

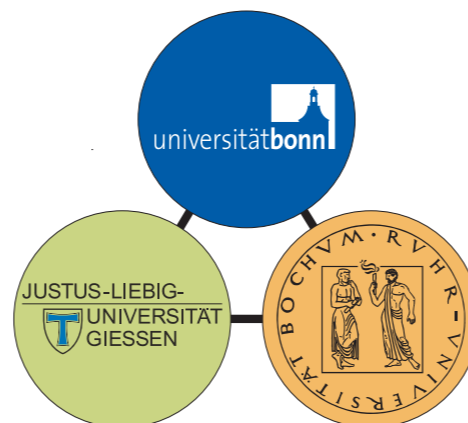
# Locating QCD's critical end point

Christian S. Fischer

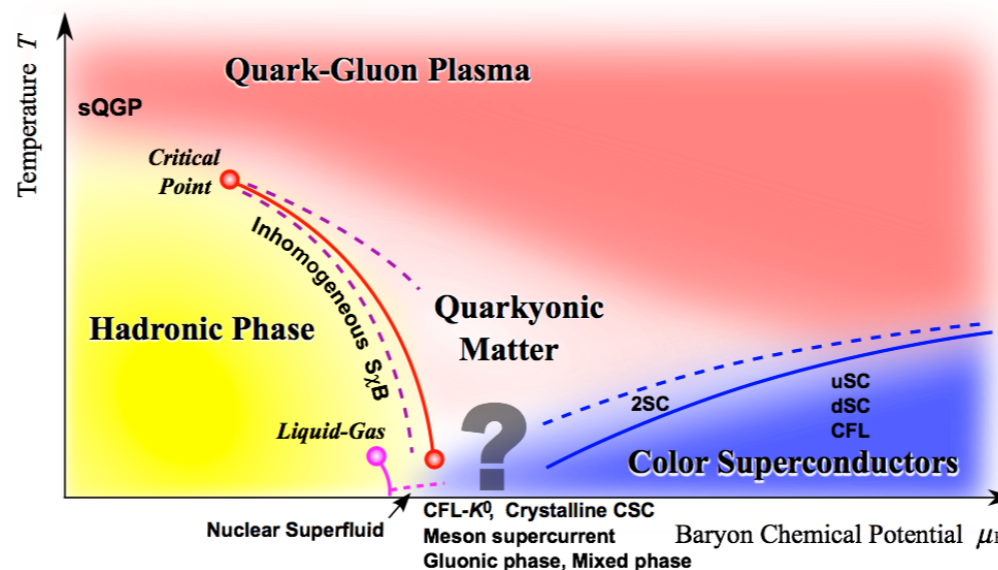
Justus Liebig Universität Gießen

CPOD 2017

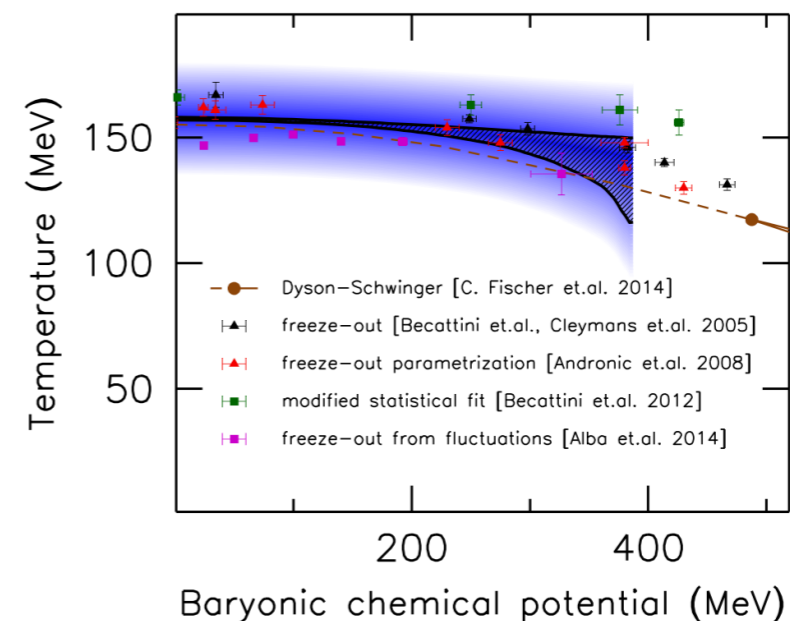
with: P. Gunkel, G. Eichmann, J. Lücker, C. Welzbacher



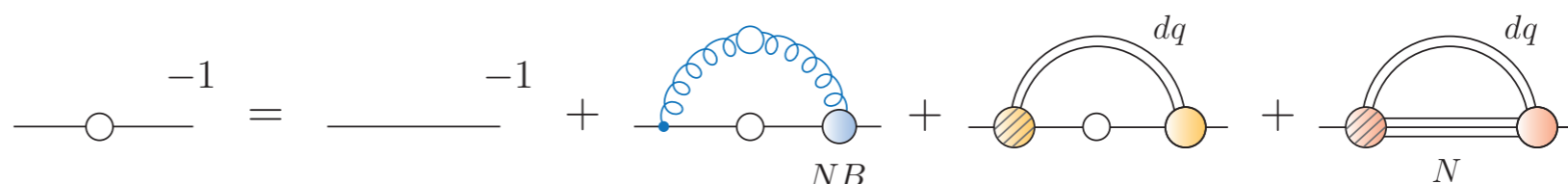
## 1. Introduction



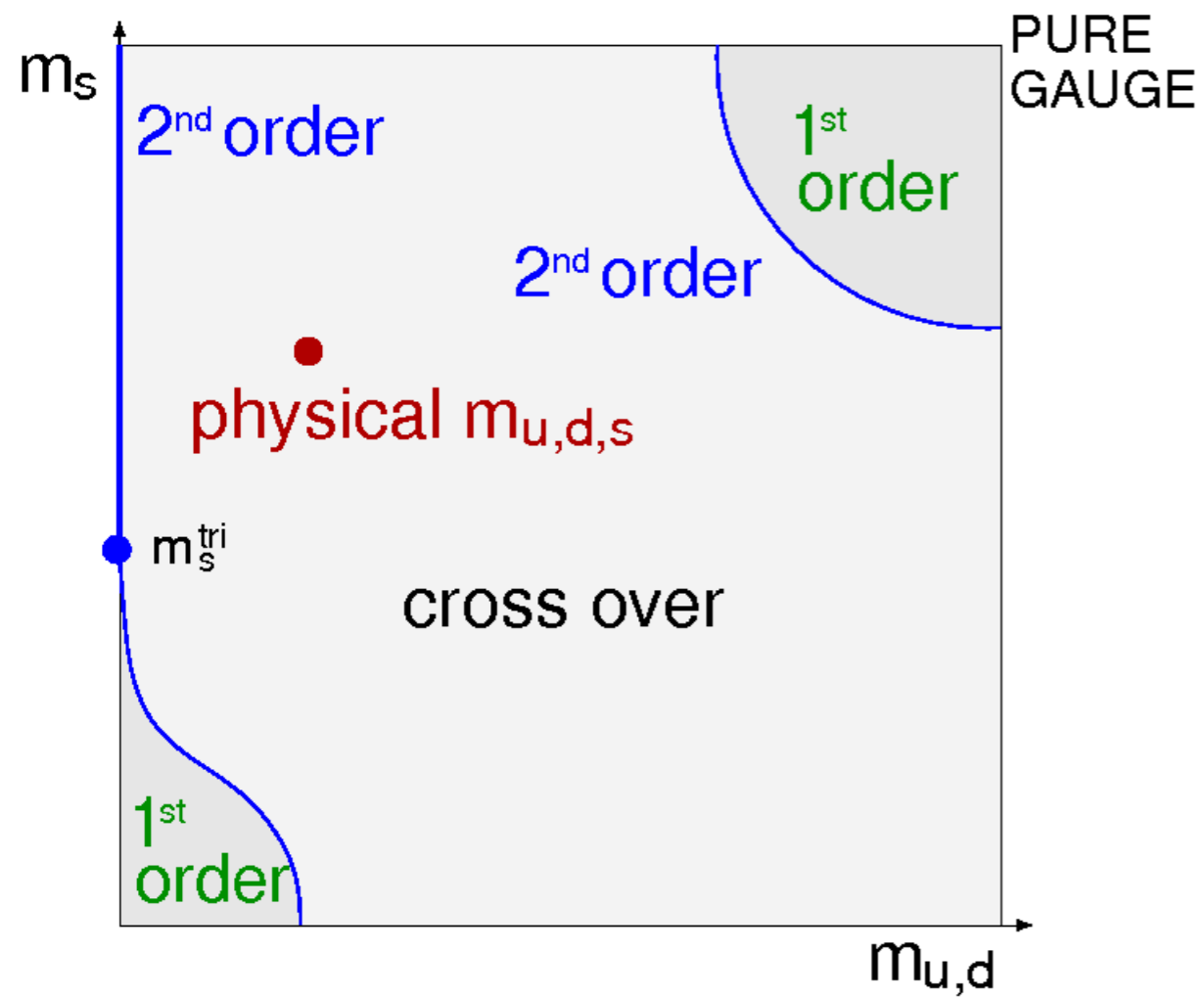
## 2. Gluons, quarks and the CEP



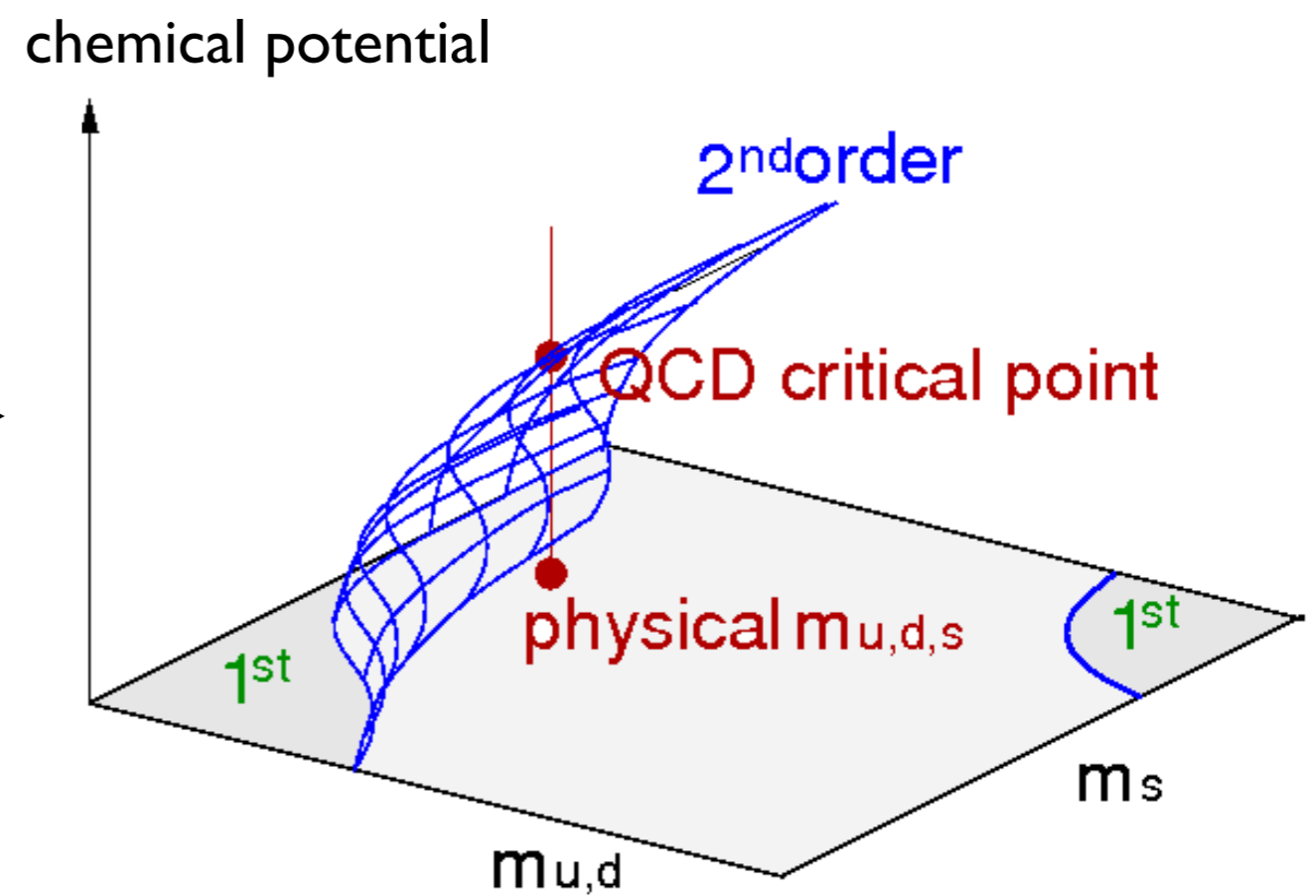
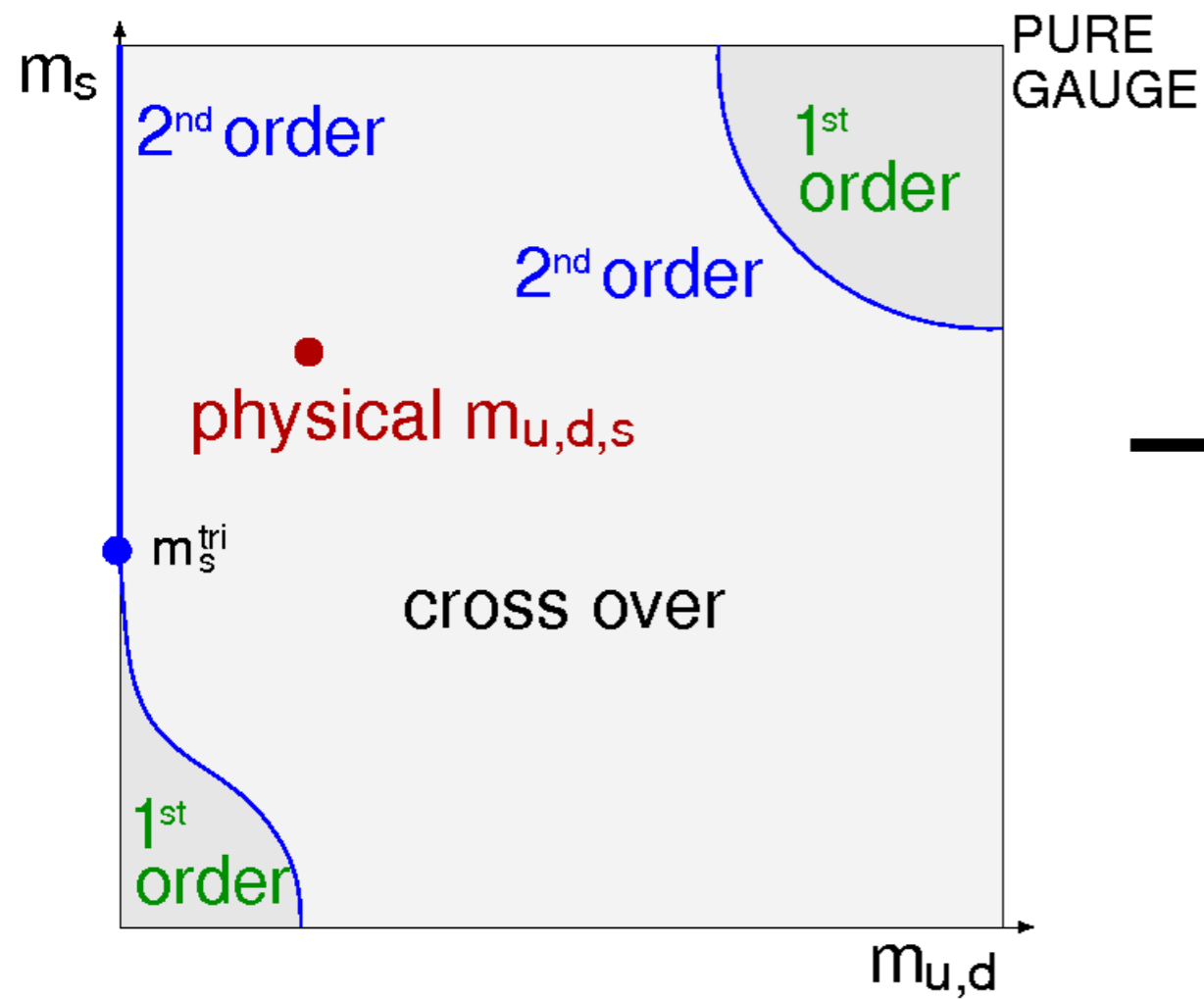
## 3. Baryon effects on the CEP



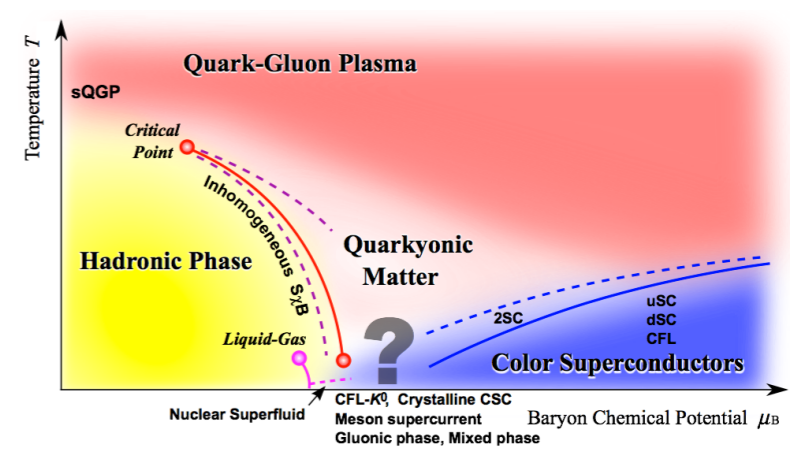
# QCD phase transitions



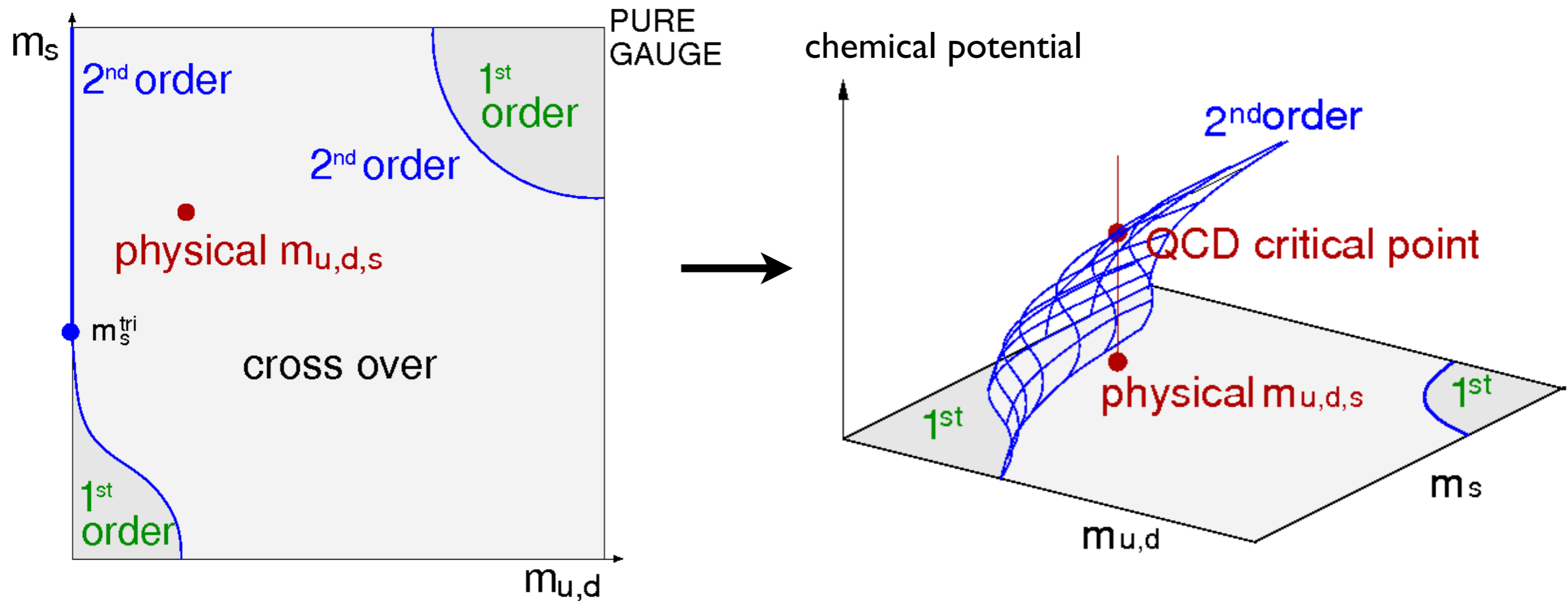
# QCD phase transitions



Is this happening ??

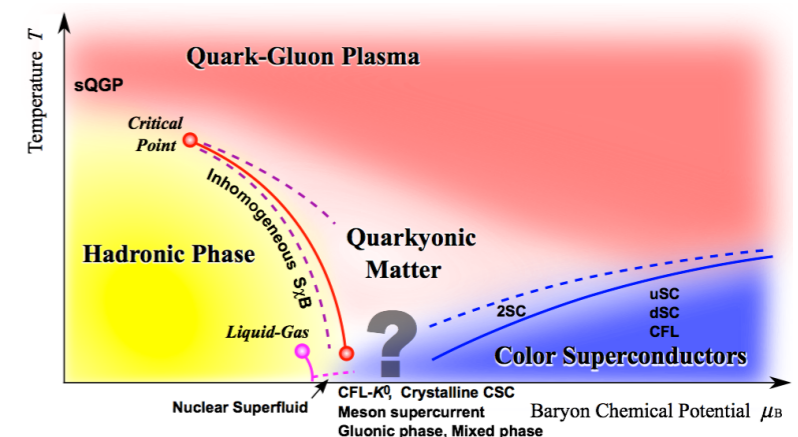


# QCD phase transitions



- Lattice-QCD
  - present: extrapolation
  - future: exact methods ?
- DSE/FRG
  - not exact, but allow for '10%-physics'

Is this happening ??



