Why are Black Hole Binaries like Quark Bound States?

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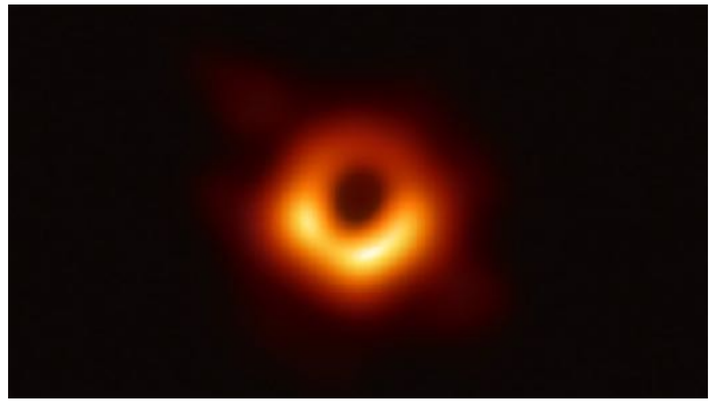
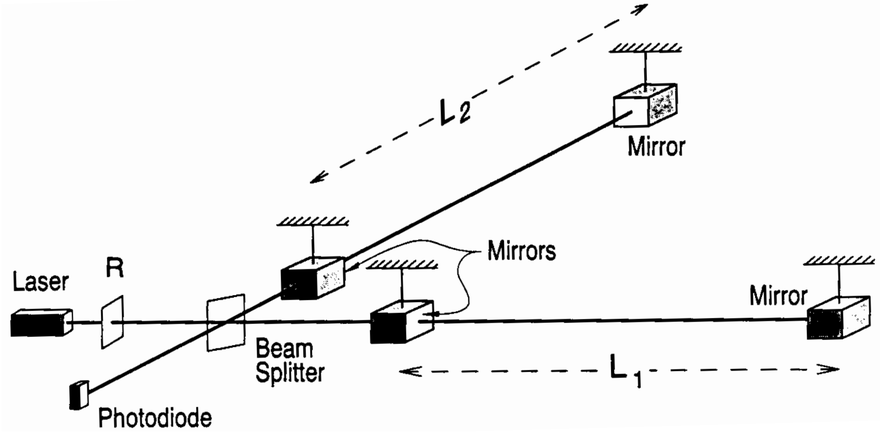
Black Holes Binaries

A black hole is a region of space where nothing can escape once past the horizon

$$r_s = 2MG$$

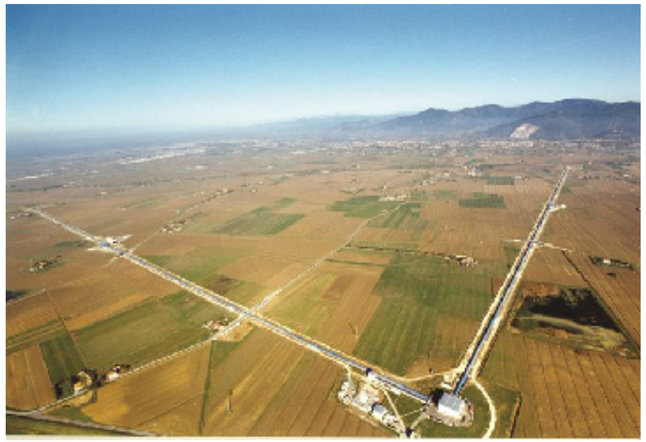


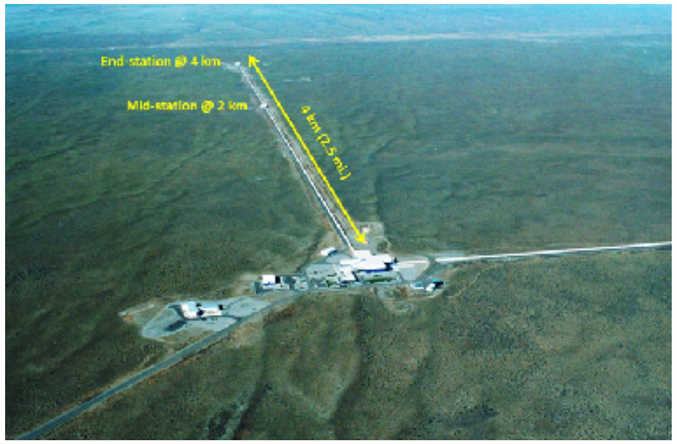
Black holes, under the correct conditions can form bound orbits, and as they orbit they emit gravitational radiation, which we can now measure via gravitational wave detectors (LIGO and Virgo) Interferometers.

