

Jet-medium interaction and Gubser flow

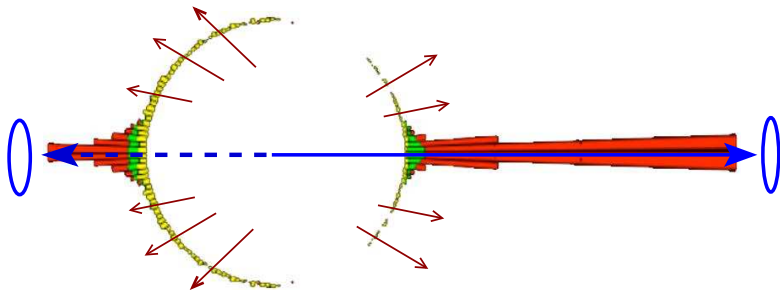
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CPOD 2017, August 10, Stony Brook



Motivation

- Experiments: jet substructures \Leftrightarrow soft yields at large cone radii.
CMS(11,16),...
- Theory: jet-medium interaction \Leftrightarrow fluid dynamics description.
Tachibana and Hirano(17), Chesler and Yaffe(08), Chen et al(17), Qin et al(09)
Betz et. al (09), Chaudhuri and Heinz(06), Casalderrey-Solana et al(05), ...

Challenge

- Understand jet-medium interaction in an **expanding** and **viscous** QGP.

Dynamical properties of the QGP medium: \hat{q} , \hat{e} , η/s , etc.

Outline

- Our (semi-analytical) approach of jet-medium interaction in hydro:
 - Gubser flow + mode-by-mode solution of linearized hydro
- Properties of jet parton going through QGP – Mach cone structure.
- Associated particle spectrum from jet-medium interaction
 - * *Dynamical* viscous suppression.
 - * *Hydrodynamical* viscous suppression.
- Summary and outlook.

Describe jet-medium interaction in fluid dynamics

Energy-momentum conservation : a jet parton + background fluid

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(\underbrace{T_{\text{hydro}}^{\mu\nu}}_{\text{background fluid}} + \underbrace{T_{\text{jet}}^{\mu\nu}}_{\text{jet parton}} + \underbrace{\delta T^{\mu\nu}}_{\text{jet-medium}} \right) = 0$$

- Determine $\delta T^{\mu\nu}$ mode-by-mode for a **viscous** medium

$$\delta \tilde{T}^{\mu\nu}(k) \longrightarrow \delta \tilde{T}_{\text{hydro}}^{\mu\nu}(k) : \quad k \lambda_{\text{mfp}} \ll 1$$

- * Long wave-length modes captured in viscous hydrodynamics.
- * Applicability of viscous hydrodynamics.
- * Separation between hard and soft (thermalized) scales.

Iancu and Wu(15)

Describe jet-medium interaction in fluid dynamics

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- Jet-medium interaction as perturbations on top of background medium

$$\begin{cases} \partial_\mu T_{\text{hydro}}^{\mu\nu} = 0 \\ \partial_\mu \delta T^{\mu\nu} = -\partial_\mu T_{\text{jet}}^{\mu\nu} = J^\nu : \text{ EoM of jet-medium interaction} \end{cases}$$

J^ν determined via the evolution of the jet parton distribution

Tachibana, Chang and Qin(17)

$$\rightarrow J^\nu = -\partial_\mu T_{\text{jet}}^{\mu\nu} = \hat{e} v_{\text{jet}}^\nu n_{\text{jet}}(\mathbf{x}, t), \quad \hat{e} = -\langle \Delta E \rangle / \langle \Delta x \rangle$$

*Contribution from broadening can be negligible for $E_{\text{jet}} \gg T$.