# Search for the CEP

- Taylor expansion ( $N_f=2$ ):

Gavai, Gupta, PRD 71 (2005) 114014  
Datta, Gavai and Gupta, NPA 904-905 (2013) 883c  
Bazavov et al., PRD 95 (2017) 054504

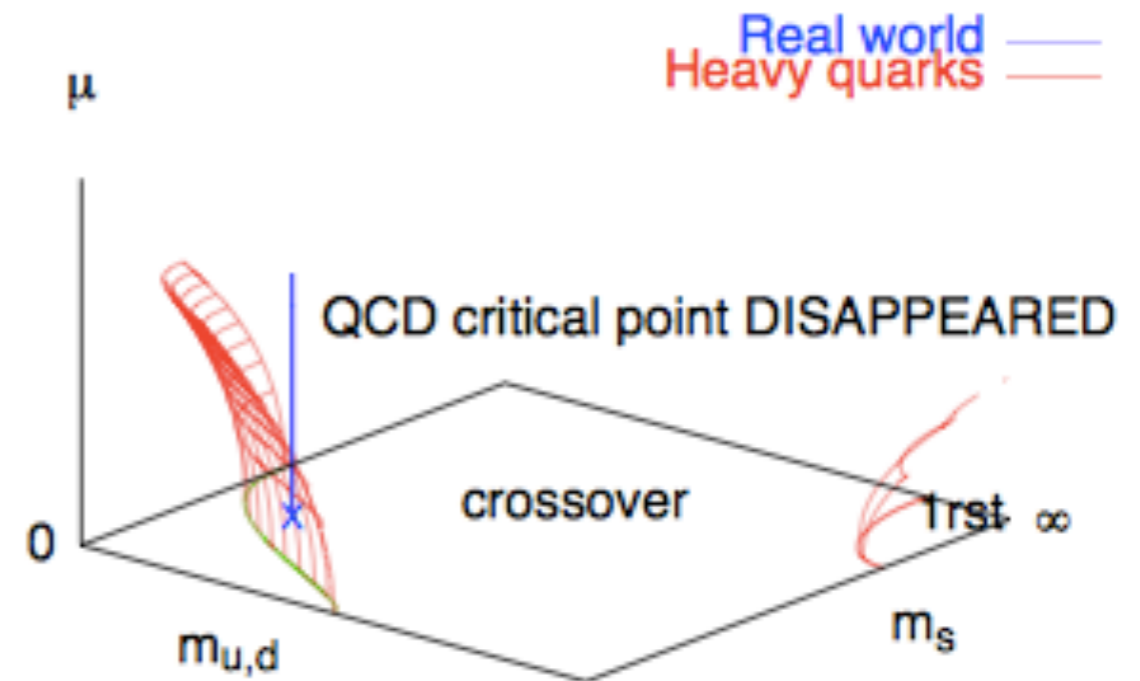
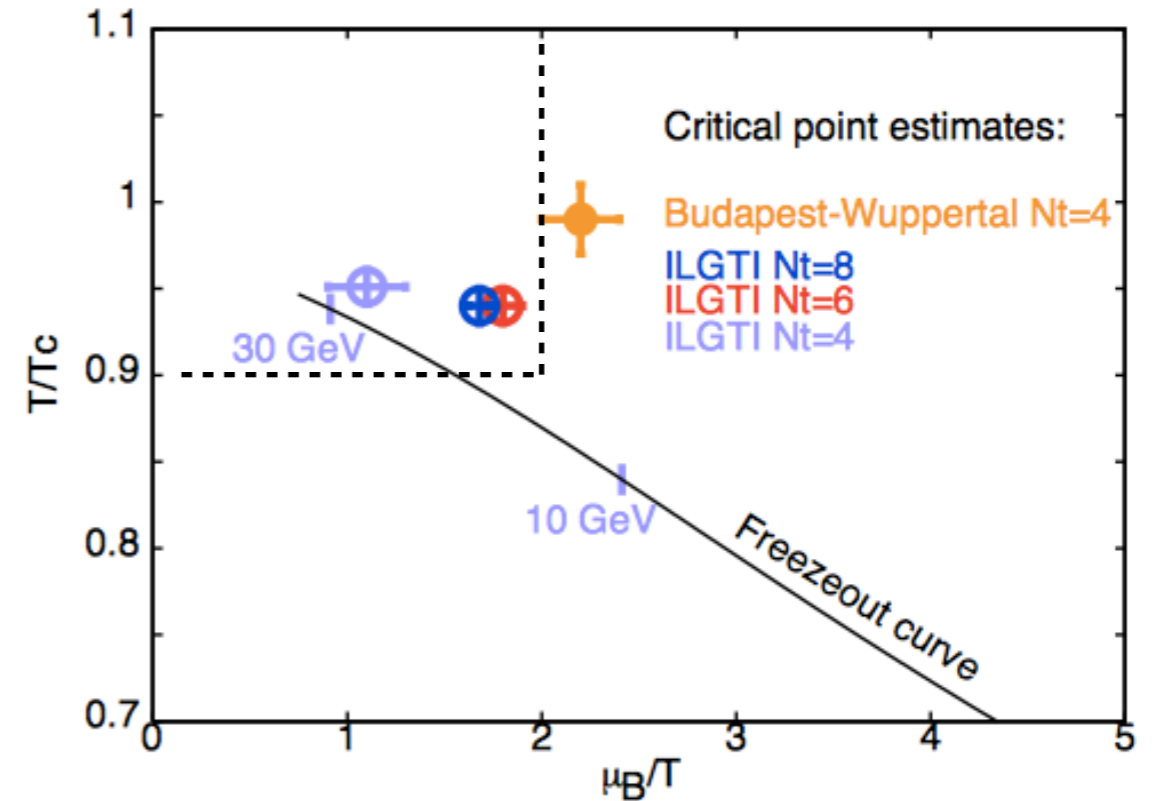
→ C. Schmidt, plenary

- Reweighting ( $N_f=2+1$ ):

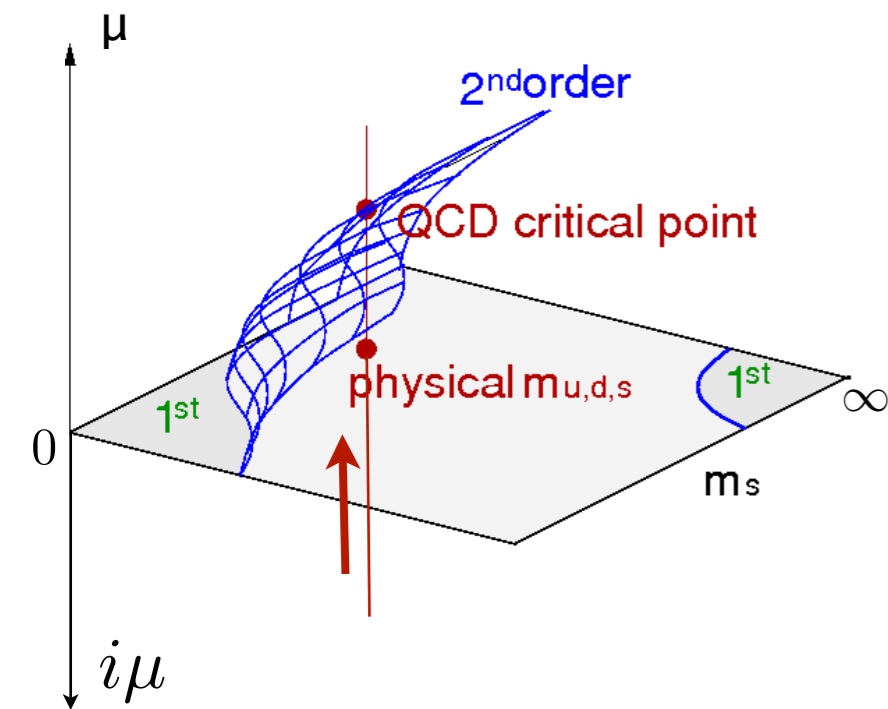
Fodor, Katz, JHEP 0404 (2004) 050

- Analytic continuation ( $N_f=3$ ):

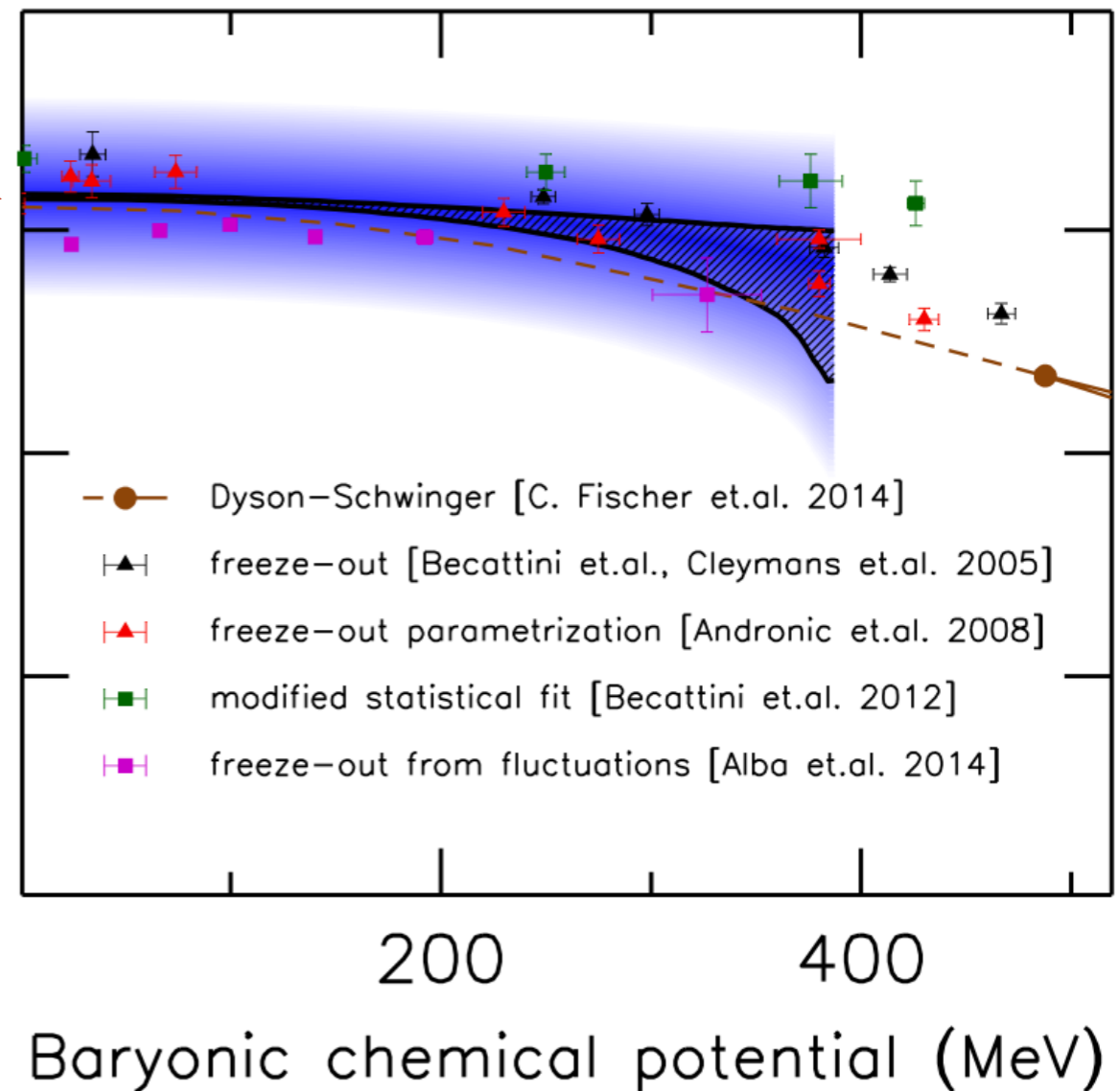
de Forcrand, Philipsen, JHEP 0811 (2008) 012;  
NPB 642 (2002) 290



# Chiral transition line from analytic continuation



Temperature (MeV)



Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB B 751 (2015) 559

→ J. Günther, plenary

## Lattice method:

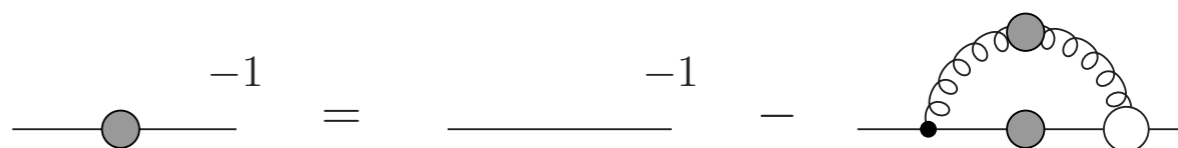
- Calc. boundary at imaginary  $\mu$  and extrapolate to real  $\mu$
- Control systematics

## Results:

- Larger curvature than previous results (but: different definitions and error budget)

Chiral order parameter:

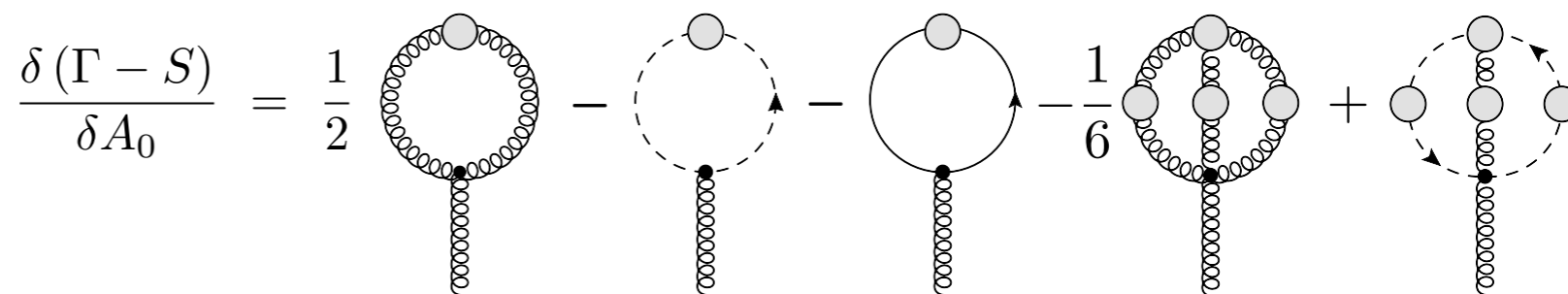
$$\langle \bar{\Psi} \Psi \rangle = Z_2 N_c \text{Tr}_D \frac{1}{T} \sum_{\omega} \int \frac{d^3 p}{(2\pi)^3} S(\vec{p}, \omega)$$



Deconfinement:

- Polyakov loop potential

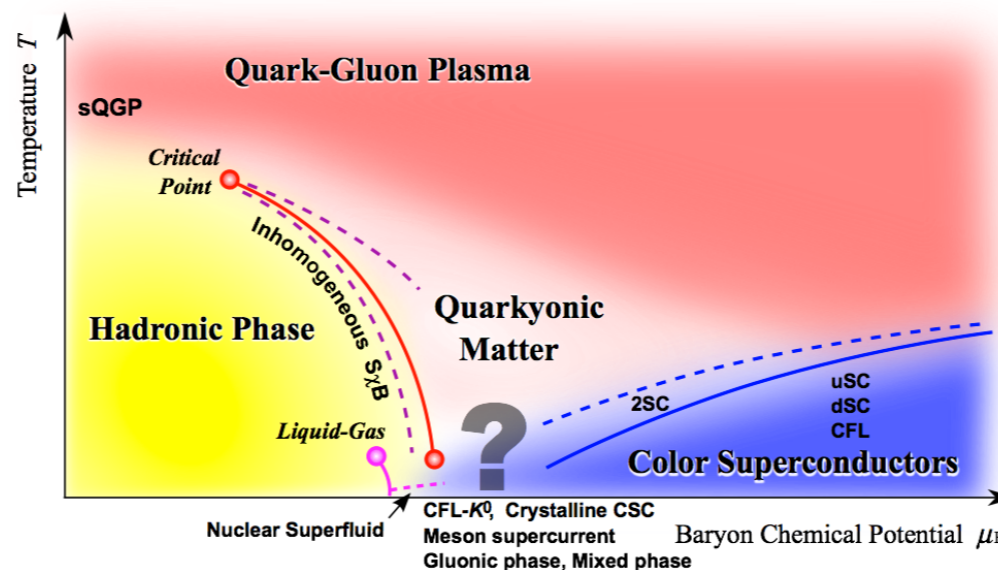
$$L = \frac{1}{N_c} \text{Tr} e^{ig\beta A_0}$$



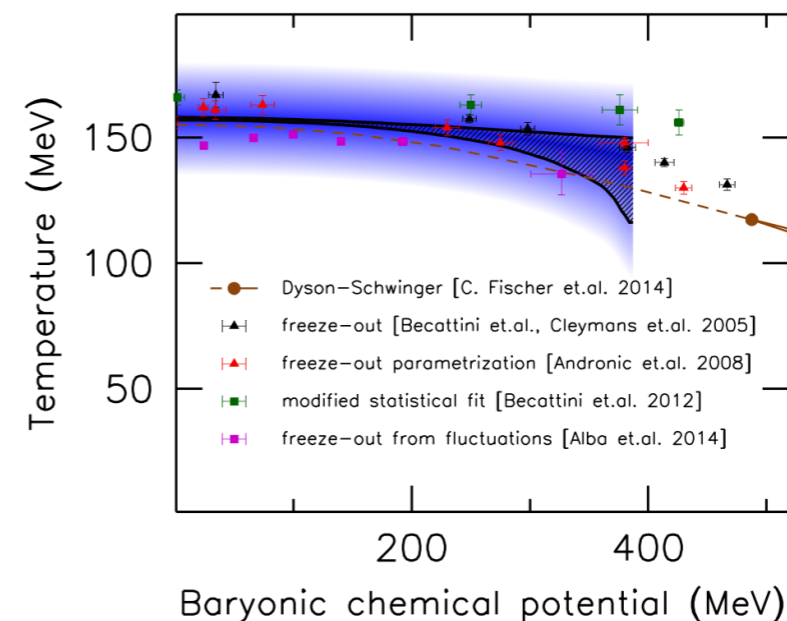
Braun, Gies, Pawłowski, PLB 684, 262 (2010)  
 Braun, Haas, Marhauser, Pawłowski, PRL 106 (2011)  
 Fister, Pawłowski, PRD 88 045010 (2013)  
 CF, Fister, Luecker, Pawłowski, PLB 732 (2013)



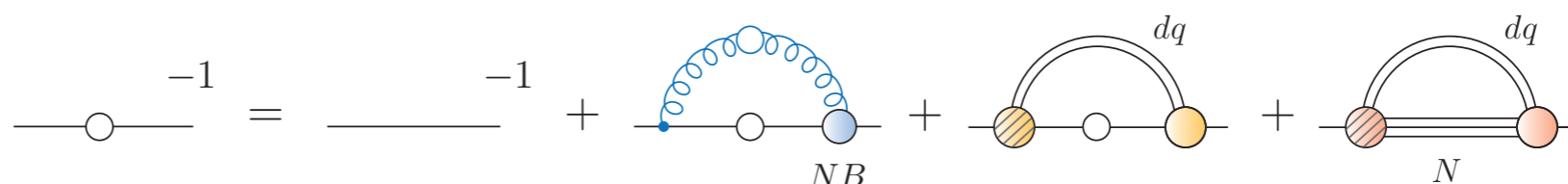
## 1. Introduction



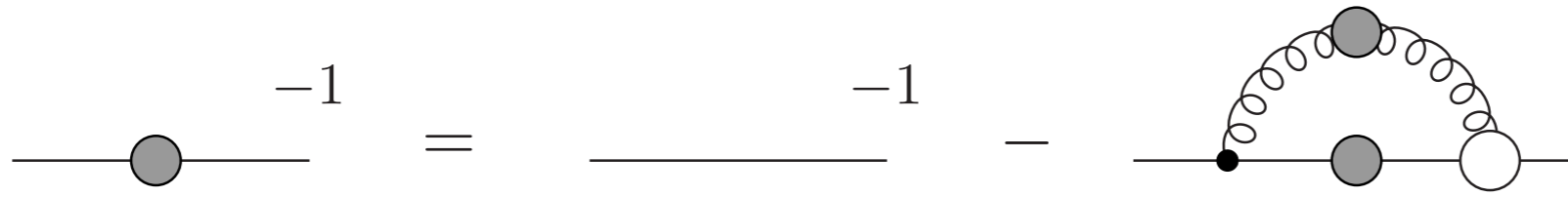
## 2. Gluons, quarks and the CEP



## 3. Baryon effects on the CEP



# The DSE for the quark propagator



The diagram illustrates the Dyson-Schwinger equation for the quark propagator. On the left, a horizontal line with a grey circle in the middle is followed by a minus sign and a superscript -1. This is set equal to a horizontal line with a minus sign and a superscript -1, minus a diagram. The diagram on the right shows a horizontal line with three vertices: a black dot on the left, a grey circle in the middle, and a white circle on the right. A curved line of small circles (representing a gluon) connects the black dot and the grey circle, with a grey circle at its top vertex.

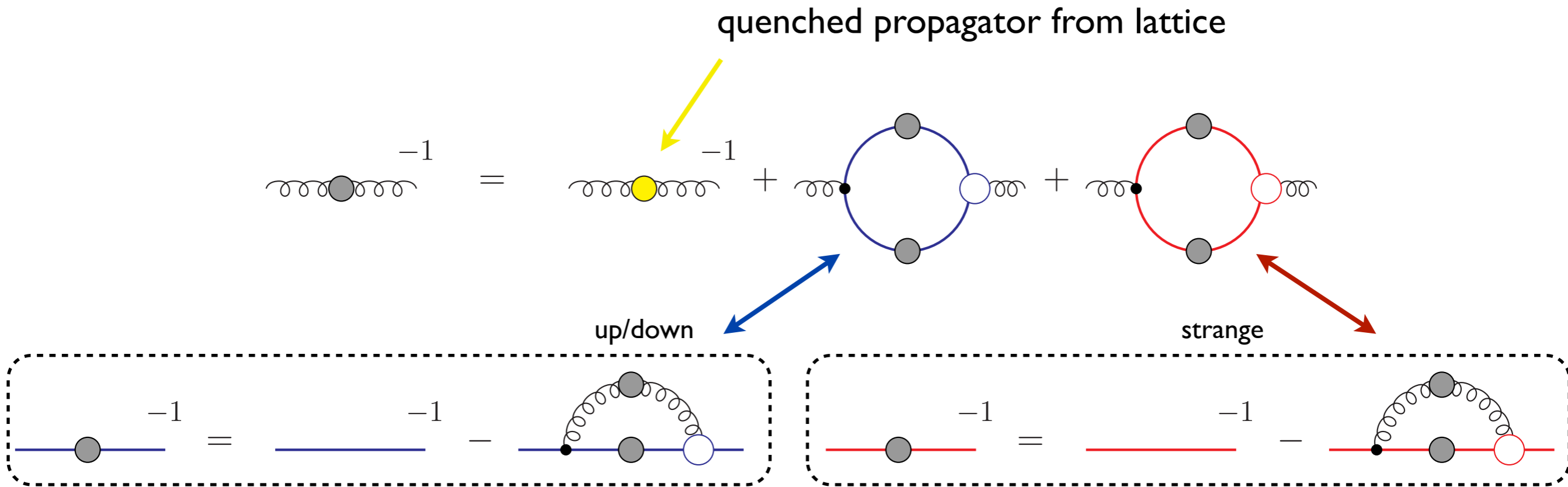
Two strategies: I. use **model** for gluon

Qin, Chang, Chen, Liu and Roberts, PRL 106 (2011) 172301  
Gutierrez, Ahmad, Ayala, Bashir and Raya, JPG 41 (2014) 075002

- ok for first insights
- not good enough for systematic study

II. **treat Yang-Mills sector explicitly**

# $N_f=2+1$ -QCD with DSEs



- quenched: without quark-loop
- $N_f=2$ : isospin symmetry
- $N_f=2+1$ : solve coupled system of  $2+3+3$  equations
- Vertex: ansatz built along STI and known UV/IR behavior  
→  $T, \mu, m$ -dependent

# Approximation for Quark-Gluon interaction

- T,μ,m-dependent vertex:

