

# Fluctuations and hadronic correlation functions from the instanton-dyons

Edward Shuryak  
Stony Brook

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# Outline

- **a (two slide) primer on hadronic correlation functions**
- **can instanton-dyon ensemble reproduce hadronic phenomenology?** (*1705.04707 and PRD, with Larsen*)
- ***another two slide primer on fluctuations in sub-volume***
- ***calculation of fluctuations using instanton-dyons*** (*1703.02434, with Larsen*)

# a primer on point-to-point correlation functions

Correlation functions in the QCD vacuum

E. Shuryak, Rev.Mod.Phys. 65 (1993) 1-46

M. -C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, PRL 70 (1993) 255

E. Shuryak., Nucl. Phys. B238, (1989), 102  
random instantons  
which chiral symmetry breaking is stronger?

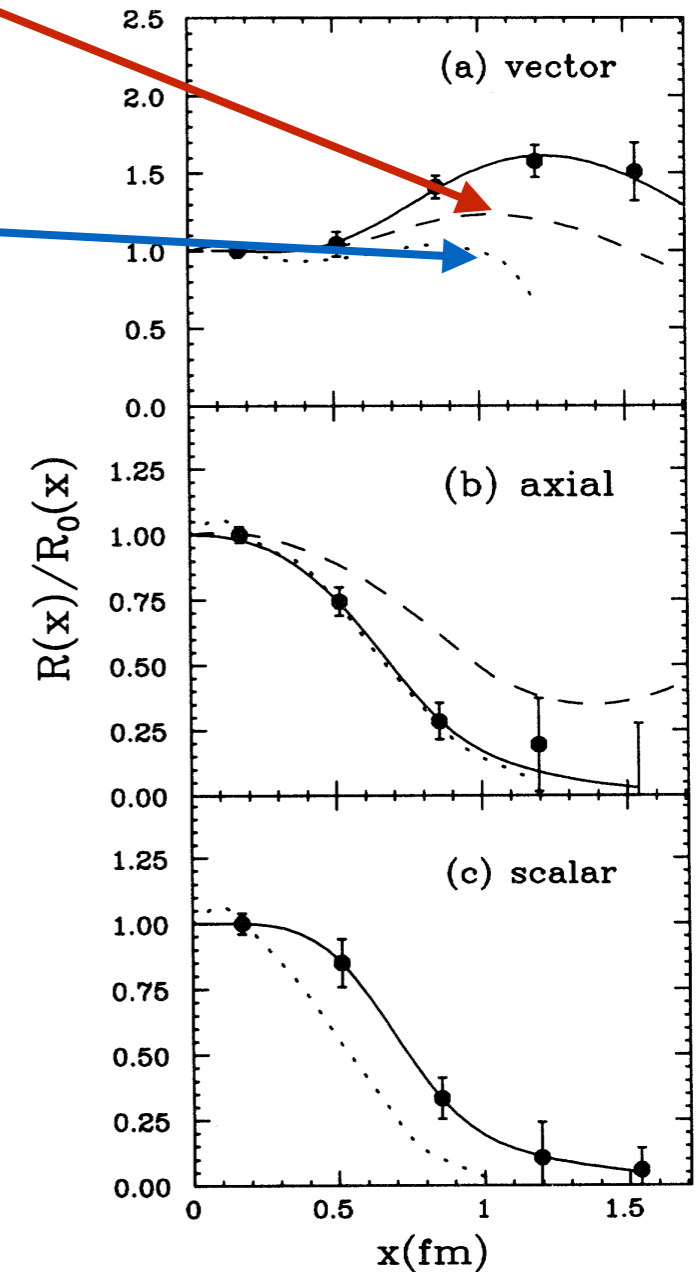
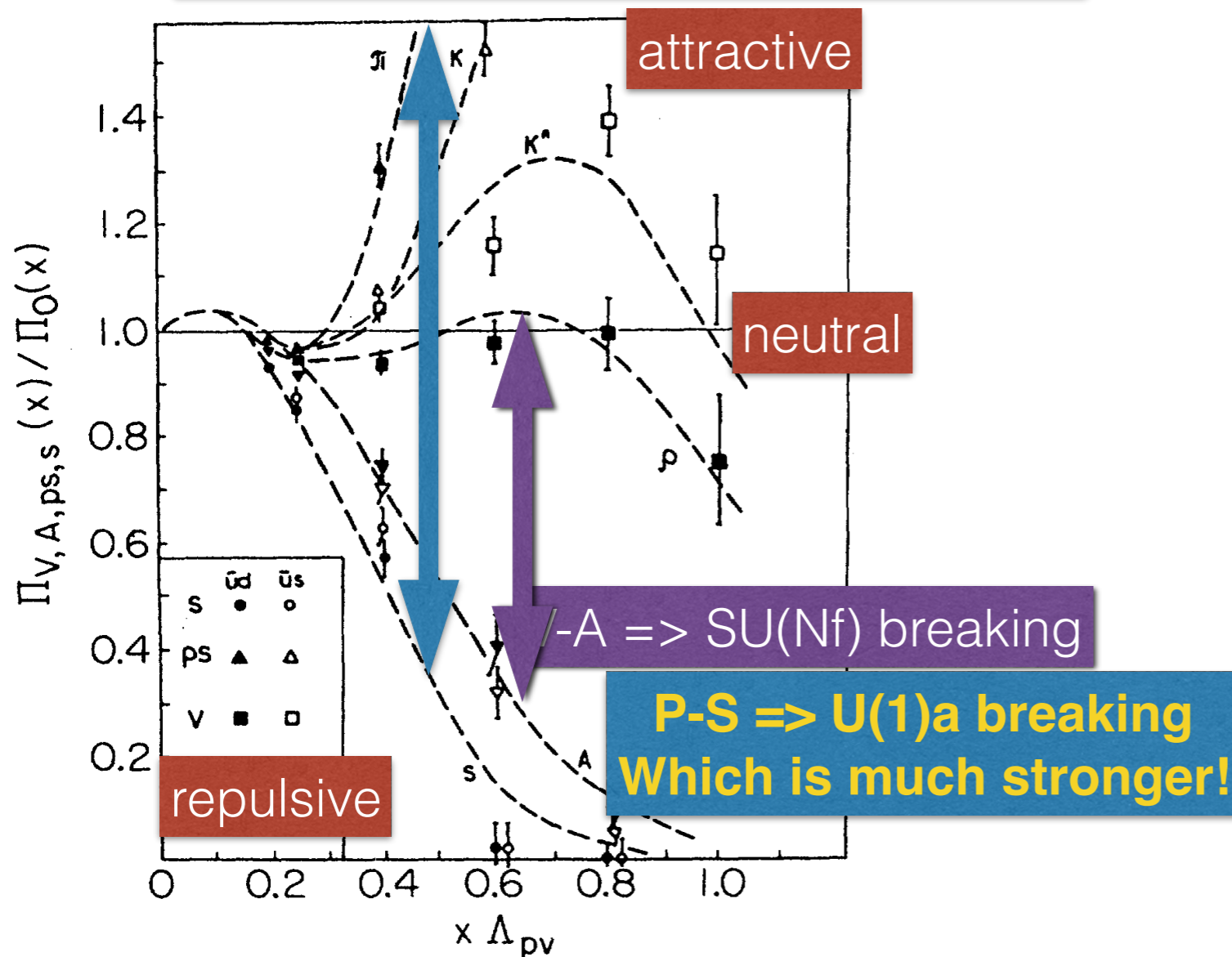
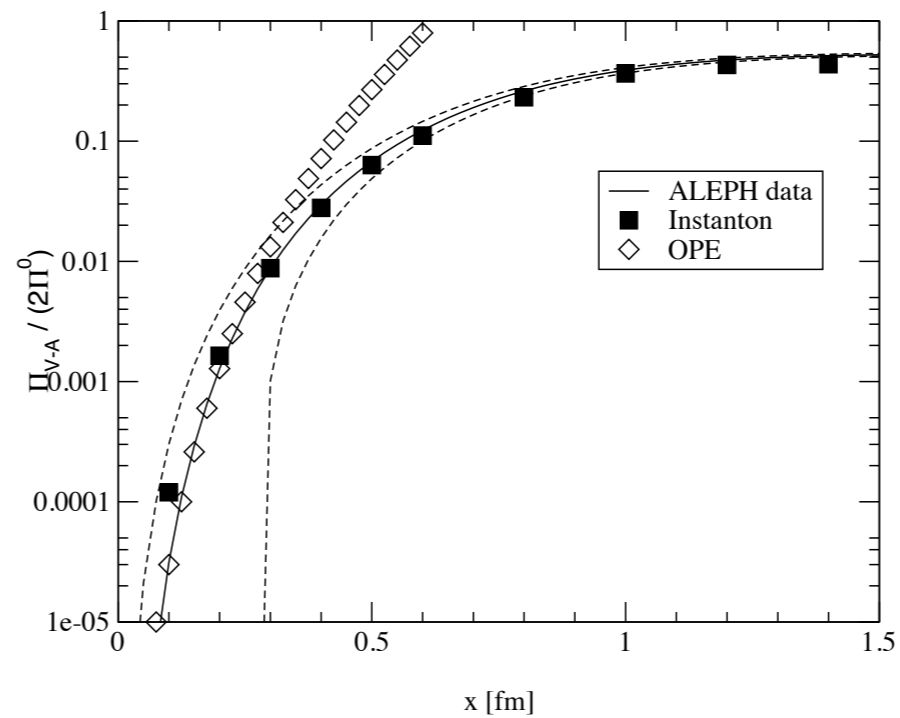