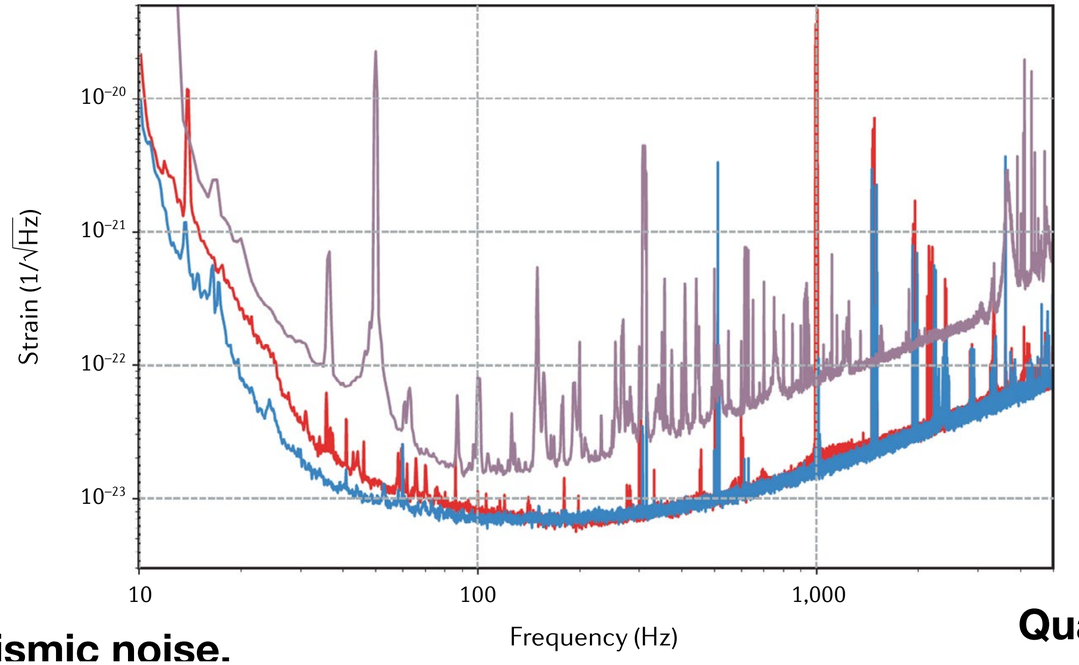


$$h = \Delta L/L \sim 10^{-21} - 10^{-22}$$

Each interferometer can resolve GW's in a frequency band 1-10^4 Hz



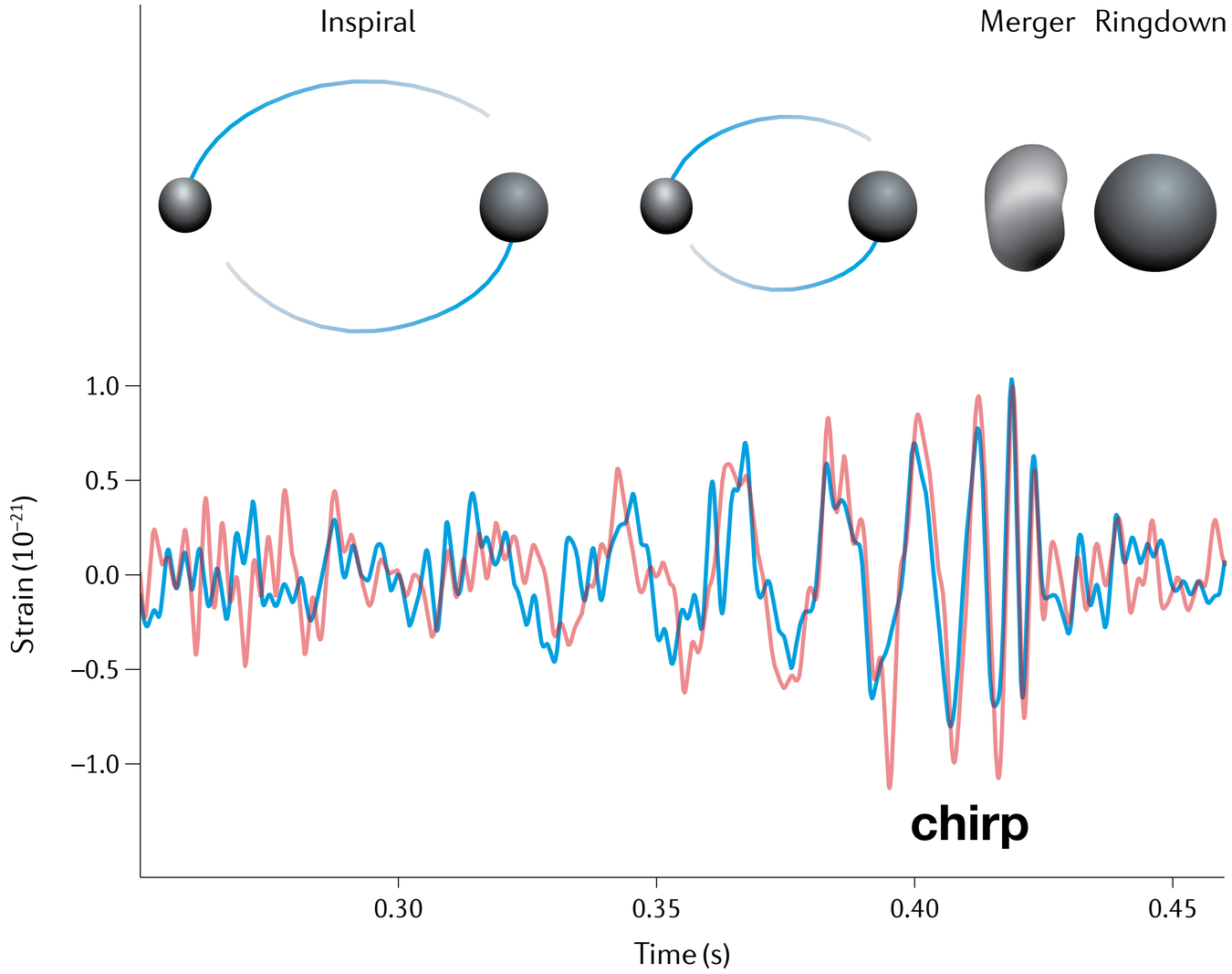




seismic noise.

Quantum noise

The Signal


Inspiral
Merger
Ringdown
1.0
0.5
0.0
−0.5
−1.0
Strain (10^{-21})
chirp
0.30
0.35
0.40
0.45
Time (s)