Abelian WTI

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times$$

$$\times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2 / \Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

perturbation theory

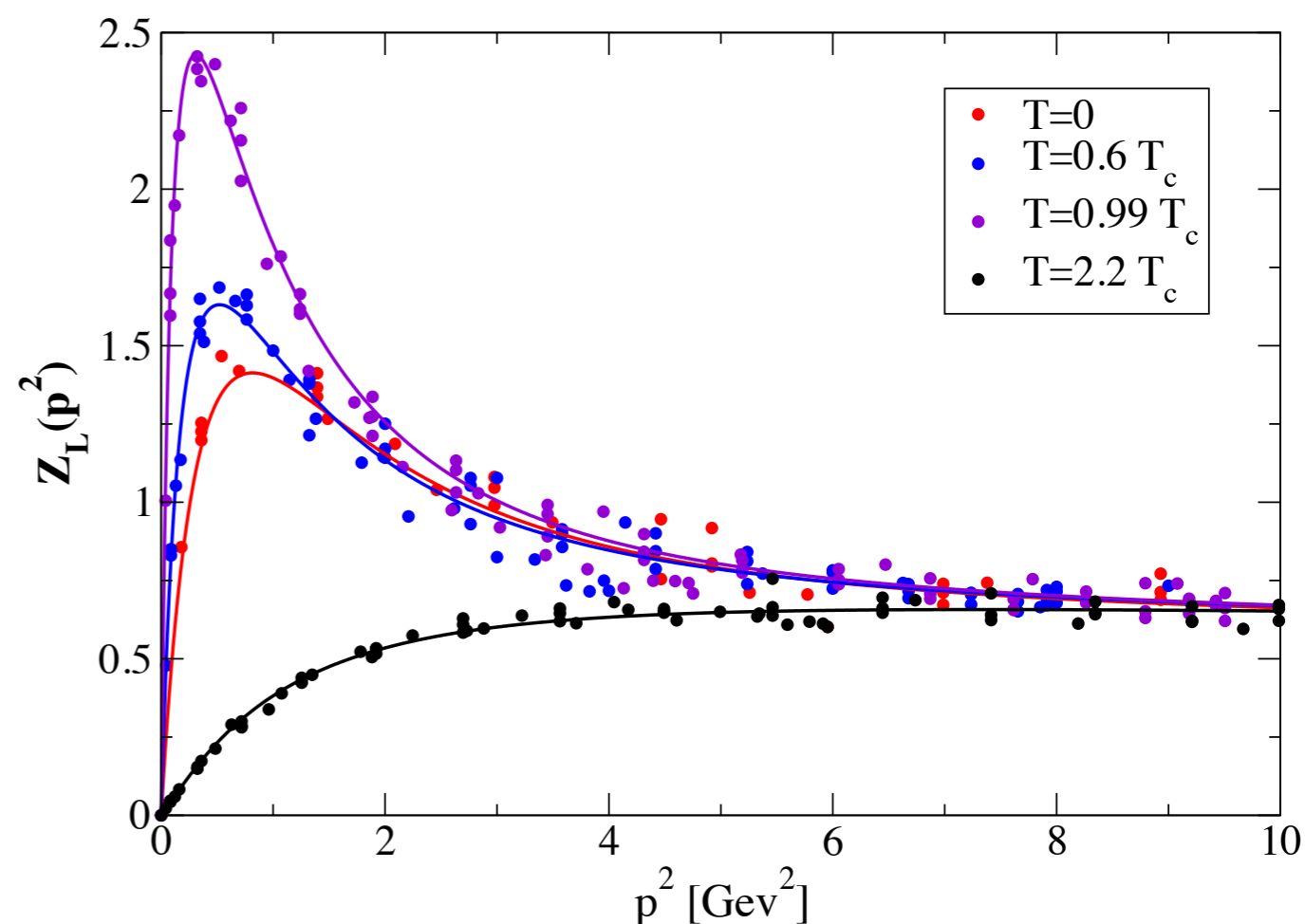
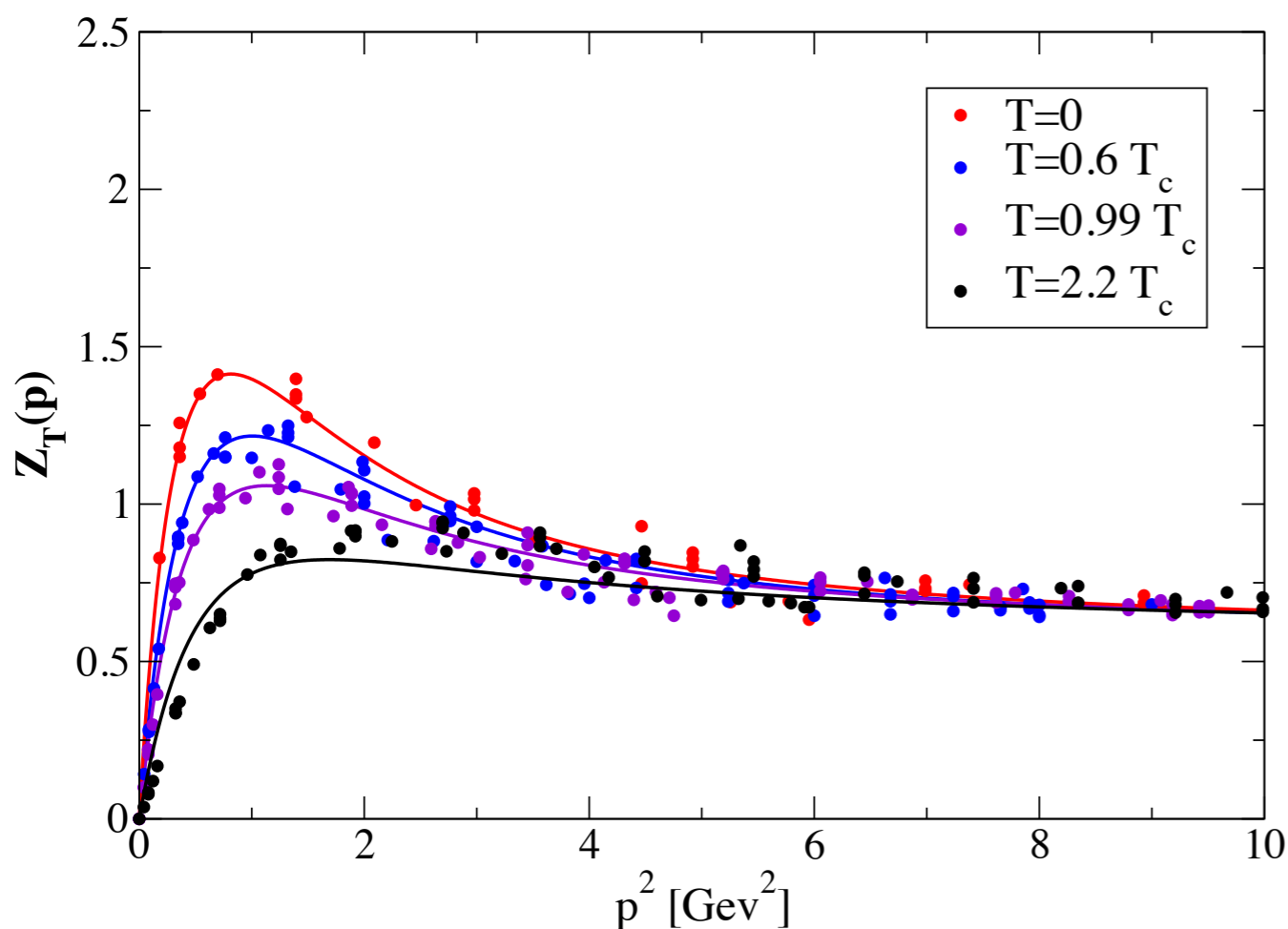
Infrared ansatz:

- d2 fixed to match gluon input
- d1 fixed via quark condensate (see later)
- correct UV (quant.) and IR-behavior (qual.)

explicit solutions at T=0: Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035  
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

# Glue at finite temperature ( $T \neq 0$ )

T-dependent gluon propagator from quenched lattice simulations:



- Crucial difference between magnetic and electric gluon
- Maximum of electric gluon near  $T_c$

Cucchieri, Maas, Mendes, PRD 75 (2007)

CF, Maas, Mueller, EPJC 68 (2010)

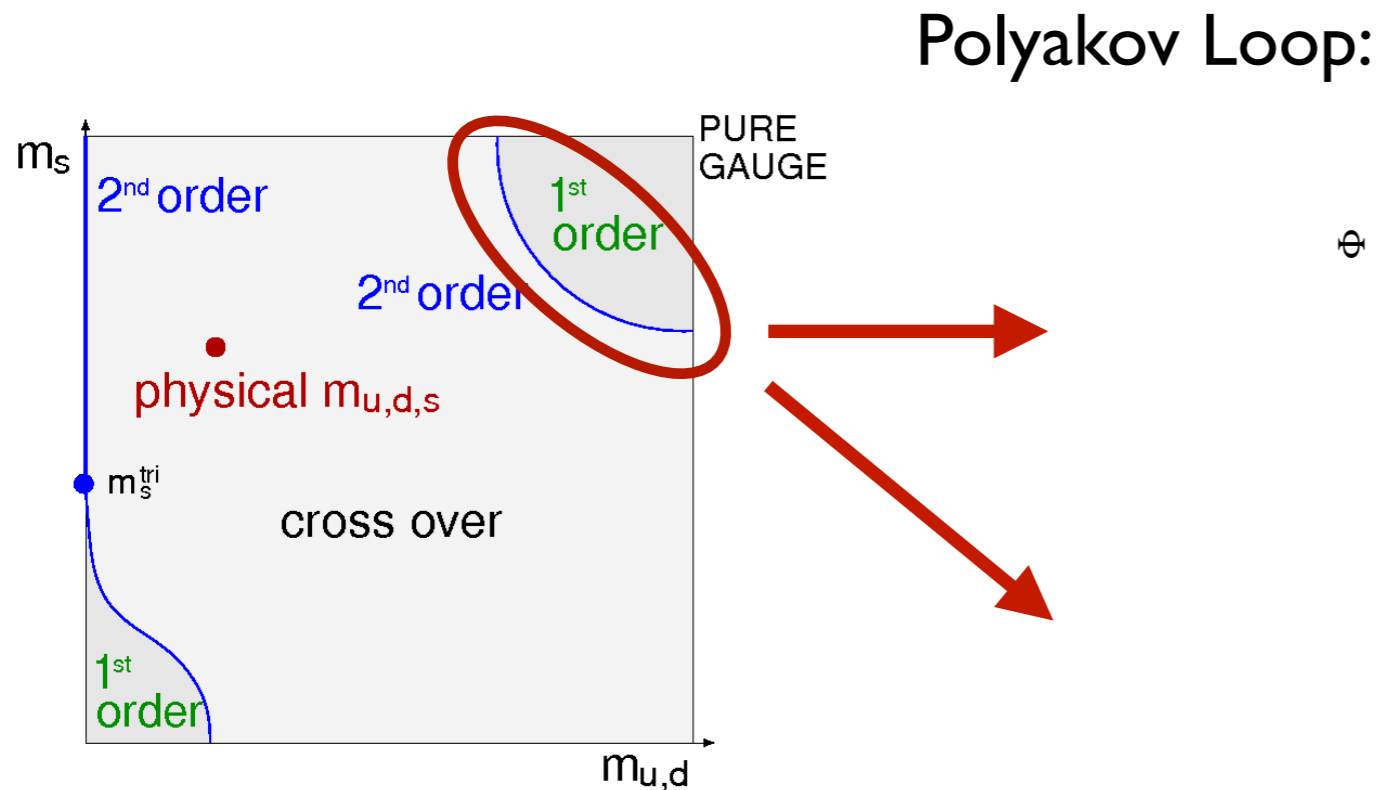
Cucchieri, Mendes, PoS FACESQCD 007 (2010)

Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker and Sternbeck, PRD 85 (2012) 034501

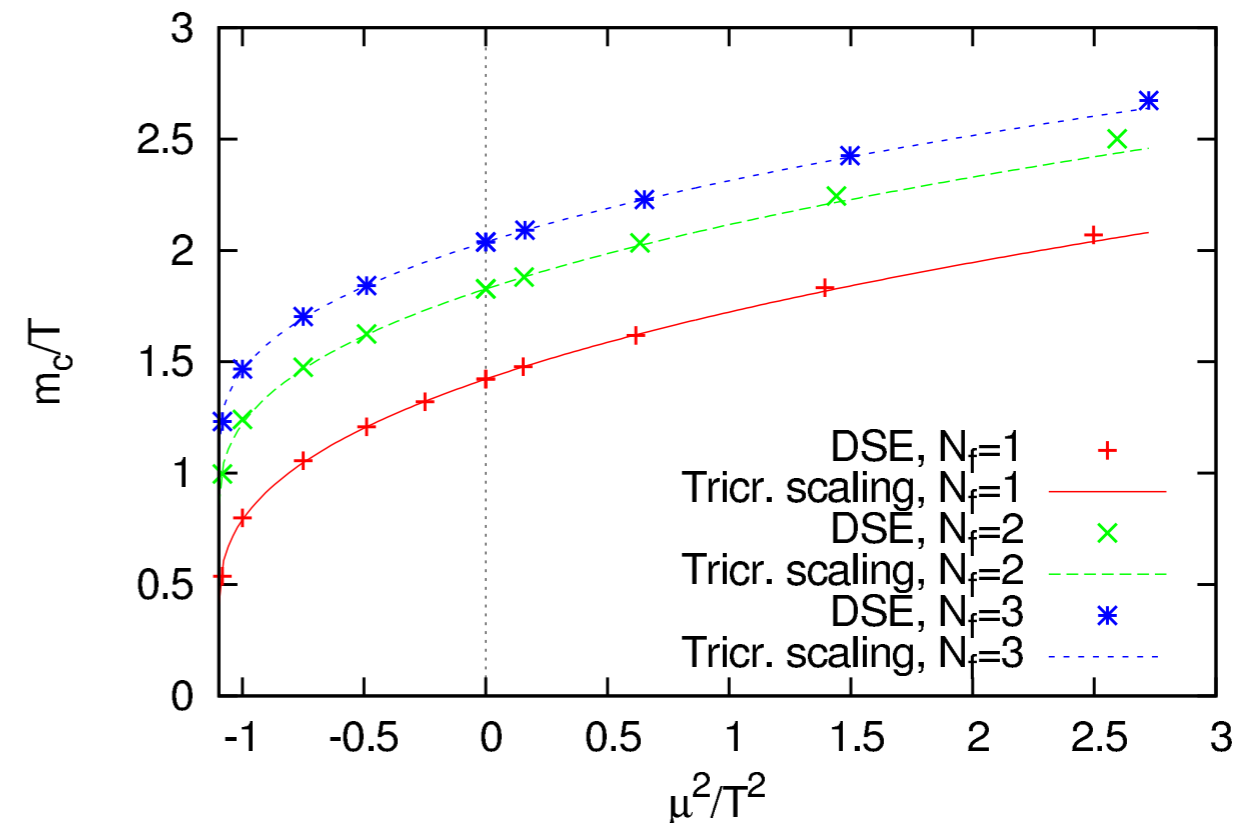
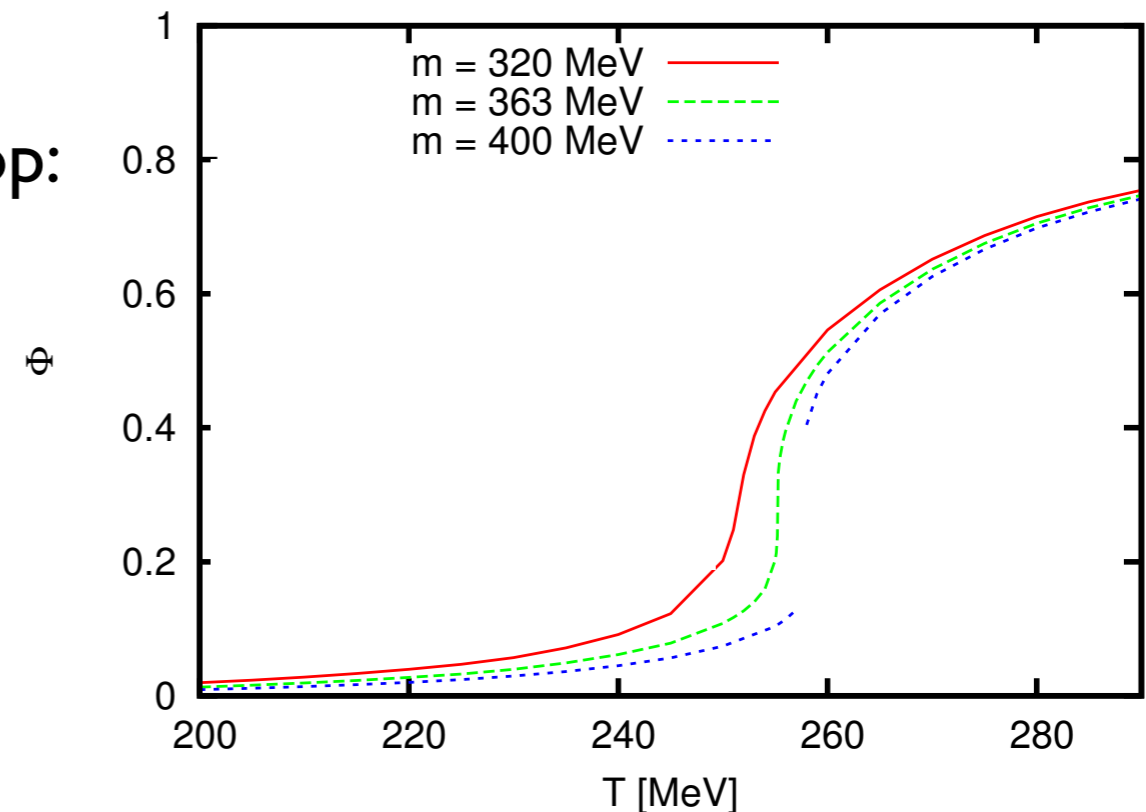
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503

FRG: Fister, Pawlowski, arXiv:1112.5440

# Critical line/surface for heavy quarks



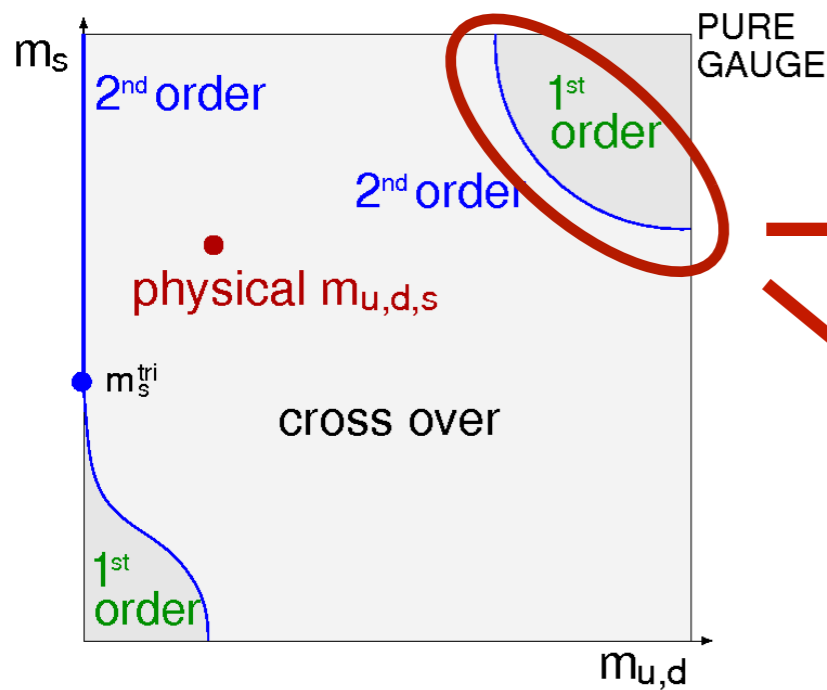
- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen



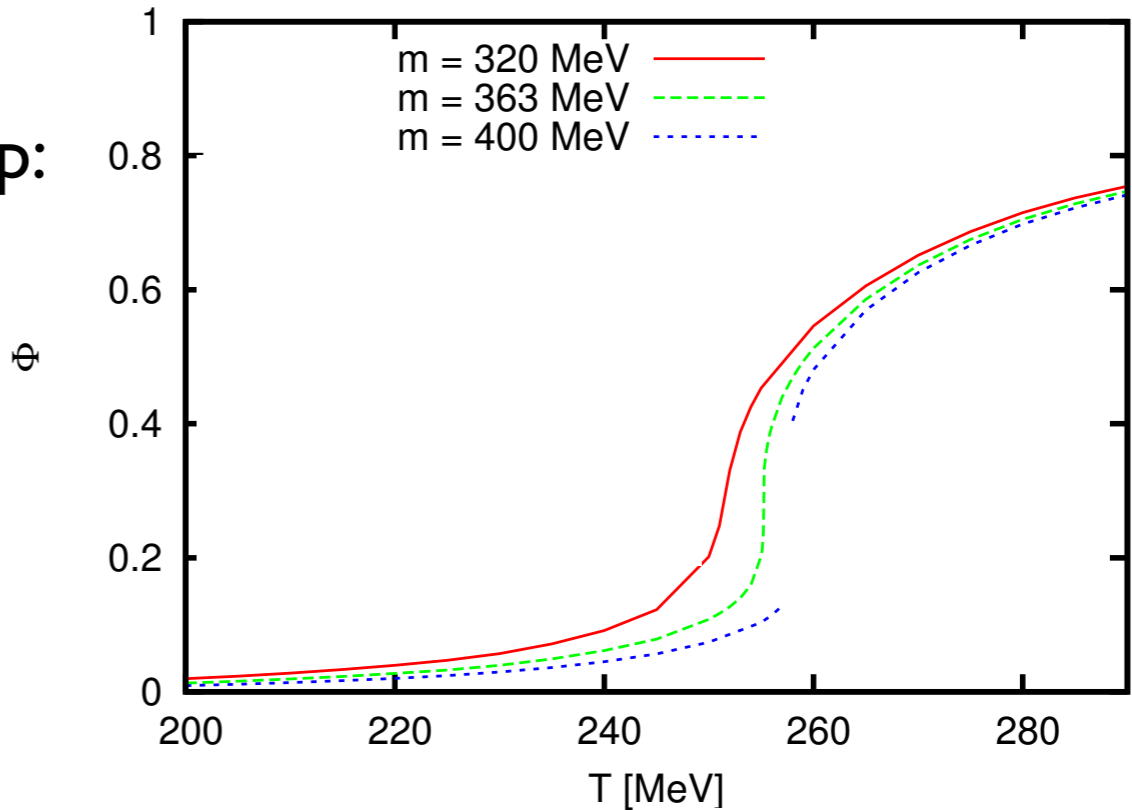
Lattice:  
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawłowski, PRD 91 (2015) 1

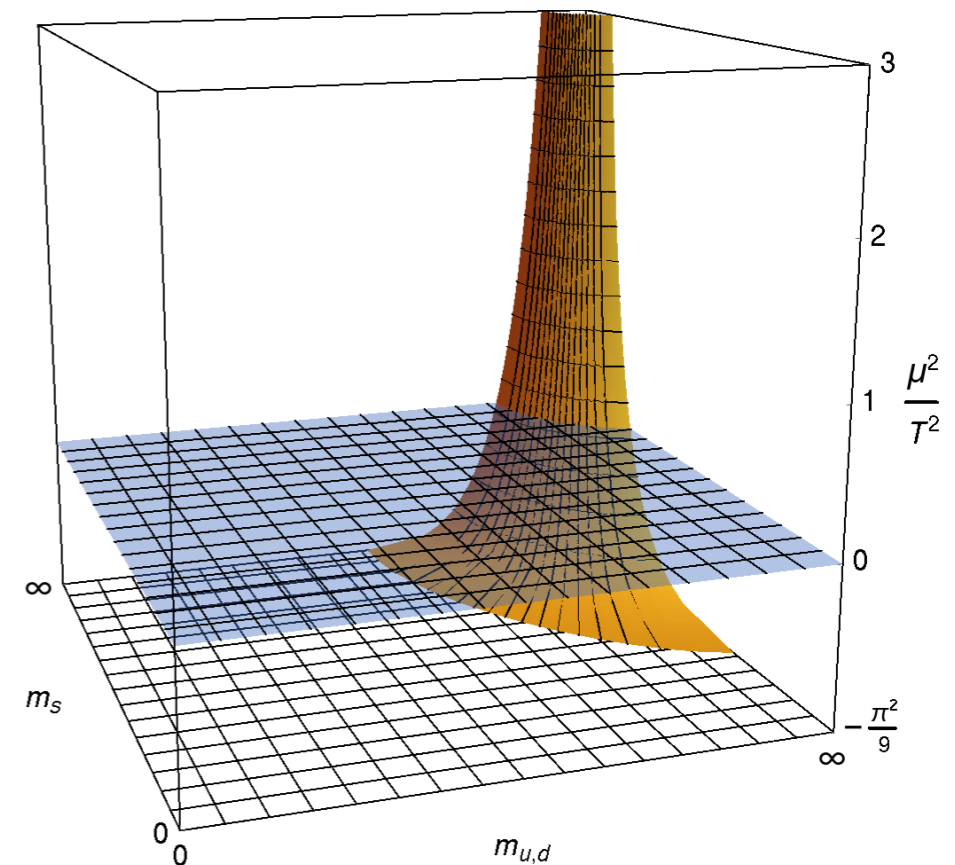
# Critical line/surface for heavy quarks



