Pre-equilibrium Chiral Magnetic Effect

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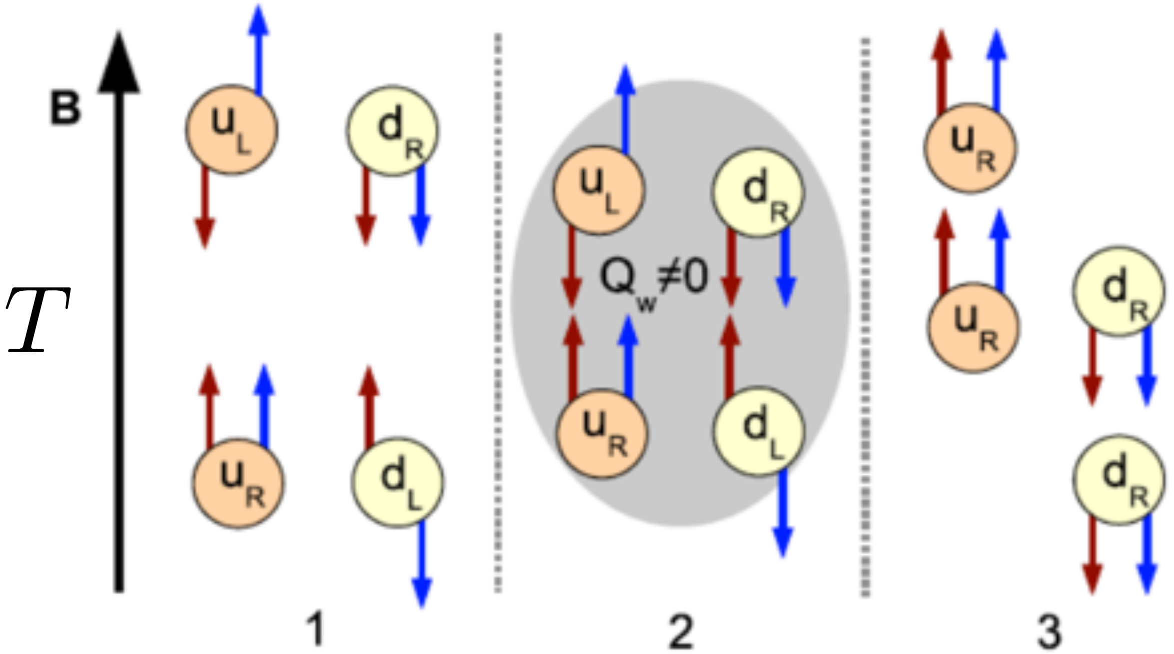
BEST Collaboration Meeting
Stony Brook University
August 7, 2017

The Standard Picture

- Chiral Magnetic Effect: $\mathbf{j}_V \sim n_5 \mathbf{B}$

  (Kharzeev, McLerran, Warringa NPA 803 2008; Fukushima, Kharzeev, Warringa PRD 78 2008)

$B \gg T$



Kharzeev Prog. Part. Nucl. Phys. 75 (2014)

Red=momentum

Blue=spin

- Chiral Separation Effect: $\mathbf{j}_a \sim n_V \mathbf{B}$

  (Son & Zhitnitsky, PRD 70, 074018; Metlitski & Zhitnitsky, PRD 72, 045011)

- Chiral Magnetic Wave: collective gapless excitation from the coupling between density waves of electric and chiral charges

  (Kharzeev & Yee, PRD 83, 085007; Newman JHEP 0601 2006)

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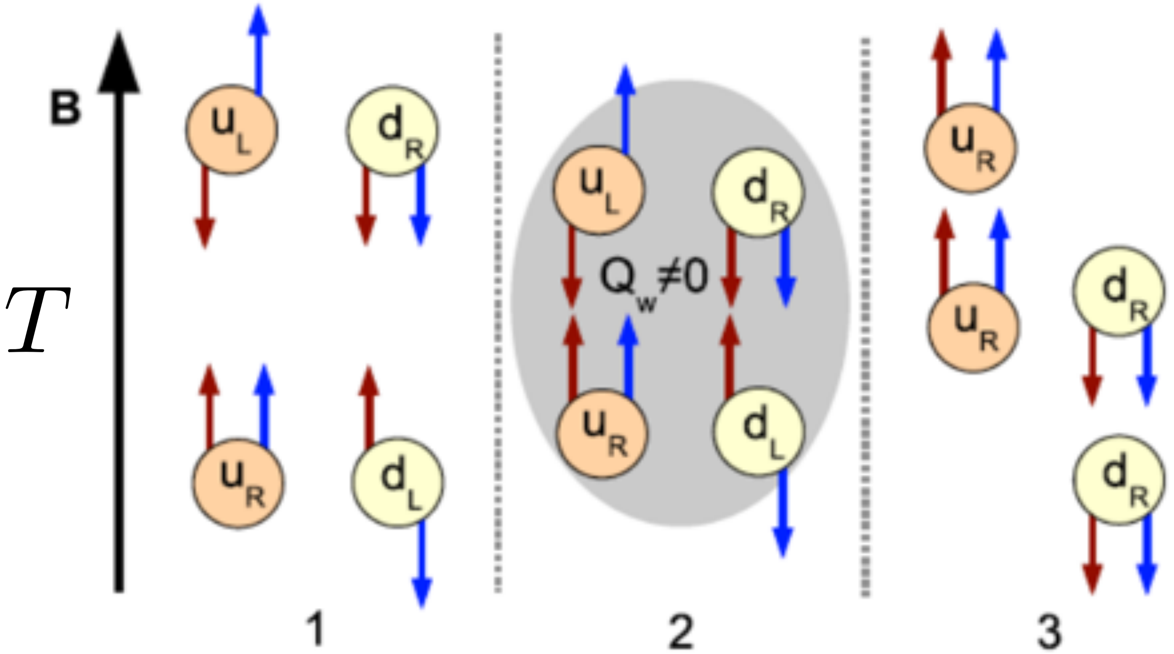