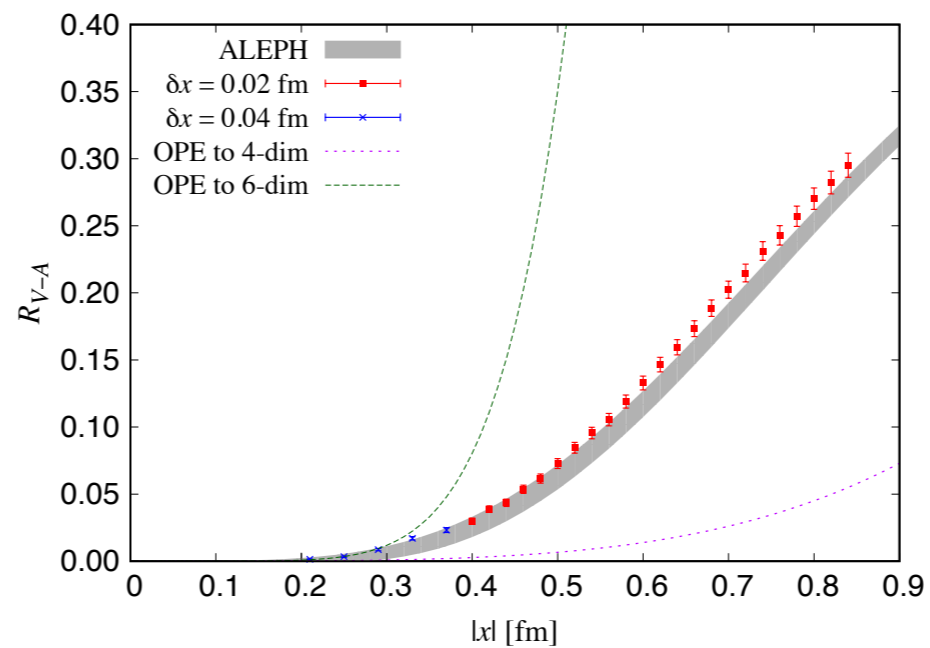
FIG. 2. Extrapolated ratio of meson correlation functions (closed circles) and fits (solid curves) as in Fig. 1. Dashed lines denote phenomenological results [4] and dotted lines show the interacting instanton approximation [5].

# V-A is the most accurately known combination reproduced by instanton liquid and lattice

T. Schafer and E. V. Shuryak, Phys. Rev. Lett. **86**, 3973 (2001) doi:10.1103/PhysRevLett.86.3973 [hep-ph/0010116].



M. Tomii *et al.* [JLQCD Collaboration] arXiv:1703.06249 [hep-lat].



# Hadronic Correlation Functions in the Random Instanton-dyon Ensemble

Rasmus Larsen and Edward Shuryak

*Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA*

It is known since 1980's that the instanton-induced 't Hooft effective Lagrangian not only can solve the so called  $U(1)_A$  problem, by making the  $\eta'$  meson heavy etc, but it can also lead to chiral symmetry breaking. In 1990's it was demonstrated that, taken to higher orders, this Lagrangian correctly reproduces effective forces in a large set of hadronic channels, mesonic and baryonic ones. Recent progress in understanding gauge topology at finite temperatures is related with the so called *instanton-dyons*, the constituents of the instantons. Some of them, called  $L$ -dyons, possess the anti-periodic fermionic zero modes, and thus form a new version of the 't Hooft effective Lagrangian. This paper is our first study of a wide set of hadronic correlation function. We found that, at the lowest temperatures at which this approach is expected to be applicable, those may be well compatible with what is known about them based on phenomenological and lattice studies, provided  $L$  and  $M$  type dyons are strongly correlated.

