# Hydrodynamic fluctuations and critical dynamics


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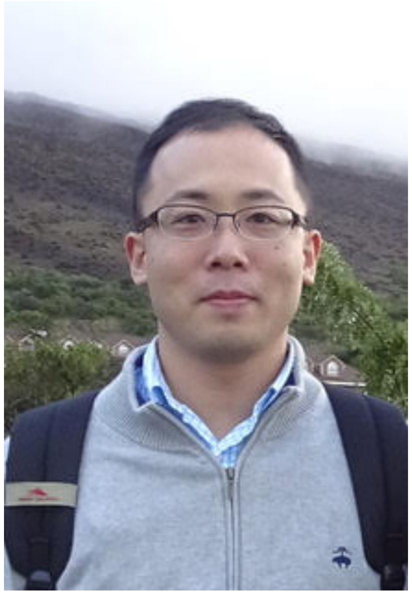
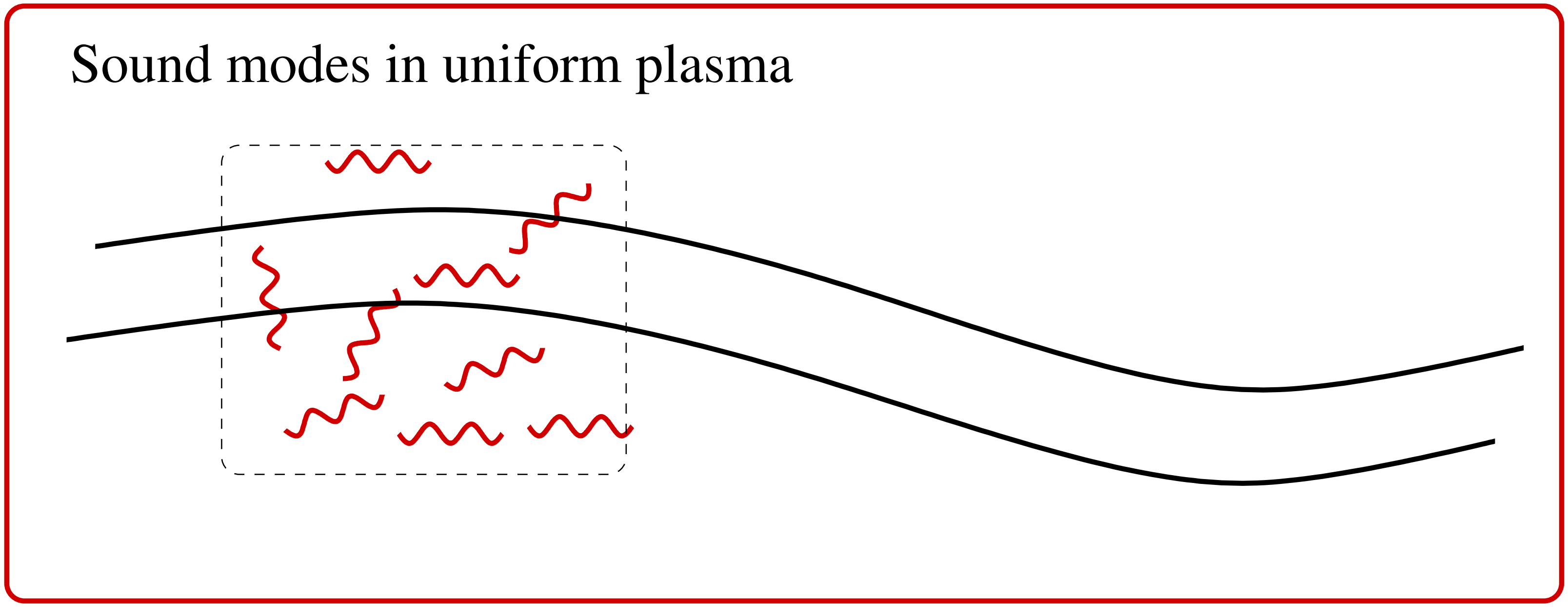
- Yukinao Akamatsu, Aleksas Mazeliauskas, DT, arXiv:1606.07742
- Y. Akamatsu, DT, Fanglida Yan, Yi Yin; in progress




# Part I: Hydrodynamic fluctuations away from the Critical Point

Thermal fluctuations:




These hard sound modes are part of the bath, giving to the pressure and shear viscosity

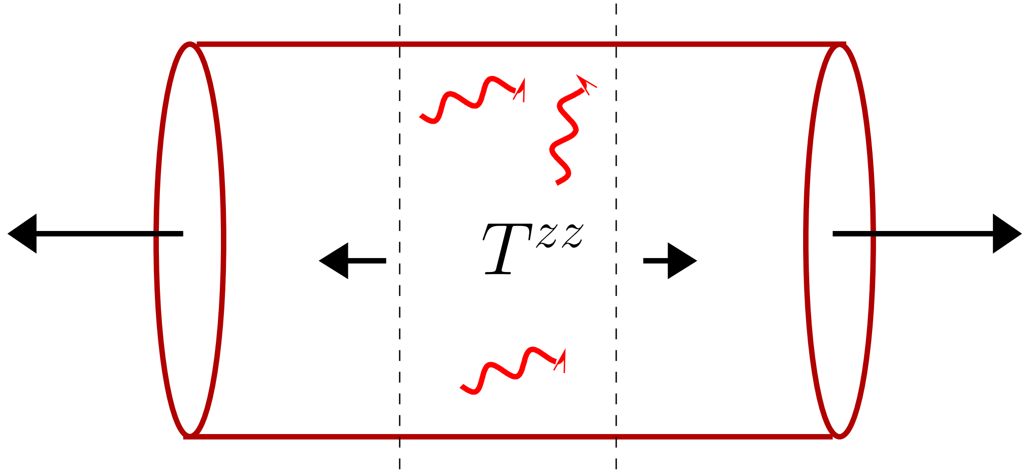
$$N_{ee}^{\rm eq}(\boldsymbol{k},t) \equiv \underbrace{\langle e^*(\boldsymbol{k},t) e(\boldsymbol{k},t)\rangle}_{\text{energy-density flucts}} = T^2 c_v$$

$$N_{gg}^{\rm eq}(\boldsymbol{k},t) \equiv \underbrace{\left\langle g^{*i}(\boldsymbol{k},t) g^j(\boldsymbol{k},t)\right\rangle}_{\text{momentum, } g^i \equiv T^{0i}} = (e+p) T \delta^{ij}$$

In an expanding system these correlators will be driven out of equilibrium.

This changes the evolution of the slow modes.

A Bjorken expansion




1. The system has an expansion rate of $\partial_\mu u^\mu = 1/\tau$
2. The hydrodynamic expansion parameter is

$$\epsilon \equiv \frac{\gamma_\eta}{\tau} \ll 1 \qquad \gamma_\eta \equiv \frac{\eta}{e+p}$$

and corrections to hydrodynamics are organized in powers of $\epsilon$

$$T^{zz} = p\Big[1 + \underbrace{\mathcal{O}(\epsilon)}_{\text{1st order}} + \underbrace{\mathcal{O}(\epsilon^2)}_{\text{2nd order}} + \ldots\Big]$$

High $k$ modes are brought to equilibrium by the dissipation and noise

## The transition regime:

- There is a wave number where the damping rate competes with the expansion

$$\underbrace{\gamma_\eta k^2}_{\text{damping rate}} \sim \underbrace{\frac{1}{\tau}}_{\text{expansion rate}}$$

and thus the transition happens for: $\gamma_\eta \equiv \eta/(e+p)$

$$k \sim k_* \equiv \frac{1}{\sqrt{\gamma_\eta \tau}}$$

need $k \gg k_*$ to reach equilibrium!