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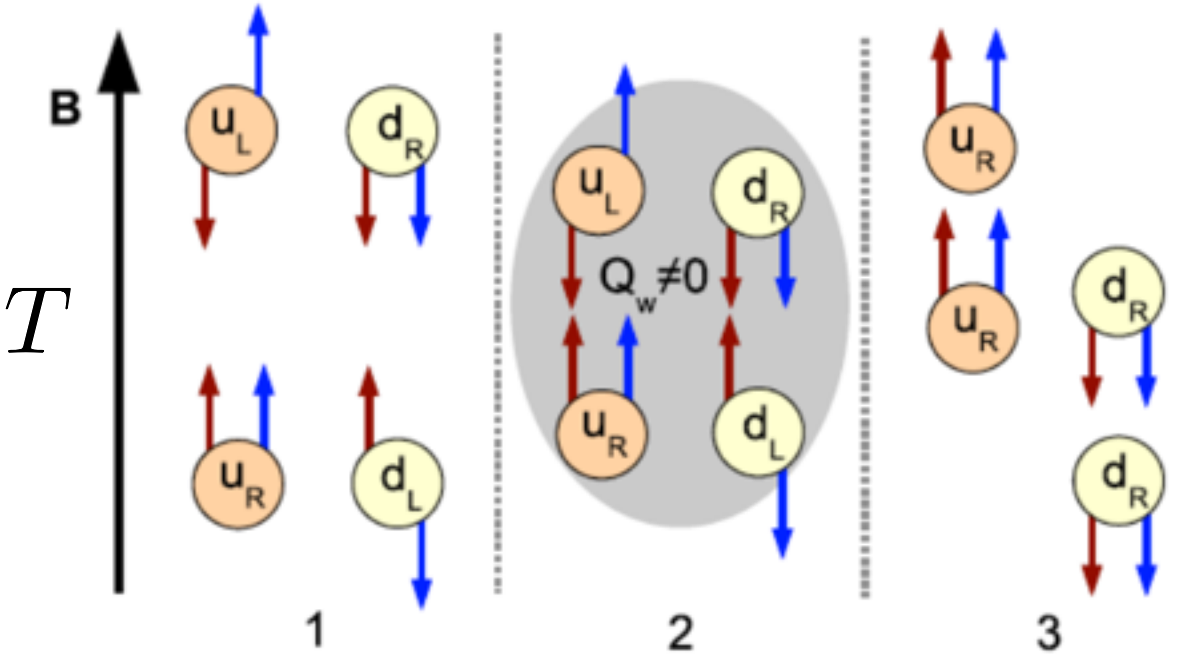

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Kharzeev Prog. Part. Nucl. Phys. 75 (2014)

- Connection between topology of QCD vacuum, nature of chiral anomaly

- Tells us about confinement and chiral symmetry transition: deconfinement and chiral symmetry restoration needed

Axial Charge in HIC $\mathbf{j}_V \sim \boxed{n_5}\mathbf{B}$

- Axial anomaly: Axial charge sourced by fluctuations in non-Abelian field strength tensor

$$\partial_\mu j_a^\mu(x) = 2im\langle\hat{\bar{\psi}}(x)\gamma_5\hat{\psi}(x)\rangle - \frac{g^2}{8\pi^2}\mathrm{Tr}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

Axial current
$j_a^\mu = (j_a^0, \mathbf{j}_5)$

Quark mass

Receives contributions from sphaleron transitions, color flux tubes, field strength fluctuations,…

Klinkhamer, Manton PRD 30 1984

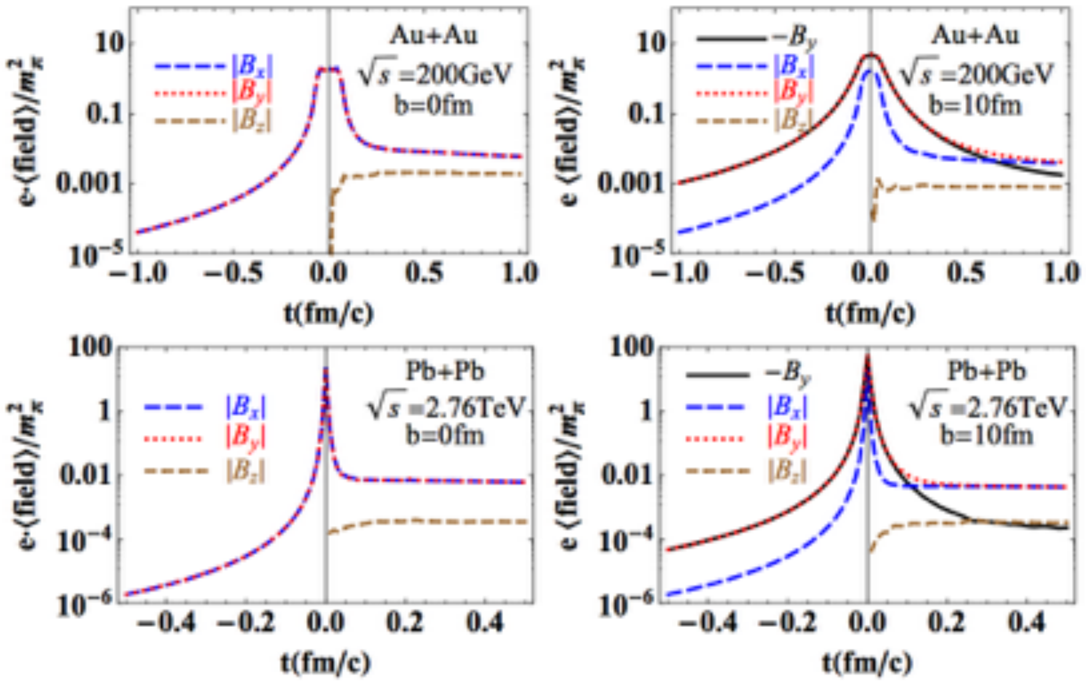
Sphaleron transition (real time topological transitions) leads to unit change of Chern-Simons number and induce axial charge imbalance

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x\langle\bar{\psi}i\gamma_5\psi\rangle$$