## Applicability limits of the instanton-dyon theory:

$$T_{max} \approx 400 \text{ MeV} : \langle P(T > T_{max}) \rangle \approx 1$$

dilute gas of instantons as seen in on the lattice in  $\chi_{top}$

$$T_{min} \approx 100 \text{ MeV} : S(T < T_{min}) < 3 \text{ too small}$$

Note that it coincides with the ranges studied in heavy ion collisions

**Does this theory at its lower range reproduce known hadronic phenomenology?**

# Yes, it can be reproduced by the dyons

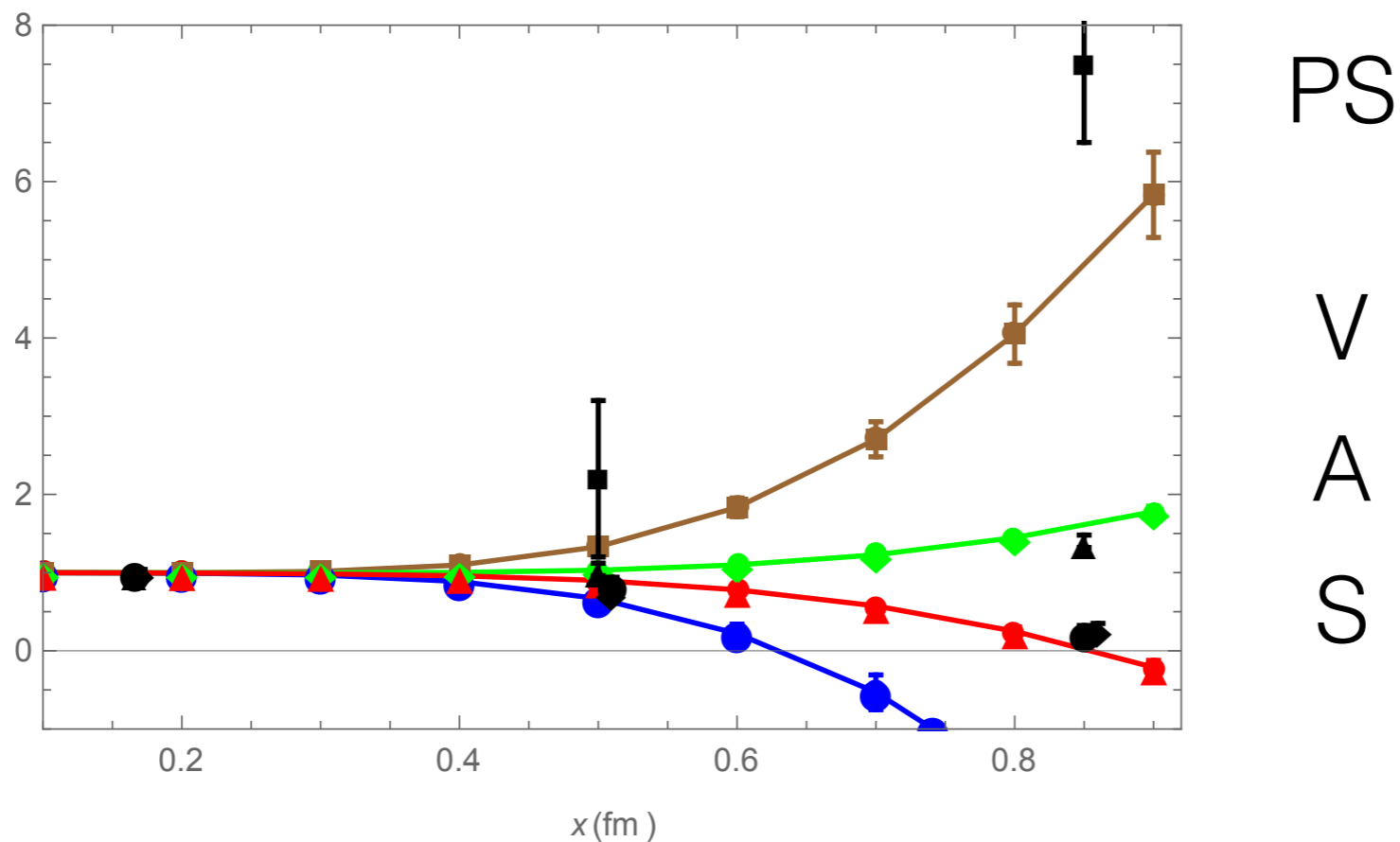


FIG. 8: (Color online) The colored points connected by lines are our results for four channels, for  $r_{12} = 0.2$ . Top to bottom: Pseudoscalar (Brown)  $\square$ , Vector (Green)  $\diamond$ , Axial vector (Red)  $\triangle$ , and Scalar (Blue)  $\bullet$ . The individual (black) points without lines are lattice data from [22], their symbols are the same as for our data.