- This is an intermediate scale $k_* \equiv 1/(\tau\sqrt{\epsilon})$, $\epsilon \equiv \eta/(e+p)\tau$

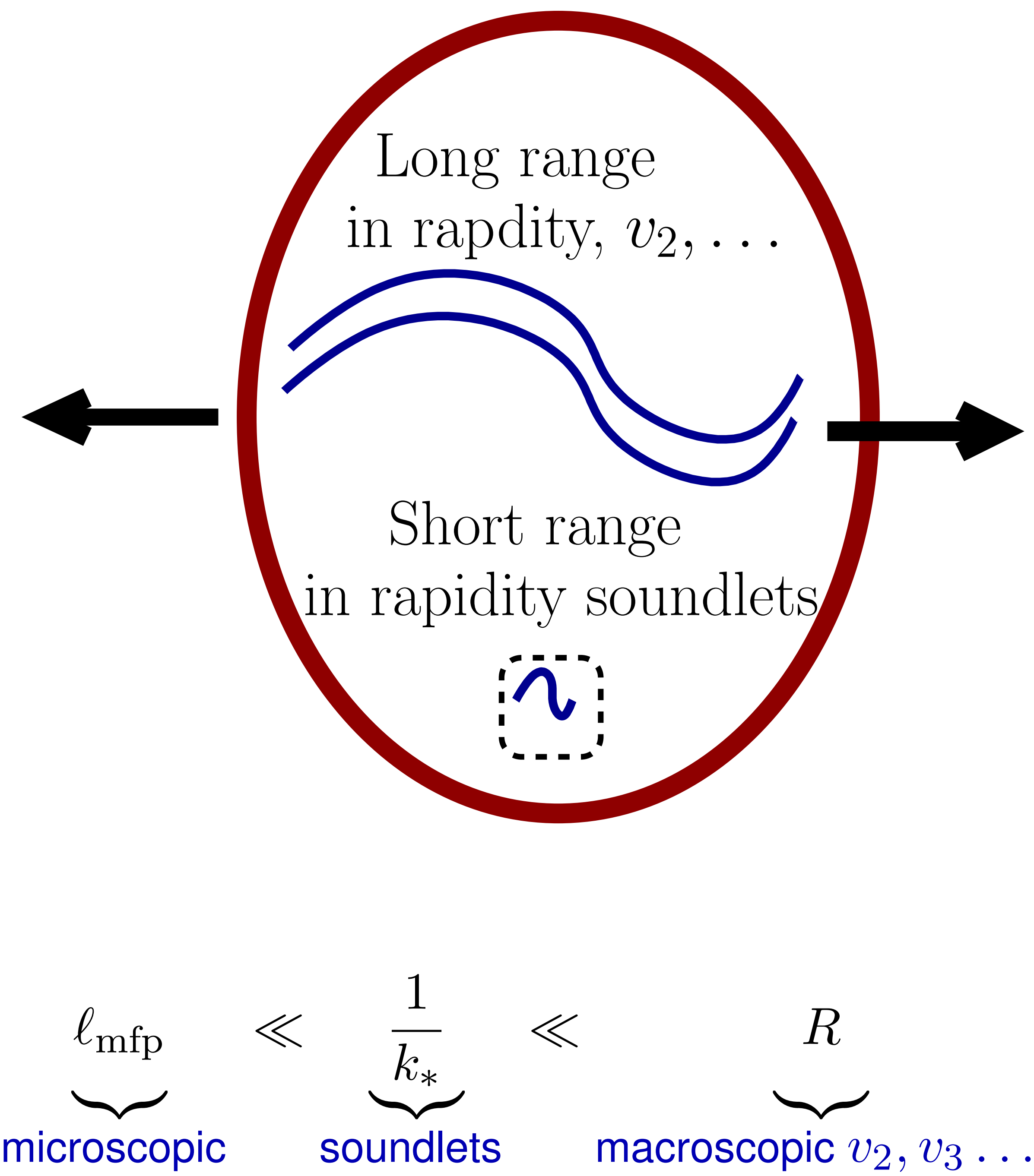
these are the same

$$\frac{1}{\tau} \ll k_* \ll \frac{1}{\ell_{\rm mfp}}$$

$$\epsilon \ll \sqrt{\epsilon} \ll 1$$

We will determine the phase-space density of sound modes with $k \sim k_*$

(using the scale separation $\epsilon \ll \sqrt{\epsilon} \ll 1$ to simplify the problem)



Long range
in rapdity, $v_2, \ldots$
Short range
in rapidity soundlets
$\ell_{\mathrm{mfp}} \ll \frac{1}{k_*} \ll R$
microscopic
soundlets
macroscopic $v_2, v_3 \ldots$

## Determining the phase space density of sound – linearized (stochastic) hydro

1. Evolve fields of linearized hydro with bare parameters $p_0(\Lambda)$, $\eta_0(\Lambda)$, $s_0(\Lambda)$ etc

$$\phi_a(\boldsymbol{k}) \equiv \Big(e(\boldsymbol{k}), g^x(\boldsymbol{k}), g^y(\boldsymbol{k}), g^z(\boldsymbol{k})\Big)$$

2. Then the equations are schematically exactly the same

$$\frac{d\phi_a(\boldsymbol{k})}{dt} = \underbrace{\mathcal{L}_{ab}(\boldsymbol{k})}_{\text{ideal} \sim c_s k} \phi_b(\boldsymbol{k}) + \underbrace{D_{ab}\phi_b}_{\text{visc} \sim -\eta_0 k^2} + \xi_a \qquad \langle \xi_a \xi_b \rangle = 2T\mathcal{D}_{ab}(\boldsymbol{k})\delta_{tt'}$$

3. Break up the equations into eigen modes of $\mathcal{L}_{ab}$, and analyze exactly same way:

$$\underbrace{\text{right moving sound}}_{\lambda_+ = +ic_s k} \qquad \underbrace{\text{left moving sound}}_{\lambda_- = -ic_s k} \qquad \underbrace{\text{two diffusion modes}}_{\lambda_T = 0}$$

So for $k$ in the z direction, work with the following linear combos (eigenvects)

$$\phi_A \equiv \Big[ \underbrace{e(\boldsymbol{k}) \pm \frac{1}{c_s} g^z(\boldsymbol{k})}_{\phi_+ \text{ and } \phi_-} \;,\; \underbrace{g^x(\boldsymbol{k})}_{\equiv \phi_{T_1}} \;,\; \underbrace{g^y(\boldsymbol{k})}_{\equiv \phi_{T_2}} \Big]$$

## The hydro-kinetic equations without exapansion

1. Compute how the phase-space density of sound (squared amplitude) evolves:

$$N_{++}(\boldsymbol{k},t) = \left\langle \phi_+^*(\boldsymbol{k},t)\phi_+(\boldsymbol{k},t)\right\rangle \qquad N_{T_1T_1} = \left\langle \phi_{T_1}^*(\boldsymbol{k},t)\phi_{T_1}(\boldsymbol{k},t)\right\rangle$$

2. The phase space distribution evolution (hydro-kinetic equation):

$$\underbrace{\frac{dN_{++}}{dt}}_{\text{phase-space}} = \underbrace{-\tfrac{4}{3}\gamma_\eta k^2 \left[N_{++} - N_{++}^{\rm eq}\right]}_{\text{damping to equilibrium}}$$

   and similar equations for $N_{--}$, $N_{T_1T_1}$ and $N_{T_2T_2}$. Here

$$N_{++}^{\rm eq} \equiv T^2 c_v = \text{equilibrium}$$

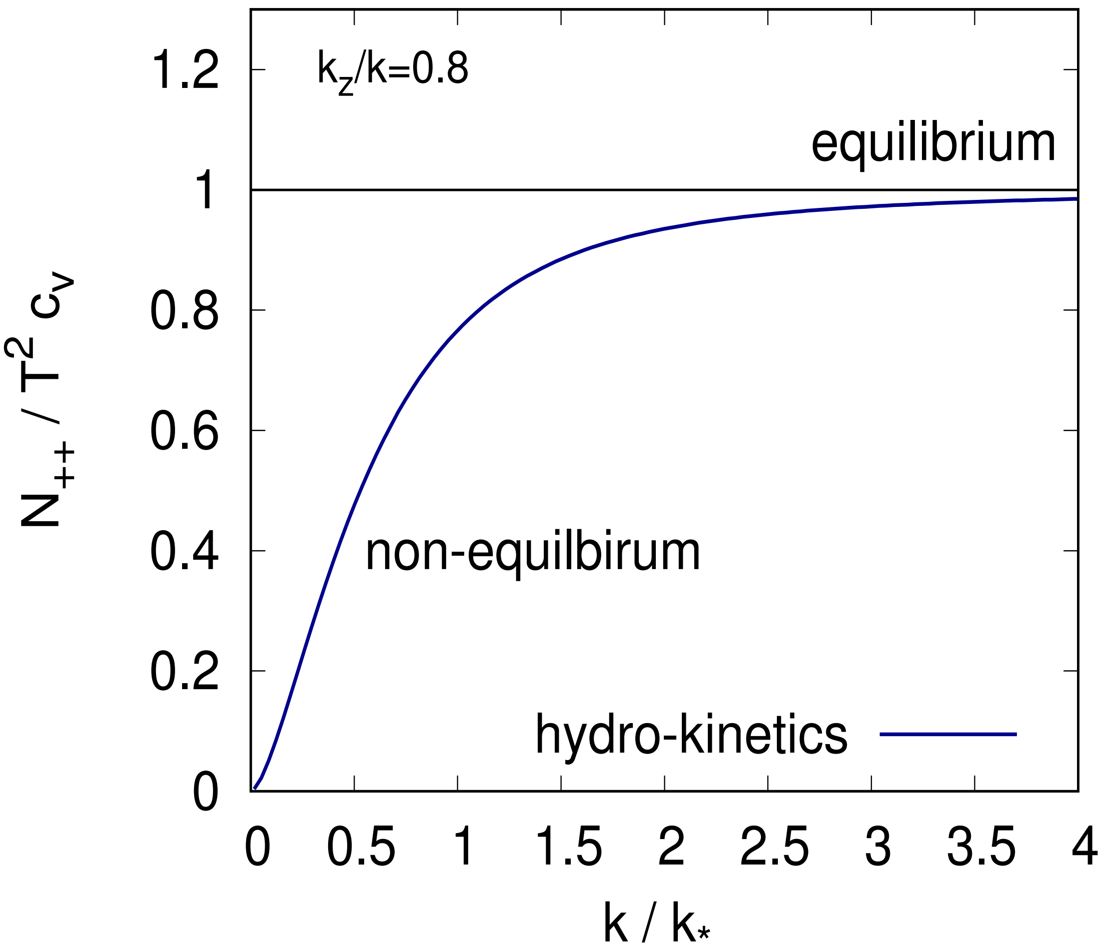
3. Neglect off diagonal components of density matrix in eigen-basis