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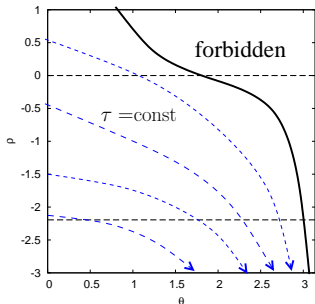
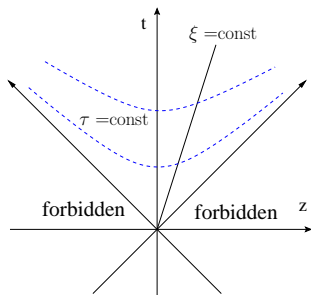
- Jet energy loss: $\hat{e} = \kappa T^2$.
Betz et al.(11), Casalduffy-Solana et al.(14), Fincar et al.(14)
- From $\hat{e} = \hat{q}/4T$ and $T^3/\hat{q} \sim 1.25 \eta/s$ Majumder, Muller and Wang(07)
 - ⎧ weakly coupled : $\kappa \approx 3/(\eta/s)$
 - ⎩ strongly coupled : $\kappa \gg 3/(\eta/s)$

* *Dynamical viscous suppression* \Leftrightarrow reduced \hat{e} in more viscous medium.

We shall consider mostly the $\kappa \approx 3/(\eta/s)$ scenario.

$\partial_\mu T_{\text{hydro}}^{\mu\nu} = 0$: Gubser's analytical solution

- Conformal medium + rotational symmetry and Bjorkn boost invariance
 Gubser (10), Gubser and Yarom(10)
 Radial and longitudinal expansion of a viscous medium \approx QGP expansion
- A new coordinate system: $(\tau, r, \phi, \xi) \leftrightarrow (\rho, \theta, \phi, \xi)$



$$ds^2 = d\rho^2 + d\xi^2 + \underbrace{\cosh^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2)}_{SO(3) \text{ symmetry}}$$

$\partial_\mu \delta T^{\mu\nu} = J^\nu$: mode-by-mode solution in expanding system

- $SO(3)$ symmetry in $(\theta, \phi) \Rightarrow$ mode expansion in spherical harmonics.
Gubser and Yarom(11), Staig and Shuryak(11), LY and Grönqvist(16)
- Reduce to EoM of hydro perturbations: $\delta T, \delta u^\mu$

$$\text{e.g. } \delta \tilde{T} = \tilde{T} \sum_{lm} \int \frac{dk_\xi}{2\pi} t^{lm}(\rho) Y_{lm}(\theta, \phi) e^{ik_\xi \xi}$$

$$\text{so that } (\delta \tilde{T}, \delta \tilde{u}^\mu) \longrightarrow \tilde{\mathcal{V}}^{lm}(\rho, k_\xi) = \underbrace{(t^{lm}, v_s^{lm}, v_\xi^{lm})}_{\text{scalar}}, \underbrace{(v_v^{lm})}_{\text{vector}}$$

and expand source J^ν into modes $\rightarrow \tilde{\mathcal{S}}^{lm}(\rho, k_\xi)$

$$\tilde{\mathcal{S}}^{lm}(\rho, k_\xi) = \begin{pmatrix} -\frac{1}{3w} c_{lm}^\rho(\rho, k_\xi) \\ -\frac{2T \tanh \rho}{3\mathcal{Y} T'} c_{lm}^s(\rho, k_\xi) \\ \frac{c_\xi^{lm}(\rho, k_\xi)}{w(T+H_0 \tanh \rho)} \\ -\frac{2T \tanh \rho}{3w T'} c_v^{lm}(\rho, k_\xi) \end{pmatrix}$$

We shall ignore k_ξ -dependence and v_ξ , so the whole system is boost invariant!

Dynamical properties of mode evolution (not very sensitive to source)

- Equation of motion of $(t^{lm}, v_s^{lm}, v_v^{lm})$

$$\partial_\rho \tilde{V}^{lm}(\rho) = \underbrace{-\Gamma(\rho, l)}_{\text{medium response}} \tilde{V}^{lm}(\rho) + \underbrace{\tilde{S}^{lm}(\rho)}_{\text{source from jet parton}},$$

* Viscosity damps mode evolution: $\sim \exp(-\Delta\rho l^2 \eta/s)$

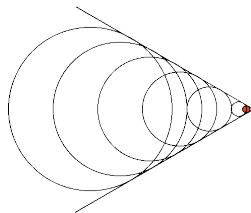
\Rightarrow *Hydrodynamical viscous suppression*

