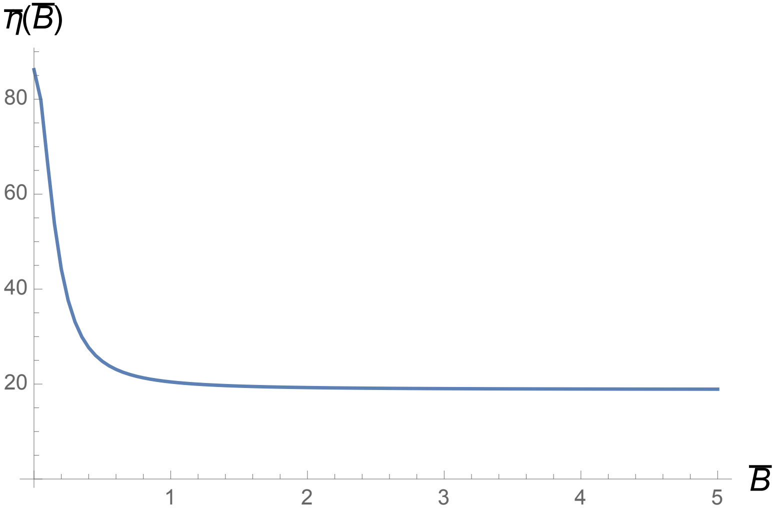
•**Shear Viscosity:**

$\eta = -\frac{1}{30\pi^2} \int_0^\infty dp\, p^5 (2(N_c^2 - 1) n_B(1 + n_B)\chi_G(p) + \sum_F 4N_c n_F(1 - n_F)\chi_+(p))$

Redefining the dimensionless quantities: $\bar{B} \equiv \frac{\bar{q}_F eB}{g^4 \log(1/g) T^2}$

•**Shear Viscosity is rewritten as:** $\eta = \bar{\eta}(\bar{B}) \frac{T^3}{g^4 \log(1/g)}$

Results



$\bar{\eta}(\bar{B}=0) = 86.46(N_f = 2)$

$\bar{\eta}(large\bar{B}) = 18.87$

Summary

•1. **We calculated electrical conductivity in the Strong B using the LLL approximations**

$$\sigma_{zz} = \sum_f e_f^2 \frac{\mathrm{dim}R}{C_2(R)} \left(\frac{e_f B}{2\pi}\right) \frac{1}{\alpha_s T} \sigma_L(\bar{m}_f)$$

$\sigma_L(\bar{m}) \to \frac{1}{\pi \bar{m}^2 \log(1/\bar{m})}$, In the small $\bar{m} \to 0$ limit
$\sigma_L(\bar{m}) \to \frac{1}{\pi \bar{m}}$ in the opposite limit ($\bar{m} \to \infty$)

• **We found that the conductivity is enhanced by large B, and small m.**

•2. **We also found the numerical result for shear viscosity in soft B.**

•**Future:Bulk viscosity in the B**

Matrix element in 1-to-2 process

$$\mathcal{M} = ig_s\epsilon_\mu(k)\left[\bar{v}(p')\gamma_{\parallel}^{\mu}t_R^a u(p)P_+\right]R_{00}(\boldsymbol{k}_\perp)e^{i\Sigma}$$

The projection operator $P_\pm = (1 \pm |eB|i\gamma_1\gamma_2)/2$.

The form factor $R_{00}(\boldsymbol{k}_\perp) = e^{-\frac{\boldsymbol{k}_\perp^2}{4|eB|}}$

$$|\mathcal{M}|^2_{colors} = 2g_s^2 C_2(R)e^{-\frac{\boldsymbol{k}_\perp^2}{2|eB|}}\frac{\boldsymbol{k}_\perp^2}{|\boldsymbol{k}|^2}\left(E_p E_{p'} + p_z p_z' + m_q^2\right) = 4C_2 g^2 m_q^2.$$

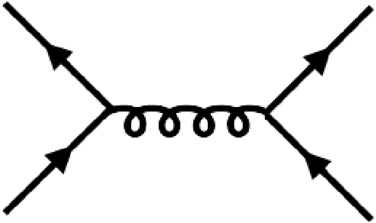
In the strictly massless limit, energy-Momentum conservation in 1+1D requires: $k^{0,z} = p^{0,z} + p'^{0,z}$

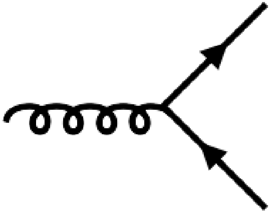
$\boldsymbol{k}_\perp^2 = (k^0)^2 - \boldsymbol{k}_z^2 = (p^0 + p')^2 - (p_z + p_z')^2 = 0$

$\Longrightarrow$ Collinear emission of gluon cannot satisfy that the gluon is transverse polarized.