Reconstructing Event Parameters

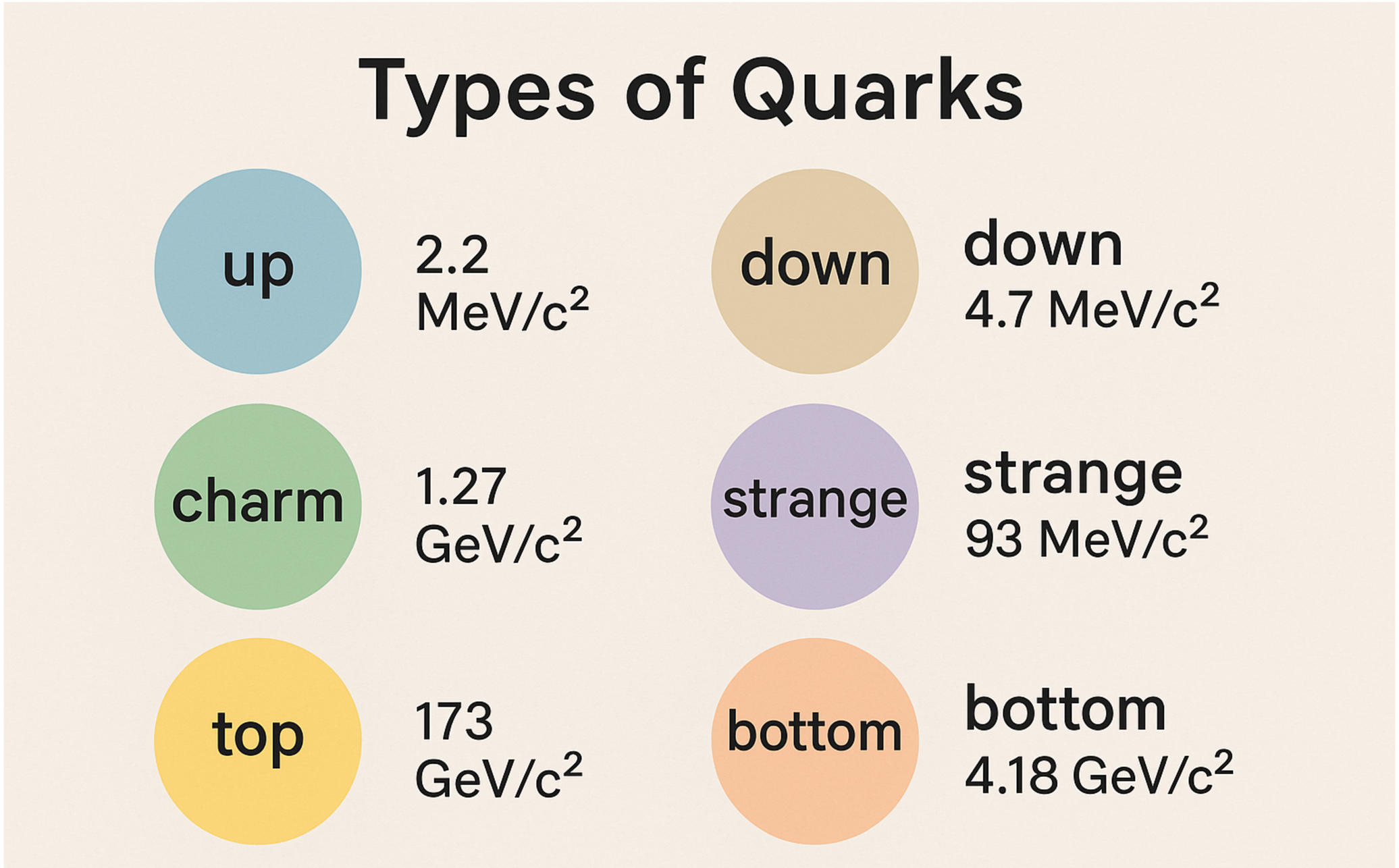
Leading order (`easy"): **Masses, Spins, Orbital plane, distance, direction**

Next to Leading order: **Internal Dynamics (gravitational susceptabilities)**

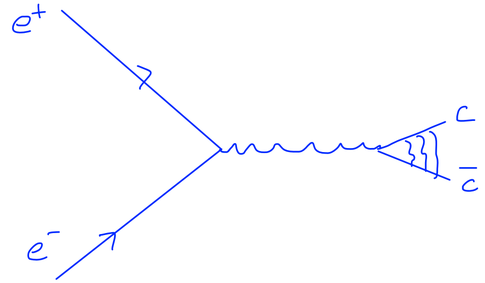
Rely about theoretical predictions to produce bank of templates (more on this later)

Since first detection in 2015 LIGO/Virgo have found hundreds of black hole inspirals, as well a neutron star inspirals. Allowed us to: 1) test GR in new ways (strong fields) 2) test theories of black hole formation 3) inform stellar evolution models via black hole statistics.....

What are heavy quark bound states (Quarkonia)



For quarks much heavier than the proton two quarks can form a hydrogen like bound state (more on that in a second)



First discovered in 1974 ``J/Psi'' or ``charmonium''

Subsequently many other states have been found

Also involving the bottom quark (tops?)

Such systems are Hydrogen like in nature with a similar spectrum.

$$V(r) = -\frac{Cg^2}{r} \qquad E_R = \frac{m_e e^4}{8h^3 c} \to \frac{m_Q g^4}{8h^3 c} \qquad r_0 = \frac{1}{e^2 m_e} \to \frac{1}{g^2 m_Q}$$

$$\frac{1}{2} m v^2 \sim \frac{e^2}{r} \to v \sim e^2$$

This system is inherently quantum mechanical because: $L \sim r m_Q v \sim \frac{\hbar}{e^2 m_Q} m_Q e^2 \sim \hbar$

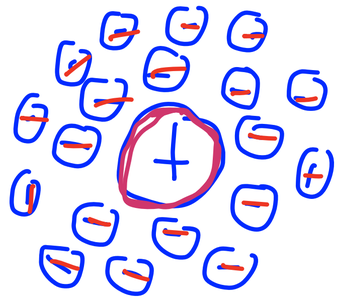
Compare to a binary black holes system $L \sim rmv \sim M_\odot R_{AU} \frac{GM_\odot}{R_{AU}} \sim 10^{2\times 57}/10^{19} \sim 10^{90}\hbar$

Why are we confining ourselves to heavy quarks only?

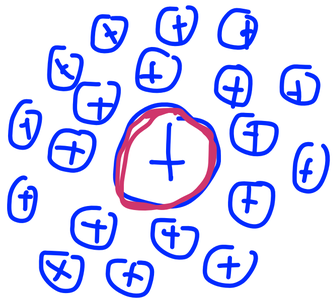
Because the spectrum of ``light quarks is nothing like Hydrogen why?"