Polyakov Loop:



- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

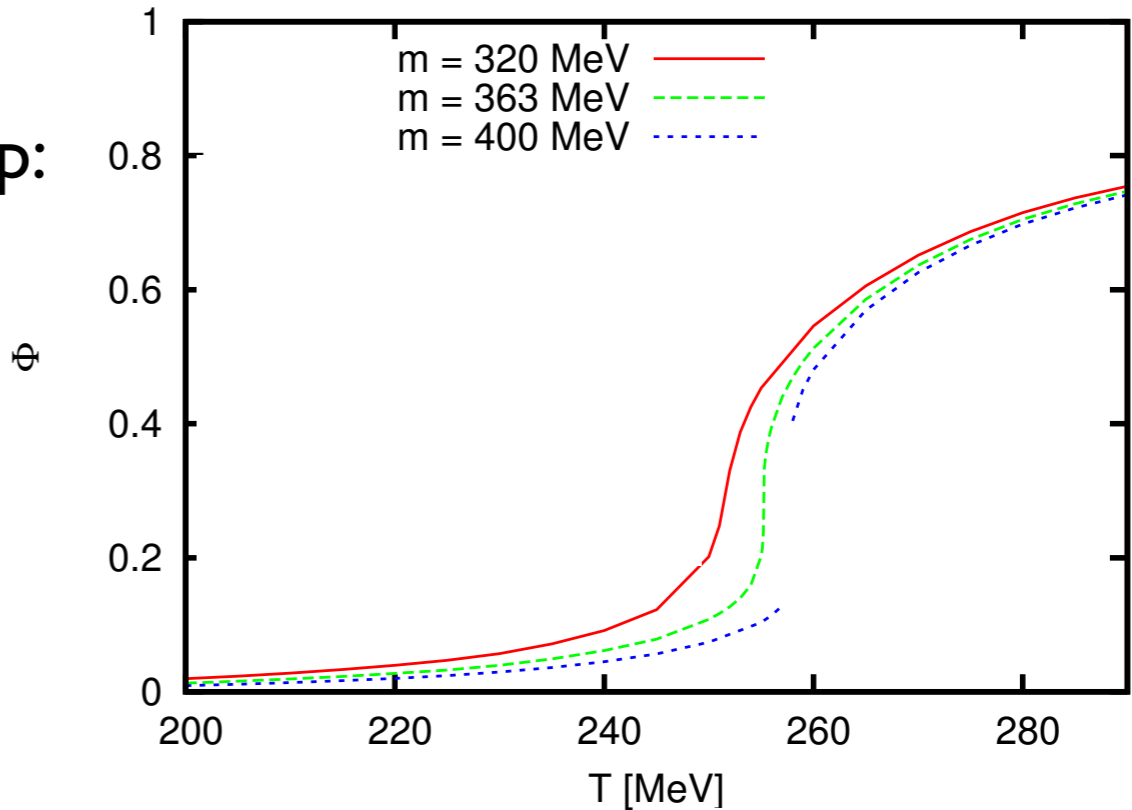
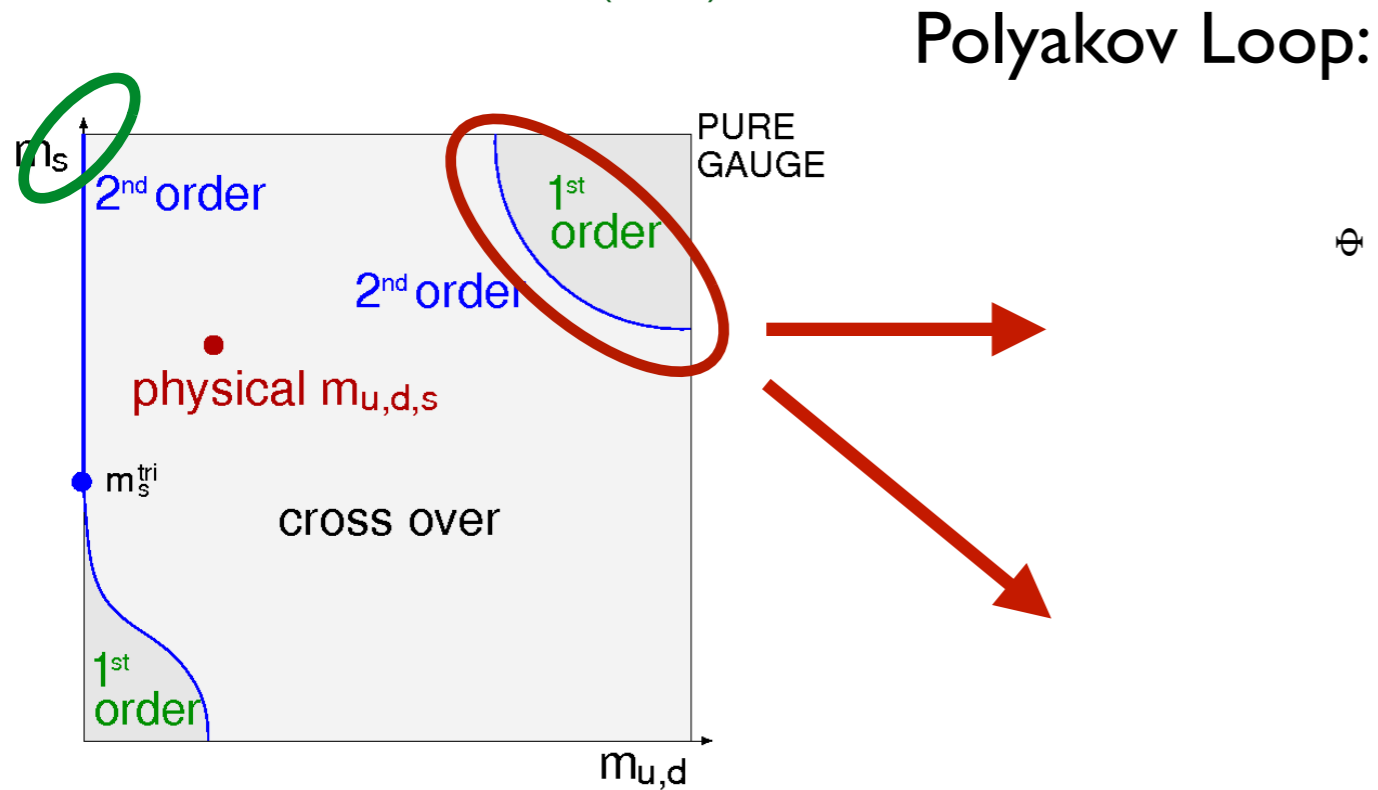


CF, Luecker, Pawlowski, PRD 91 (2015) 1

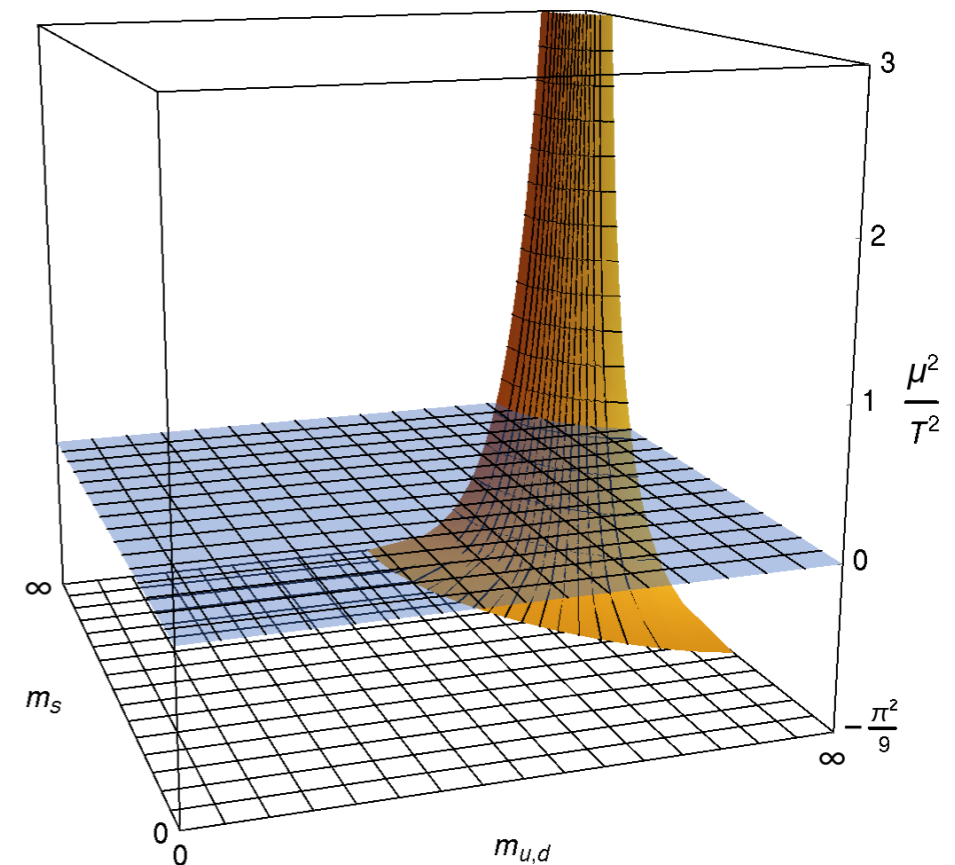
Lattice:  
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

# Critical line/surface for heavy quarks

Nf=2: CF and Mueller, PRD 84 (2011) 054013



- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen



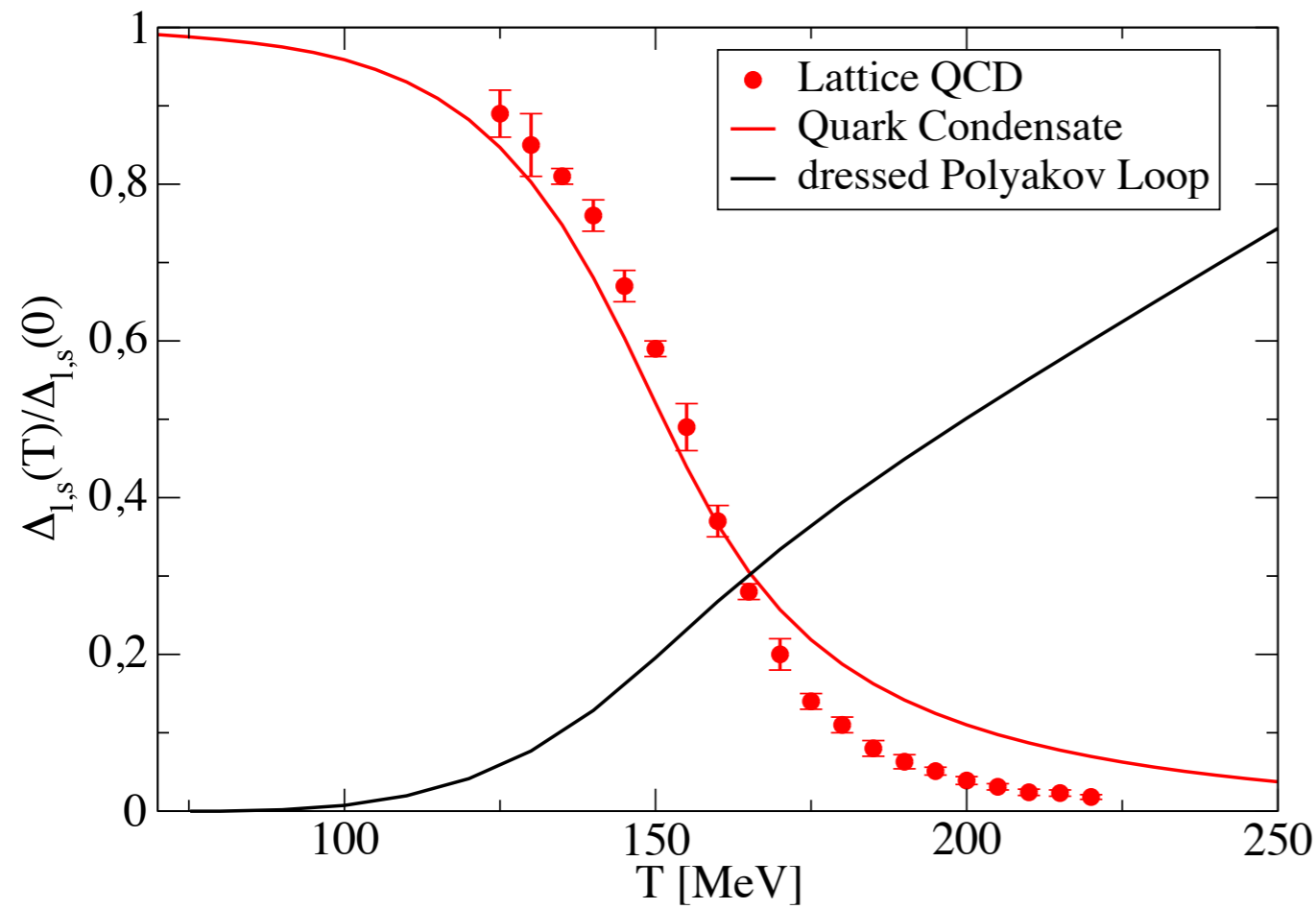
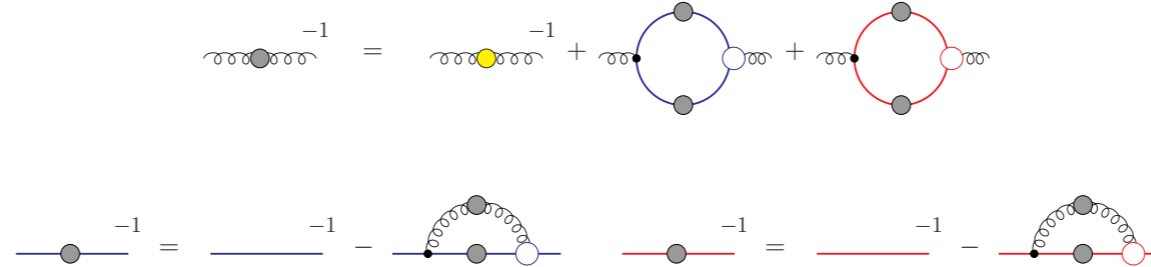
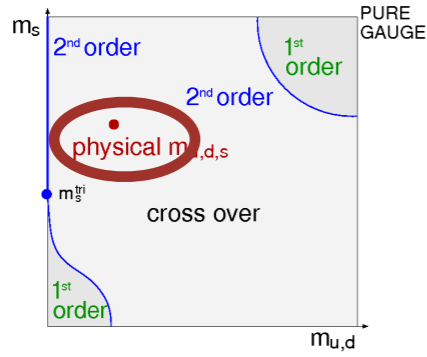
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Lattice:

Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042



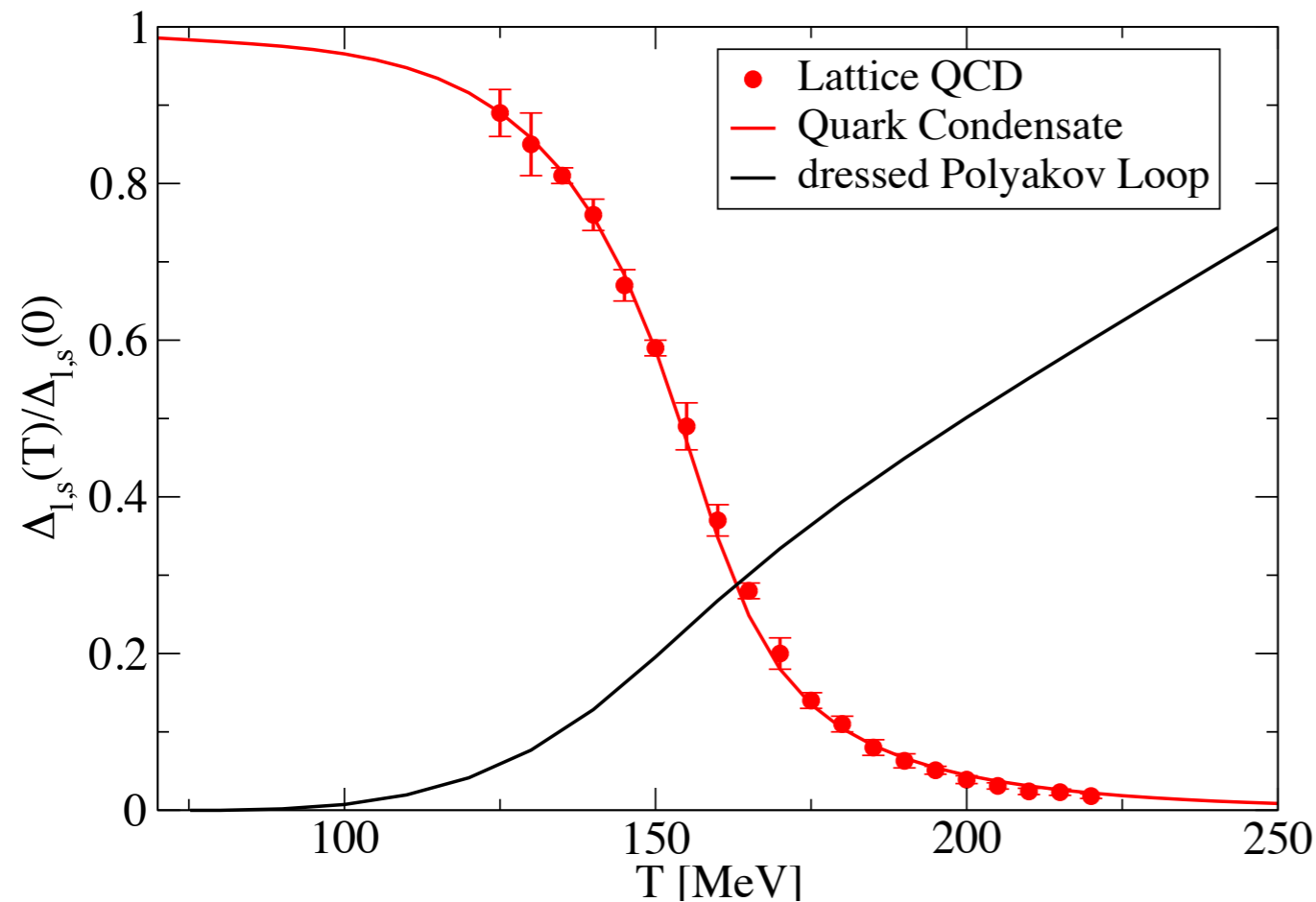
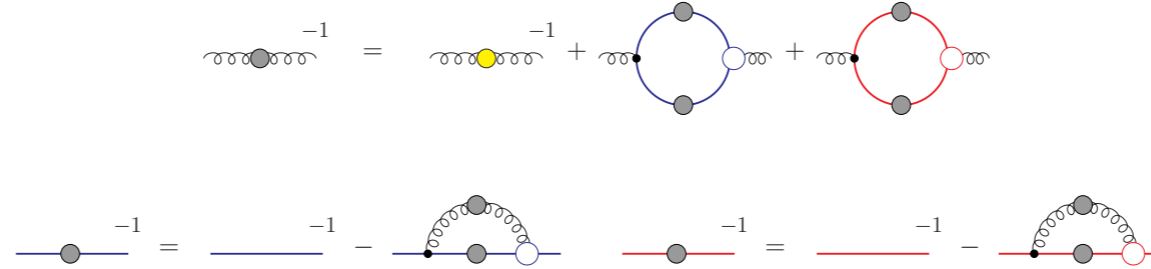
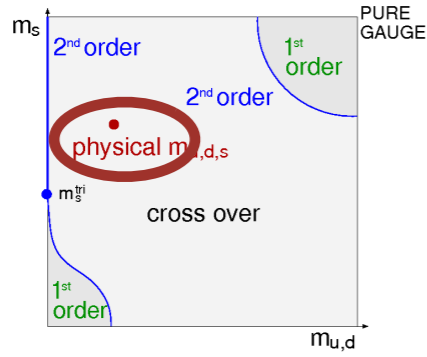
# $N_f=2+1, \mu=0$ , physical point



Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073

DSE: CF, Luecker, PLB 718 (2013) 1036,  
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