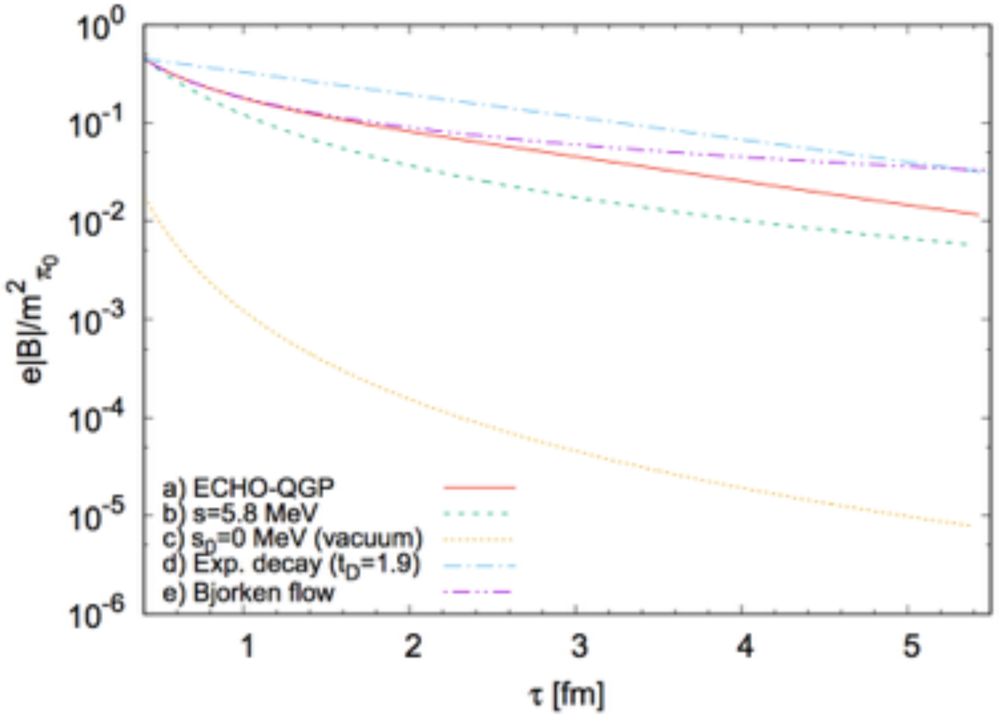
Magnetic Field in HIC $\mathbf{j}_V \sim n_5 \boxed{\mathbf{B}}$

- Non-central collisions creates *initially* strong magnetic field

$$e\mathbf{B} \sim (m_\pi)^2 \sim 10^{12}\mathrm{T} \qquad t \sim 0.1 - 0.2\,\frac{\mathrm{fm}}{\mathrm{c}}$$



Deng and Huang PRC 85 (2012)



Inghirami et al EJP C76 (2016)

Greatest fraction of anomalous chiral effects should be at very initial times, when magnetic field is strongest

(Skokov,Illarionov,Toneev IJMP A24 2009; Deng and Huang PRC85 2012; McLerran and Skokov NPA929 2014; Tuchin et al PRC91 2015,…)

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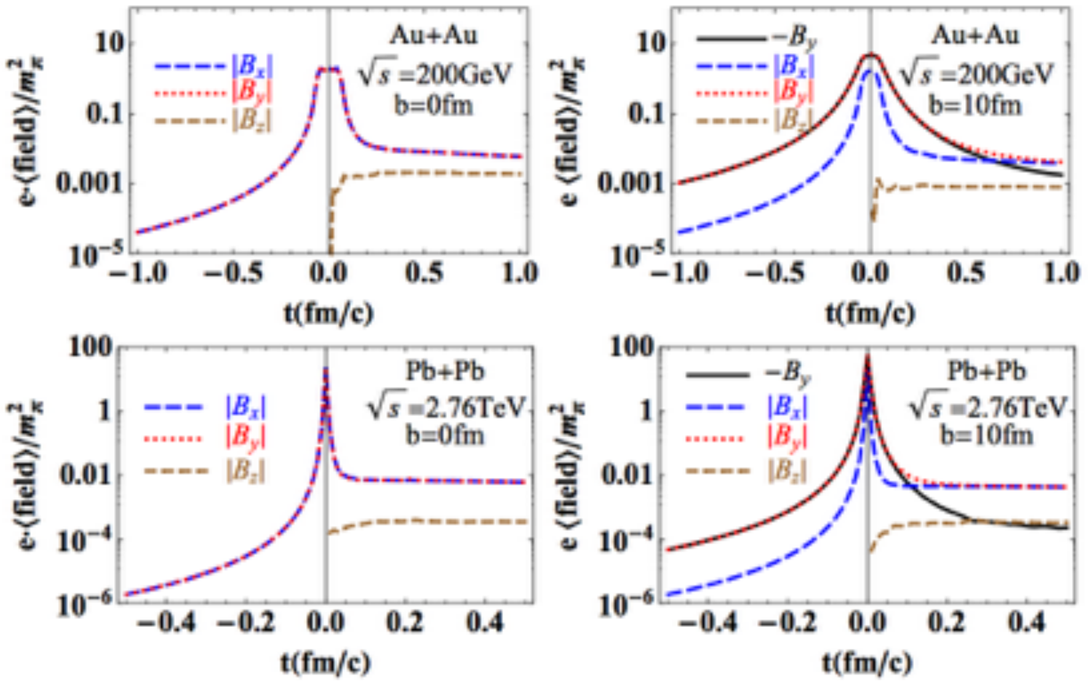

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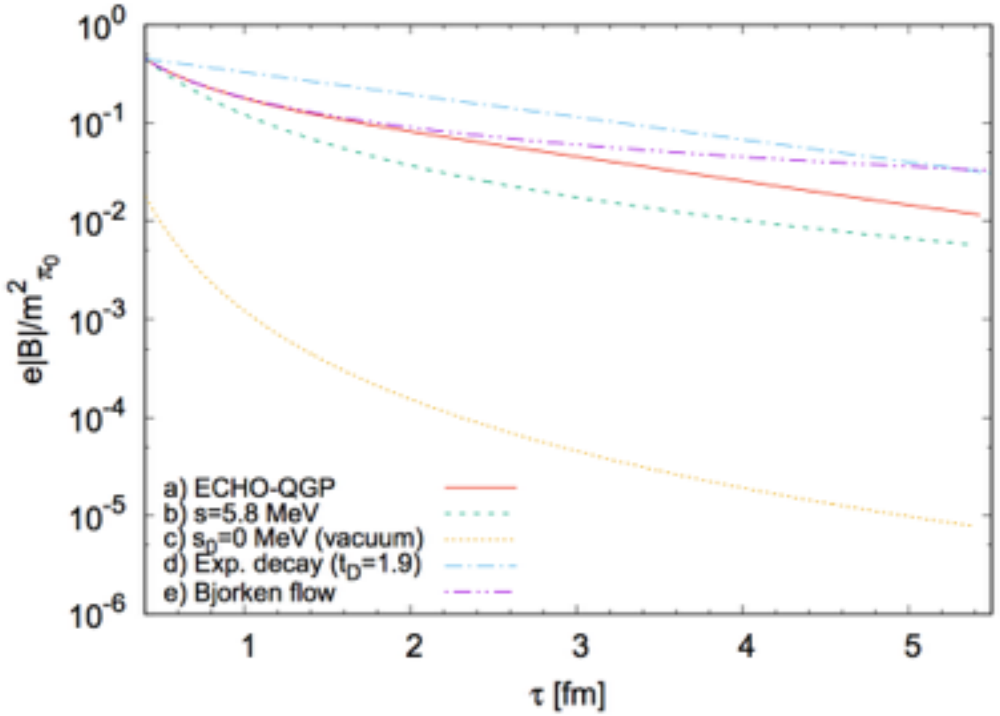