**But with a heavy (and very non-trivial) price:  
L and M1,M2 dons must be well correlated,  
To make the fermionic zero modes well localized**

$$r_{LM} = |\vec{r}_L - \vec{r}_M| = \pi\rho^2 T$$

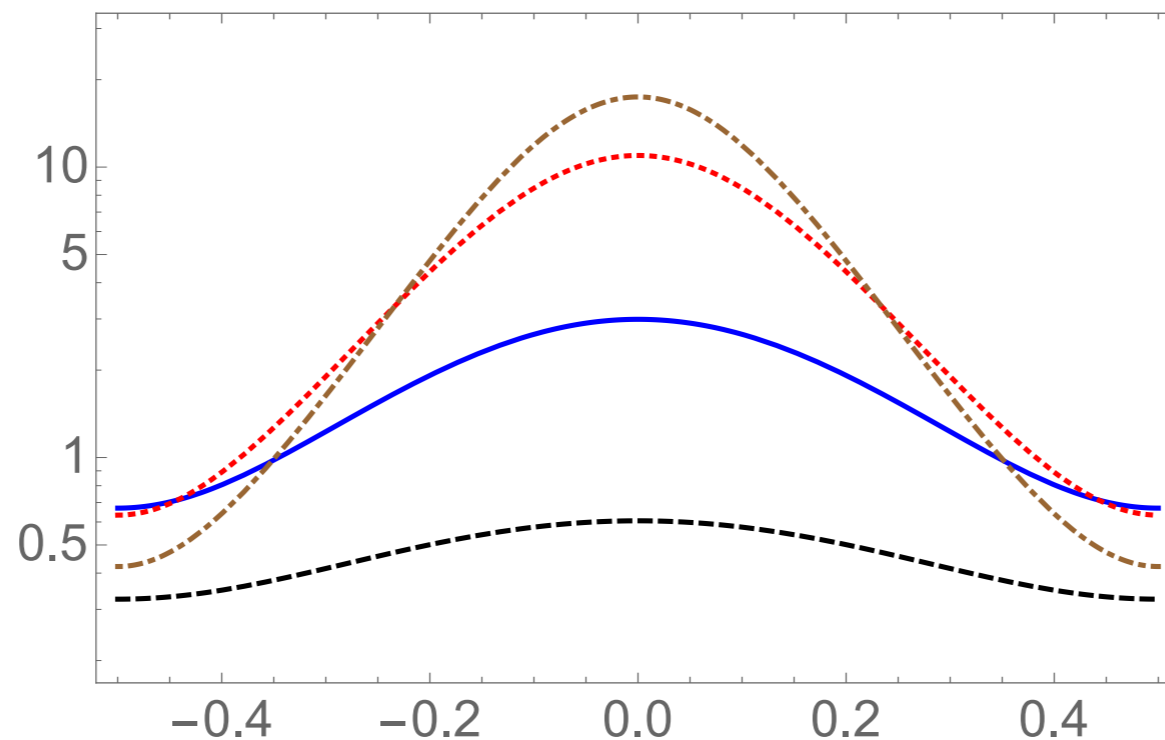


FIG. 5: (Color online) The time dependence of the zero mode densities, at  $r = 0$ , for the SU(2) caloron at confining holonomy  $v = \bar{v} = \pi$ . The lowest (black dashed) curve is at relative distance 1, the next (blue solid) is 0.5, then (red dots) 0.2 and (brown dash-dotted) one 0.1. The time and distances are in units such that  $\beta = 1/T = 1$ .