Now we will do the same for a Bjorken expansion

# The distribution of sound modes during Bjorken expansion:

$$\underbrace{\frac{\partial}{\partial\tau}N_{++}}_{\text{phase-space-dist of sound}} = -\frac{1}{\tau}\underbrace{\left[2 + c_s^2 + \frac{k_z^2}{k_\perp^2 + k_z^2}\right]}_{\text{expansion}} N_{++} \underbrace{- \frac{4}{3}\gamma_\eta\left(k_\perp^2 + k_z^2\right)\left[N_{++} - \frac{T^2 c_v}{\tau}\right]}_{\text{damping to equilibrium}},$$

- Solve:




consequences discussed by Mazeliauskas

See also Pak-Huang Lau for EFT discussion

This (non-equilibrium) distribution of sound modes has consequences . . .

The distribution of sound modes during Bjorken expansion:

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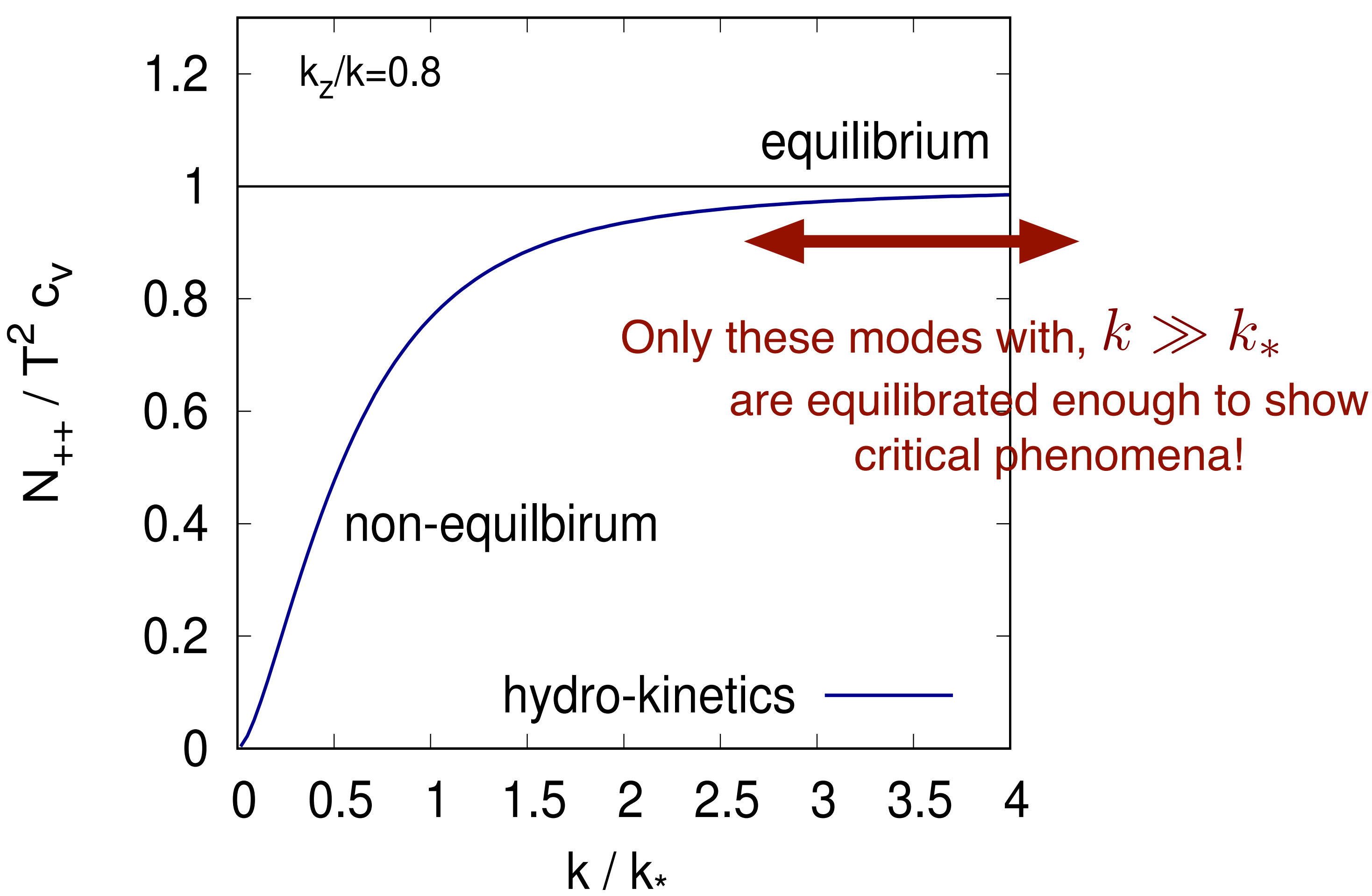
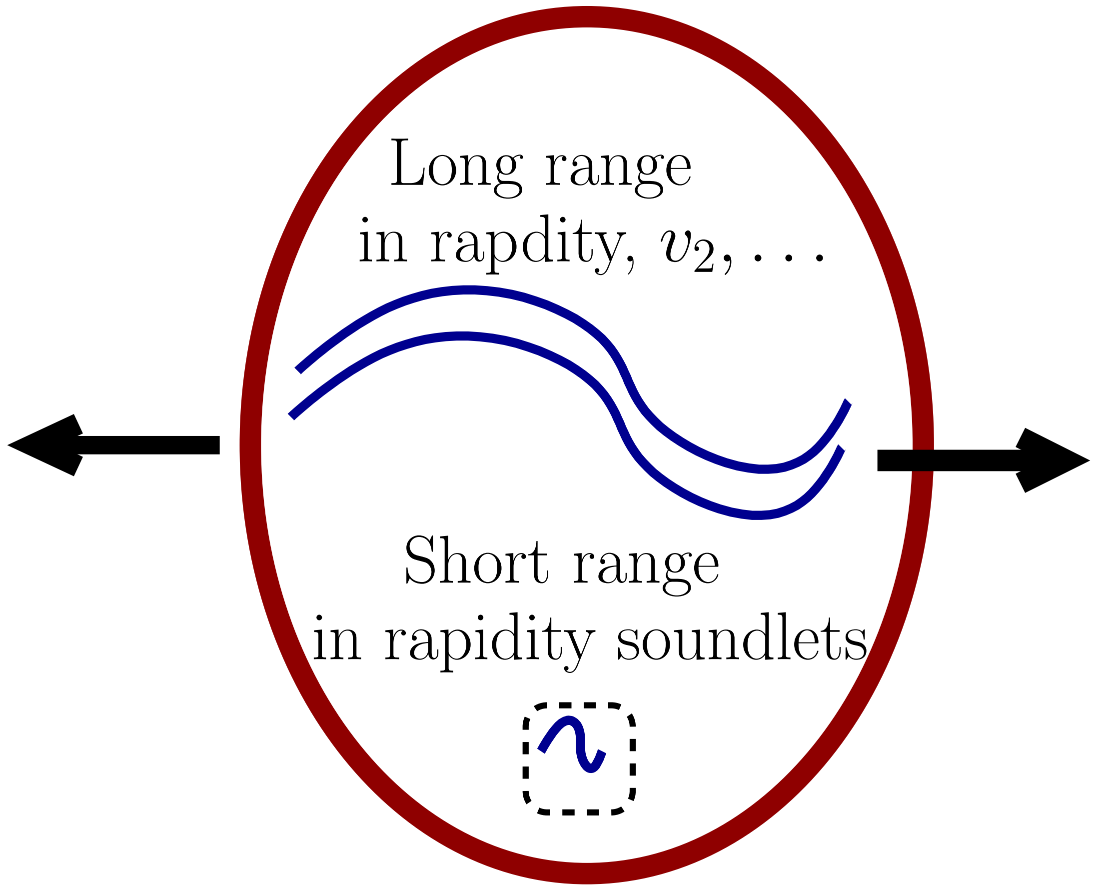



This (non-equilibrium) distribution of sound modes has consequences . . .