Collision terms from the 2-to-2 scattering process

- $C^{s}_{q\bar{q}->q\bar{q}}(\chi_{+}(p_z)) \equiv C_{q\bar{q}->g}(\chi_{+}(p_z))$





- $C^{t}_{qq->qq/\bar{q}\bar{q}->\bar{q}\bar{q}}(f_{+}(p_z)) \equiv 0$

- For Quark-antiquark t-channel Scattering and Quark-gluon t-channel Scattering are subleading

Action Functional

$$\bar{\eta}[\bar{\chi}_+, \bar{\chi}_-, \bar{\chi}_G] \equiv \int_0^\infty d\bar{p}\ (\mathcal{I}_+ + \mathcal{I}_- + \mathcal{I}_G + \mathcal{I}_{\text{mix}} + \mathcal{S})$$

$$\mathcal{I}_+ = -\frac{4N_cN_F}{30\pi^2}\left(\frac{1}{8\pi}\bar{m}_D^2 C_F n_F(1-n_F)(\bar{p}^2([\bar{p}^2\bar{\chi}_+]')^2 + 6\bar{p}^4\bar{\chi}_+^2) + 2\bar{\gamma}\bar{p}^5 n_F(1+n_B)\bar{\chi}_+^2\right)$$

$$\mathcal{I}_- = +\frac{4N_cN_F}{30\pi^2}\left(\frac{1}{8\pi}\bar{m}_D^2 C_F n_F(1-n_F)(\bar{p}^2([\bar{p}^2\bar{\chi}_-]')^2 + 6\bar{p}^4\bar{\chi}_-^2) + 2\bar{\gamma}\bar{p}^5 n_F(1+n_B)\bar{\chi}_-^2\right)$$

$$\mathcal{I}_G = -\frac{2(N_c^2-1)}{30\pi^2}\left(\frac{1}{8\pi}\bar{m}_D^2 C_A n_B(1+n_B)(\bar{p}^2([\bar{p}^2\bar{\chi}_G]')^2 + 6\bar{p}^4\bar{\chi}_G^2) + 2\bar{\gamma}\frac{2N_cN_F}{(N_c^2-1)}\bar{p}^5 n_F(1+n_B)\bar{\chi}_G^2\right)$$

$$\mathcal{I}_{\text{mix}} = -\frac{1}{15\pi^2}4N_cN_F(-2\bar{\gamma}\bar{p}^5 n_F(1+n_B)\bar{\chi}_+\bar{\chi}_G + \bar{B}\bar{p}^5 n_F(1-n_F)\bar{\chi}_+\bar{\chi}_-)$$

$$\mathcal{S} = -\frac{1}{15\pi^2}(4N_cN_F\bar{p}^5 n_F(1-n_F)\bar{\chi}_+ + 2(N_c^2-1)\bar{p}^5 n_B(1+n_B)\bar{\chi}_G)$$

Collision terms

Writing δf **as**: $\delta f^a = n_{F/B}(1 \mp n_{F/B})\chi^a\,, \quad a = q, \bar{q}, g$

the linearized collision term is obtained by: $\mathcal{C}^a[\delta f(\boldsymbol{p})] = -\frac{(2\pi)^3}{\nu_a}\frac{\delta \mathcal{I}[\chi]}{\delta \chi^a(\boldsymbol{p})}$

where $\mathcal{I} = \mathcal{I}^I + \mathcal{I}^{II}$ with:

Type 1 t-channel soft gluon exchanges

$$\mathcal{I}^I = \frac{Tm_D^2 g^2 \log(1/g)}{16\pi} \sum_a C_a \nu_a \int_{\boldsymbol{p}} n_a(\boldsymbol{p})(1 \mp n_a(\boldsymbol{p})) \left(\frac{\partial \chi^a(\boldsymbol{p})}{\partial \boldsymbol{p}^i}\right)^2 -$$
$$\frac{g^4 \log(1/g)}{16\pi d_A} \left(\sum_a C_a \nu_a \int_{\boldsymbol{p}} n_a(\boldsymbol{p})(1 \mp n_a(\boldsymbol{p})) \left(\hat{\boldsymbol{p}} \cdot \frac{\partial \chi^a(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)\right)^2 -$$
$$\frac{g^4 \log(1/g)}{16\pi d_A} \left(\sum_a C_a \nu_a \int_{\boldsymbol{p}} n_a(\boldsymbol{p})(1 \mp n_a(\boldsymbol{p})) \frac{\partial \chi^a(\boldsymbol{p})}{\partial \boldsymbol{p}^i}\right)^2$$

Type 2 t-channel soft fermion exchanges

$$\mathcal{I}^{II} = \gamma \sum_{a=q,\bar{q}} \nu_a \int_{\boldsymbol{p}} \frac{1}{|\boldsymbol{p}|} n_F(\boldsymbol{p})(1 + n_B(\boldsymbol{p})) \left(\chi^a(\boldsymbol{p}) - \chi^g(\boldsymbol{p})\right)^2 +$$
$$\frac{16\gamma}{T^2} \sum_{a=q} \nu_a \int_{\boldsymbol{p}} \frac{1}{|\boldsymbol{p}|} n_F(\boldsymbol{p})(1 + n_B(\boldsymbol{p})) \left(\chi^a(\boldsymbol{p}) - \chi^g(\boldsymbol{p})\right) \int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|} n_F(\boldsymbol{k})(1 +$$
$$n_B(\boldsymbol{k})) \left(\chi^{\bar{a}}(\boldsymbol{k}) - \chi^g(\boldsymbol{k})\right) -$$
$$\frac{8\gamma}{T^2} \sum_{a=q,\bar{q}} \nu_a \left(\int_{\boldsymbol{p}} \frac{1}{|\boldsymbol{p}|} n_F(\boldsymbol{p})(1 + n_B(\boldsymbol{p})) \left(\chi^a(\boldsymbol{p}) - \chi^g(\boldsymbol{p})\right)\right)^2$$