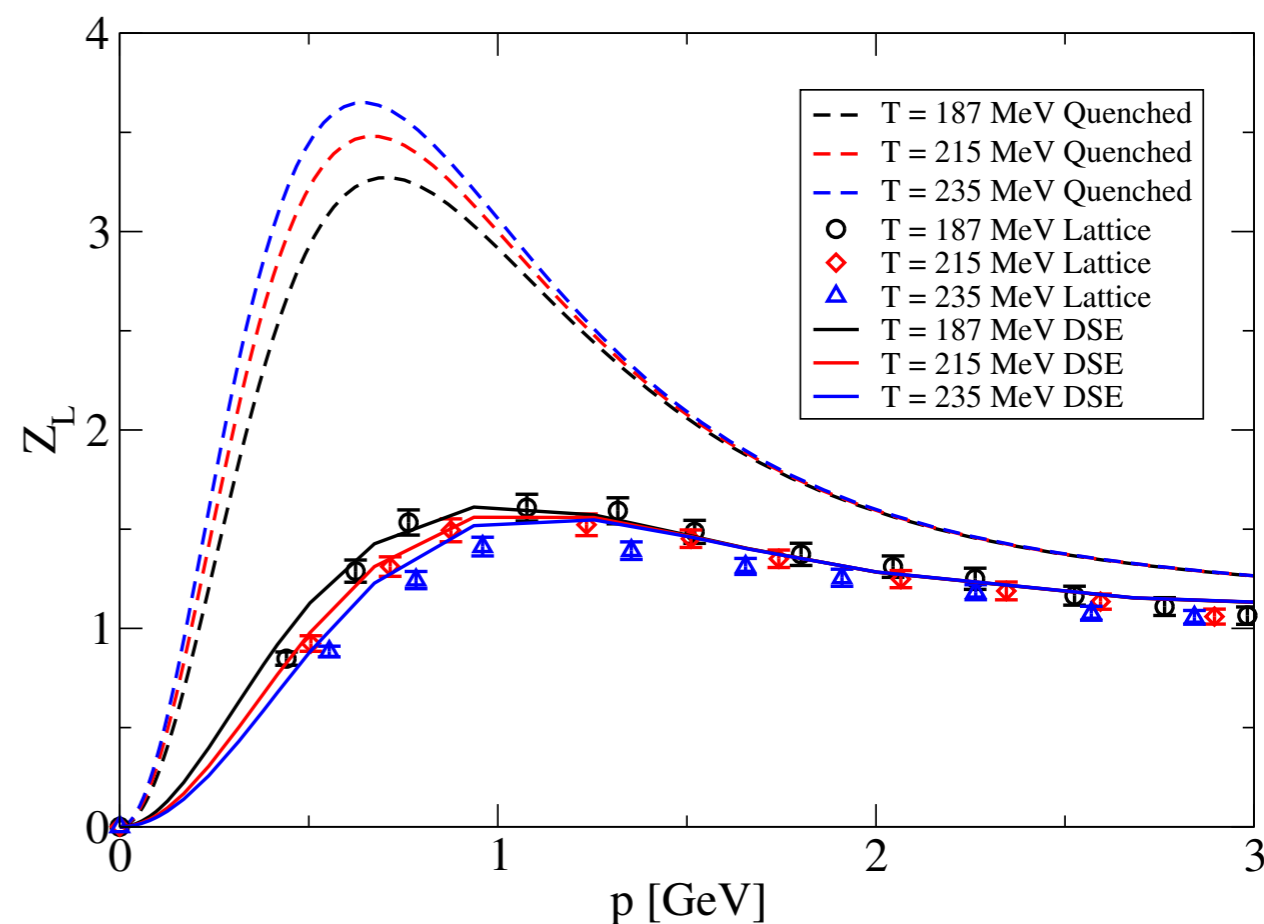
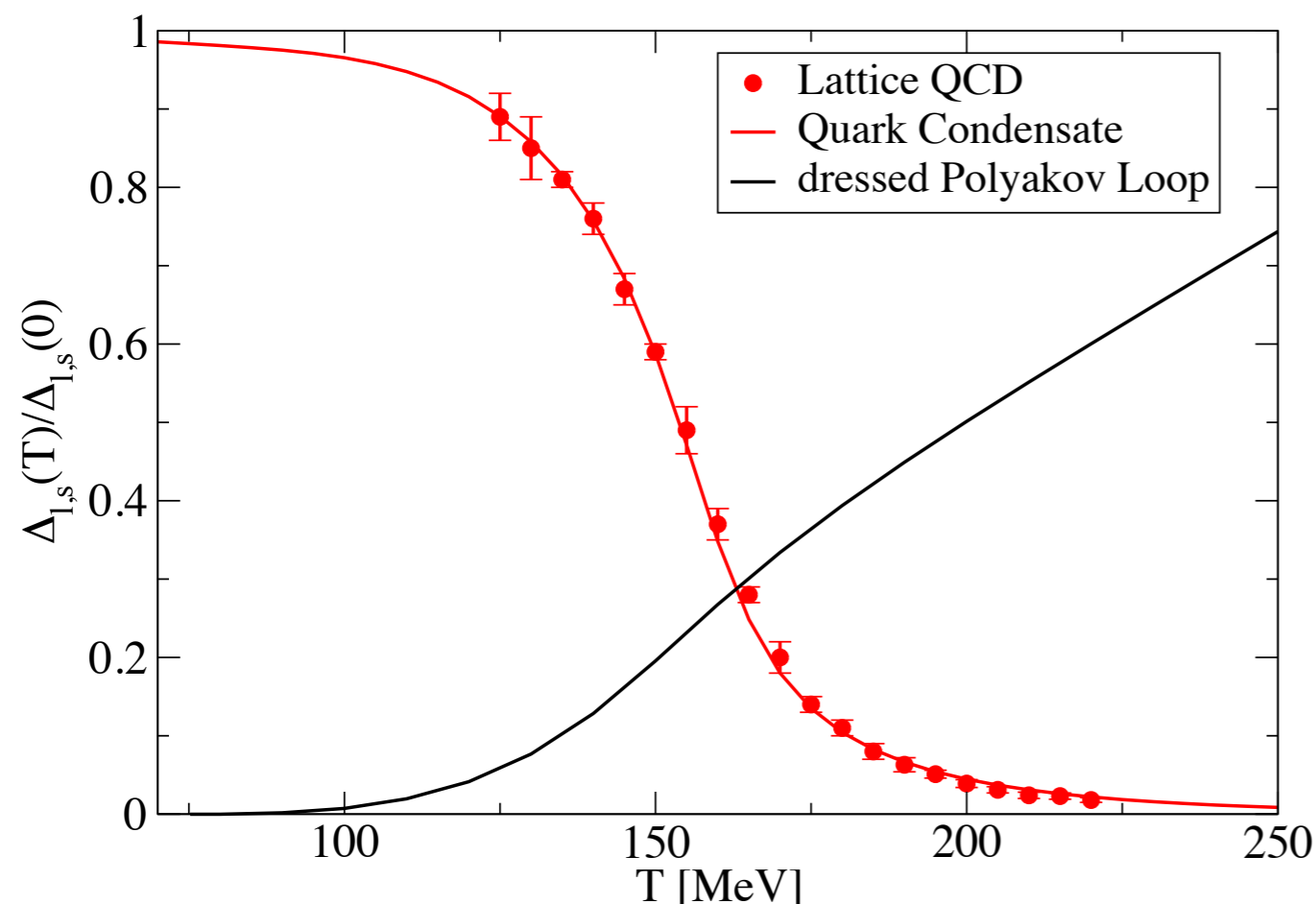
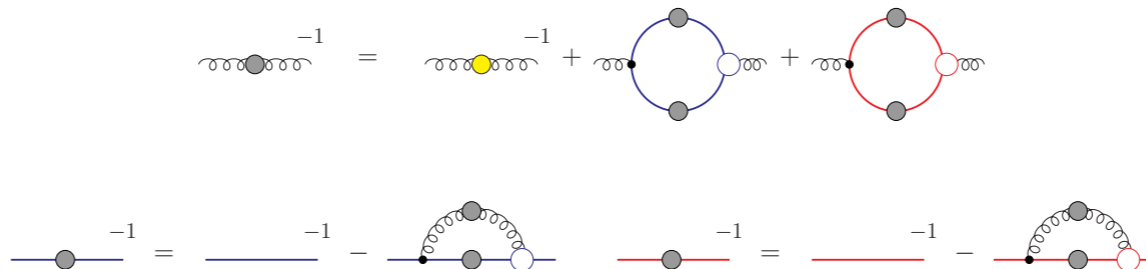
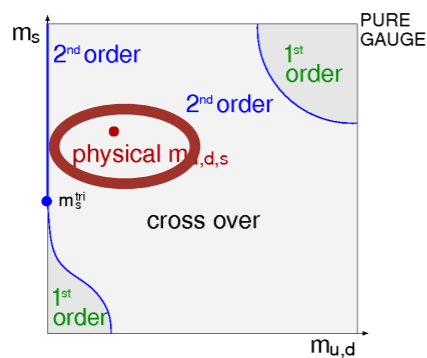
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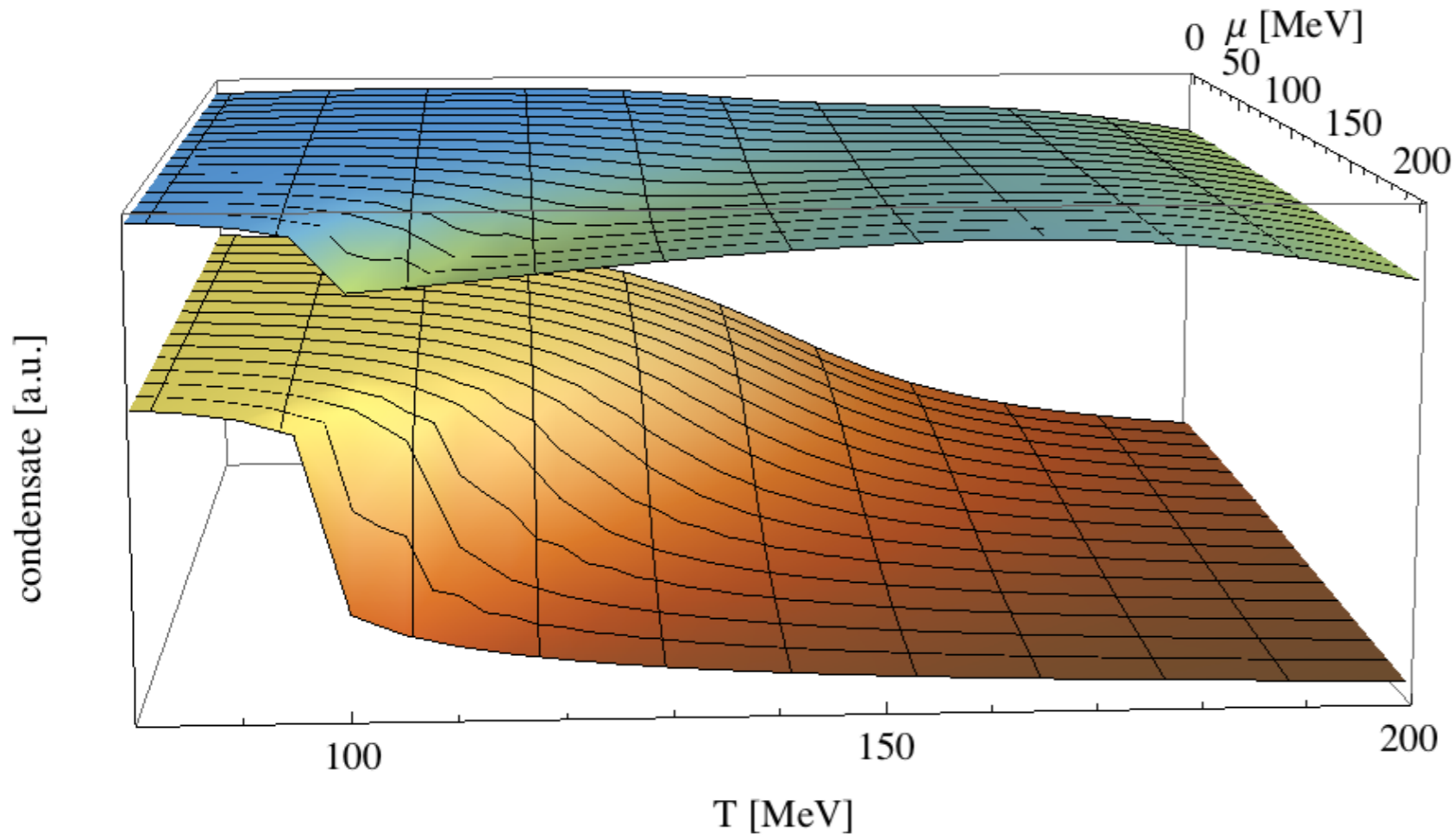


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 DSE: CF, Luecker, PLB 718 (2013) 1036,  
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Lattice: Aouane, *et al.* PRD D87 (2013), [arXiv:1212.1102]  
 DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

● quantitative agreement: DSE prediction verified by lattice

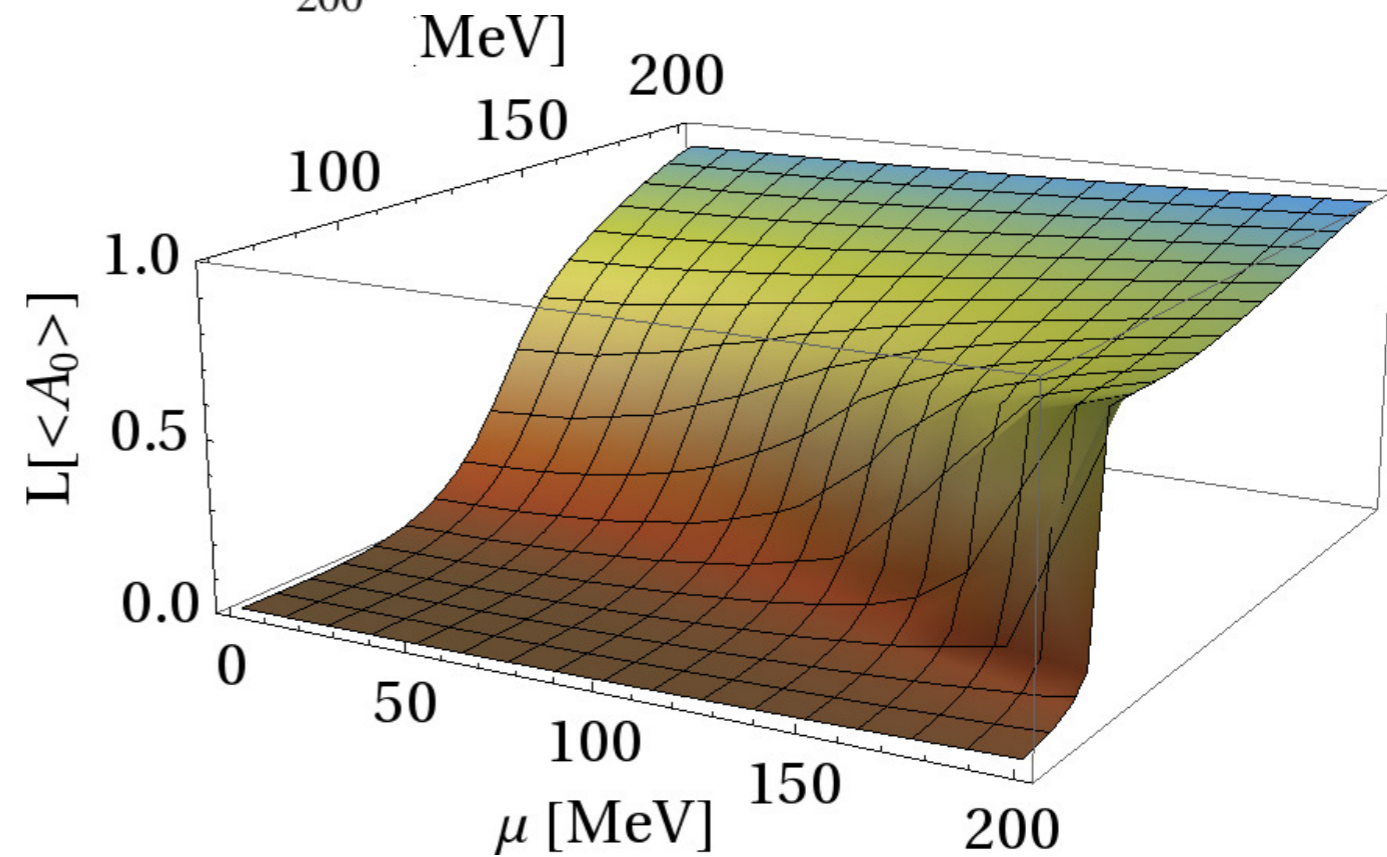
# Nf=2+1: Condensate and dressed Polyakov Loop



Quark condensate

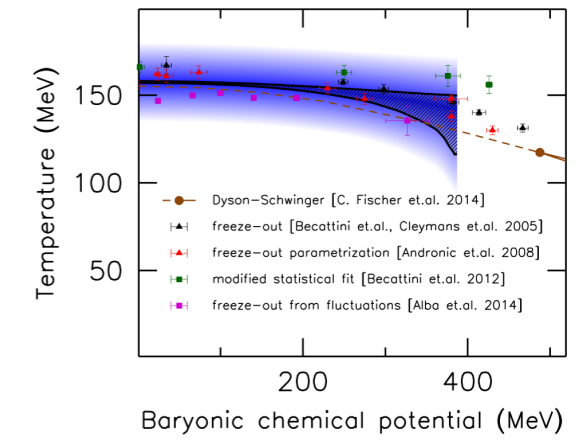
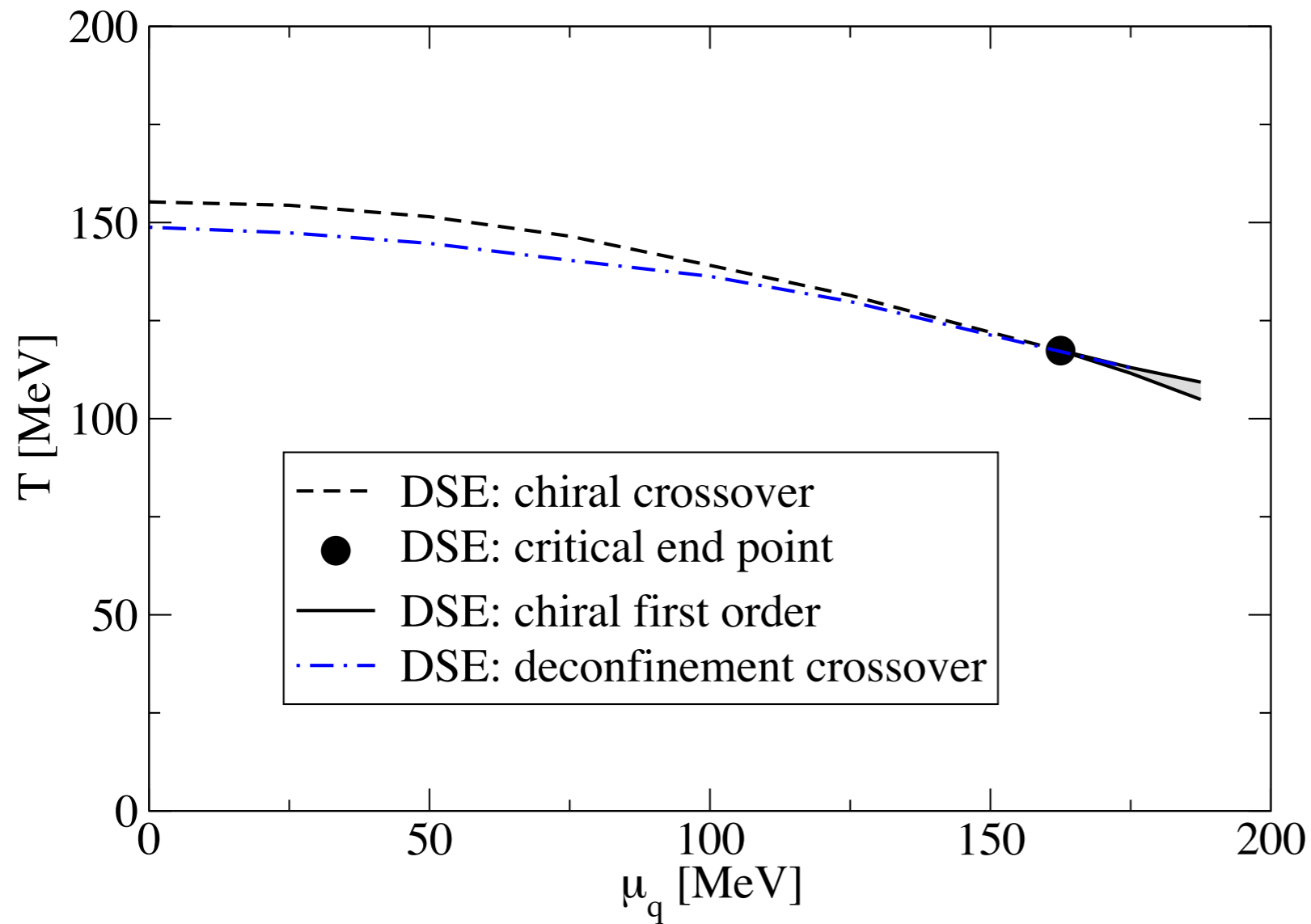
Polyakov-Loop

$$L = \frac{1}{N_c} \text{tr} e^{ig \int A_0}$$

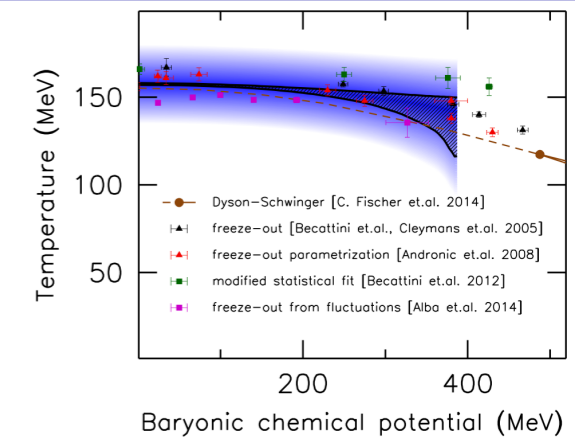
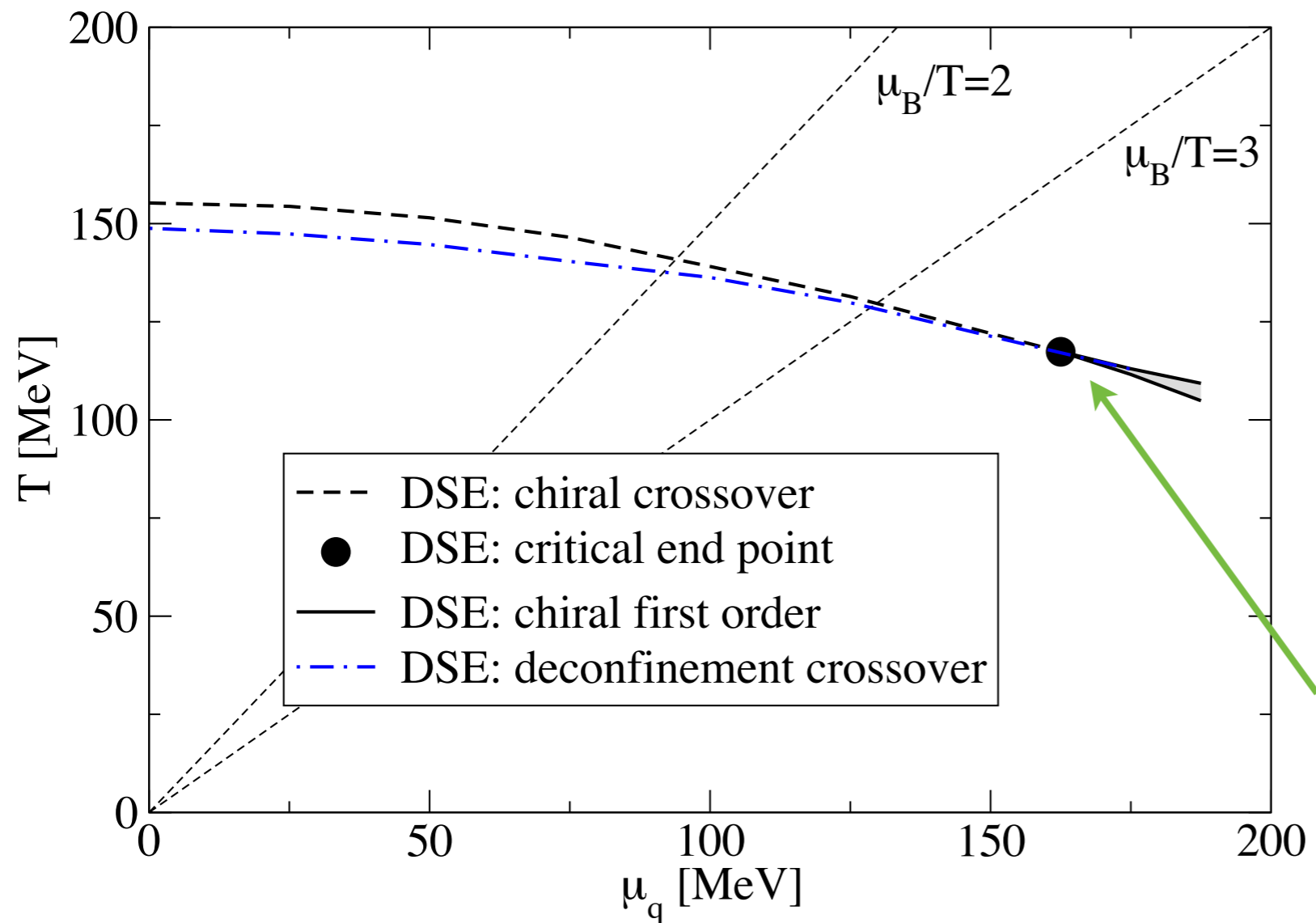