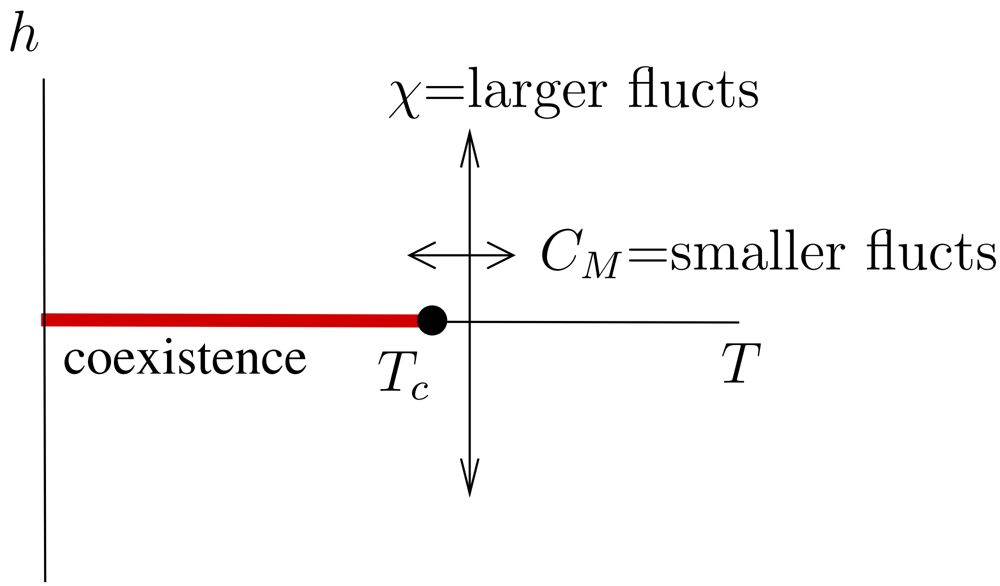
# Part II: Hydrodynamic fluctuations transiting the Critical Point




What happens to the soundlets while transiting the critical point?

# Ising Model Fluctuations:






- Thermodynamic variables and their equilibrium fluctuations

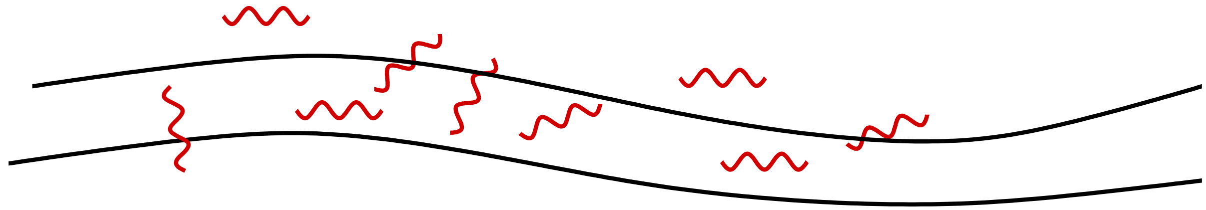
$$x^A \equiv \underbrace{(\mathcal{M}, \delta e_{\rm is})}_{\text{magnetization and energy density}} \qquad \mathcal{X}_{\rm is}^{AB} = \underbrace{\langle \delta x^A \delta x^B \rangle}_{\text{fluctuations}}$$

- Largest and smallest fluctuations, $\det \mathcal{X}_{\rm is} = \chi C_M$

$$\chi \equiv \mathcal{X}_{\rm is}^{11} = \frac{\partial \overline{\mathcal{M}}}{\partial h} = \text{largest fluctuations} = \delta T_{\rm is}^{-\gamma} \qquad \gamma = 1.2$$

$$C_M \equiv \mathcal{X}_{\rm is}^{22} - \frac{(\mathcal{X}_{\rm is}^{12})^2}{\mathcal{X}_{\rm is}^{11}} = \text{smallest fluctuations} = \delta T_{\rm is}^{-\alpha} \qquad \alpha = 0.1$$

QCD hydrodynamic fluctuations:



1. Thermodynamic variables and their conjugates

$$x^a = \underbrace{e(\boldsymbol{k}), n(\boldsymbol{k}),\ g^i(\boldsymbol{k})}_{\text{energy, density, momentum}} \qquad \delta X_a(\boldsymbol{k}) = -\frac{\partial S}{\partial x^a} = \underbrace{-\beta, \hat{\mu},\ \beta u^i}_{\text{conjugates}}$$

2. We will study

$$\mathcal{X}^{ab}(k,t) = \left\langle x^a(k) x^b(-k) \right\rangle \Big|_{\text{equilibrium}}$$

3. Also study pressure fluctuations:

$$\delta p = p^a \delta X_a \qquad (p^e, p^n) = (T(e+p),\ Tn)$$

which determine the speed of sound

$$\left\langle (\delta p)^2 \right\rangle = T(e+p)c_s^2 = \underbrace{p^a \mathcal{X}^{-1}_{ab} p^b}_{\text{“nice little formula”}}$$

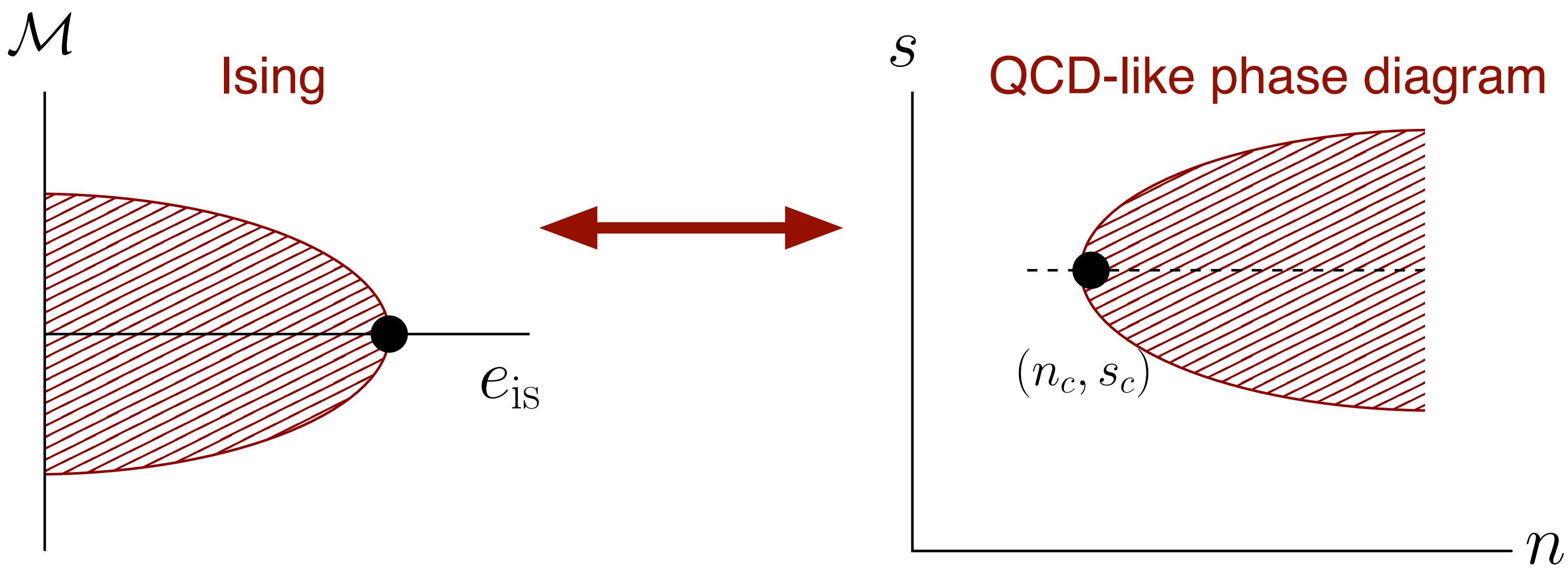
# From QCD to Ising and back

Onuki phase transition dynamics

Assume a linear relation between reduced parameters, e.g. $\left(\frac{\delta T_{\rm is}}{T_{\rm isc}}, h\right) \Leftrightarrow \left(\frac{\delta\mu}{\mu_c}, \frac{\delta T}{T_c}\right)$

$$\underbrace{\delta x^A}_{\text{Ising fields}} = M^A{}_b \underbrace{\delta x^b}_{\text{QCD fields}}$$

Thermodynamic conjugates obey the inverse linear map, $X_{\rm is} = M^{-1} X_{\rm QCD}$




We will take the simplest mapping:

$$\underbrace{\delta\mathcal{M}}_{\text{magnetization}} = M^h_e \underbrace{\frac{\delta s}{s_c}}_{\text{entropy}}$$

# Hydrodynamic Fluctuations and Dynamics with Baryon Number

Linearized equations of motion for $e$, $g$, and now $n$

$$\frac{dx^a(\boldsymbol{k})}{dt} = \underbrace{\mathcal{L}^{ab}(\boldsymbol{k})X_b(\boldsymbol{k})}_{\text{ideal}} + \underbrace{\Lambda^{ab}X_b}_{\text{viscosity+conductivity}} + \underbrace{\xi_a}_{\text{noise}}$$

1. New diffusive (zero) mode for the *entropy per baryon* fluctuations

$$\delta\sigma \equiv \delta e - \frac{e+p}{n}\,\delta n = Tn\,\delta\left(\frac{s}{n}\right)$$

which satisfies $\langle \delta p \delta\sigma \rangle = 0$.