# P,V,A,S point-to-point correlation functions from random instanton-dyon ensemble

M. Tomii et al. [JLQCD Collaboration],  
arXiv:1703.06249

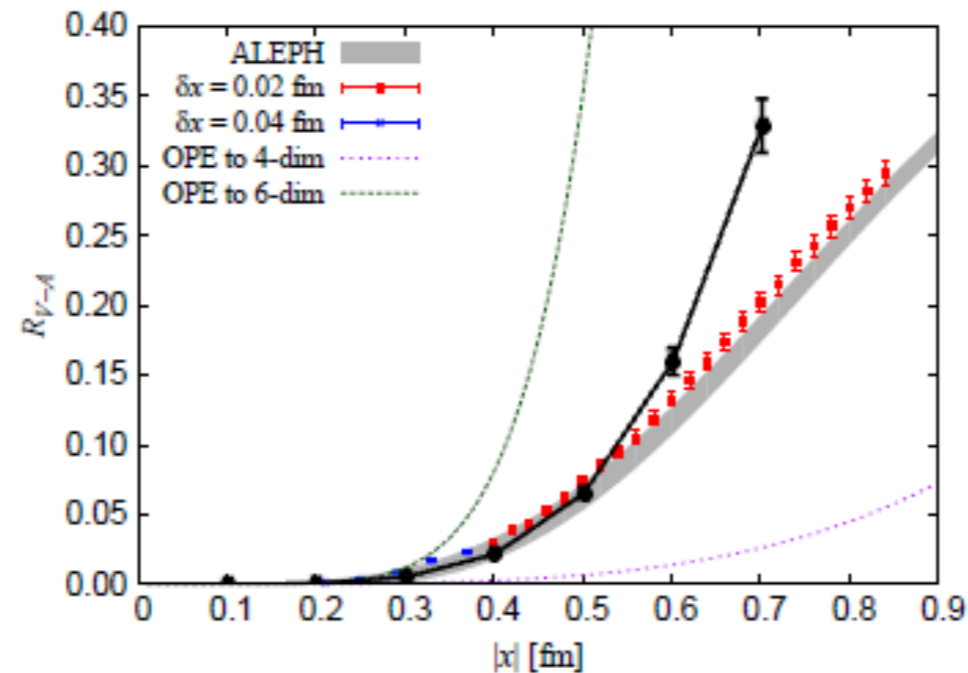
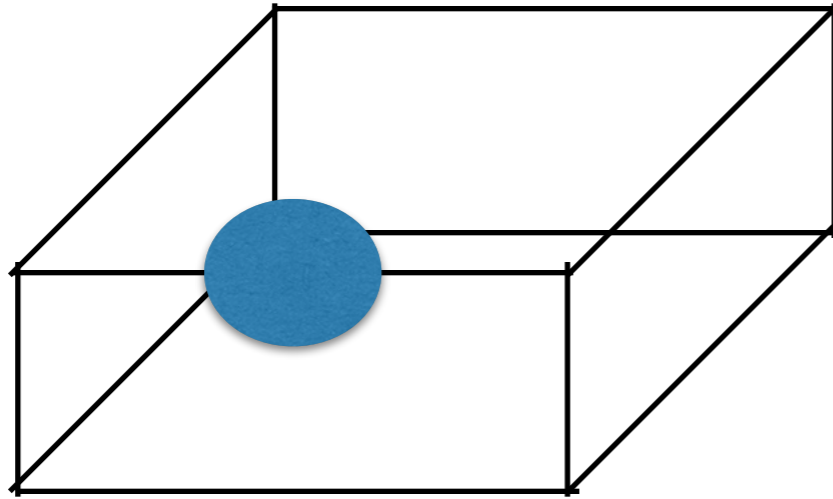


FIG. 7: (Color online) The normalized vector minus axial vector difference  $(V(x) - A(x))/(2K_0(x))$  channels versus the distance  $x$  ( $fm$ ). The narrow shadowed region corresponds to ALEPH data, the red and blue dots correspond to the lattice data [26], for two lattice spacings indicated on the plot. Our results for  $r_{12} = 0.2$  are shown by (black)  $\bullet$ .

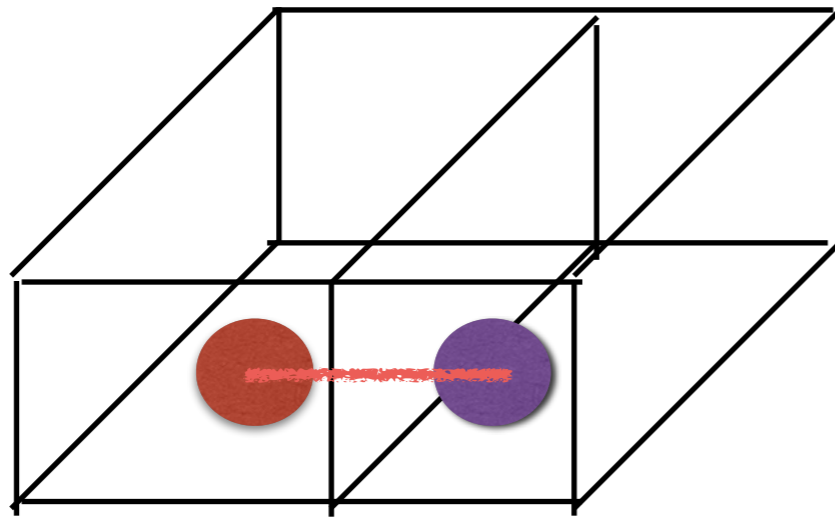
The  $r_{LM}$  parameter of the model is fitted from the V-A combination which is well known from ALEPH data and lattice. Its value tells us that L and  $M_1, M_2$  are well correlated low T