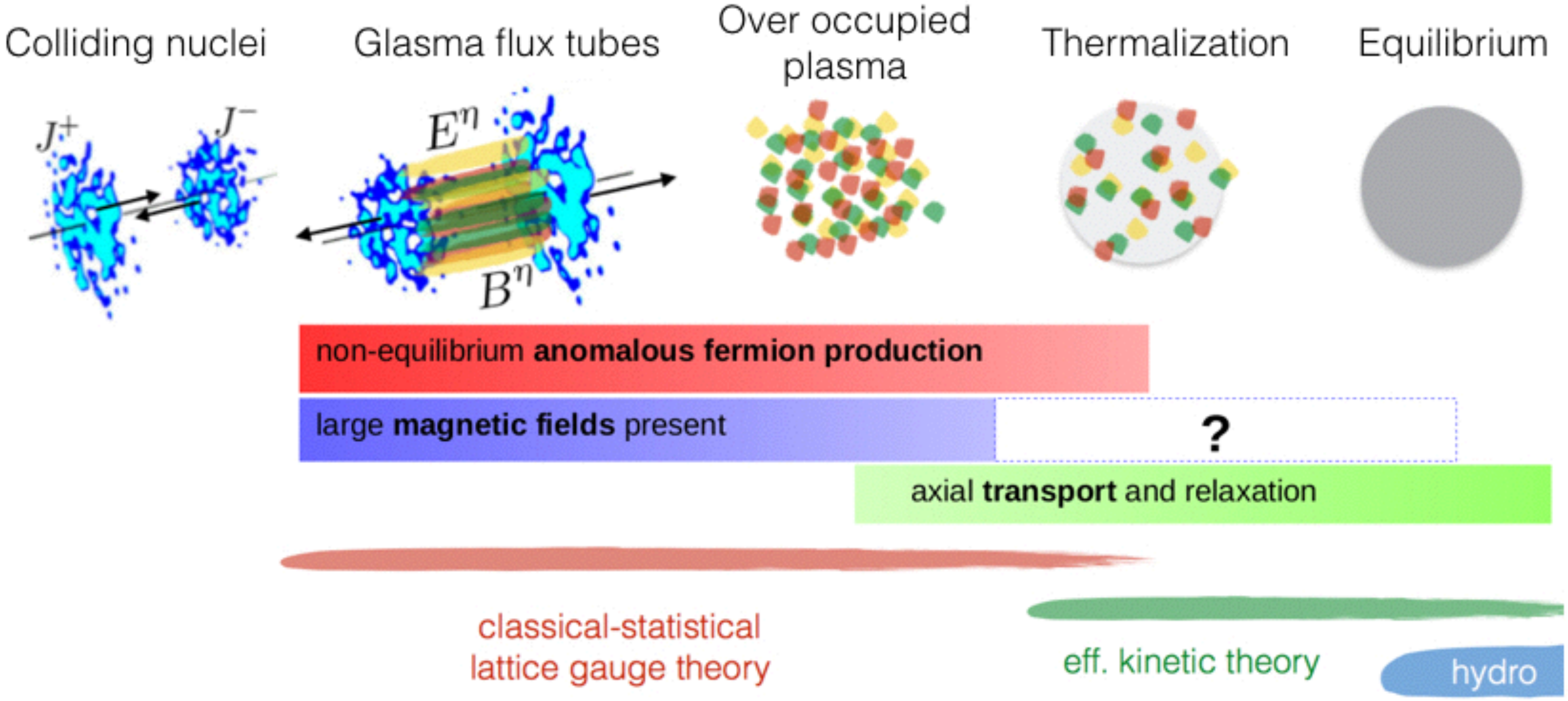
Inghirami et al EJP C76 (2016)

Current studies underway on magnetic field behavior in numerical MHD

Hirono, Kharzeev, MM + Inghirami et al. (ECHO-QGP-MHD)

Theorists Overview

Goal: Want to *quantitatively* study anomalous transport (CME/CSE/CMW…) in heavy ion collisions



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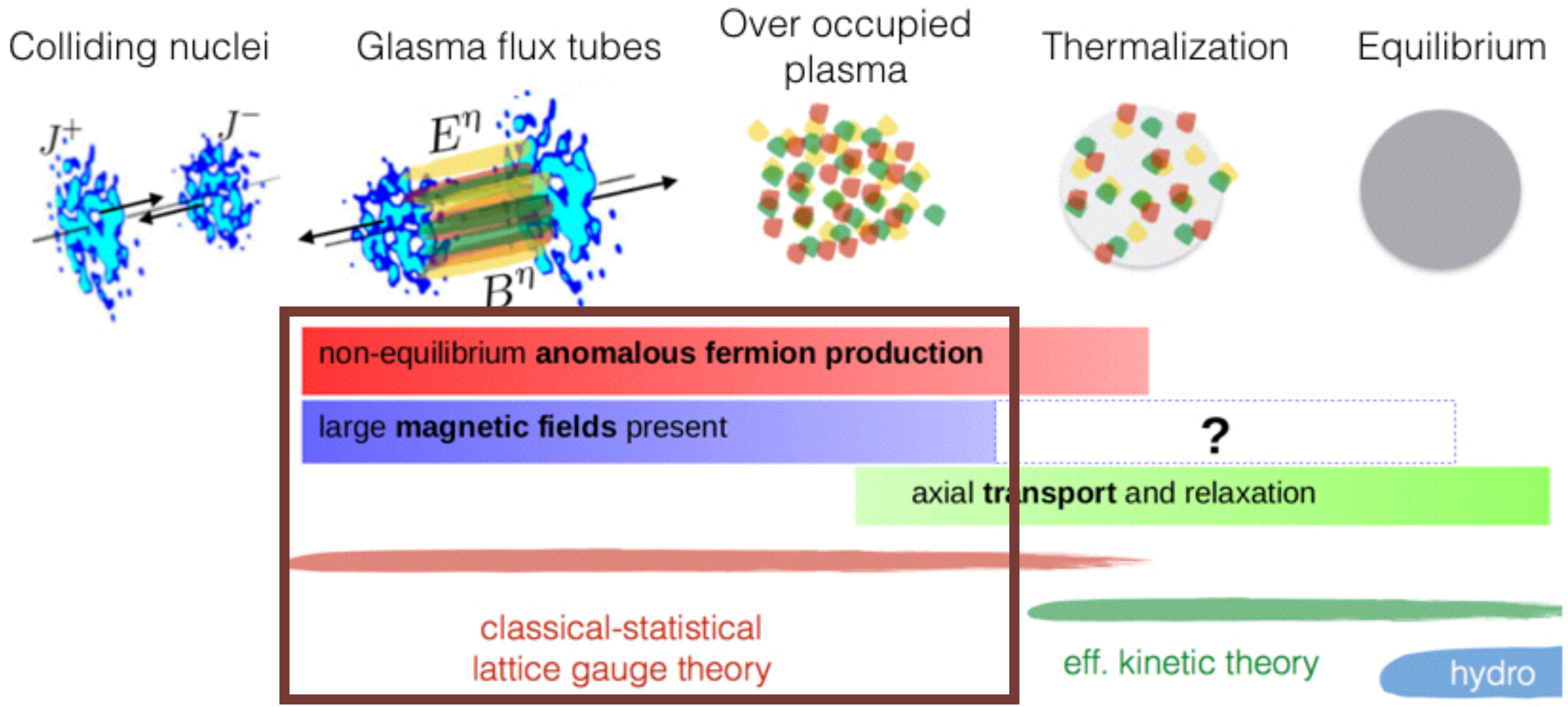