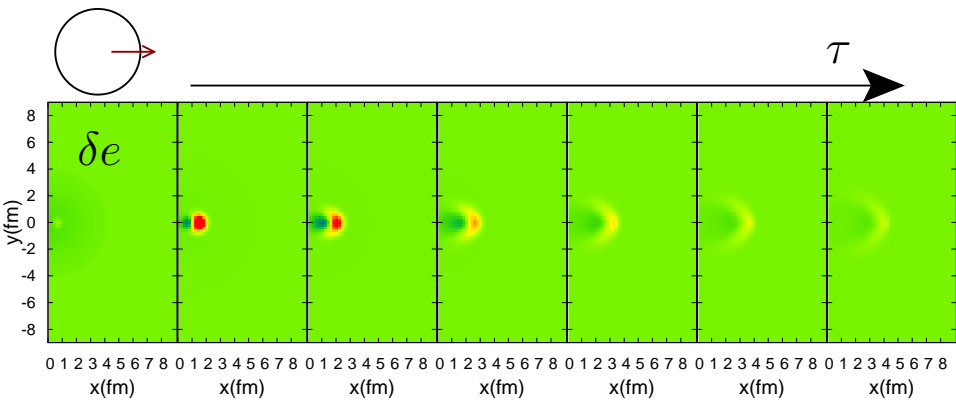
* Eigen-modes of medium response:

v_v^{lm} : diffusive

(t^{lm}, v_s^{lm}) : sound propagation

Sound propagation \Rightarrow **Mach cone**

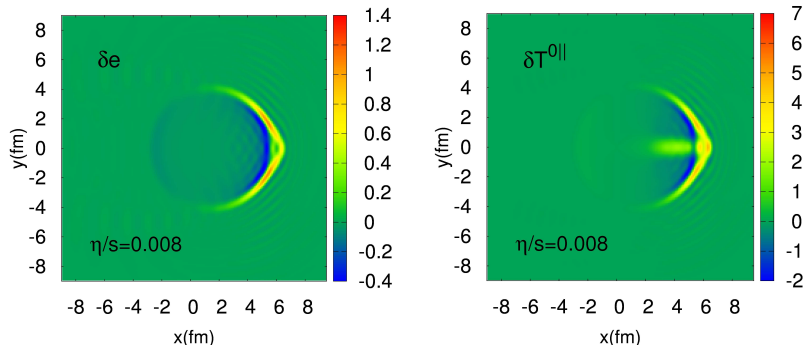




$$\eta/s = 1/4\pi$$

Mach cone structure and jet-medium interaction

“ideal” fluid ($\tau = 6.0$ fm/c):



- Common features of mach cone structure: shock, depletion, etc.

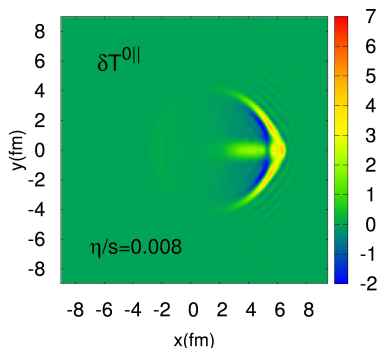
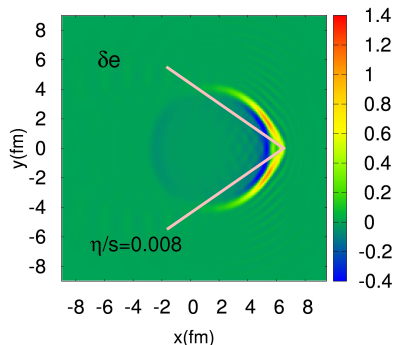
* Distorted by medium expansion: $\theta^M > 2 \sin^{-1}(c_s/c)$

- δe has *only* scalar modes, $\delta T^{0||}$ has of scalar and vector modes.

* A diffusive wake in $\delta T^{0||}$ along the path of jet parton. Chesler and Yaffe(08)

