Recent progress in understanding Confinement and Chiral symmetry from Instanton-dyons

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arXiv:1511.02237 [PhysRevD] , arXiv:1605.07474 [PhysRevD]

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Overview

- Motivation and method
- Introduction
  - Properties of dyons and 2-point interactions
- Gas of Dyons (Ensemble)
  - Find configurations that minimize free energy density f
- Fermions
  - Obtain chiral condensate from eigenvalue distribution
- Results

Motivation and method

- Objective:
  - Understand chiral symmetry breaking and confinement transitions
- Method:
  - Approximate path integral with instanton-dyons contribution to the path integral (Semi-classical method)
- Tools:
  - Metropolis algorithm used to obtain the free energy
- Results:
  - Density of dyons, chiral condensate and Polyakov loop as a function of coupling constant/temperature

Instanton-dyons 1

- Calculate path-integral by expanding around classical solutions (local minimums)
- Work with 2 colors $N_c = 2$
- Require solution to have a non-zero expectation value of $< A_4^3 >\equiv 2\pi T\nu$, holonomy ν

$$
\begin{aligned}
Polyakov\ \ loop\ \ P &= \frac{1}{N_c} tr(Path[\exp(i\oint A_4 d\tau)]) \\
&= \cos(\pi\nu)
\end{aligned}
$$

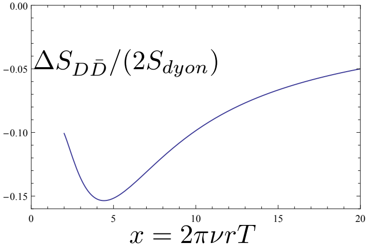
- Solution found by Lee, Lu[hep-th/9802108] and Kraan, van Baal [arXiv:hep-th/9806034]
- SU(2) Caloron composed of 2 **dyons** each a solution
- Dyons called M and L dyons

Dyon properties

- M dyons depends on ν and L dyons on $1-\nu$

	M	$\bar{M}$	L	$\bar{L}$
$\frac{g^2 S_{cl}}{8\pi^2}$	ν	ν	$1-\nu$	$1-\nu$
Q_T	ν	$-\nu$	$1-\nu$	$\nu-1$
e	1	1	-1	-1
m	1	-1	-1	1

- Coulomb like interaction, with opposite sign on electric charges
- Attractive case obtained from gradient flow

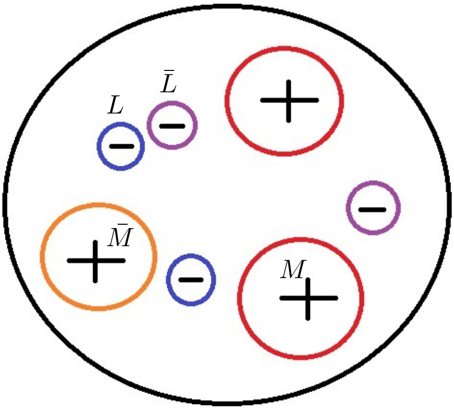


$$\Delta S = \frac{8\pi^2\nu}{g^2}\left(-e_1 e_2 \frac{1}{x} + m_1 m_2 \frac{1}{x}\right)$$
$$x = 2\pi\nu rT$$

- Lack of states within $x < 2$ simulated as a hard core

Ensemble on 3-sphere

- Simulation done on 3-sphere with 64 dyons



- Size of M dyons scales as $\frac{1}{\nu}$
- Size of L dyons scales as $\frac{1}{1-\nu}$
- Action of M dyons are $\nu 8\pi^2/g^2$
- Action of L dyons are $(1-\nu)8\pi^2/g^2$

Free Energy Density

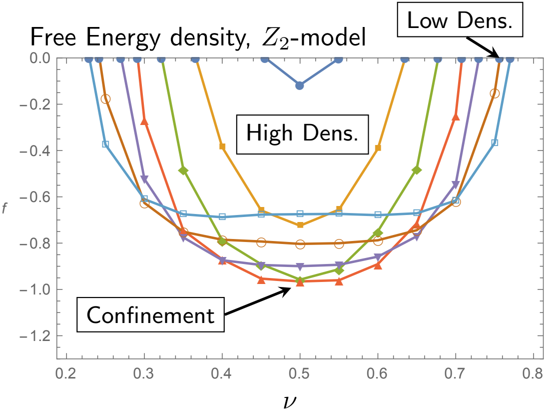
- At volume $V \to \infty$, the dominating configuration is the parameters that minimizes free energy density

$$
\begin{aligned}
f &= \frac{4\pi^2}{3}\nu^2\bar{\nu}^2 - 2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{\bar{\nu}} e}{n_L}\right] \\
&\quad + \Delta f_{Interactions} \\
d_\nu &= \Lambda\left(\frac{8\pi^2}{g^2}\right)^2 e^{-\frac{\nu 8\pi^2}{g^2}} \nu^{\frac{8\nu}{3}-1}/(4\pi) \\
\frac{8\pi^2}{g^2(T)} &= b \cdot ln\left(\frac{T}{\Lambda}\right),\ b = \frac{11}{3}N_c - \frac{2}{3}N_F
\end{aligned}
$$