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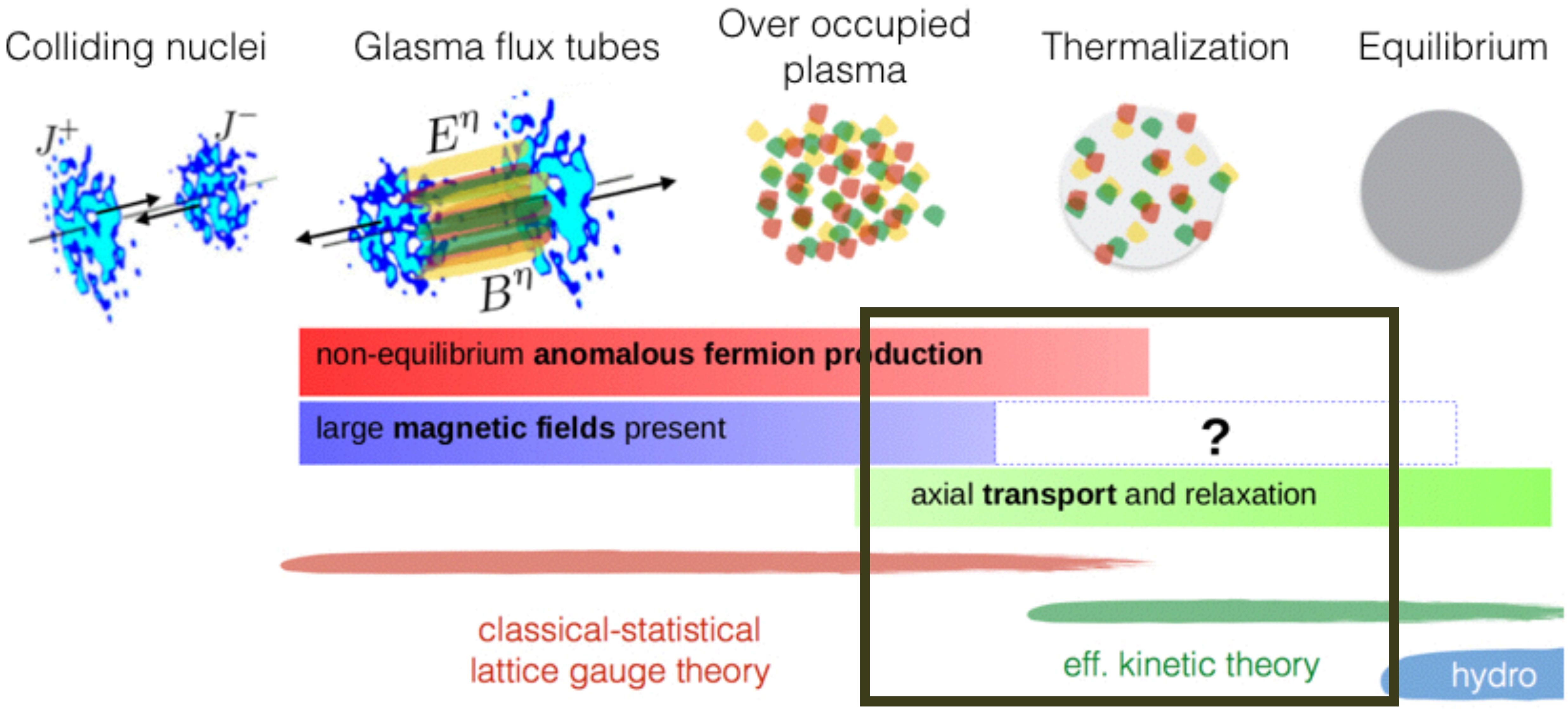

Colliding nuclei

Glasma flux tubes

Over occupied plasma

Thermalization

Equilibrium

non-equilibrium **anomalous fermion production**

large **magnetic fields** present

?