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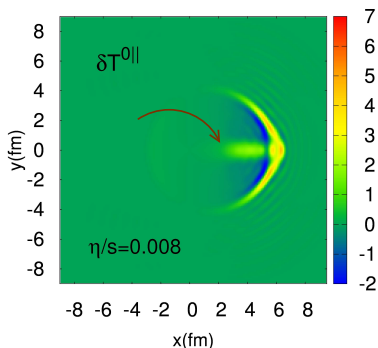
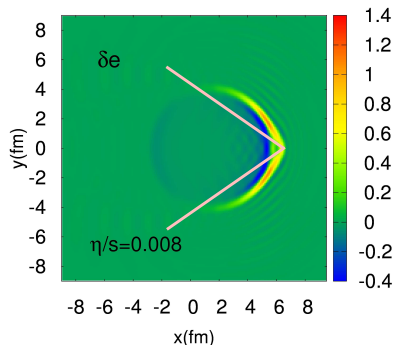
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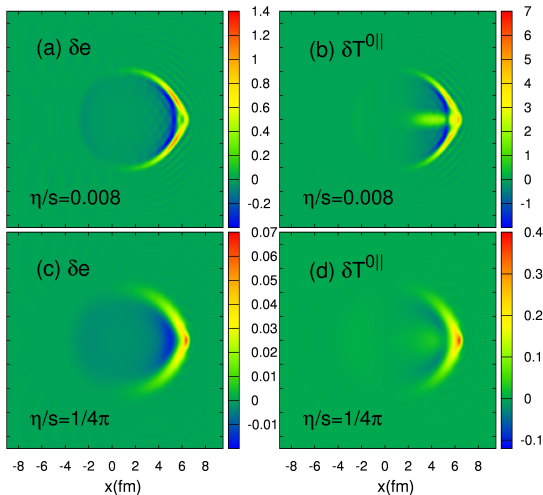
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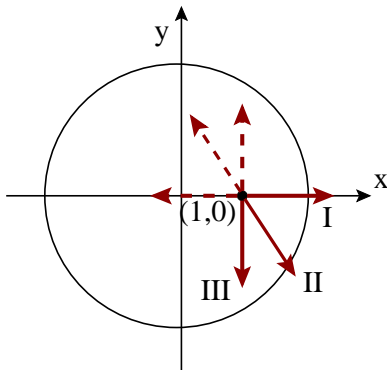
Viscous effects on the cone structure



* *Dynamical* viscous suppression:
an **overall** suppression of the cone
structure proportional to $1/(\eta/s)$
i.e., a factor 1/10.

* *Hydro* viscous suppression:
suppresses and smears the cone
structure, *e.g.*, the diffusive
wave gets damped and broadened.

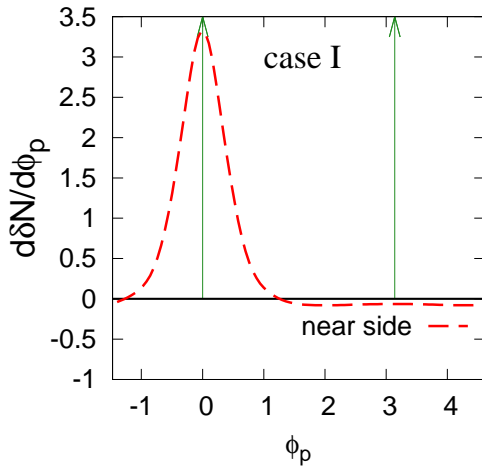
Particle spectrum and di-jet in heavy-ion collisions



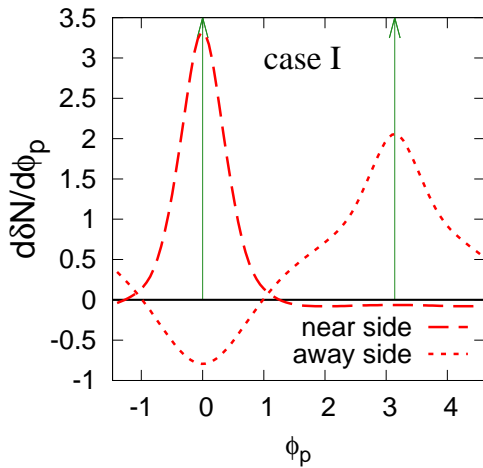
- Ultra-central Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV, di-jet (photon-jet).
- ‘Knife-shaped’ dijet partons (boost invariant) with sufficiently high energy.

$$J^\mu(t, \mathbf{x}) = \hat{e}v_{\text{jet}}^\mu n_{\text{jet}}(t, \mathbf{x}) = \hat{e}v_{\text{jet}}^\mu \delta^{(2)}(\mathbf{x}_\perp - \mathbf{v}_{\text{jet}}\Delta\tau)/\tau$$