2. Fluctuations of $\sigma$ obey a relaxation type equation, $N^{\sigma\sigma} = \langle \delta\sigma(\boldsymbol{k},t)\delta\sigma(-\boldsymbol{k},t)\rangle$

$$\frac{dN^{\sigma\sigma}}{dt} = -\underbrace{\frac{2T(e+p)\lambda k^2}{\mathcal{X}^{\sigma\sigma}}}_{\text{relaxation controlled by } \lambda \equiv \text{conductivity}} \left[N^{\sigma\sigma} - \mathcal{X}^{\sigma\sigma}\right],$$

where $\mathcal{X}^{\sigma\sigma} = c_s^2 \det \mathcal{X}^{ab}$ is the static susceptibility for $\delta\sigma$.

## Mapping entropy/baryon fluctuations onto the ising

(Onuki - Phase transition dynamics)

1. The speed of sound approaches zero like the smallest ising susceptibility

$$T(e+p)c_s^2 = p^a \mathcal{X}^{-1}_{ab} p^b$$

$$\text{sound} = p^A \mathcal{X}^{-1}_{AB} p^B \simeq \underbrace{\left(T_{\rm is}\frac{dp}{dT_{\rm is}}\right)^2 \frac{1}{C_M}}_{\text{the smallest susceptibility}}$$

2. The susceptibility matrix also transforms $\det\mathcal{X} = (\det M)^2 \det\mathcal{X}_{\rm is} \propto \chi C_M$

3. The fluctuation in $\sigma$ diverge as the largest susceptibility

$$\mathcal{X}^{\sigma\sigma} = c_s^2 \det \mathcal{X} \propto \underbrace{\chi}_{\text{largest ising susceptibility}}$$

The fluctuations in the entropy per baryon diverge maximally like $\chi$

(independently of how the mapping to the ising variables is done!)

## Summary of equation for fluctuations in the specific entropy $\sigma \equiv n\delta(s/n)$

$$\frac{d\bar{N}^{\sigma\sigma}(k,t)}{dt} = -\frac{2\lambda_{\rm eff}k^2}{\chi(k)}\left[\bar{N}^{\sigma\sigma} - \chi(k)\right]$$

1. Definitions:

$$\bar{N}^{\sigma\sigma} = \underbrace{N^{\sigma\sigma}}_{\text{flucts of } \sigma} \times \text{mapping parameters}$$

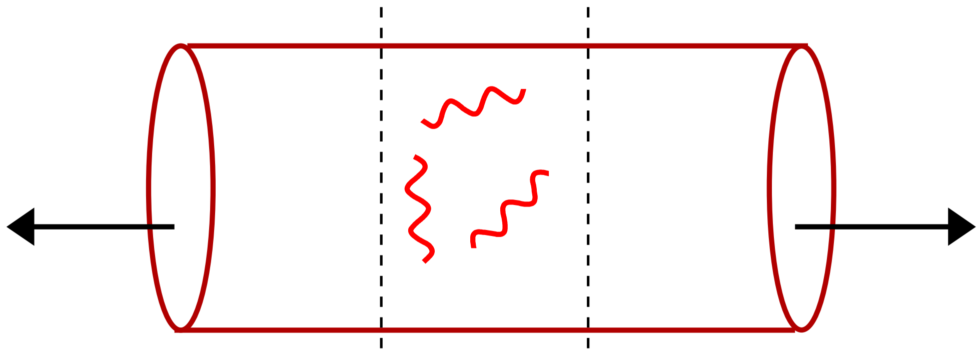
$$\lambda_{\rm eff} = \underbrace{\lambda}_{\text{conductivity}} \times \text{mapping parameters}$$

2. Model susceptibility near critical point as a function of $k$ with correlation length $\xi$ (critical-exponent $\eta = 0.02$)

$$\bar{N}^{\sigma\sigma}(\boldsymbol{k},t)\Big|_{\rm equil} = \chi(k) = \underbrace{\frac{\chi_o(\xi/\ell_o)^{2-\eta}}{1+(k\xi)^{2-\eta}}}_{\text{susceptibility } \chi(\boldsymbol{k})}$$

We will solve this equation to monitor the equilibration of various wavenumbers

## Transiting the critical point



1. Pass right through the critical point at late time $\tau = \tau_Q$, define $t \equiv \tau - \tau_Q$:

$$\partial_\tau \overline{n} = -\frac{n_c}{\tau_Q} \qquad \Longrightarrow \qquad \frac{\delta\overline{n}}{n_c} = -\frac{t}{\tau_Q}$$

$$\partial_\tau \overline{s} = -\frac{s_c}{\tau_Q} \qquad \qquad \frac{\delta\overline{s}}{s_c} = -\frac{t}{\tau_Q} + \underbrace{\Delta}_{\text{set to zero}}$$

Set $\Delta = 0$ to go directly through the critical point.

2. The (ising) reduced $T_{\rm is}$ and correlation length behaves $a\nu \equiv \nu/(1-\alpha) \simeq 0.71$

$$\delta T_{\rm is} \propto \left(\frac{|t|}{\tau_Q}\right)^{1-\alpha} \qquad \text{and} \qquad \xi = \ell_o \left(\frac{\tau_Q}{|t|}\right)^{a\nu}$$

## Dynamical critical exponents

Son and Stephanov most helpful

1. The fluctuations of $\delta\sigma \equiv n\delta(s/n)$ satisfy:

$$\partial_t \bar{N}^{\sigma\sigma} = -\frac{2\lambda_{\rm eff} k^2}{\chi(k)} \left[\bar{N}^{\sigma\sigma} - \chi(k)\right]$$

$$= -\frac{2\lambda_{\rm eff}}{\chi_o \ell_o^2 (\xi/\ell_o)^{4-\eta}} (k\xi)^2 (1 + (k\xi)^{2-\eta}) \left[\bar{N}^{\sigma\sigma} - \chi(k)\right]$$

2. Then the equilibration time for $k\xi \sim 1$ :

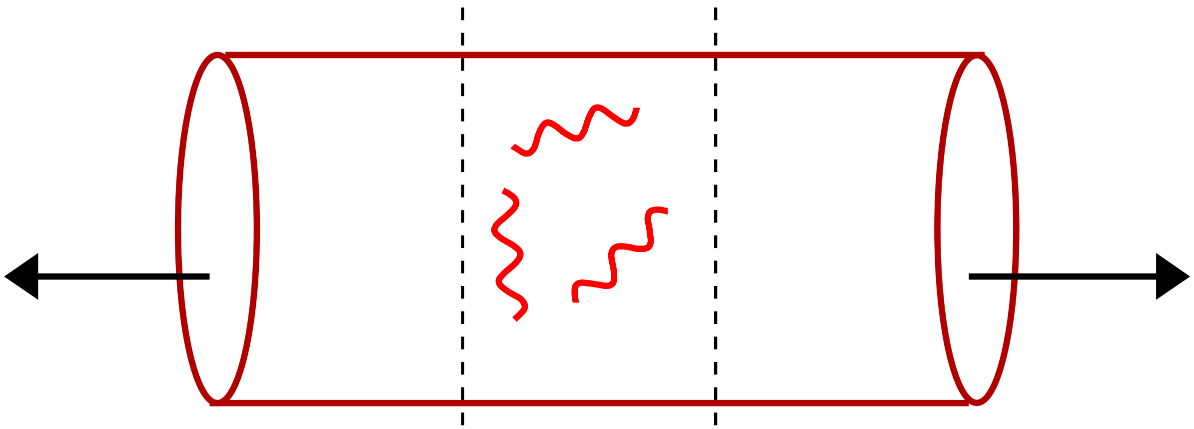
$$\underbrace{\tau_{\rm eq}(\xi) \equiv \tau_{R_o} \left(\frac{\xi}{\ell_o}\right)^z}_{\text{equilibration time}} \quad \text{with} \quad \underbrace{z \equiv 4 - \eta}_{\text{dynamic critical exponent}} \quad \text{and} \quad \underbrace{\tau_{R_o} \equiv \frac{\chi_o \ell_o^2}{\lambda_{\rm eff}}}_{\text{micro relax-time}}$$