# Why study fluctuations using sub-volumes?



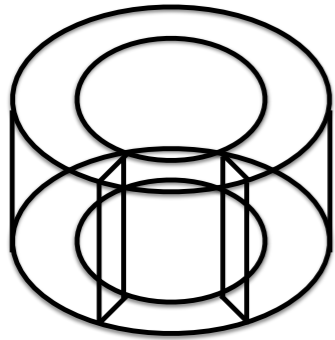
instanton in a box  
 $E, M=0 \Rightarrow Q$  integer  
 $\chi = \langle Q^2 \rangle$



dyon and anti-dyon  
have non-integer  $Q$   
and nonzero  $E, M = \pm 1$

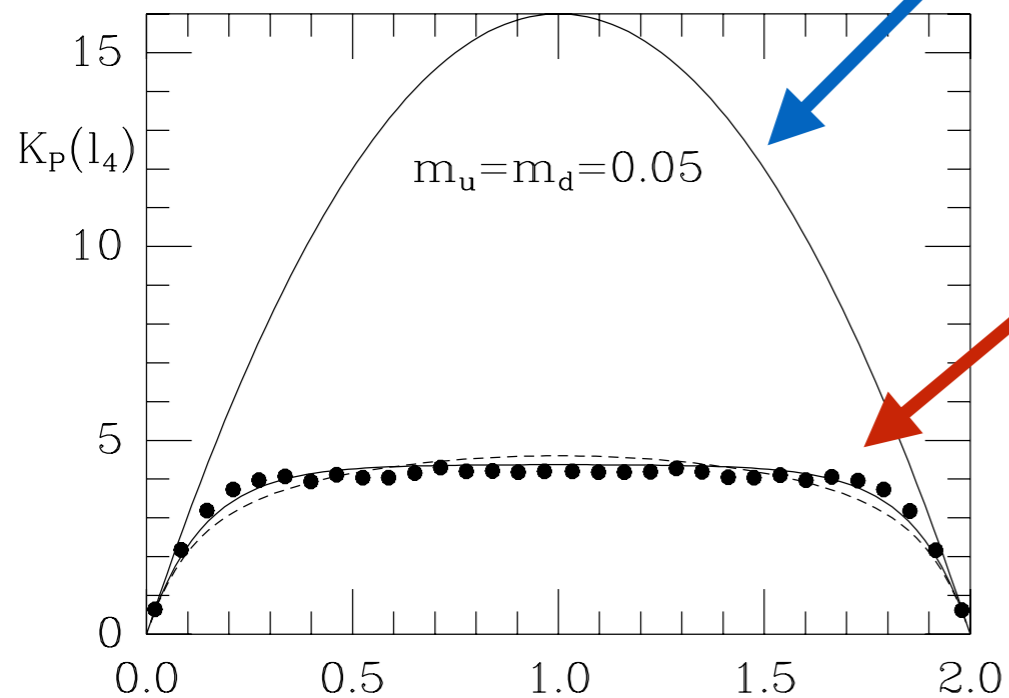
the  $Q$  quantization theorem is not violated  
because of the (invisible) Dirac string

# Topological fluctuations, using the $Q=0$ ensemble: The “slab method”



E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **52**, 295 (1995) doi:10.1103/PhysRevD.52.295 [hep-lat/9409020].

Random  
Instanton  
ensemble



Interacting  
Instanton  
ensemble  
is locally  
well neutralized

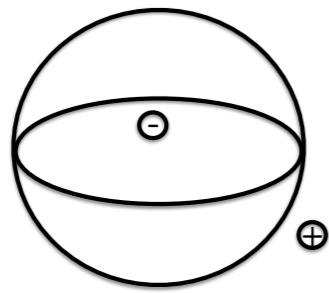
The lines provided fit to the eta' mass

# Correlations and fluctuations of the gauge topology at finite temperatures

Rasmus Larsen and Edward Shuryak

*Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA*

Instanton-dyons are topological solitons – solutions of Yang-Mills equations – which appear at non-trivial expectation value of  $A_0$  at nonzero temperatures. Using the ensembles of those, generated in our previous work, for 2-color and 2-flavor QCD, below and above the deconfinement-chiral phase transition, we study the correlations between them, as well as fluctuations of several global charges in the sub-volumes of the total volume. The determined correlation lengths are the finite- $T$  extension of hadronic masses, such as that of  $\eta'$  meson.



The interacting  
Instanton-dyon ensemble  
simulated on the  $O(3)$  sphere.  
If cut and projected to flat 3-d,  
it produces **interior** and **exterior**  
**of an ordinary  $O(2)$  sphere**