CF, Fister, Luecker, Pawłowski, PLB 732 (2014) 273

# $N_f=2+1$ : phase diagram



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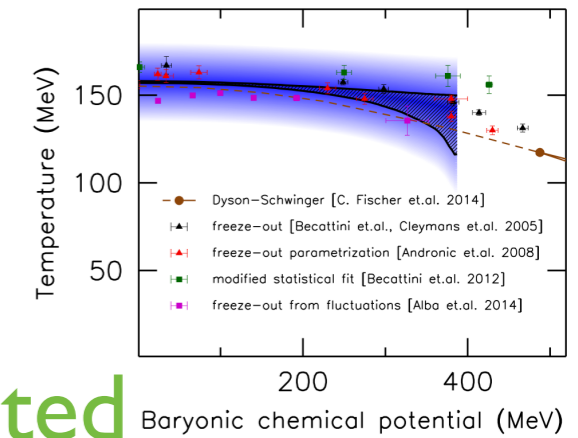
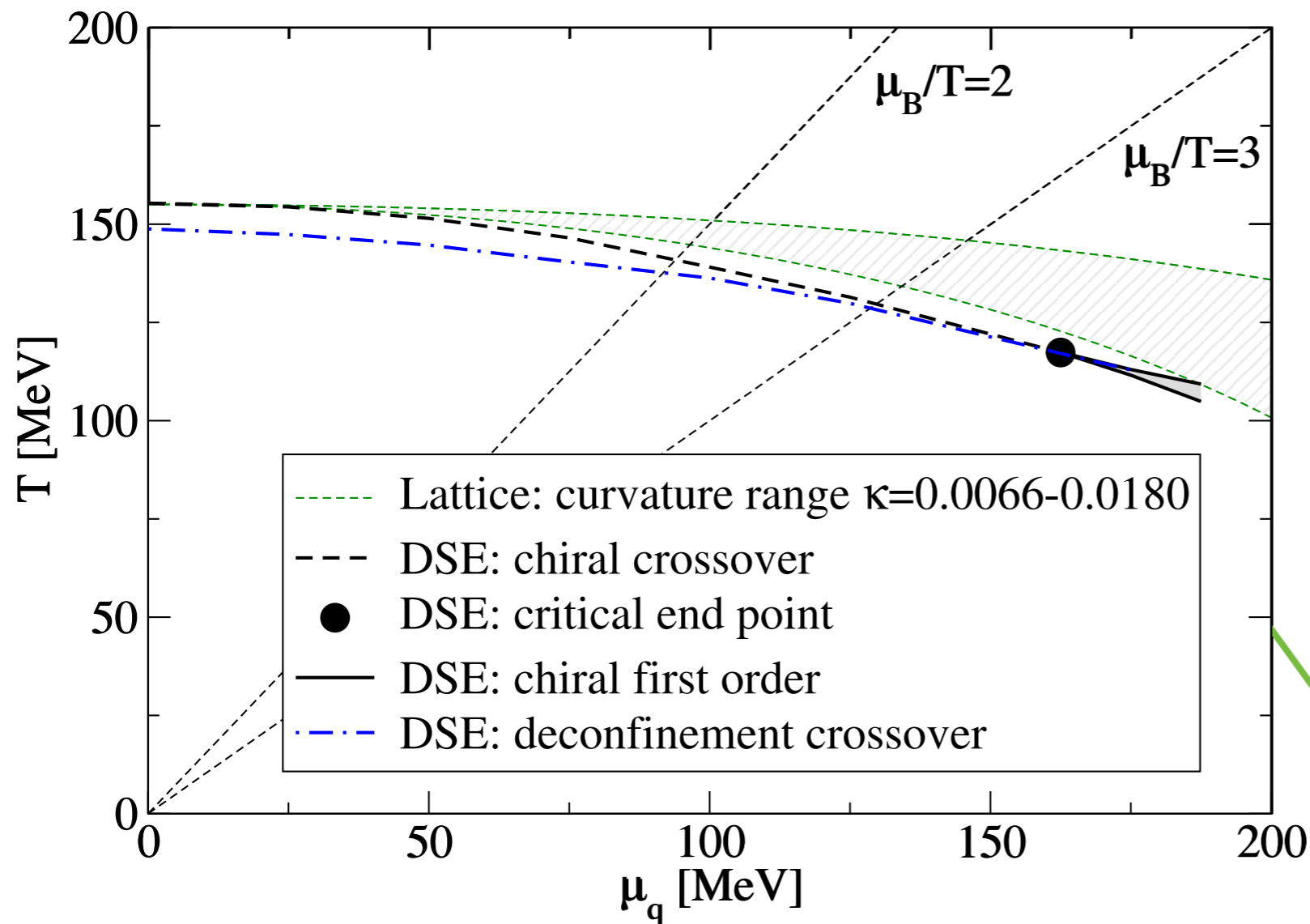


**CEP at large  $\mu$**

CF, Luecker, PLB 718 (2013) 1036,  
 CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273  
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- combined evidence of FRG and DSE: no CEP at  $\mu_B/T < 2$

# $N_f=2+1$ : phase diagram



Extrapolated curvature from lattice

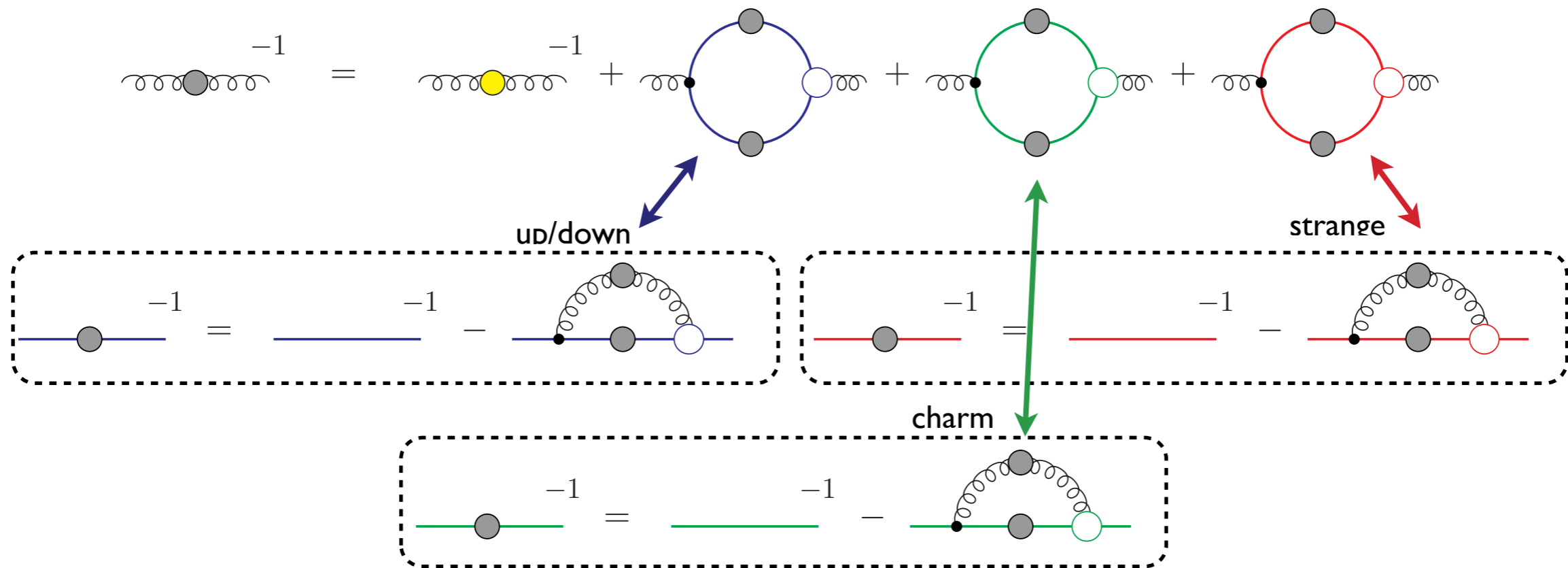
Kaczmarek et al. PRD 83 (2011) 014504,  
 Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001  
 Cea, Cosmai, Papa, PRD 89 (2014), PRD 93 (2016)  
 Bonati et al., PRD 92 (2015) 054503  
 Bellwied et al. PLB 751 (2015) 559

CEP at large  $\mu$

CF, Luecker, PLB 718 (2013) 1036,  
 CF, Fister, Luecker, Pawłowski, PLB 732 (2014) 273  
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

● combined evidence of FRG and DSE: no CEP at  $\mu_B/T < 2$

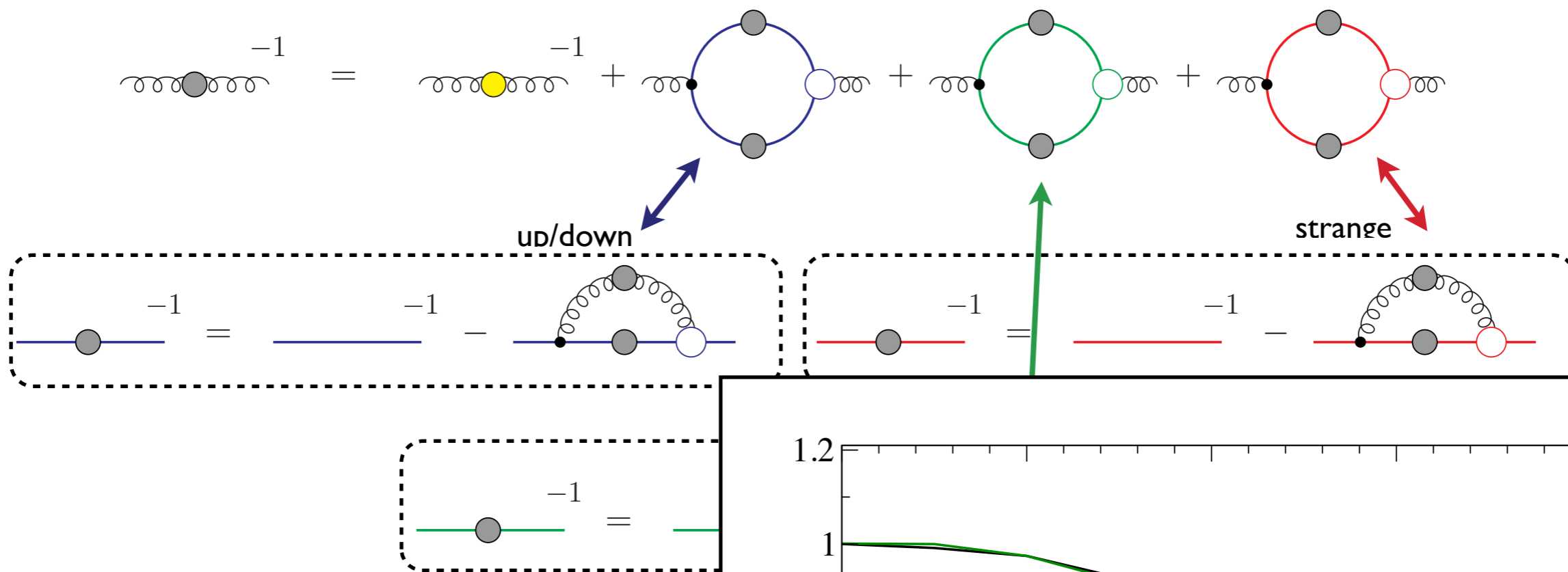
# $N_f=2+1+1$ : effects of charm



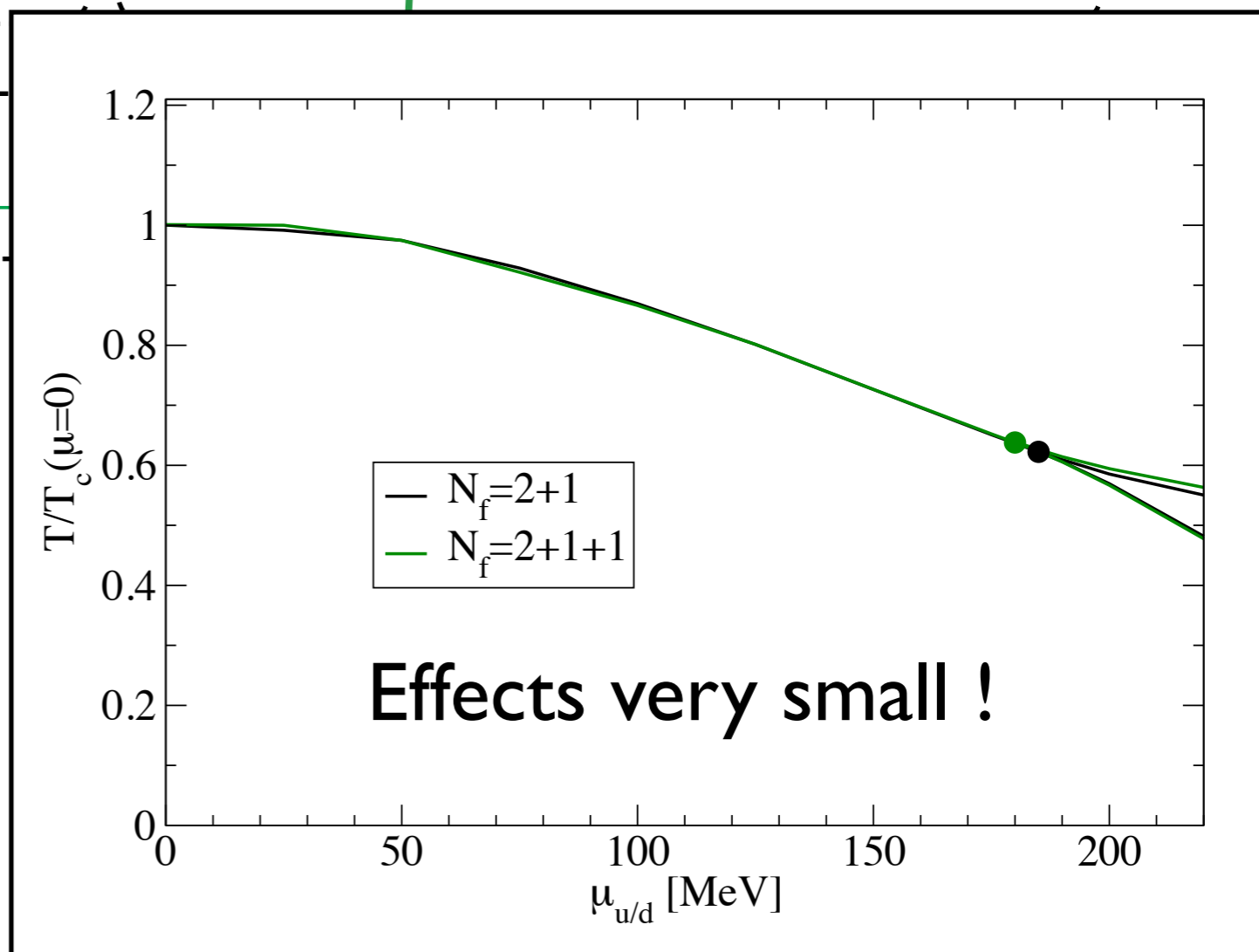
- Physical up/down, strange and **charm quark masses**
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*



# $N_f=2+1+1$ : effects of charm



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CF, Luecker, Welzbacher, PRD 90 (2014) 034022

# Location of CEP in freeze-out landscape

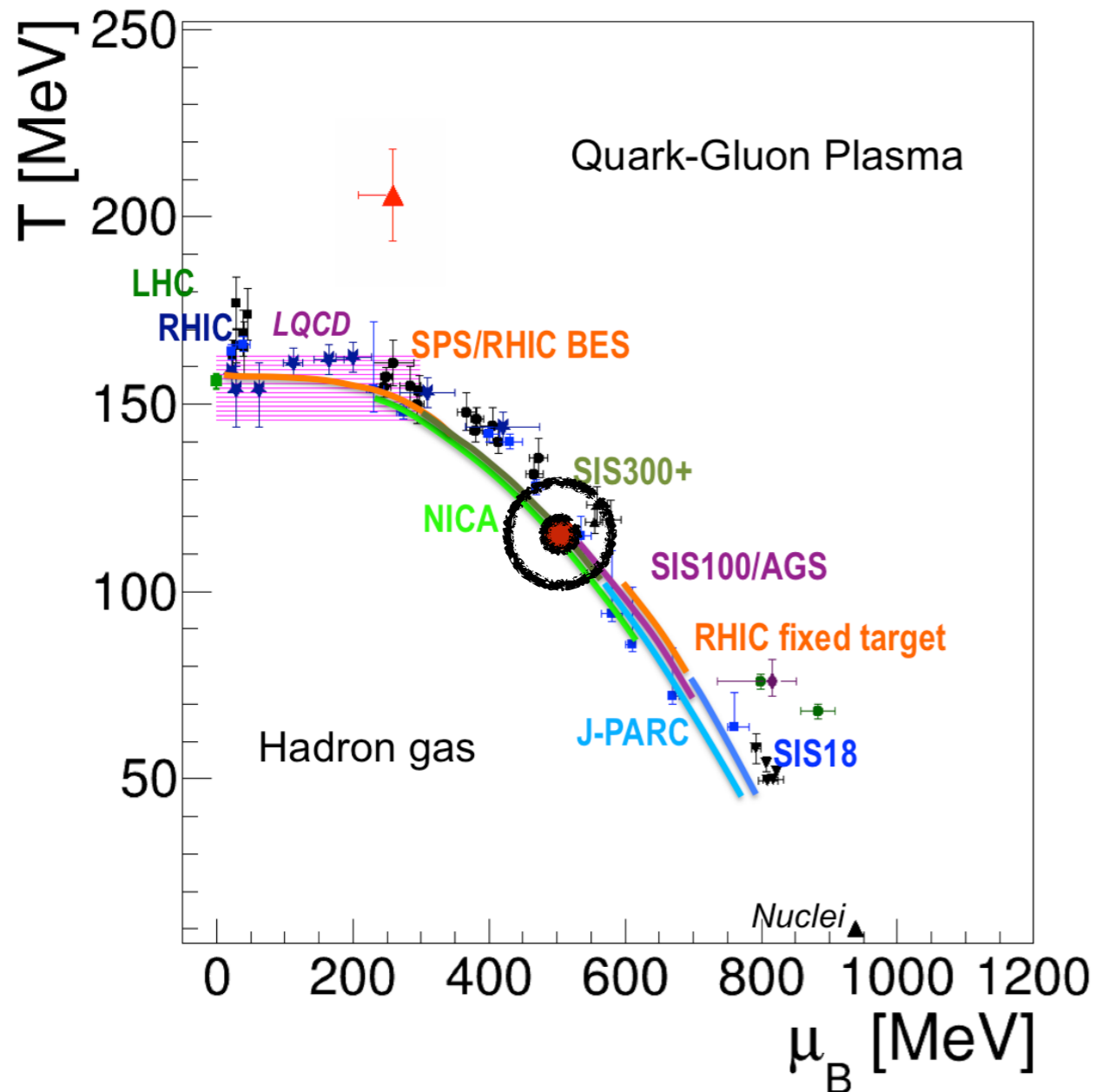


Figure taken from talk of T. Galatyuk, Erice 2016

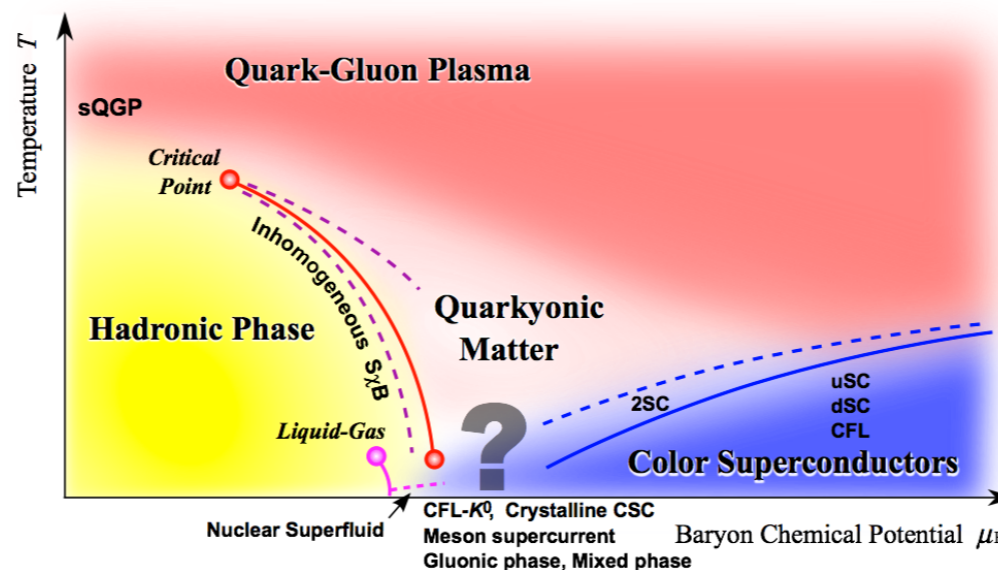
## Caveats:

- inhomogeneous phases
- effects of baryons ?
- ...