- Free energy density contains 3 items
  - The **GPY potential** that prefer **trivial** Holonomy
  - The **entropy** due to the dyons moving around
  - ($\Delta f_{Interactions}$) **Corrections** to the energy due to the interactions of the dyons
- GPY potential [Gross, Pisarski, Yaffe, Rev. Mod. Phys. 53, 43]
- d_ν[Diakonov, Gromov, Petrov, Slizovskiy: arXiv:hep-th/0404042]

Finding the dominating Configuration

- We minimize free energy in the following parameters:
  - Density of M dyons n_M and L dyons n_L
  - Holonomy ν
  - Screening mass describing the fall off of the fields



- Dimensionless density : $(0.47, \bullet)$, $(0.37, \blacksquare)$, $(0.30, \blacklozenge)$, $(0.24, \blacktriangle)$, $(0.20, \blacktriangledown)$, $(0.16, \circ)$, $(0.14, \square)$, $(0.12, \lozenge)$, $(0.10, \triangle)$
- The minimization is done as a function of Action $8\pi^2/g^2$

$$S = \frac{8\pi^2}{g^2(T)} = b \cdot ln\left(\frac{T}{\Lambda}\right), \quad b = \frac{11}{3}N_c - \frac{2}{3}N_F$$

Fermions

- Will focus on the cases of two fermion flavors

$$\Delta S_{fermion} = -\log\left(\det[\not{D}_u]\right) - \log\left(\det[\not{D}_d]\right)$$

- The subscript denotes the flavor of the quark
- We will look at the cases:
  - Standard $SU(2)$ QCD with 2 anti-periodic fermions
  - Deformed QCD with imaginary chemical potential
    - Z_2-symmetric QCD with one periodic and one antiperiodic fermion

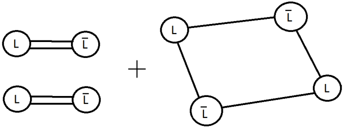
Dirac determinant

- Expand the Dirac operator in subspace of fermionic zero-modes

$$\underbrace{<i|}_{\text{Dyon zero mode}} \quad \not{D} \quad \underbrace{|j>}_{\text{Antidyon zero mode}} \quad \equiv T_{ij}$$

- The determinant of the Dirac operator approximated as closed loops of hopping over zero modes [Shuryak, Verbaarschot:Nucl.Phys. B341 (1990) 1-26]

$$Det \begin{vmatrix} 0 & T_{ij} \\ -T_{ji} & 0 \end{vmatrix} = \sum_{\text{Closed loops}}$$



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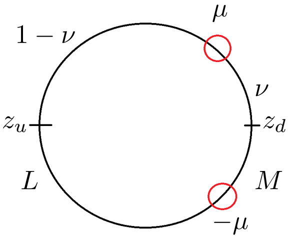
Zero Modes

- Periodicity condition affect the zero modes

$$\begin{aligned} \psi(t+1/T) &= \psi(t)\exp(i2\pi z) \\ P &= \frac{1}{2}tr(\exp[i2\pi \times diag(\mu,-\mu)]) \end{aligned}$$

- Zero mode behavior described by angles on S_1



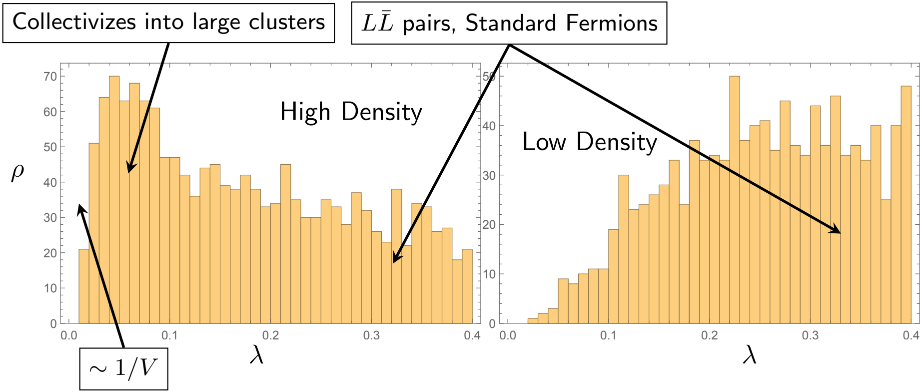
- $T_{i,j} \propto \exp(-2\pi r T|z_i - \mu|)$
- Periodic zero modes on M dyons and anti periodic on L dyons