Because E+M is weak, the strong force is strong!!!! (Almost always)

In E+M the coupling gets weaker at long distances due to ``screening"

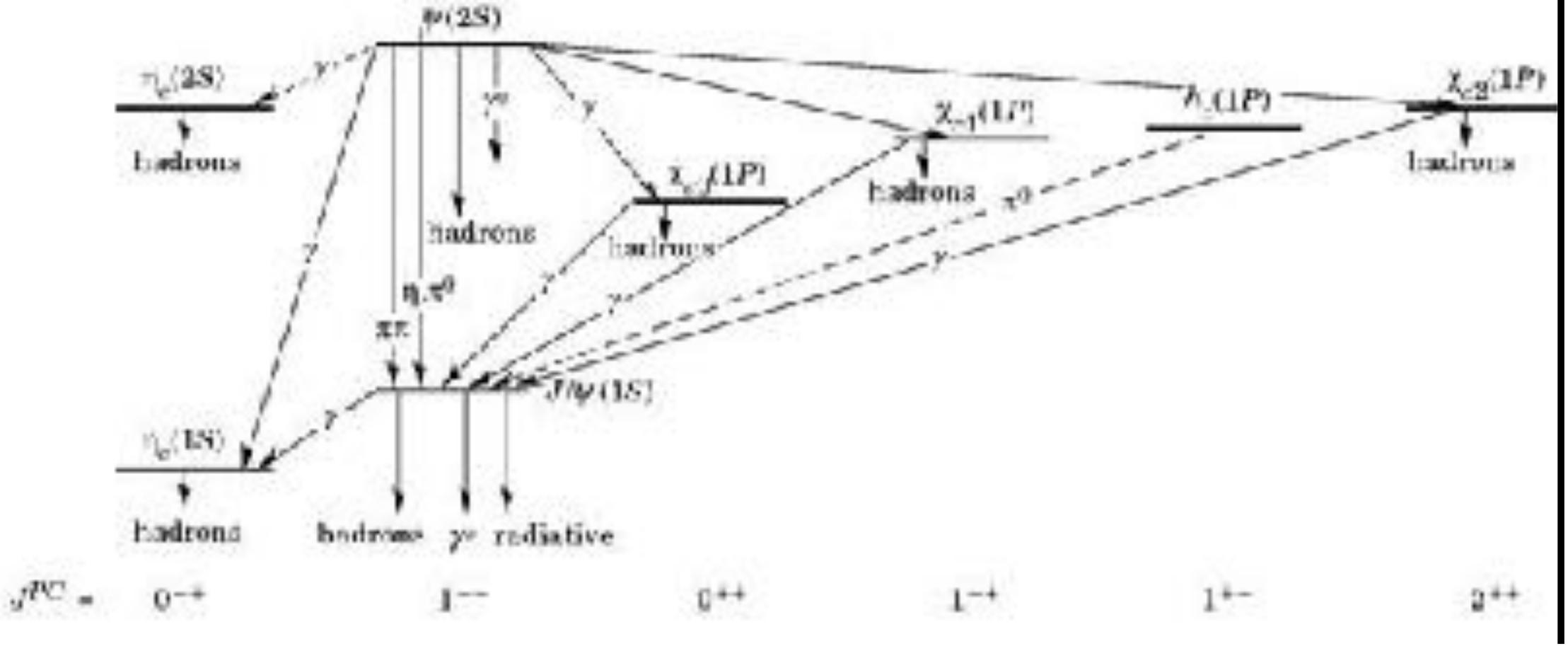


SCREENING

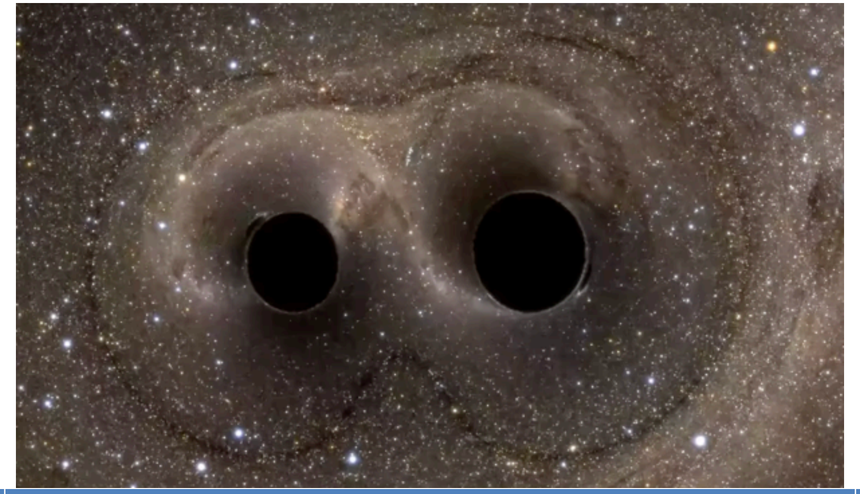


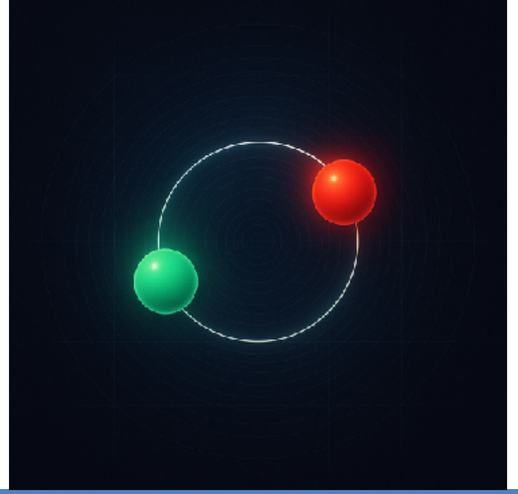
ANTI-SCREENING

In the strong force as the mass of the quarks gets larger the radius of the orbit gets smaller and the coupling weaker. Thats why only heavy Quark bound states look like Hydrogen which in term looks like planetary orbits



These states are pretty well understood in vacuum. But right now they are playing a crucial Role in understanding the quark-gluon plasma.





