Chiral vortical effect for bosons

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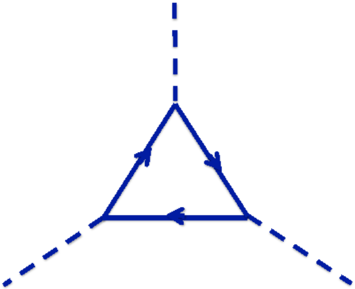
Stony Brook, August, 2017



$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi = \frac{1}{2\pi^2}E\cdot B$$

In medium it results in

$$J_\mu^V = \sigma_B^V B_\mu + \sigma_\omega^V \omega_\mu \quad , \quad J_\mu^A = \sigma_B^A B_\mu + \sigma_\omega^A \omega_\mu$$

$$\sigma_B^V = \frac{\mu_A}{2\pi^2} \quad , \quad \sigma_\omega^V = \frac{\mu_V \mu_A}{\pi^2}$$

$$\sigma_B^A = \frac{\mu_V}{2\pi^2} \quad , \quad \sigma_\omega^A = \left(\frac{\mu_V^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6} \right)$$

and the conductivities appear to be pretty universal.

Let's concentrate on the CVE which gives an additional contribution to the axial charge:

$$J_5^\mu = \sigma_\omega^A \omega^\mu \quad , \quad Q_{fh} \sim \int \sigma_\omega^A \, (v \cdot \Omega) \, d^3x \, ,$$

which survives **in the absence of the EM fields**

$$\begin{array}{ccc} \partial_\mu J_A^\mu = C_a E \cdot B & & \partial_\mu J_A^\mu = 0 \\ & \xrightarrow{E,B \to 0} & \\ J_\mu^A = \sigma_\omega^A \omega_\mu + \sigma_B^A B_\mu & & J_\mu^A = \sigma_\omega^A \omega^\mu \end{array}$$

and we naively allow $Q_{fh} \to Q_5^{(0)}$.

- A simple explanation may be obtained in an EFT description[1]

$$\delta S = \int \mu \bar{\psi} \gamma^0 \psi d^4 x \Rightarrow \int \mu u_\mu \bar{\psi} \gamma^\mu \psi d^4 x$$

where we introduced a slowly varying boost field.

- Then, the axial charge conservation is modified

$$\partial_t \left(Q_5^{(0)} + \frac{C_a}{2} \int_x A \cdot B + \int_x \sigma_B^A \, v \cdot B + \int_x \sigma_\sigma^A \, v \cdot \Omega \right) = 0 \, .$$

where helicities are known to correspond to the linkage of the flow and/or field lines.

[1] AS, V.I. Shevchenko, V.I. Zakharov, 2011

- If the generalized axial charge picture is valid it may considerably enrich the dynamics, for instance through **the new instabilities**[1]

$$Q_5^{(0)} \to \mathcal{H}_{mh}, \mathcal{H}_{fmh}, \mathcal{H}_{fh}$$

- However it is **NOT** obvious that there is a unified conservation. Indeed, in ideal hydrodynamics all helicities are conserved separately

$$\sigma_E \to \infty \ , \ \eta \to 0 \Longrightarrow \partial_t \mathcal{H} = 0$$

while one has to suggest **a microscopic mechanism** to transfer the medium motion to fermionic modes in the presence of the dissipation.

[1] A. Avdoshkin, V. Kirilin, AS, V.I. Zakharov, 2014

- Recently we found **an example** of such a mechanism[1] with an intermediate generation of $\mathcal{H}_{mh}$

$$\int \sigma_\omega^A \, v \cdot \Omega \; d^3x \to \int \; A \cdot B d^3x \to N_A$$

- The first step may be seen as a consequence of **a novel chiral effect** – CVE for photons

$$\langle K^\mu \rangle \sim T^2 \omega^\mu \,,$$

which results in a helicity transfer along the vorticity.

[1] A. Avkhadiev, AS, 2017

But let's firstly review the thermal contribution to the common CVE (tCVE). In holography it is related to the **gravitational axial anomaly**[1]:

$$d = 5 \qquad\qquad d = 4$$

$$\chi \int \epsilon_{MNPQR} A^M R^{BNP}_{A} R^{AQR}_{B} \quad \Longrightarrow \quad \nabla_\mu J^\mu_5 \sim \chi R \tilde{R}$$

where χ controls the anomalous coefficient.

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\boldsymbol{J^\mu_5 \sim \chi T^2 \omega^\mu}$$

[1] K. Landsteiner et al., 2011

However the relation is less transparent on the field theory side.

- Indeed, the gravitational anomaly appears to be **of higher order in derivatives**:

$$\partial_\mu J_5^\mu = C E \cdot B \sim O(\partial^2) \qquad \nabla_\mu J_5^\mu = C_g R\tilde{R} \sim O(\partial^4)$$

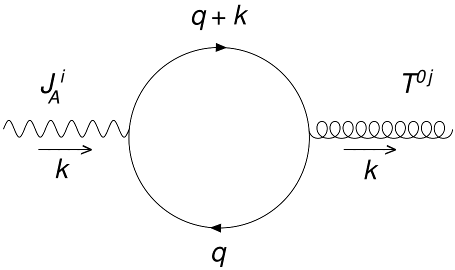
$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$J_5^\mu \sim \mu^2 \omega^\mu \sim O(\partial) \qquad J_5^\mu \sim T^2 \omega^\mu \sim O(\partial)$$

  and cannot play the same role as the axial anomaly in EM fields at least within hydrodynamics.