axial **transport** and relaxation

classical-statistical lattice gauge theory

eff. kinetic theory

hydro

Adapted from S.Schlichting/N. Mueller 2016

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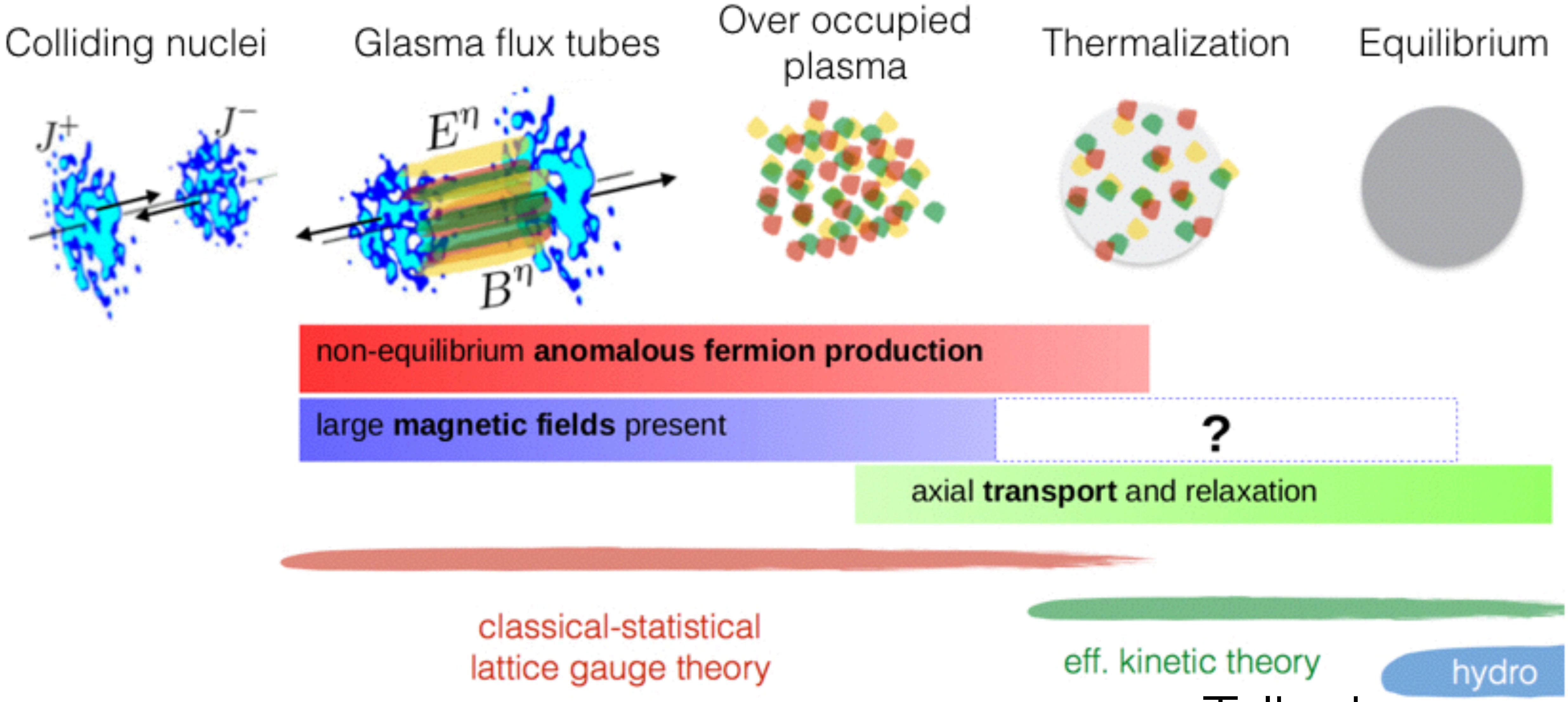

Talks by
Y. Hirono, S. Shi

Adapted from S.Schlichting/N. Mueller 2016

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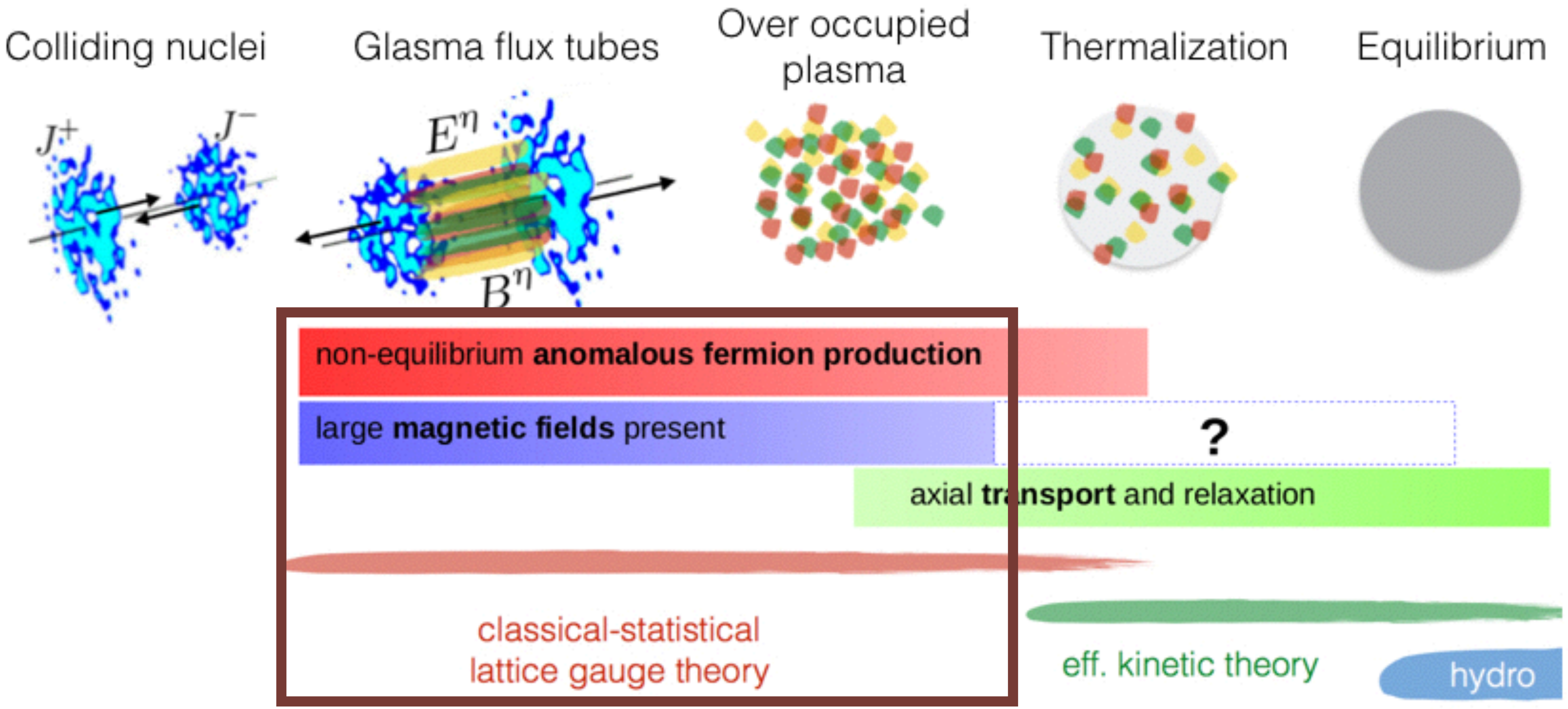

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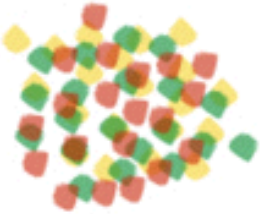
First focus on earliest times, classical description applicable