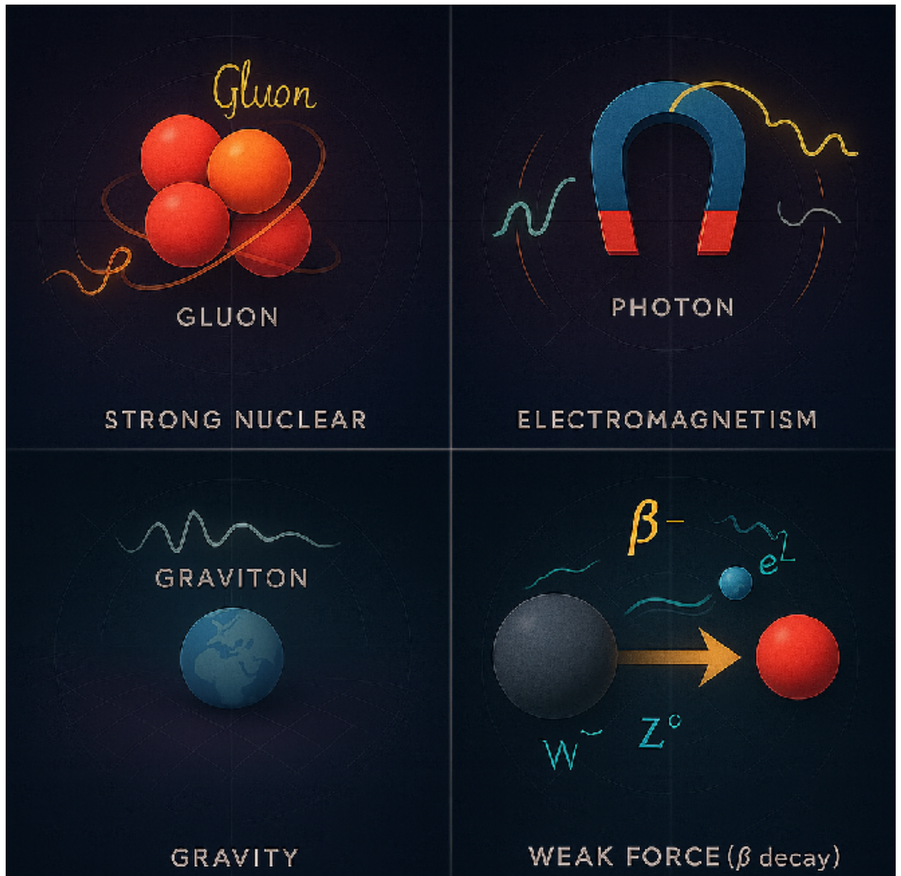
Property	Black Holes	Quark-bound state
Constituents	Curved spacetime region (baryonic matter collapsed) Event horizon present	Heavy quark–antiquark pair (e.g., $c\bar{c}$, $b\bar{b}$)
Binding Force	Gravity (general relativity)	Strong force (QCD)
Typical Size	Stellar-mass: ≈30 km SMBH (10^6 M⊙): ≈3×10^7 km	Charmonium: ≈0.4 fm (4×10^{-16} m) Bottomonium: ≈0.2 fm (2×10^{-16} m)
Mass Scale	3–10 M⊙ (stellar) up to 10^9 M⊙ (galactic centers)	3 GeV/c^2 (J/ψ) – 10 GeV/c^2 (Υ)
Lifetime / Stability	Classically stable; quantum Hawking evaporation times >10^{66} yr (stellar BH)	Short-lived; decay via gluons or electromagnetism ~10^{-20} s

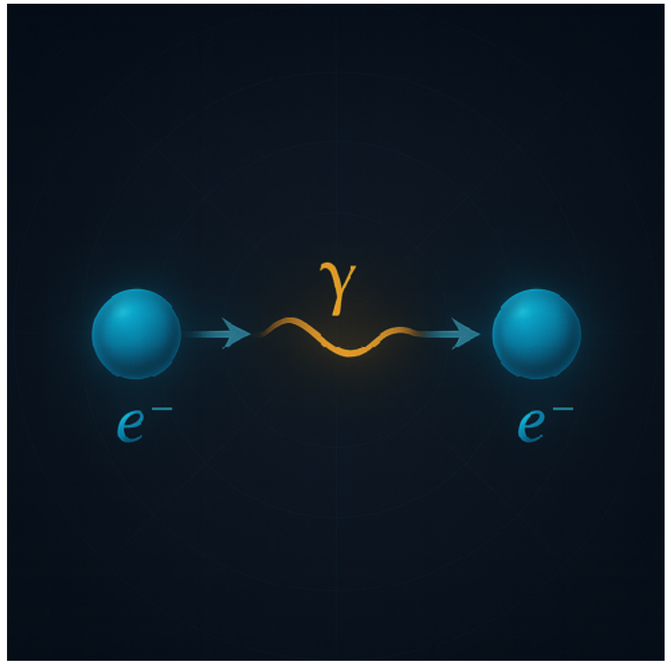
It stretches the imagination that these two phenomena are in anyway related!

But systems with very disparate scales can behave in very similar ways if the underlying dynamics are based on the same basic principles, even if they are manifestations of different forces.

What are the basic principles that govern these two phenomena?

Fundamental forces of Nature, each has an associated force carrier





Coulombs' law is a result of the exchange of a photon between electrons

We will see that all of the force carriers are manifestations/realization of the same class of objects.