The equation to be solved is :

$$\partial_t \bar{N}^{\sigma\sigma}(\boldsymbol{k}, t) = -\frac{2(k\xi)^2(1 + (k\xi)^{2-\eta})}{\tau_{\rm eq}(\xi)} \left[\bar{N}^{\sigma\sigma}(\boldsymbol{k}, t) - \chi(\boldsymbol{k}, t)\right]$$

# Kibble-Zurek Scaling of Equation

Kible,Zurek; Berdnikov, Rajagopal; Mukherjee, Venugopalan, Yin



$$\xi(t) = \ell_o \left(\frac{\tau_Q}{|t|}\right)^{a\nu}$$

$$\tau_{\rm eq}(\xi) = \tau_{R_o} \left(\frac{\xi(t)}{\ell_o}\right)^{z}$$

1. There is a timescale, $t = t_{\rm kz}$, where the relaxation rate can't keep up with $\xi(t)$

$$\underbrace{\frac{1}{\tau_{\rm eq}(\xi(t_{\rm kz}))}}_{\text{relaxation rate}} = \underbrace{\frac{\partial_t \xi(t_{\rm kz})}{\xi(t_{\rm kz})} = \frac{a\nu}{t_{\rm kz}}}_{\text{rate-of change of } \xi(t)}$$

2. Find a Kibble-Zurek time scale, $t_{\rm kz}$, and length, $\ell_{\rm kz} = \xi(t_{\rm kz})$

$$t_{\rm kz} \equiv \tau_{R_o} \left(\frac{\tau_Q}{\tau_{R_o}}\right)^{a\nu z/(a\nu z+1)} \simeq \tau_{R_o} \left(\frac{\tau_Q}{\tau_{R_o}}\right)^{0.74} \gg \tau_{R_o}$$

$$\ell_{\rm kz} = \ell_o \left(\frac{\tau_Q}{\tau_{R_o}}\right)^{a\nu/(a\nu z+1)} \simeq \ell_o \left(\frac{\tau_Q}{\tau_{R_o}}\right)^{0.19} \gg \ell_o$$

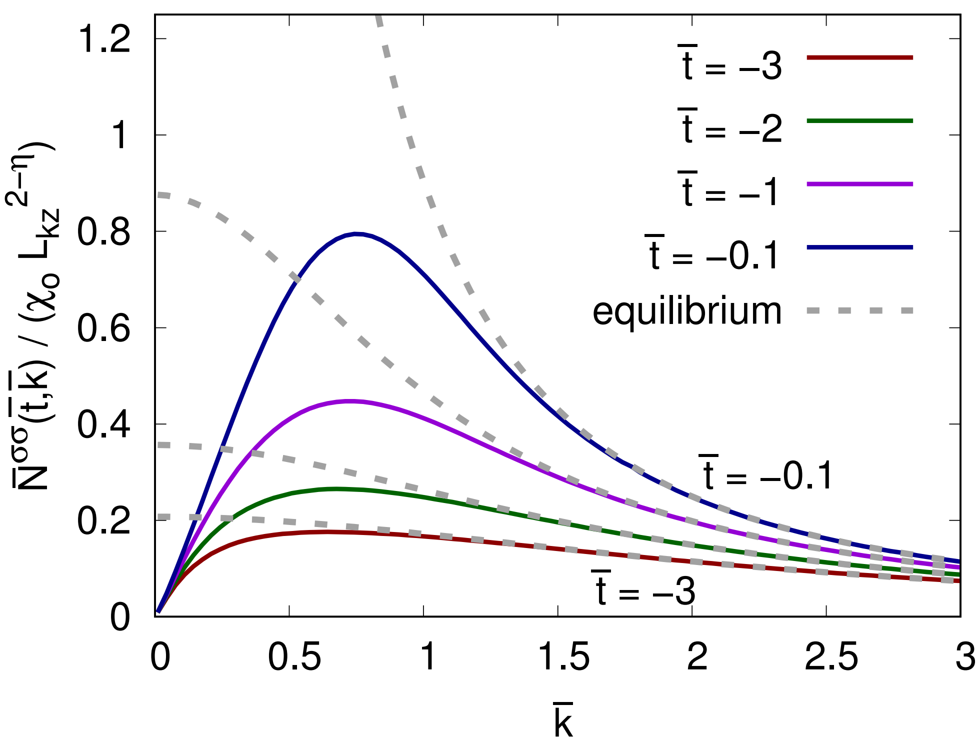
## Kibble-Zurek rescaled equation:

1. Measure all lengths, wavenumbers, and times in terms of $\ell_{\rm kz}$ and $t_{\rm kz}$

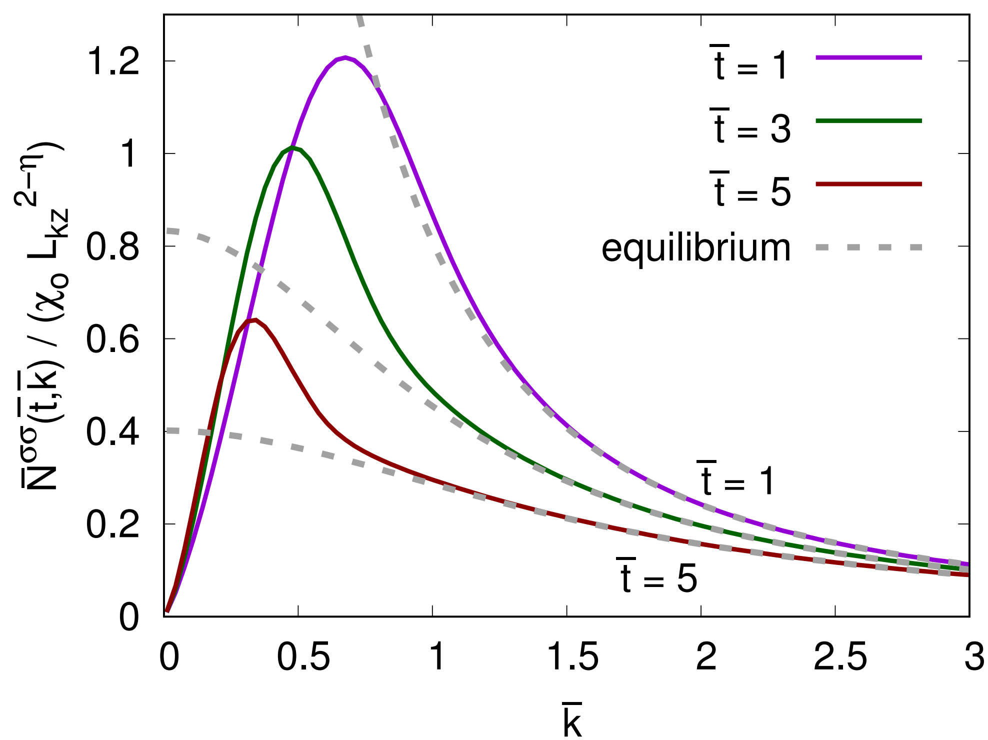
$$\bar{t} = \frac{t}{t_{\rm kz}} \quad \text{and} \quad \bar{k} = k\ell_{\rm kz} \quad \text{and} \quad \bar{\xi} = \frac{\xi}{\ell_{\rm kz}}$$

2. Also rescale the correlator, $\bar{N}^{\sigma\sigma} \to \bar{N}^{\sigma\sigma}/\chi_o \ell_{\rm kz}^{2-\eta}$, motivated by equilibrium:

### Above Critical Point




### Below Critical Point



## Summary of Scales

1. The small parameter is the ratio of microscopic length to system size:

$$\epsilon = \frac{\tau_{R_o}}{\tau_Q} = \frac{\text{micro scale}}{\text{macro scale}} \simeq \frac{1}{7}$$

2. Hierarchy of scales:

$$\underbrace{k_{\rm hydro}}_{\sim v_2} \ll \underbrace{k_*}_{\text{hyd-kinetics}} \ll \underbrace{k_{\rm kz}}_{\text{longest critical fluct}} \ll \underbrace{\frac{1}{\ell_o}}_{\text{microlength}}$$