Adapted from S.Schlichting/N. Mueller 2016

Simulating early times after HIC

Over occupied plasma



Earliest time dynamics described by classical Yang-Mills $D_\mu F^{\mu\nu} = j^\nu$

McLerran, Venugopalan PRD49 2233 (1994)

Non-perturbatively large gluon phase space density $f(p \sim Q_s) \sim \frac{1}{\alpha_s}$

Sphaleron production included in this framework

Well established non-equilibrium lattice gauge theory techniques

Venugopalan, Kransitz NPA 237 (1999), Berges, Scheffler, Sexty, PRD 77 034504 (2008)

Track right hand side of anomaly by measuring Chern-Simons number

$$\frac{dN_{CS}}{dt} = \frac{g^2}{8\pi^2} \int d^3x \; E_i^a(\mathbf{x}) B_i^a(\mathbf{x})$$

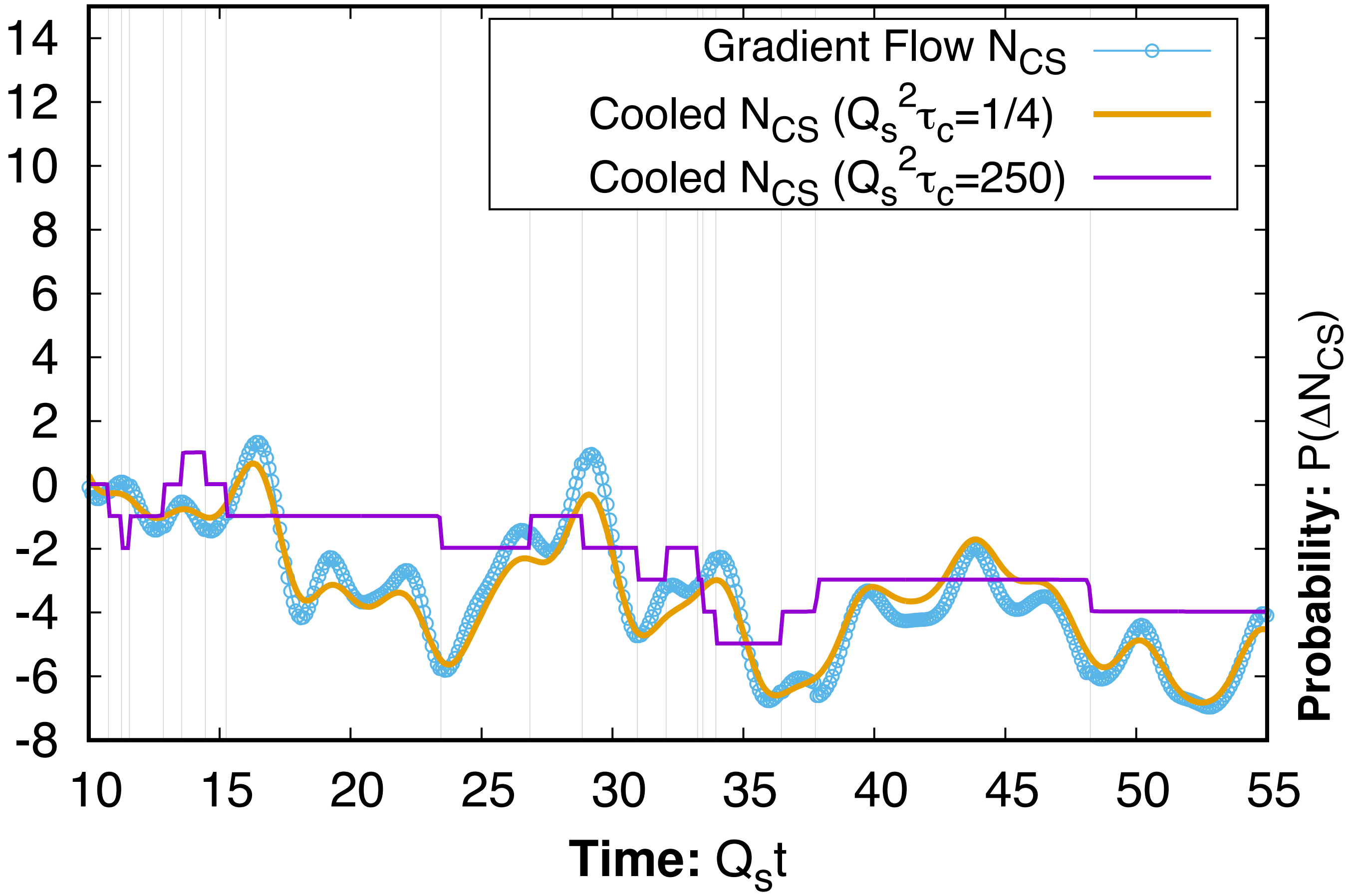
Use cooling to remove short range fluctuation and isolate topological transitions