Ultimate Dream of Phsysics is to ``Unify'' the forces into one Theory of Everything

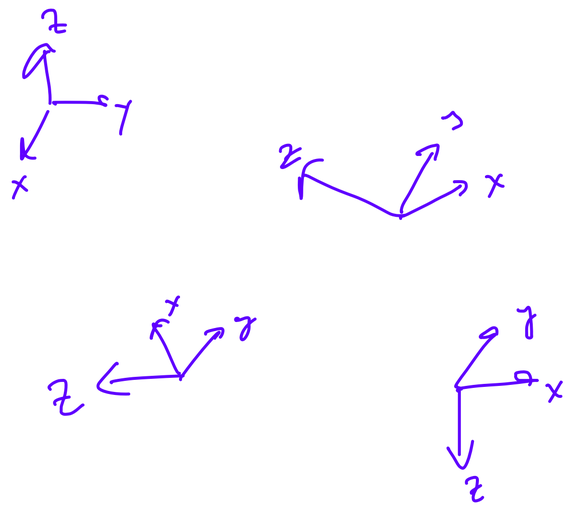
While we are not there yet, but we have shown that all forces are based on the same FUNDAMENTAL PRINCIPLE, and that sameness will allow us to understand how black hole binaries are intimately related (in a quantifiable way) to heavy quark bound states.

All four forces are based on the Principle of Gauge Invariance.

Principle of Gauge Invariance

Coordinate system are a man made artifial crutch and nothing should ever depend upon that choice!

In fact I am free to break up my coordinate system into a whole bunch of coordinate systems in separate regions:



But surely there has to be SOME relations between coordinate systems at different points! We must add some structure to the theory which tell us how the coordinate systems are related.

Let us define: $R_{ij}(\vec{x}, \vec{x} + \delta\vec{x})$

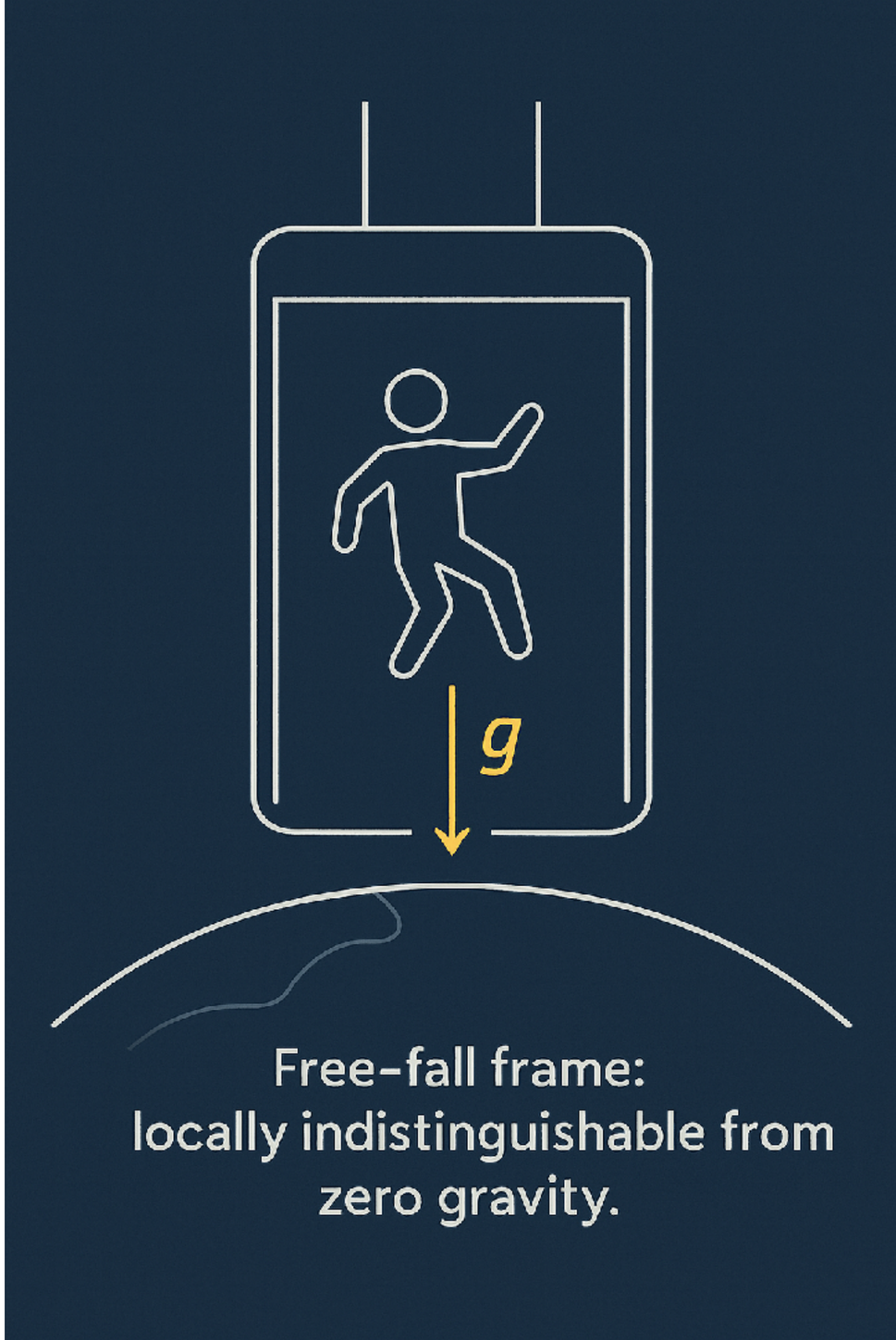
As the rotation matrix which relates coordinate systems at nearby points $\vec{x}$ and $\delta\vec{x}$ And lets call this the ``CONNECTION FIELD".

Every theory based on the gauge principle MUST have a connection, and this connection is nothing but the force carrier!

In gravity we can get some physical understanding for the gauge principle: the coordinate systems we drew are coordinates in space/time and the gauge principle is equivalent to the equivalence principle, which states that

At any point in space/time we can go a reference frame (coordinate system) where we can turn off gravity.

But how do we understand these coordinate systems in the case of the other forces?