$$\chi_Q = \langle Q^2 \rangle$$

$$\chi_M = \langle M^2 \rangle$$

$$\chi_E = \langle E^2 \rangle - \langle E \rangle^2$$

$$\chi_S = \langle S^2 \rangle - \langle S \rangle^2$$

All <1  
Which means  
Partial local  
Neutralization

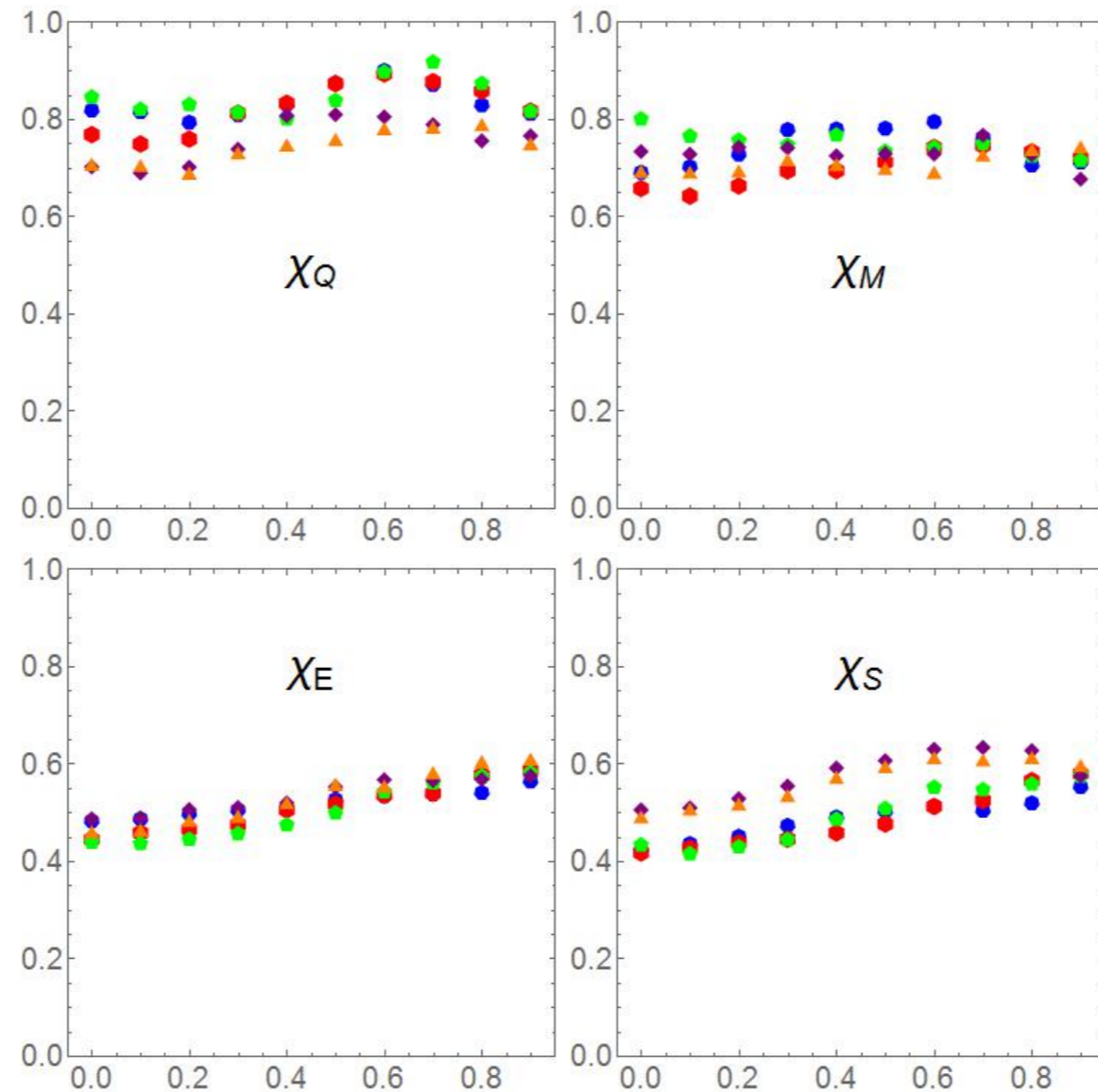


FIG. 7: (Color online) The normalized fluctuations of the topological charge  $Q$ , the magnetic charge  $M$ , the electric charge  $E$  and the action  $S$ , as a function of subvolume cut  $\cos(\psi_{cut})$ . Because of symmetry of the distributions, only one half of it is shown. The l.h.s.,  $\cos(\psi_{cut}) = 0$  corresponds to cutting the sphere into two equal halves, the r.h.s. at  $\cos(\psi_{cut}) \rightarrow 1$  corresponds to cutting off a very small part. The different points corresponds to different temperatures as explained in Fig. 1.

# Summary

Instanton-dyon ensembles  
At  $T=100$  MeV can reproduce  
Mesonic and baryonic  
Correlation functions

But this is only possible if  
L and M1,M2 dyons are quite well correlated,  
Making a (slightly deformed) instantons

Fluctuations in interacting dyon ensemble studied  
And reveal to which extent quantum numbers  
Are locally neutralized