Chiral Condensate

- The Banks-Casher relation for the chiral condensate tells us that

$$|<\bar{\psi}\psi>| \quad = \quad \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty}$$

- For finite volume we need to look at eigenvalue distribution around 0



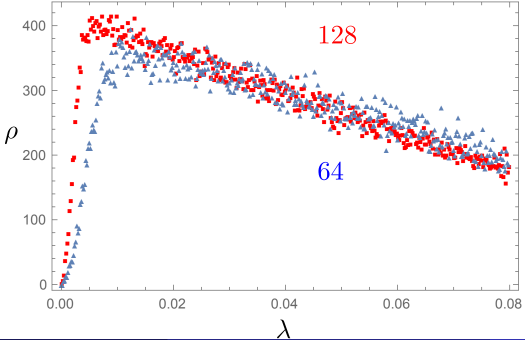
- Done for 64 and 128 dyons

Eigenvalue Distribution

- The slope of the eigenvalue distribution dependent on the number of flavors N_f [Smilga, Stern, Phys. Lett. B 318, 531 (1993)],[Osborn, Toublan, Verbaarschot, hep-th/9806110]

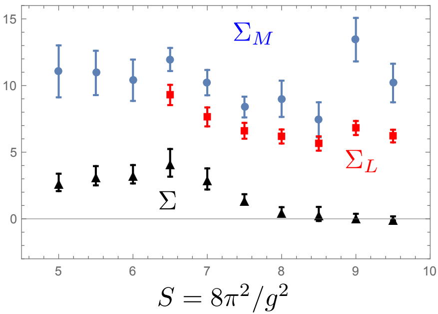
$$\rho(\lambda) \quad = \quad \frac{\Sigma}{\pi}\left(1 + \frac{|\lambda|\Sigma(N_f^2 - 4)}{32\pi N_f F_c^4} + \ldots\right)$$

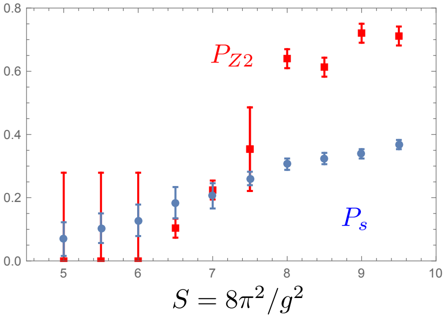
- For standard boundary conditions we get flat for $N_f = 2$
- Z_2 fermions behaves as two independent $N_f = 1$ gases



Results

- (left) Chiral condensate for M (Σ_M), L (Σ_L) for Z_2 QCD, compared to standard fermions (Σ)
- (right) Polyakov loop for Z_2 QCD P_{Z2} and standard fermions P_s

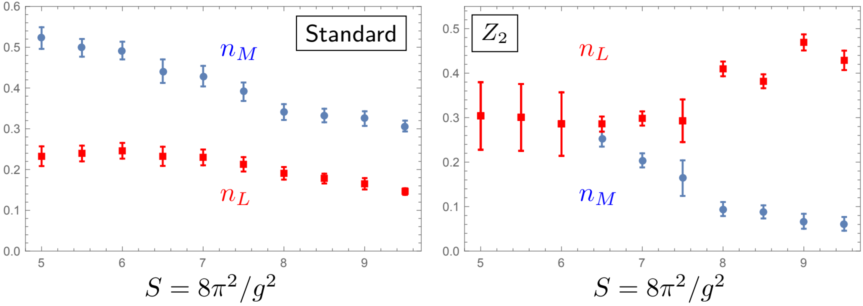




- Chiral condensate stay non-zero for Z_2-symmetric QCD
- Similar to SU(3) results [T. Misumi, T. Iritani and E. Itou, arXiv:1508.07132]

Results 2

- Dyon densities for standard fermions (left) and Z_2 QCD (right)



- Action S related to temperature through running coupling constant

$$S = \frac{8\pi^2}{g^2(T)} = b \cdot ln\left(\frac{T}{\Lambda}\right), \; b = \frac{11}{3}N_c - \frac{2}{3}N_F$$

Summary

- Repulsive core drove the Polyakov loop towards the confining value
- Standard fermions gave a transition from zero to non-zero chiral condensate
- Chiral transition dependent on change in the Polyakov loop
- Compared to Z_2-symmetric QCD with 1 periodic and 1 antiperiodic flavor
- Z_2-symmetric QCD: Center symmetry restored and phase transitions changed
- Z_2-symmetric QCD:: Chiral symmetry never restored