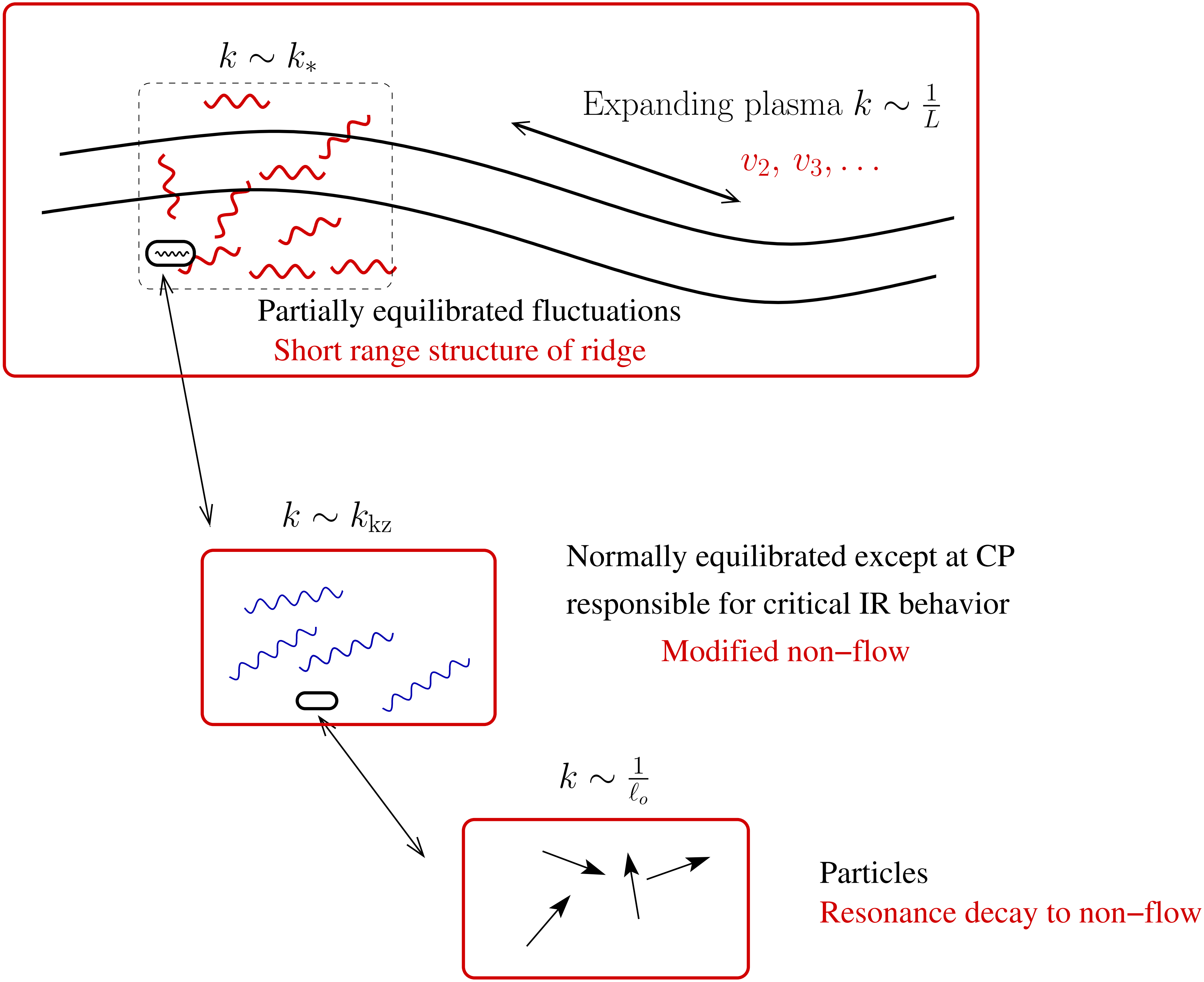
which are of relative order

$$\epsilon \ll \sqrt{\epsilon} \ll \epsilon^{0.18} \ll 1 \qquad \text{or} \qquad 0.14 \ll 0.38 \ll 0.70 \ll 1$$

3. The duration of the KZ regime is short compared to $\tau_Q$ (parametrically only)

$$\tau_{R_o} \ll t_{\rm kz} \ll \tau_Q \qquad \text{or} \qquad \epsilon \ll \underbrace{\epsilon^{0.26}}_{\sim 0.6} \ll 1$$

May not have a clear separation of scales in practice

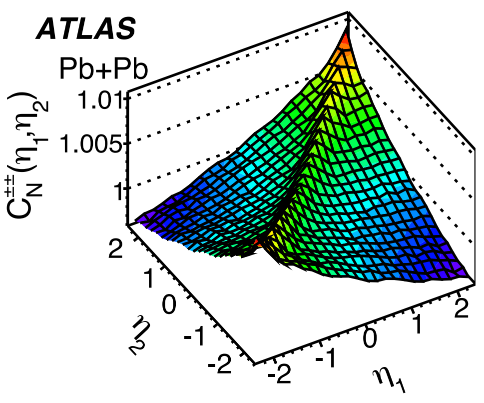


$k \sim k_*$
Expanding plasma $k \sim \frac{1}{L}$
$v_2, \; v_3, \ldots$
Partially equilibrated fluctuations
Short range structure of ridge
$k \sim k_{\rm kz}$
Normally equilibrated except at CP
responsible for critical IR behavior
Modified non–flow
$k \sim \frac{1}{\ell_o}$
Particles
Resonance decay to non–flow

Real correlation functions at high energies (see W. Llope)

$$C(\eta_1, \eta_2) = \frac{\left\langle \frac{dN}{d\eta_1} \frac{dN}{d\eta_2} \right\rangle}{\left\langle \frac{dN}{d\eta_1} \right\rangle \left\langle \frac{dN}{d\eta_2} \right\rangle}$$

Correlation function

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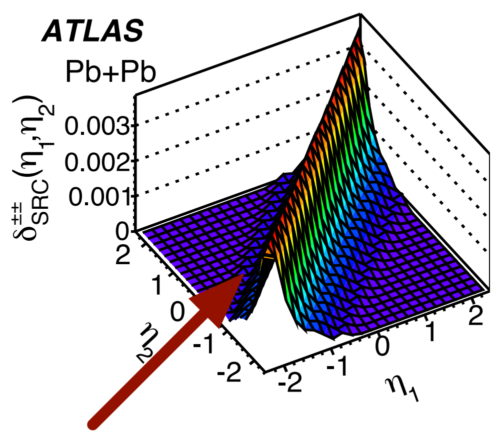
Short Range = "Non-flow"

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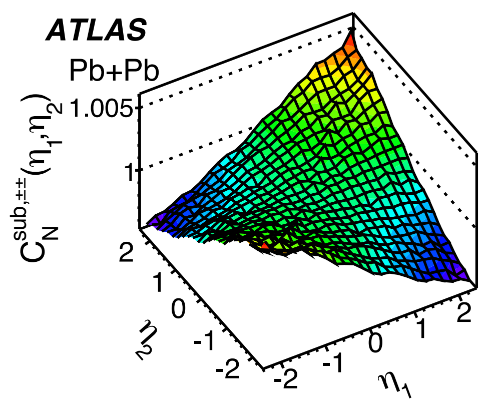
Long range rapidity flucts




Find the CP in here
at lower energy

Look for short range in $\eta$ entropy/baryon correlations

with significant higher-point cumulants

## Summary

1. Away from the critical point, for wavenumbers of order

$$k \gtrsim k_* \equiv \sqrt{\frac{e+p}{\eta\tau}}$$

   the system transitions to equilibrium.

2. Clarified where critical fluctuations are relevant

$$\underbrace{k_{\rm hydro}}_{\sim v_2} \ll \underbrace{k_*}_{\text{hyd-kinetics}} \ll \underbrace{k_{\rm kz}}_{\text{longest critical fluct}} \ll \underbrace{\frac{1}{\ell_o}}_{\text{microlength}}$$

3. Encouraged experimentalists to study non-flow...

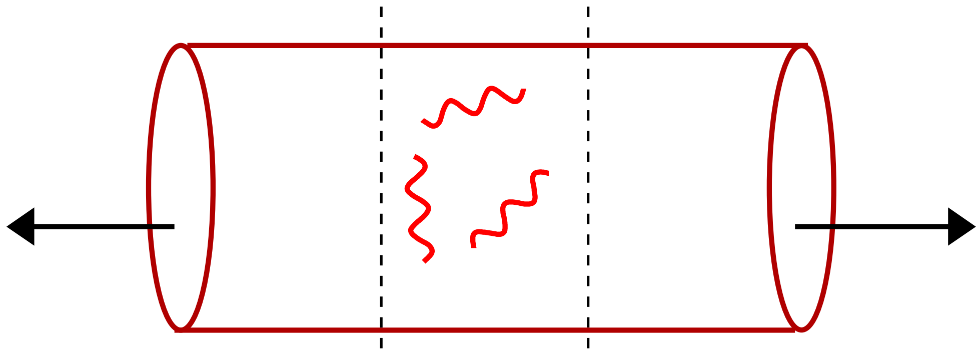
   (a) Look for significant short-range entropy/baryon higher point cumulants

4. Left out of the talk due to time: What happens when you miss the critical point?

$$\Delta \equiv \frac{n_c}{s_c}\delta(\overline{s}/\overline{n}) = \text{a finite detuning}$$

# Backup I

## Transiting close to the critical point



1. Pass close to the critical point at late time $\tau = \tau_Q$.

2. The "detuning" $\Delta$ acts like a magnetic field regulating critical dynamics

$$\Delta = \underbrace{\frac{n_c}{s_c}\delta\left(\overline{s}/\overline{n}\right)}_{\text{a small detuning}}$$

The detuning limits the rate of change of critical fluctuations

## Transiting close to the critical point:

- The "detuning" $\Delta = \frac{n_c}{s_c}\delta(\overline{s}/\overline{n})$ regulates the critical dynamics.