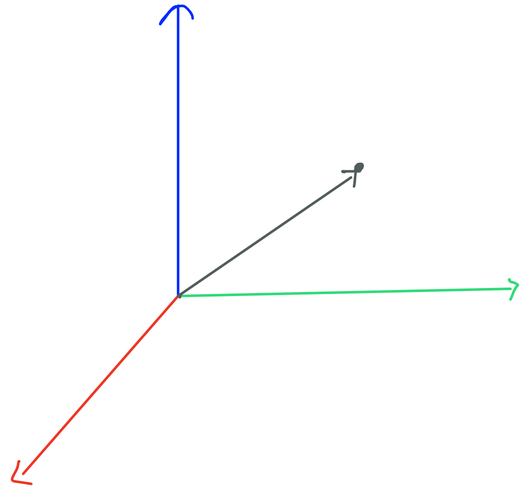


Lets consider the strong force. Lets say that every particle can come in three different colors.

It could even come in some linear combinations of colors. Next lets define a vector space spanned by three colors such that each particles

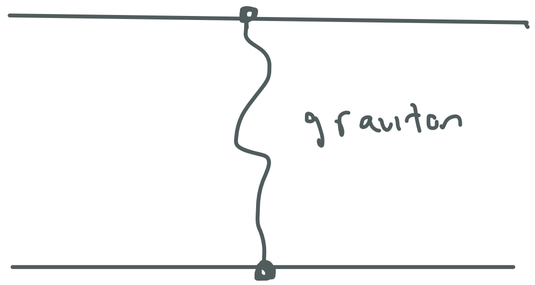
Color state is represented by a vector

According to our gauge principle we are free to choose a different orientation for our ``coordinate system'' at each point in space!



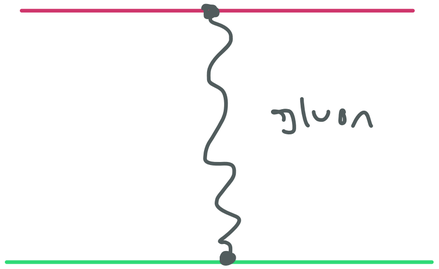
That means a blue particle for me might be a red one for you! Thats right. But once we introduce a connection (the gluon) we can compare colors in a sensible way.

The strong and gravitational force both arise due to the exchange of the connection field (*).



$V(r) = \frac{G M_1 M_2}{r}$

planetary orbits



$V = \frac{g_1 g_2 C}{r}$

HEAVY QUARK BOUND STATES.

(*) Technically the connection is actually the derivative of what we call the graviton.

So we have established a connection between these two systems, but its been kinda vague and fuzzy, but where is the scientific pay off?

In our to extract the most information from gravitational wave observatiosn we need PRECISION predictions. Suppose we want to know more than the mass, e.g. what is the object were looking at made of? e.g. if its a neutron star or a dark matter star?

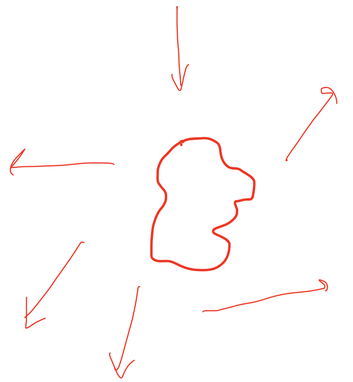
Stretching under a tidal deformation does tell us about its compostion:

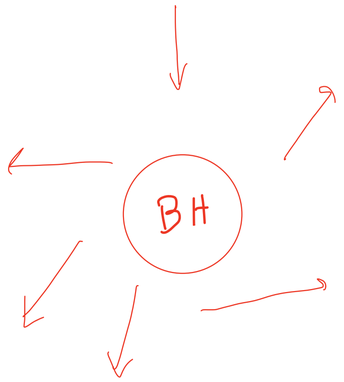
$$\delta x = \chi \partial \partial V$$

χ Is known as the love number

One of the most remarkable predictions of GR is that for black hole the Love number vanishes!

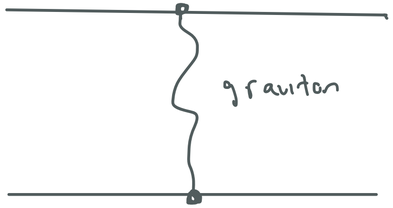
That is truly bizarre: You cant stretch a black hole using any type of static field





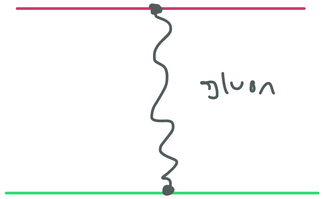
But this is a tiny effect ~10^(-6), and in order to extract from the data we need very precise prediction

Lets come back to our force Law



$V(r) = \frac{G M_1 M_2}{r}$

planetary orbits

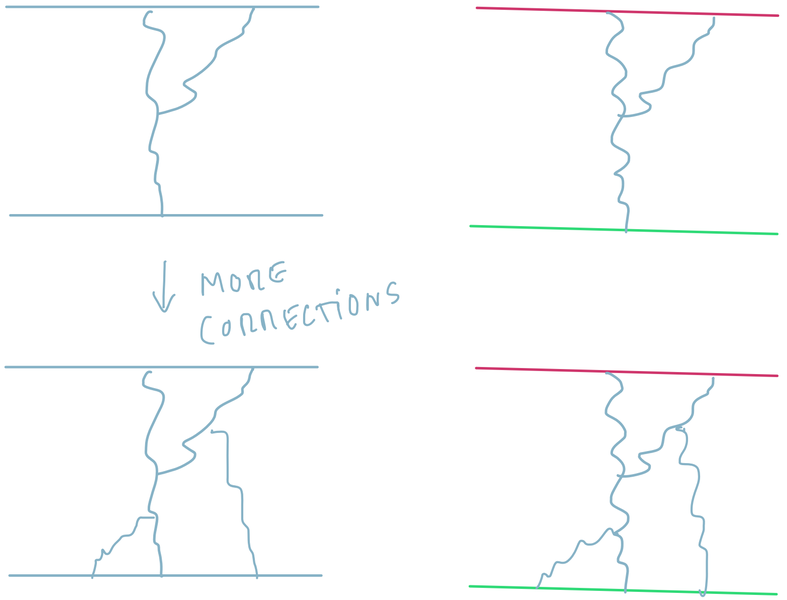


$V = \frac{g_1 g_2 C}{r}$

HEAVY QUARK BOUND STATES.