- In the weakly interacting limit, the tCVE can be obtained by the Kubo formula with gravitational perturbation $g_{0i} \sim v_i$:

$$J^\mu_A(x) = \sigma^A_\omega \omega^\mu \quad , \quad \sigma^A_\omega = \lim_{q\to 0} \frac{-i}{q_k} \epsilon_{ijk} \left\langle J^i_A T^{0j} \right\rangle |_{\omega=0}$$



K. Landsteiner et al., 2011

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- The coefficient in front of the gravitational anomaly and the coefficient in the conductivity **coincide**

$$\nabla_\mu J_5^\mu = \frac{\#}{768\pi^2} \cdot \epsilon^{\mu\nu\rho\lambda} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\lambda} \quad , \quad J_5^\mu = \frac{\#}{12} T^2 \omega^\mu$$

K. Landsteiner et al., 2011

Despite the discussion mentioned above there is **one particular** field theoretical **setup** where the anomalous origin of the tCVE is transparent and it requires two ingredients:

- the tCVE as a P-odd contribution to the Hawking radiation[1]:

$$\frac{dN}{dtd\omega do} \sim \frac{\omega^2}{T^2}\left(1 - \frac{L\Omega}{4T}\cos\theta\right)$$

- the relation of the Hawking radiation with the anomalies of the effective $1+1$d theory at the horizon[2] (the chiralness of the dimensionally reduced theory at the horizon).

[1] A. Vilenkin, 1978-1979

[2] S. Robertson, F. Wilczek, 2006

Let's summarize:

- The tCVE can be seen as a polarization effect in a rotating thermal radiation.
- It has an analog in the Hawking radiation of a rotating BH.
- It is related to the gravitational anomaly in the holographic considerations.
- There is a connection of the tCVE and the global anomalies.

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  One expects polarization effects for any non-zero spin.
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Let's summarize:

- **One expects polarization effects for any non-zero spin.**

- It has an analog in the Hawking radiation of a rotating BH. **The spin-gravity interaction is "trivial" and it is natural to expect a similar effect for $s \neq \frac{1}{2}$**
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- **One expects polarization effects for any non-zero spin.**

- **The spin-gravity interaction is "trivial" and it is natural to expect a similar effect for $s \neq \frac{1}{2}$**
- It is related to the gravitational anomaly in the holographic considerations.
  There are well known examples of gravitational anomalies for theories with $s > \frac{1}{2}$
- There is a connection of the tCVE and the global anomalies.

Gravitational anomaly for photons[1]:

$$F'_{\mu\nu} = F_{\mu\nu}\cos\alpha + \tilde{F}_{\mu\nu}\sin\alpha$$

$$\Downarrow$$

$$K^\mu = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta \quad , \quad \nabla_\mu K^\mu = \frac{1}{2}F\tilde{F} \quad , \quad \langle\nabla_\mu K^\mu\rangle_{\text{naive}} = 0$$

$$\Downarrow$$

$$\langle\nabla_\mu K^\mu\rangle_{\text{triangle}} = -\frac{1}{96\pi^2}R\tilde{R}$$

[1] A.D. Dolgov et al., 1988

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- **One expects polarization effects for any non-zero spin.**

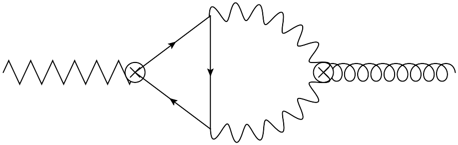
- **The spin-gravity interaction is "trivial" and it is natural to expect a similar effect for $s \neq \frac{1}{2}$**

- **There are well known examples of gravitational anomalies for theories with $s > \frac{1}{2}$**
- There is a connection of the tCVE and the global anomalies. **The relation of some chiral effects to the global anomalies could be extended to $s \neq \frac{1}{2}$ (some examples for $d \neq 4$ in the literature).**

Let's summarize:

- **One expects polarization effects for any non-zero spin.**
- **The spin-gravity interaction is "trivial" and it is natural to expect a similar effect for $s \neq \frac{1}{2}$**
- **There are well known examples of gravitational anomalies for theories with $s > \frac{1}{2}$**
- **The relation of the tCVE to the global anomalies could be extended to $s \neq \frac{1}{2}$ (some examples for $d \neq 4$ in the literature[1]).**

[1] S.D. Chowdhury, J.R. David, 2016



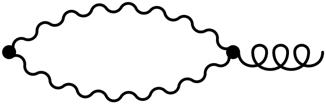
$$J_A^\mu = C\epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

$$\Downarrow$$

$$J_A^\mu = \frac{T^2}{6}\left(1 + \frac{e^2}{4\pi^2}\right)\omega^\mu$$

D.-F. Hou at el., 2012; S. Golkar, D. T. Son, 2012

Combining these insights one expects a bosonic analogue of CVE (bCVE)



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[1] A. Avkhadiev, AS, 2017

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$$\sigma_b = \lim_{q\to 0} \frac{-i}{q_k} \epsilon_{ijk} \left\langle K^i_A T^{0j} \right\rangle |_{\omega=0}$$

$$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

$$T^{\mu\nu} = F^{\mu\lambda} F^\nu_{\ \lambda} - g^{\mu\nu} \frac{1}{4} F^2$$

$$\Downarrow$$

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- Photons can have non-trivial topological phase[1] and one may think about **a generalization of the chiral kinetic theory**[2].

[1] V. Liberman et al., 1992; K. Bliokh et al., 2004; M. Onoda et al., 2004;
[2] N. Muller, R. Venugopalan, 2017; N. Yamamoto, 2017;

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- Possible **phenomenological applications** in condensed matter, primordial plasma, QGP, etc.

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