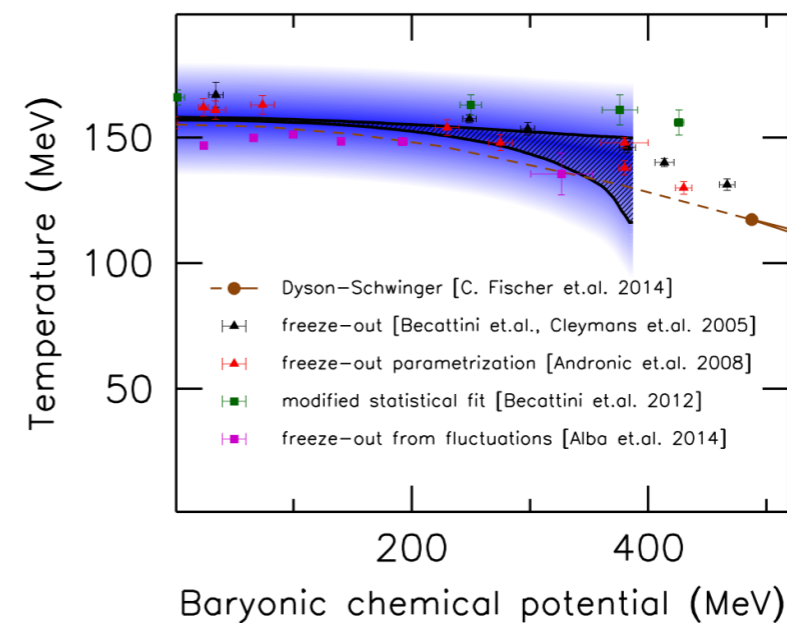
Müller, Buballa and Wambach, PLB 727 (2013) 240

Nc=2: Brauner, Fukushima and Hidaka, PRD 80 (2009) 74035  
Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

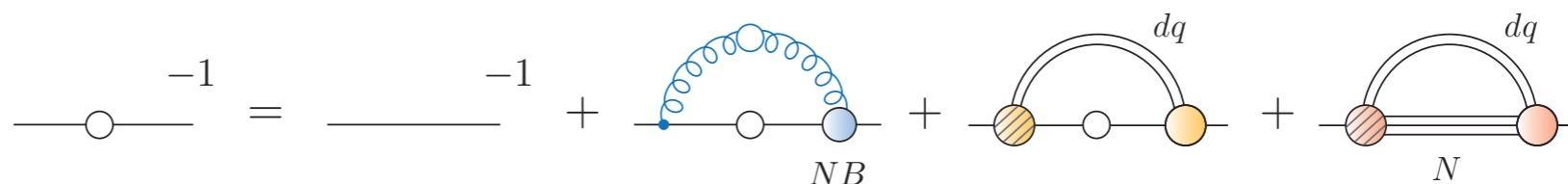
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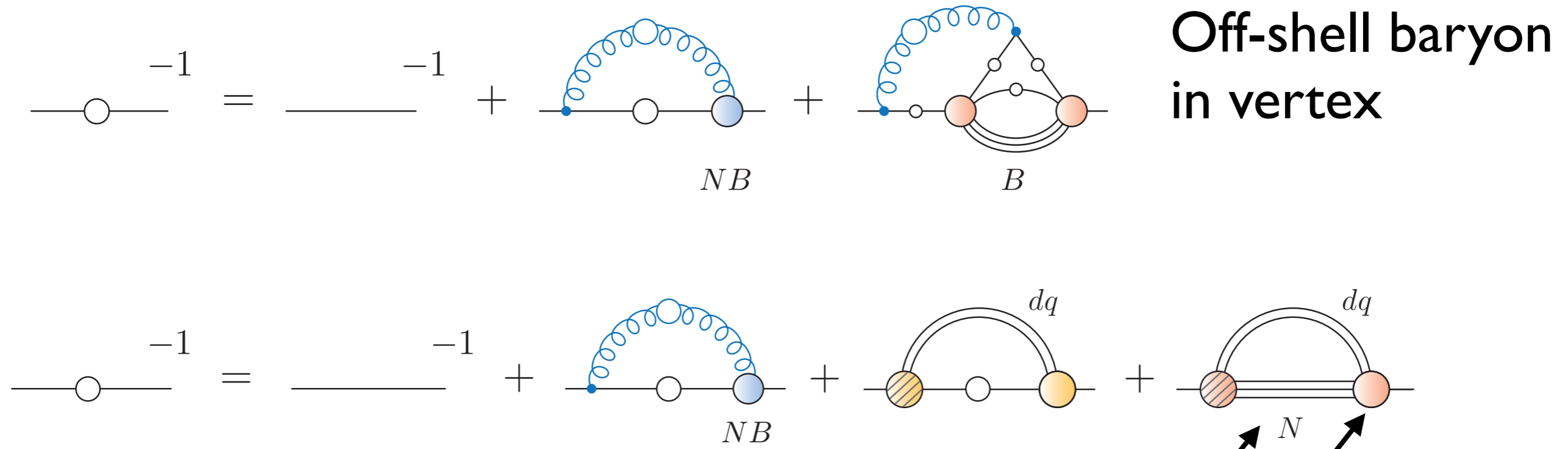
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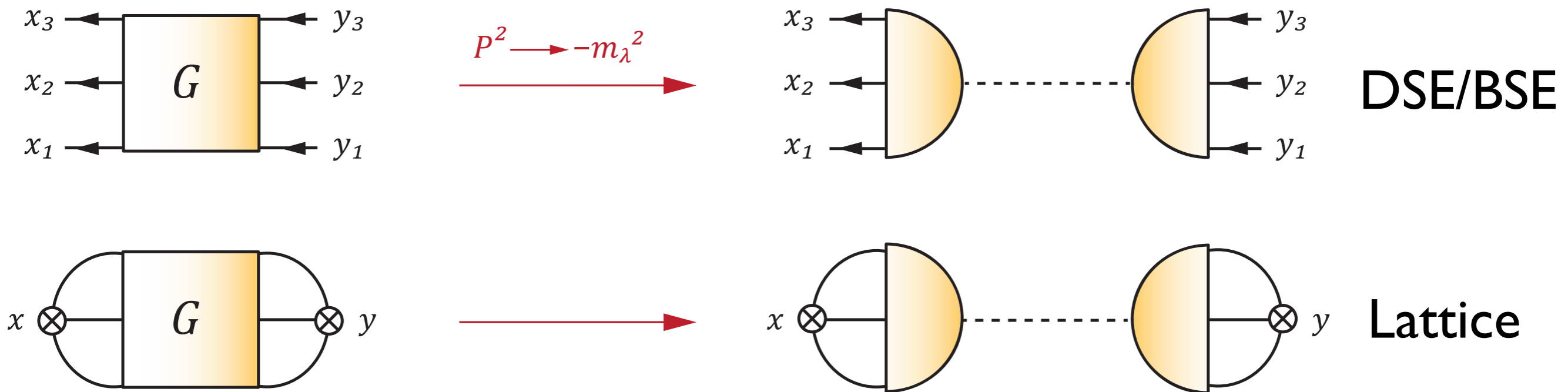


# Baryon effects onto quark

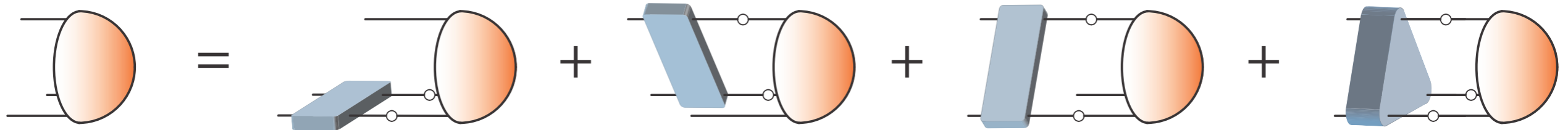


- Dependence on  $T$  and  $\mu$  via  $\gamma$ -propagators  
-wave functions
- Exploratory calculation: use wave functions from  $T=\mu=0$

# Extracting spectra from correlators

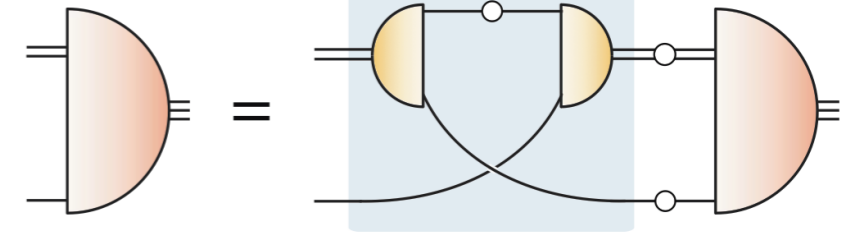
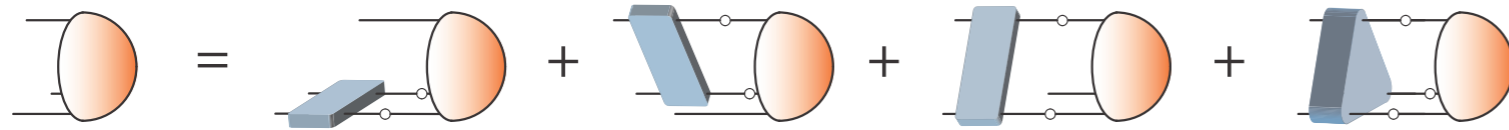


**BSE for baryons** (derived from equation of motion for G)

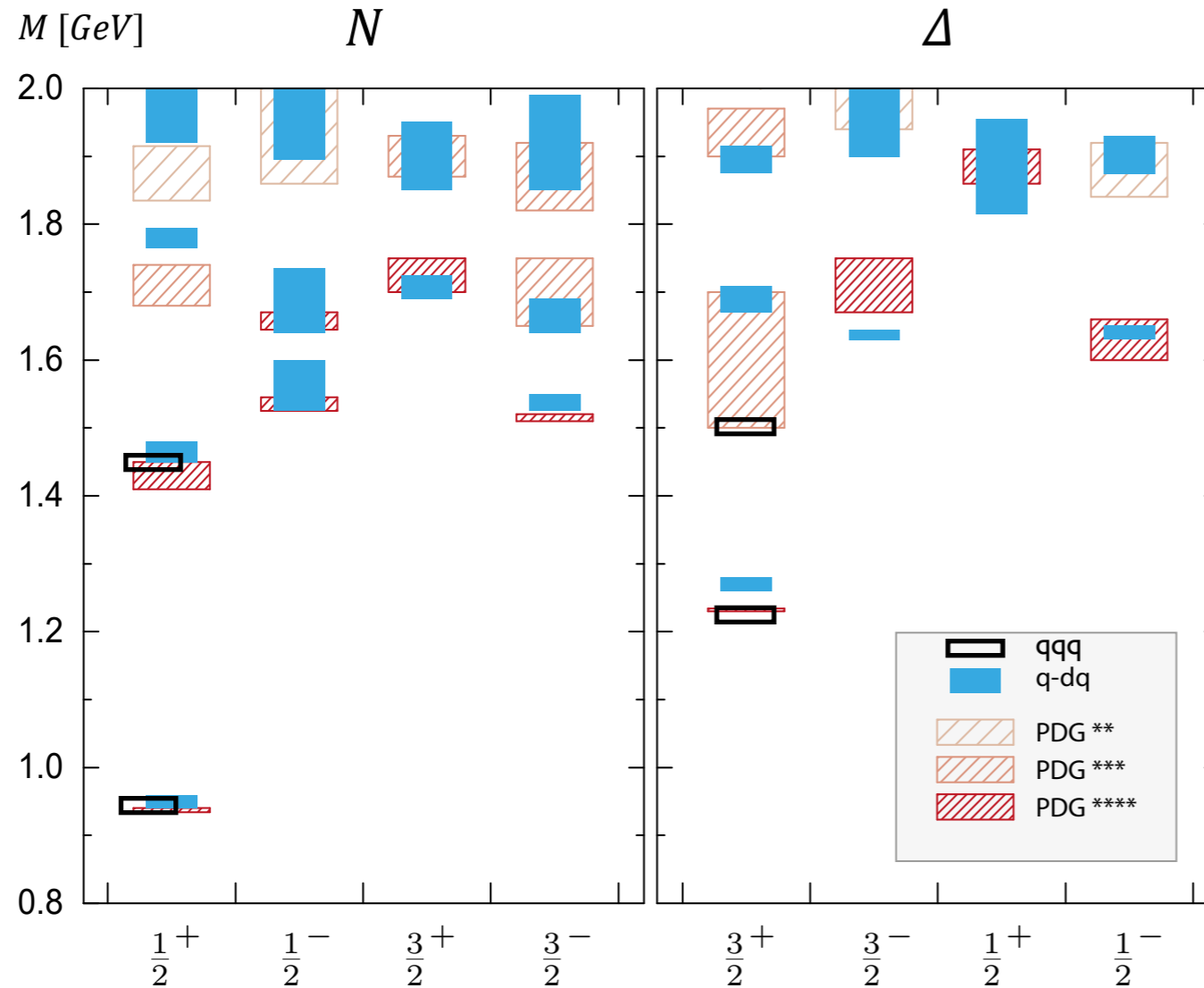
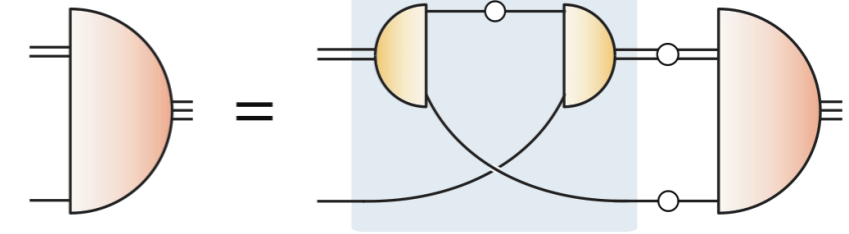
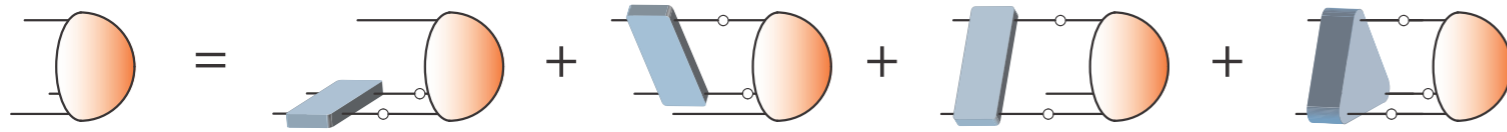


- exact equation for baryon ‘wave function’

# Vacuum: Light baryon spectrum



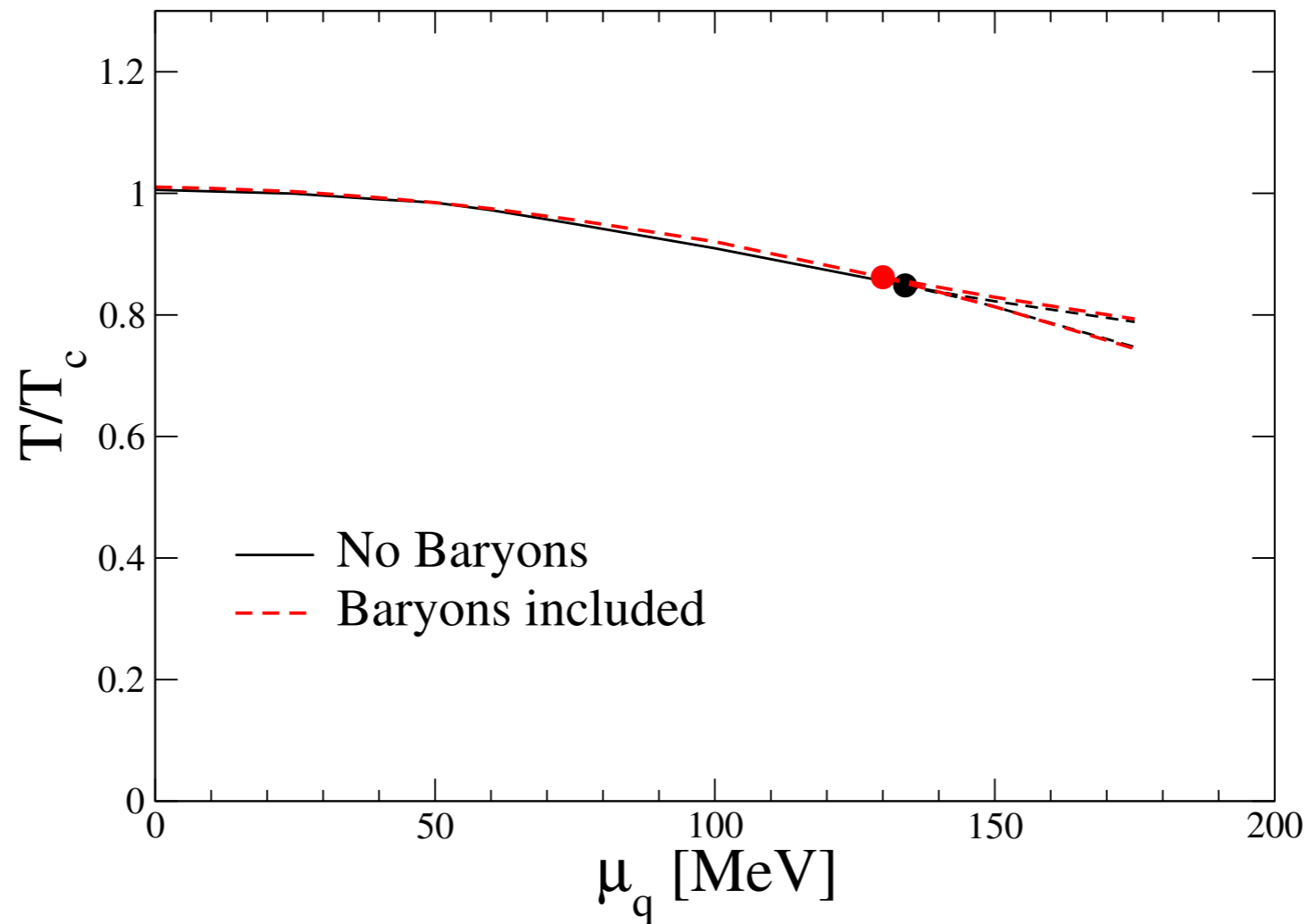
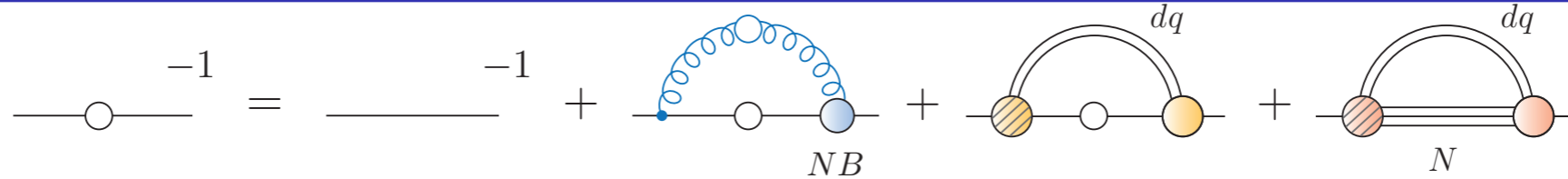
# Vacuum: Light baryon spectrum



Eichmann, CF, Sanchis-Alepuz, 1607.05748  
 Eichmann, Sanchis-Alepuz, Williams, Alkofer,  
 CF, PPNP in press [1606.09602]

- Three-body and diquark-quark approach agree qualitatively
- Spectrum in one-to-one agreement with experiment
- Correct level ordering (wo. coupled channel effects) !

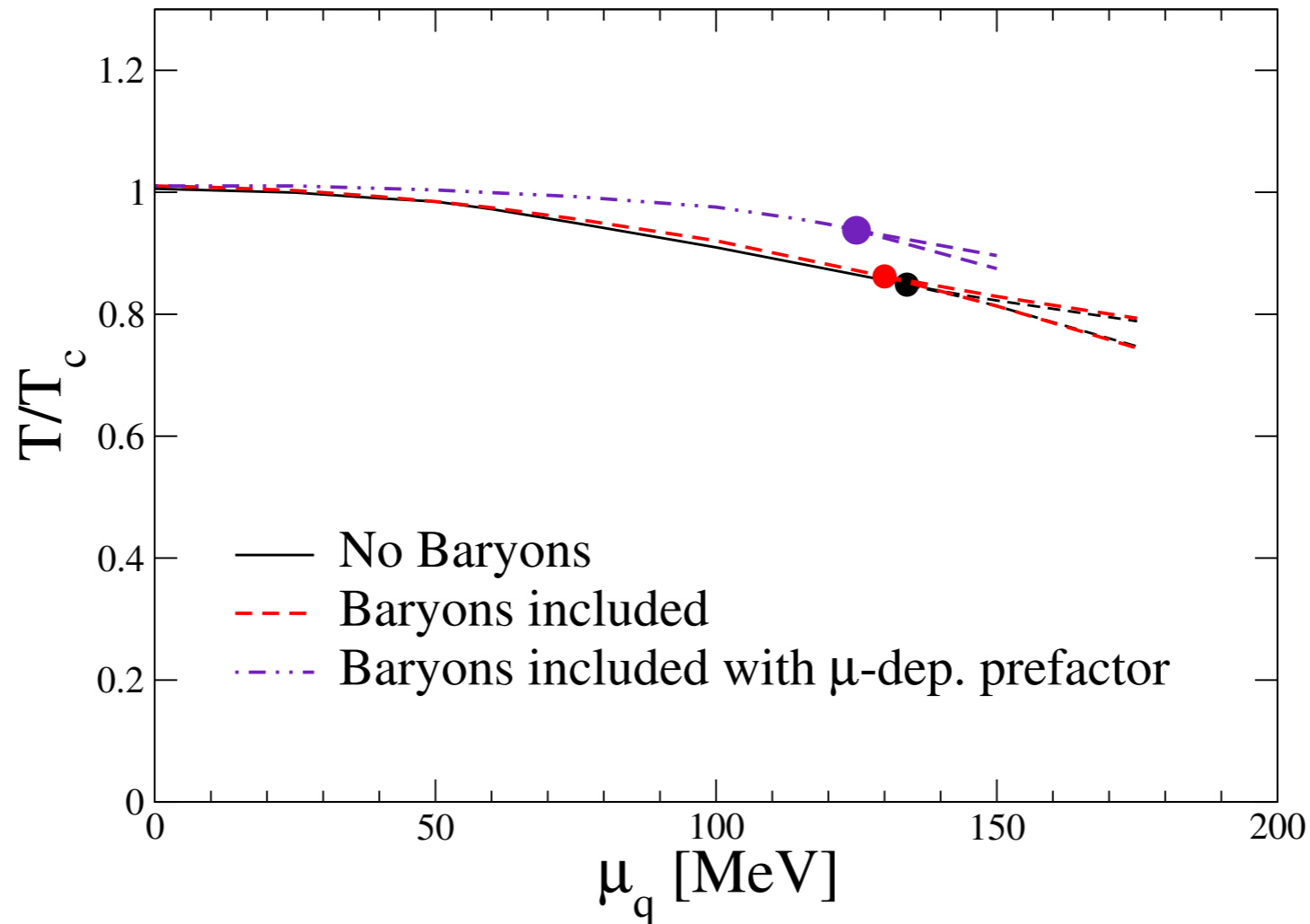
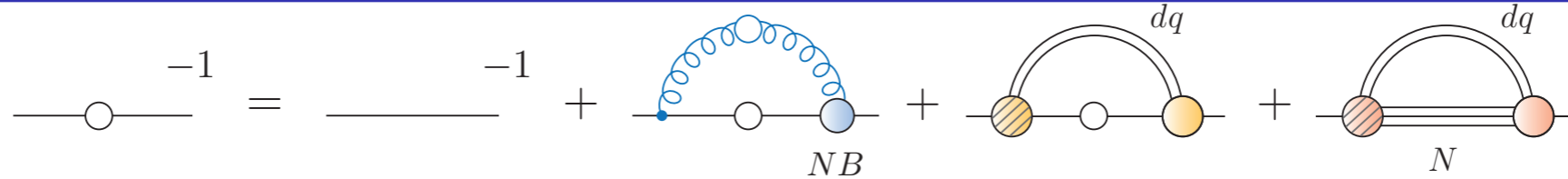
# Baryon effects on the CEP - results ( $N_f=2$ )



- Zero chemical potential: no effects after rescaling
- CEP: almost no effects



# Baryon effects on the CEP - results ( $N_f=2$ )



- Zero chemical potential: no effects after rescaling
- CEP: almost no effects
- But: strong  $\mu$ -dependence of baryon wave function may change situation...

Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

## QCD with finite chemical potential:

- back-reaction of quarks onto gluons important
- $N_f=2+1$  and  $N_f=2+1+1$ : CEP at  $\mu_c/T_c > 3$
- charm quark does not influence CEP
- Baryon effects may or may not be significant for CEP...