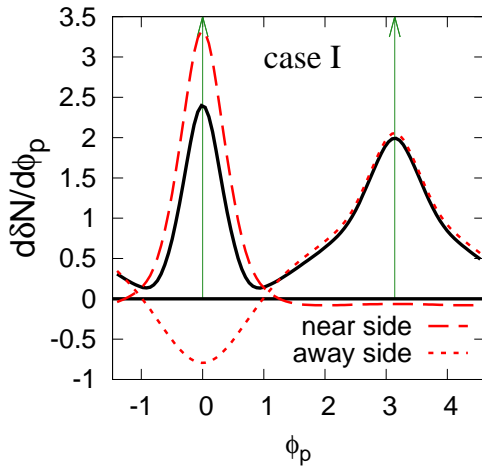
- Cooper-Frye freeze-out at constant $\tau = 6.0$ fm/c.



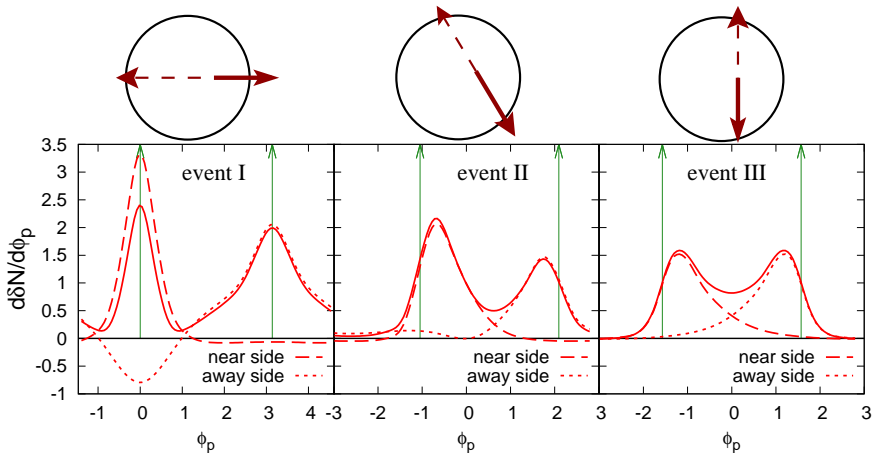
$$\frac{\eta}{s} = \frac{1}{4\pi}$$



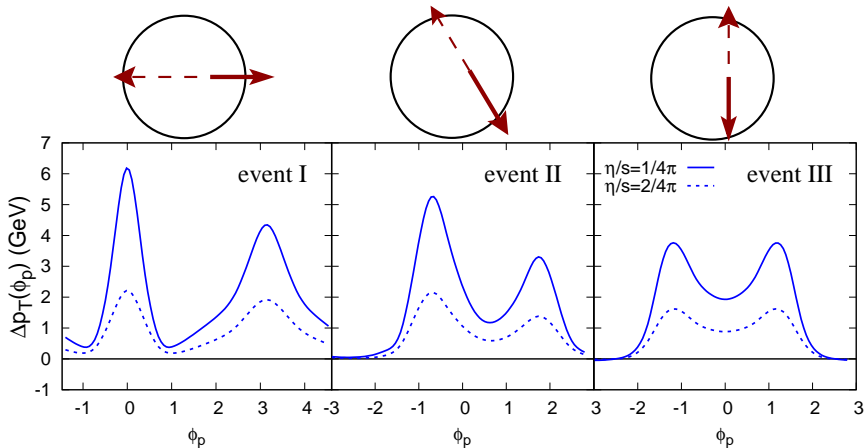
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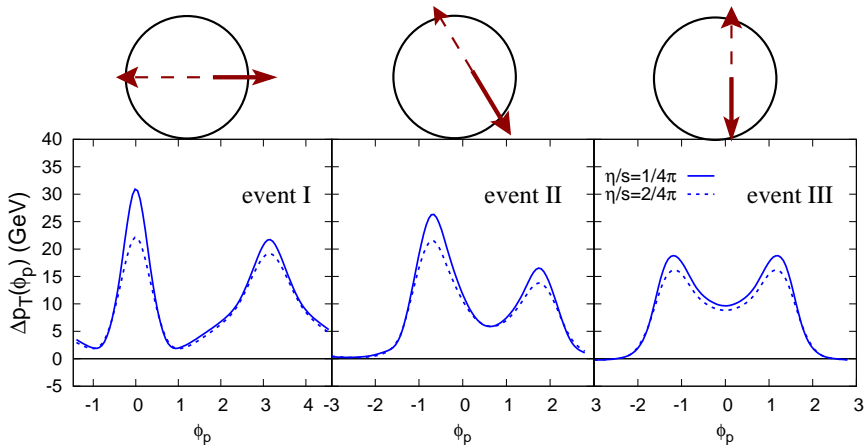