These objects are called "Feynman diagrams" and are representations of integrals.

This is only the largest pieces but there are smaller corrections



For example at one loop order we have

Leads to Einstein-Infeld-Hoffman Potential

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

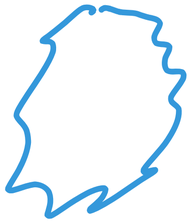
Note we generate velocity dependent potentials as well as higher powers of 1/r.

As we go to higher order in non-linearities things get VERY messy very quickly. Especially in gravity (compared to QCD), as non-linearities are much messier.

It turns out that the gravity calculations are much more difficult than the QCD calculations, but there is a deep relation between the two that comes from string theory:

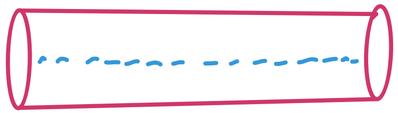
In string theory,

a graviton is excitation of a closed string



a gluon is excitation of an open string

Closed string propagation



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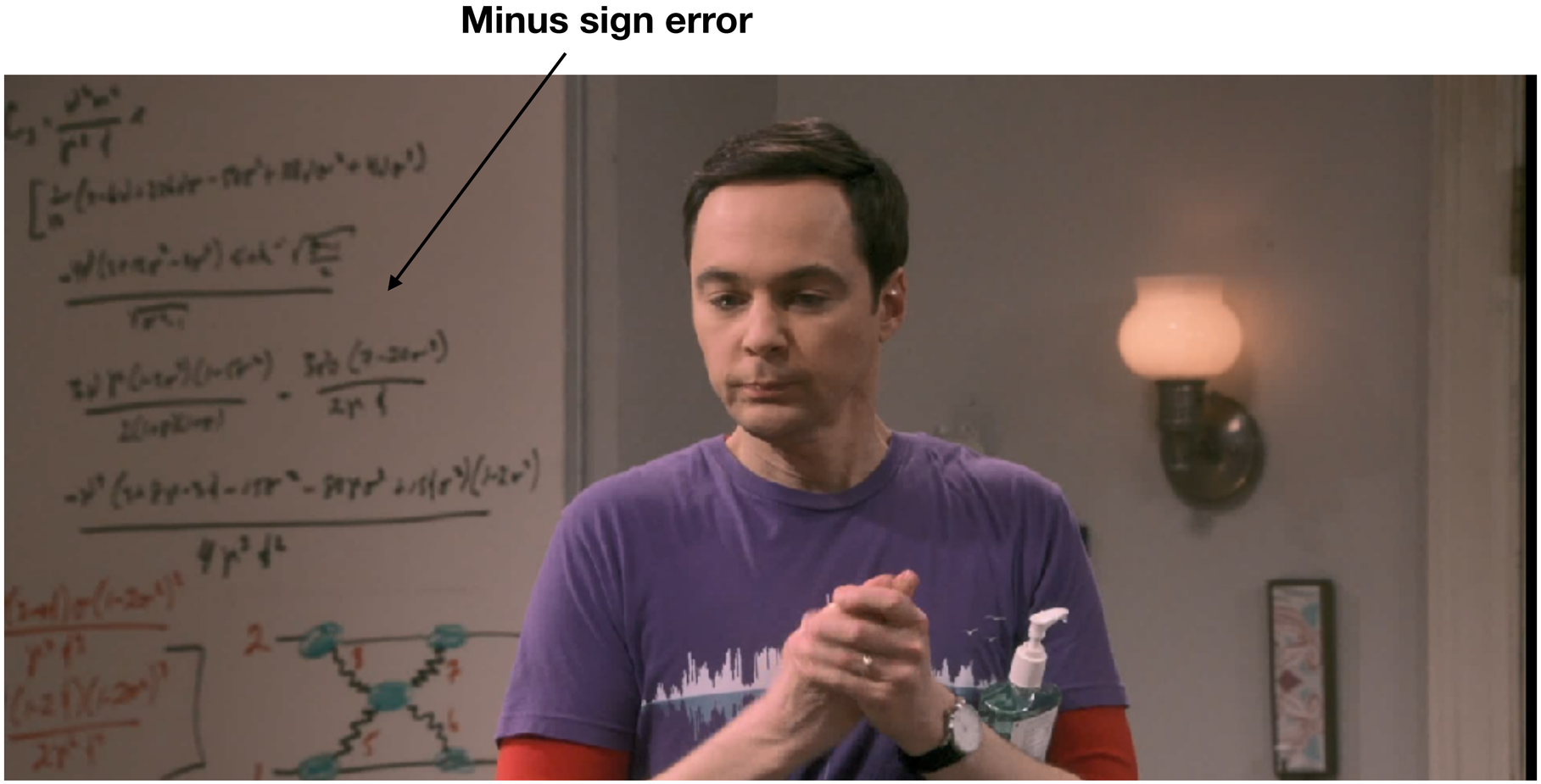
Product of two open string propagations

The Gravity contributions to the potential are given by the square of the QCD contributions! (KLT relations, BCJ relations)

These relations have been used to calculate to very high order (3 loops)! A lot of work. But still need higher precision to be able to extract the Love numbers from the data. This is an very active field of research.



There is much more theory that needs to be done.

Lessons Learned: The laws of physics like to repeat themselves, why?

My guess: There are certain set of fundamental principles that all of the laws of nature must obey.

Causality

Laws of Relativity

Locality (no action at a distance)

It seems that forming logically consistent systems based on these constraints is much more constraining then one might think! Especially if we insist that nature includes long range forces.