Work in progress: - mesons and baryons at finite  $T$  and  $\mu$

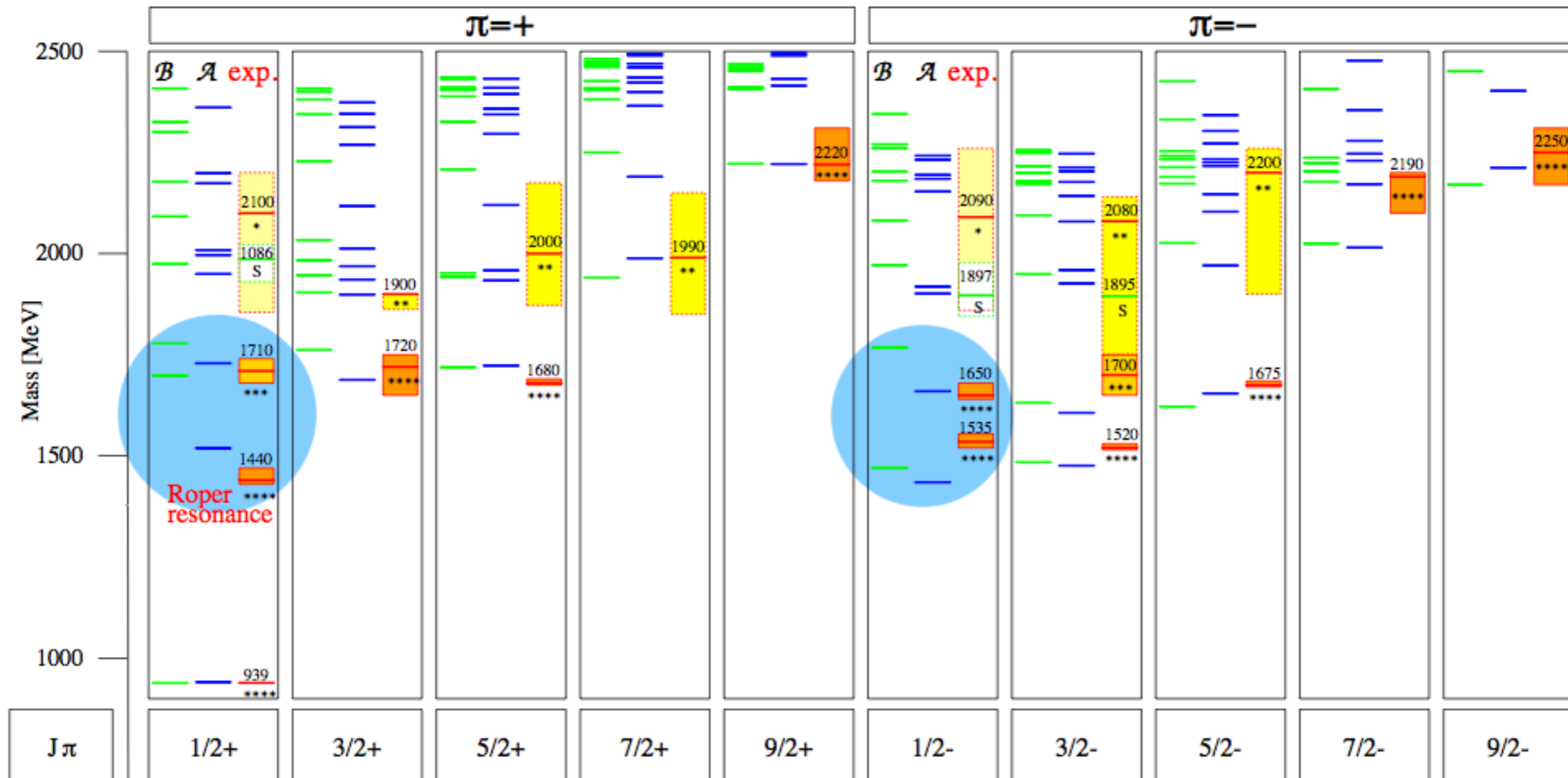
Gunkel and CF, work in progress

- quark spectral functions

CF, Pawłowski, Rothkopf and Welzbacher, arXiv:1705.03207 [hep-ph]



# Baryons: Quark model



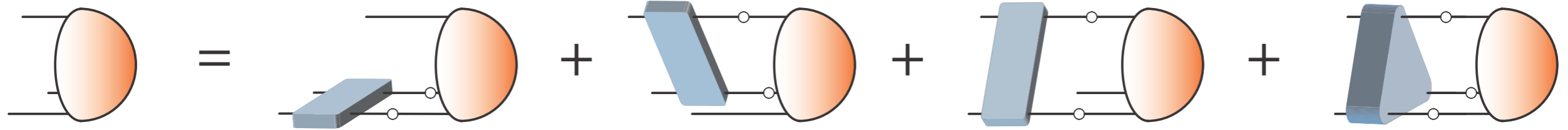
Loring, Metsch, Petry, EPJA 10 (2001) 395

- ‘missing resonances’ - **three-body vs. quark-diquark**

- level ordering:  $N_{\frac{1}{2}}^{\pm}$  vs.  $\Lambda_{\frac{1}{2}}^{\pm}$

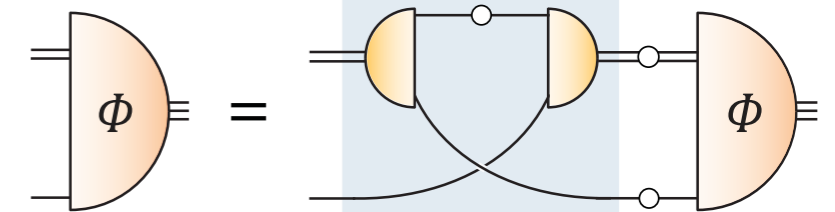
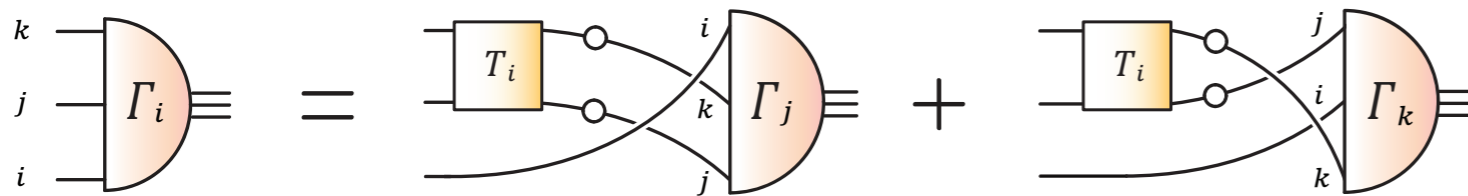
# Vacuum: Baryons from BSEs

**BSE for baryons** (derived from equation of motion for G)



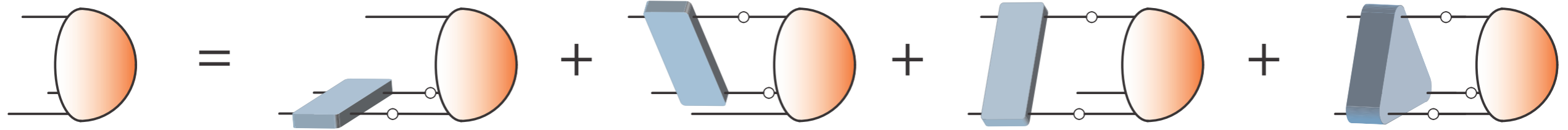
**Faddeev equation** (no three-body forces)

**Diquark-quark**



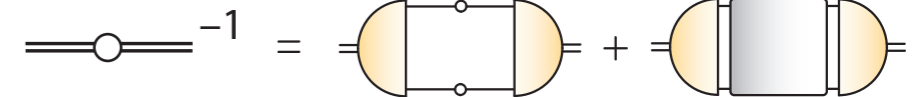
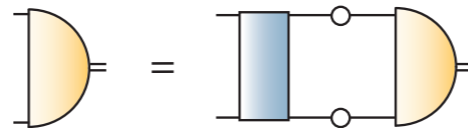
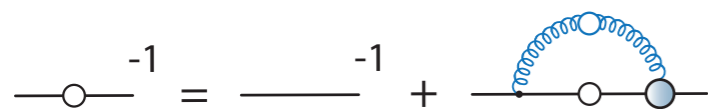
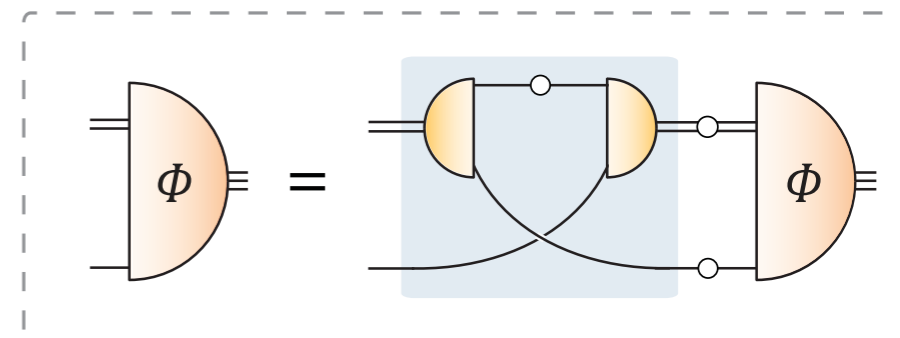
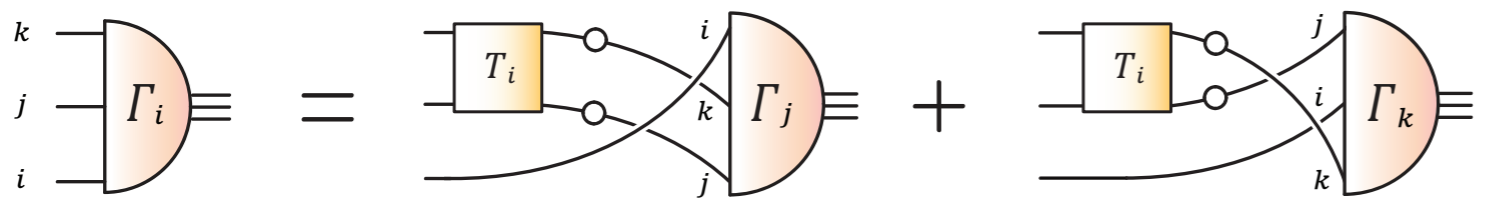
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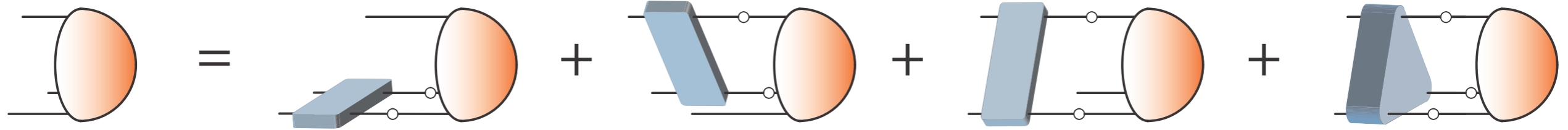
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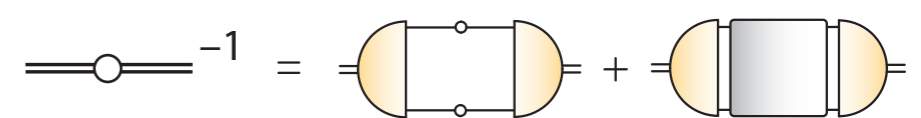
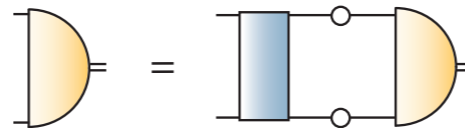
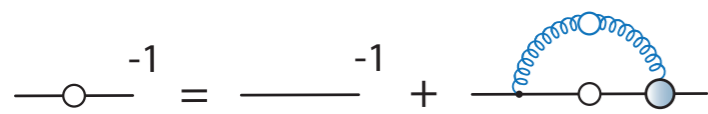
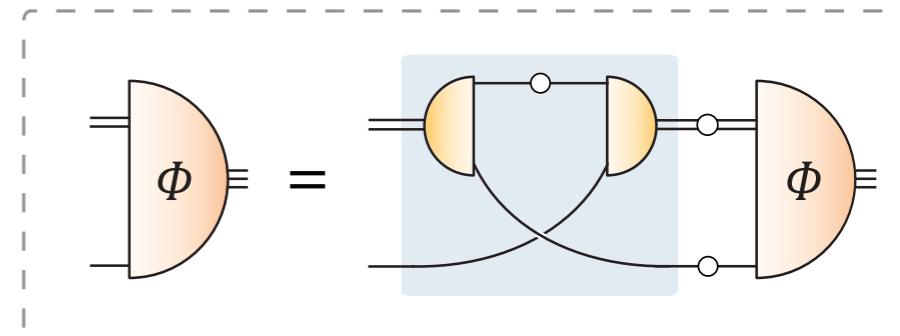
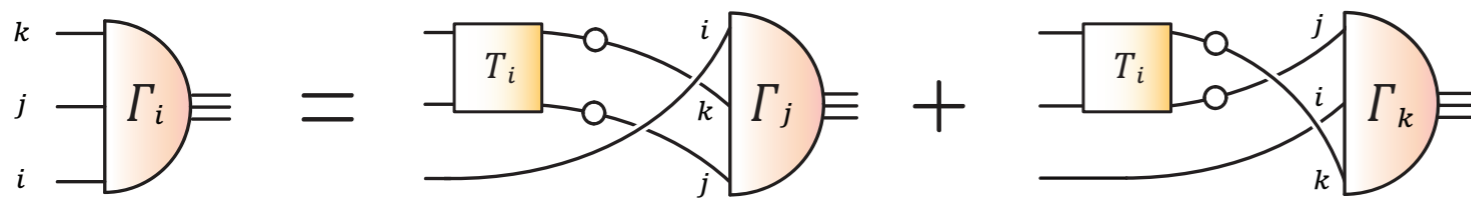
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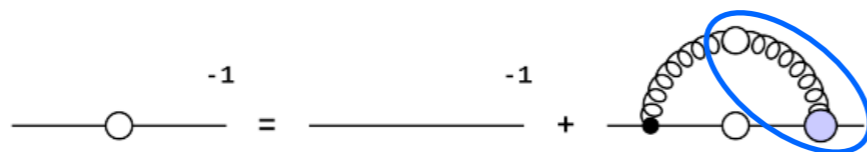


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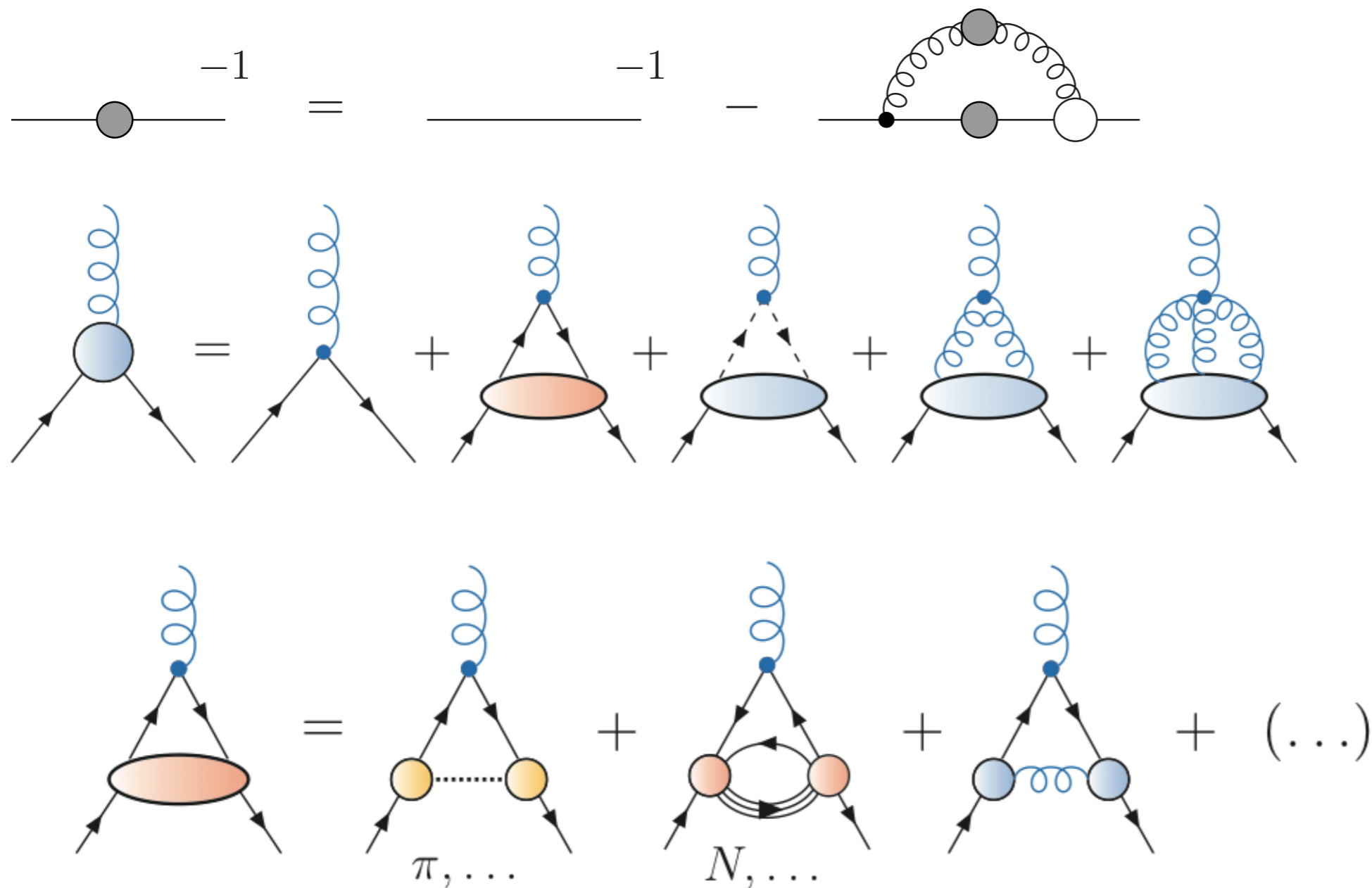


● Input: Non-perturbative quark, quark-gluon interaction (RL)



$$\alpha(k^2) = \pi\eta^7 \left( \frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left( \frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

# Baryon effects onto quark I



- ‘Off-shell baryons’ do affect quark condensate...



# Vacuum: DSE/Faddeev landscape

	Quark-diquark			Three-quark		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
$N, \Delta$ masses	✓	✓	✓	✓	✓	...
$N, \Delta$ em. FFs	✓	✓	✓	✓		
$N \rightarrow \Delta \gamma$	✓	✓	✓	...		
Roper	✓	✓		...		
$N \rightarrow N^* \gamma$	✓	✓		...		
$N^*(1535), \dots$	...	...		...	...	
$N \rightarrow N^* \gamma$	...	...				

Roberts et al

Oettel, Alkofer  
Roberts, Bloch  
Segovia et al.

Eichmann, Alkofer  
Nicmorus, Krassnigg

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Williams

Eichmann, N\*-Workshop, Trento 2015

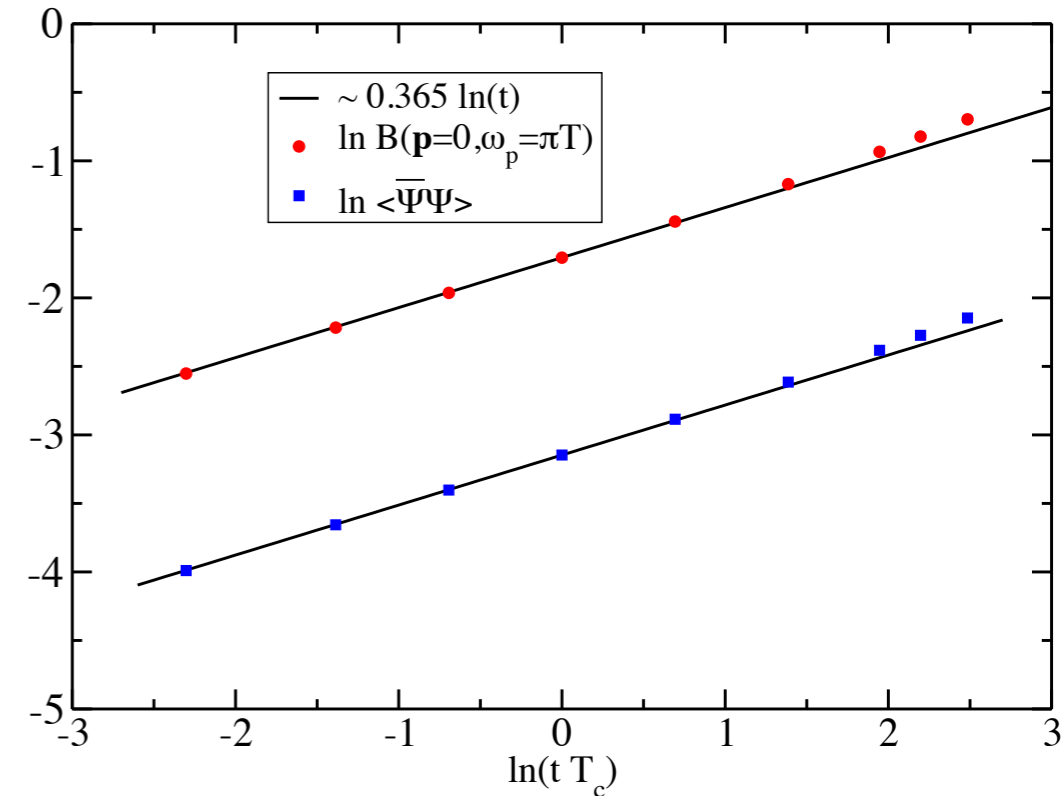
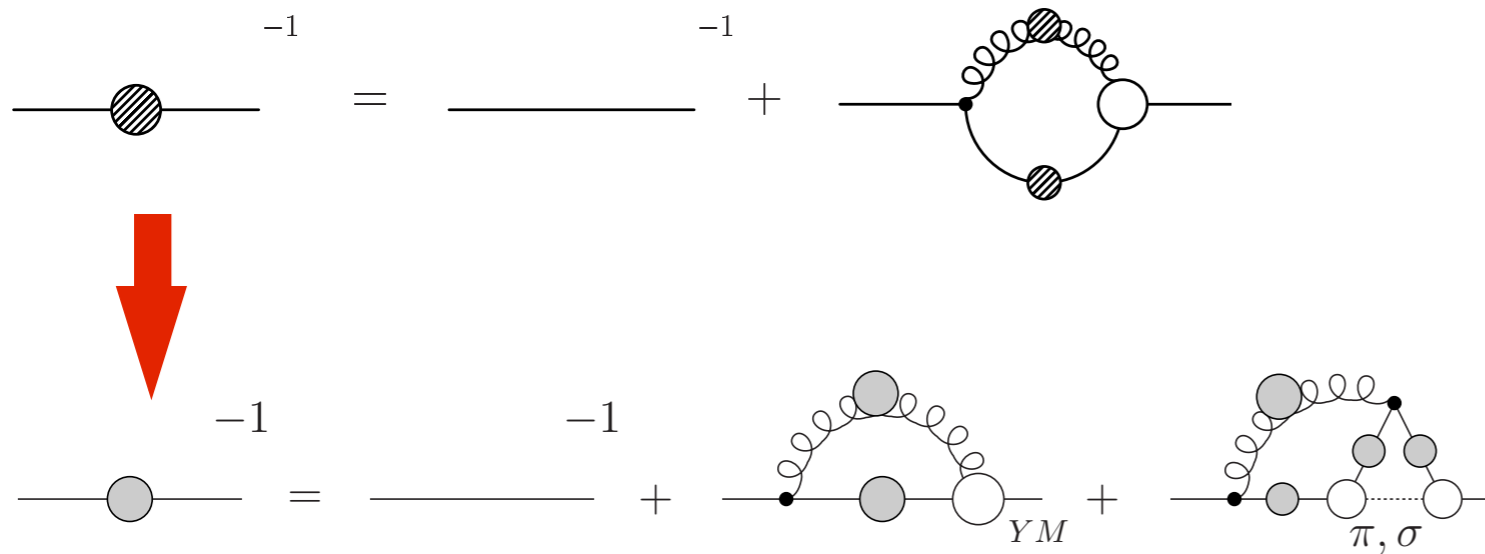
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Roper	✓	✓		...		
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$N^*(1535), \dots$	...	...		...	...	
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 Sanchis-Alepuz, CF  
Williams

Eichmann, N\*-Workshop, Trento 2015

# Critical scaling from DSEs



- Need to take meson part of vertex explicitly into account

- $T=0$ : meson cloud corrections of order of 10-20 %

CF, Williams, PRD 78 (2008) 074006

- $T=T_c$ : meson corrections are dominant !

- Critical scaling:  $\langle \bar{\Psi} \Psi \rangle(t) \sim B(t) \sim t^\nu/2$

$$f_{\pi,s}^2 \sim t^\nu \quad (t = (T_c - T)/T_c)$$

CF and Mueller, PRD 84 (2011) 054013

# Approximation for Quark-Gluon interaction

- T,μ,m-dependent vertex:

Abelian WTI

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times$$

$$\times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2 / \Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

perturbation theory

Infrared ansatz:

- d2 fixed to match gluon input
- d1 fixed via quark condensate (see later)
- correct UV (quant.) and IR-behavior (qual.)

CF, Pawłowski, PRD 80 (2009) 025023  
Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035  
Williams, Fischer, Heupel, PRD 93 (2016) 034026

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Williams

Eichmann, N\*-Workshop, Trento 2015

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$N, \Delta$ $N, \Delta$ $N \rightarrow \dots$ Roper $N \rightarrow i$					✓	...
$N^*(1535), \dots$ $N \rightarrow N^* \gamma$	...	...	...	...	...	...
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

Eichmann, N\*-Workshop, Trento 2015

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Eichmann, N\*-Workshop, Trento 2015

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