  The timescale for this regulation is determined by the scaling equation of state:

$$t_{\rm cross} \sim \underbrace{\tau_Q}_{\text{only timescale}} \times \underbrace{\Delta^{(1-\alpha)/\beta}}_{\text{only dimensionless number}}$$

- We will remain in equilibrium if the system is sufficiently detuned

$$t_{\rm cross} \gg t_{\rm kz}$$

- Find that

$$\Delta \gg \left(\frac{\tau_{R_o}}{\tau_Q}\right)^{\beta/(\nu z+1-\alpha)}$$

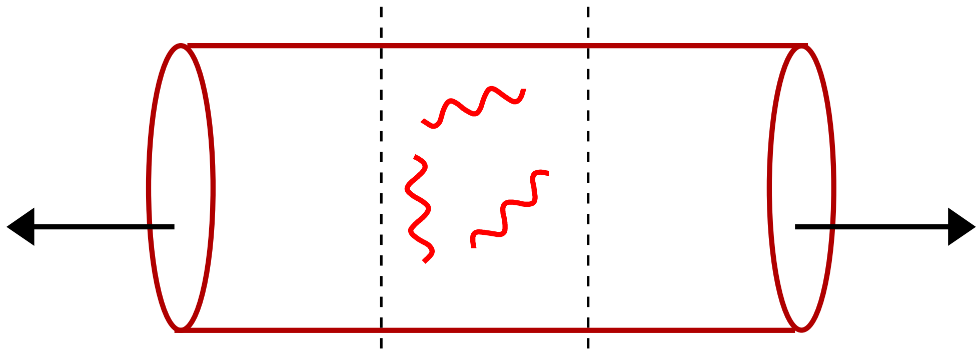
  or

$$\Delta \gg \underbrace{\left(\frac{\tau_{R_o}}{\tau_Q}\right)^{0.096}}_{\text{A very small power}}$$

The Kibble-Zurek mechanism is probably the dominant regulator of critical dynamics since the power 0.096 is small.

# Backup II

## Transiting close to the critical point



1. Pass close to the critical point at late time $\tau = \tau_Q$, define $t \equiv \tau - \tau_Q$:

$$\partial_\tau \overline{n} = -\frac{n_c}{\tau_Q} \qquad \Longrightarrow \qquad \frac{\delta \overline{n}}{n_c} = -\frac{t}{\tau_Q}$$

$$\partial_\tau \overline{s} = -\frac{s_c}{\tau_Q} \qquad\qquad \frac{\delta \overline{s}}{s_c} = -\frac{t}{\tau_Q} + \underbrace{\Delta}_{\text{small}}$$

2. The "detuning" $\Delta$ acts like a magnetic field regulating critical dynamics

$$\Delta = \underbrace{\frac{n_c}{s_c}\delta\left(\overline{s}/\overline{n}\right)}_{\text{a small detuning}}$$

## Time-scale for the maximal equilibrium fluctuations:

see Berdnikov,Rajagopal hep-ph/9912274

1. The correlation length is a function of the scaling variable, $\xi = \bar{h}^{-\nu/\beta\delta} f(z)$

$$\underbrace{z}_{\text{scaling-var}} = \underbrace{\bar{\tau}}_{\text{reduced } T_{\text{is}}} \times \underbrace{\bar{h}^{-1/\beta\delta}}_{(\text{reduced field})^{-1/\beta\delta}}$$

2. The correlation length is maximal for $z \sim 1$. With

$$\frac{\delta n}{n_c} \sim -\frac{t_{\text{cross}}}{\tau_Q} \quad \text{and} \quad \frac{\delta s}{s_c} \sim \Delta - \frac{t_{\text{cross}}}{\tau_Q}$$

we find the timescale for the maximal correlation length

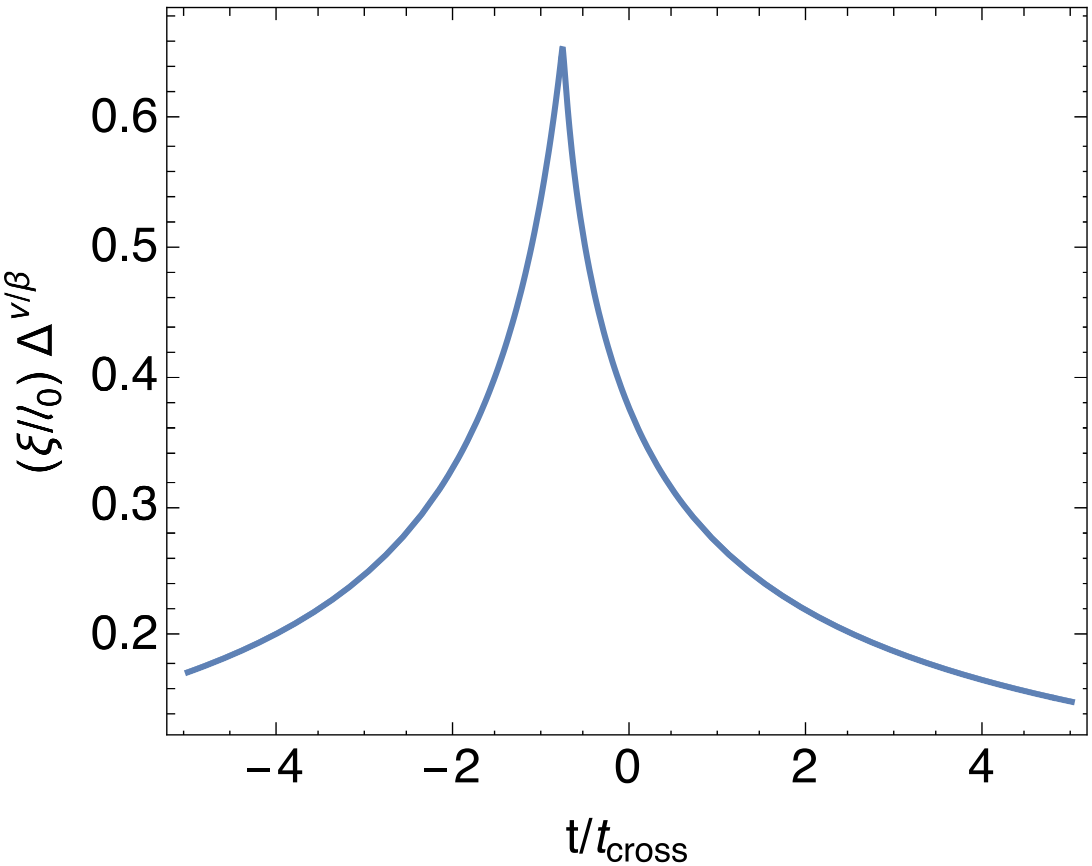
$$t_{\text{cross}} \sim \underbrace{\tau_Q}_{\text{only timescale}} \times \underbrace{\Delta^{(1-\alpha)/\beta}}_{\text{only dimensionless number}}$$

For $t \sim t_{\text{cross}}$ the correlation length is regulated by the detuning $\Delta$

The correlation length:

numerical data Engels,Fromme,Seniuch, cond-mat/0209492

$$t_{\text{cross}} \propto \tau_Q \Delta^{(1-\alpha)/\beta}$$




If the system is sufficiently detuned (i.e. $t_{\text{cross}} \gg t_{kz}$) we remain in equilibrium

## Comparing the Kibble-Zurek and crossing time-scales

1. We will remain in equilibrium for

$$t_{\text{cross}} \gg t_{\text{kz}}$$

2. Find that

$$\Delta \gg \left(\frac{\tau_{R_o}}{\tau_Q}\right)^{\beta/(\nu z+1-\alpha)}$$

or

$$\Delta \gg \underbrace{\left(\frac{\tau_{R_o}}{\tau_Q}\right)^{0.096}}_{\text{A very small power}}$$

The Kibble-Zurek mechanism is probably the dominant regulator of critical dynamics since the power 0.096 is small.