$$\frac{\eta}{s} = \frac{1}{4\pi}$$



$$\frac{\eta}{s} = \frac{1}{4\pi} \text{ and } \frac{2}{4\pi}$$

- * *Dynamical* viscous suppression (overall reduction) \Rightarrow dominant!
- * *Hydrodynamical* viscous suppression \Rightarrow not very important
- * Viscous correction in $\delta f \Rightarrow$ negligible.



fixed $\kappa = \frac{20\pi}{3} \gg 3/(\eta/s)$, for a strongly coupled system

- * \hat{e} is independent of η/s , no *dynamical* viscous suppression.
- * Totally induced by *hydro* viscous suppression.

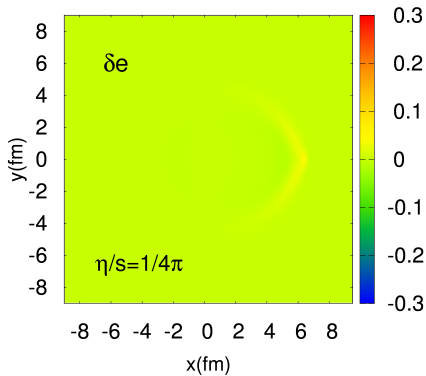
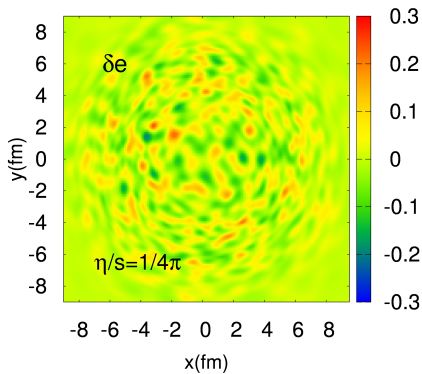
Summary:

- A formalism to study jet-medium interaction mode-by-mode.
- Expansion and dissipation change Mach cone, and particle spectrum.
 - * *Dynamical* viscous suppression: $\hat{e} \propto 1/(\eta/s)$
 - * *Hydrodynamical* viscous suppression: $\sim \exp(-k^2 \Delta t \eta/sT)$
 - * How parton loses energy to the medium is important.

Outlook:

- To compare and check viscous hydrodynamic simulations.
- Interplay with thermal noise:

$$\partial_\mu \delta T^{\mu\nu} = J^\nu - \partial_\mu S^{\mu\nu}, \quad \langle S^{\mu\nu} S^{\alpha\beta} \rangle \sim 2\eta T$$

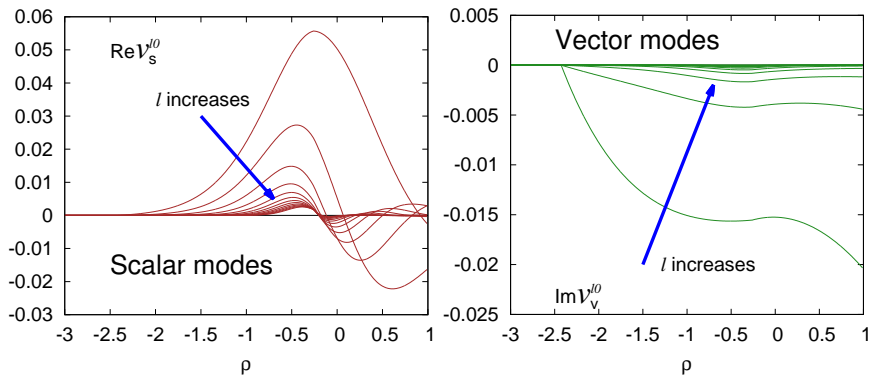


$$\tau = 6 \text{ fm}/c$$

$$\eta/s = 1/4\pi$$

Back-up

$$\eta/s = 1/4\pi$$



- Scalar modes exhibit sound wave propagation, especially at late time.
- Vector modes are diffusive.

Mode summation converges up to $l < 35$.