Neglect longitudinal expansion and use SU(2) for simplicity

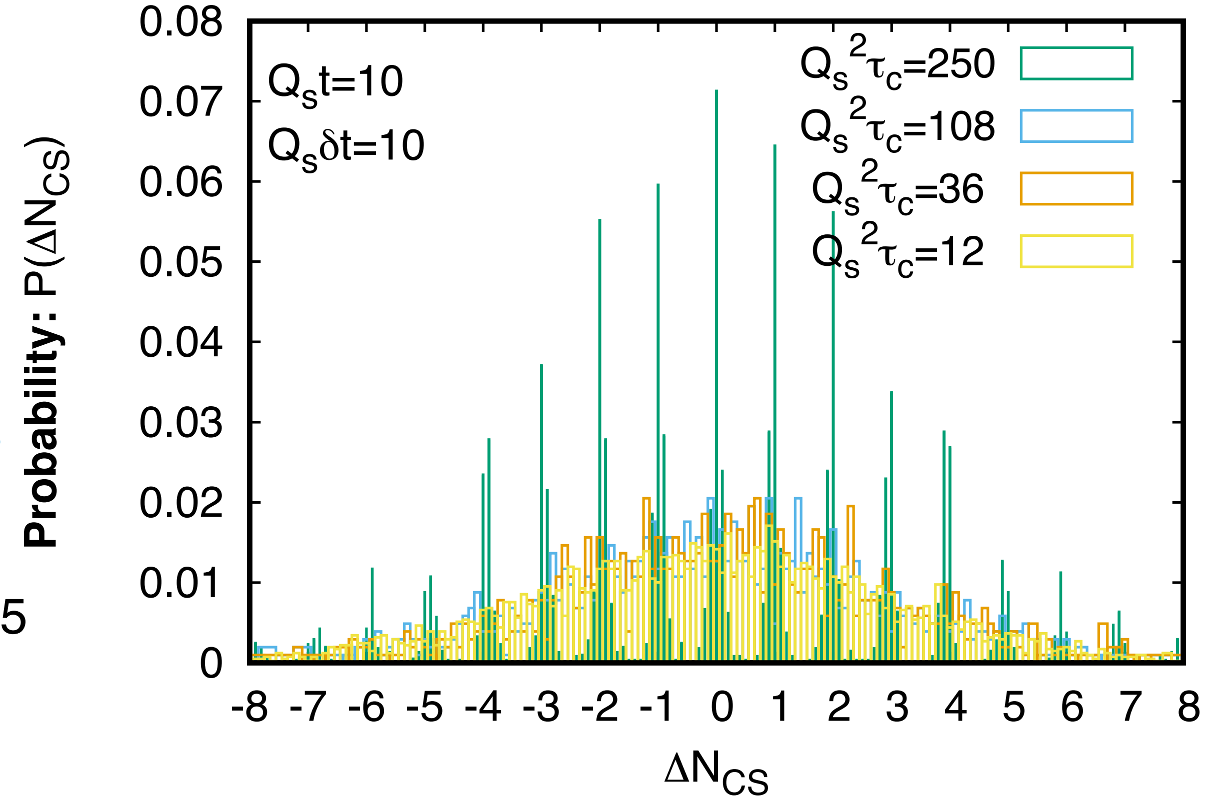
Sphalerons in the glasma

Detect integer changes in Chern-Simons number for single configuration

Histogram Chern-Simons diffusion of many configurations shows transitions between different topological sectors occurs



MM, Schlichting, Venugopalan PRD 93 2016



Can isolate transitions from background

Significant number on order of few $1/Q_s$

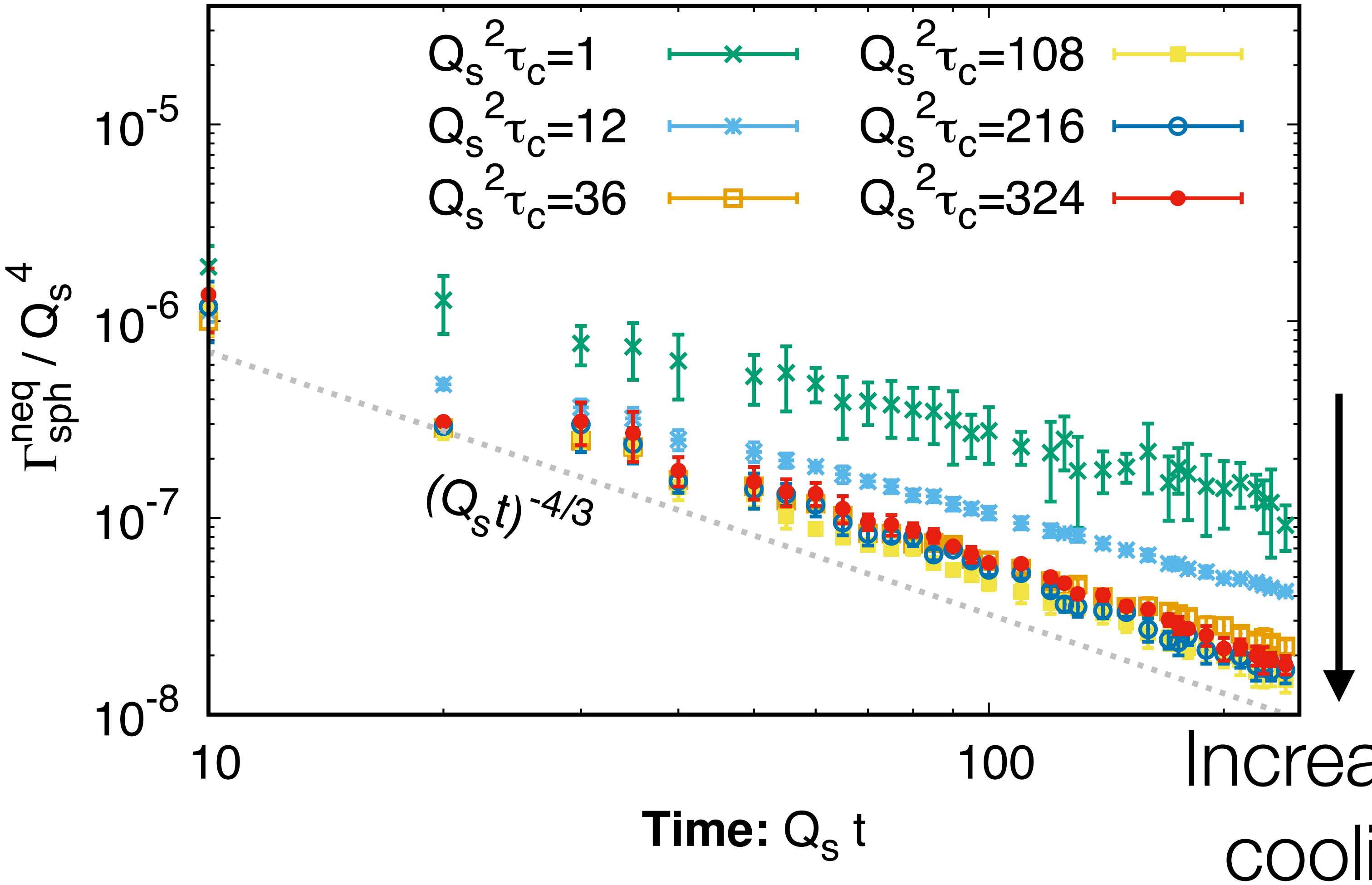
Non-equilibrium sphaleron transition rate

Strongly time dependent, largest are early times

Contributions from field strength fluctuations

Non-markovian, not random walk like thermal equilibrium

MM, Schlichting, Venugopalan PRD 93 2016



Increased cooling

From dynamical separation of scales, non-equilibrium sphaleron transition rate controlled by modes of order of magnetic screening scale

MM, Schlichting, Venugopalan PRD 93 2016

Same scaling exponent expected for SU(2) and SU(3)

Berges, MM, Schlichting PRL 118 (2017)

Expect qualitatively similar features when extending to longitudinal expanding, relevant to HIC - **Work in progress**

Sphalerons in non-Abelian plasmas

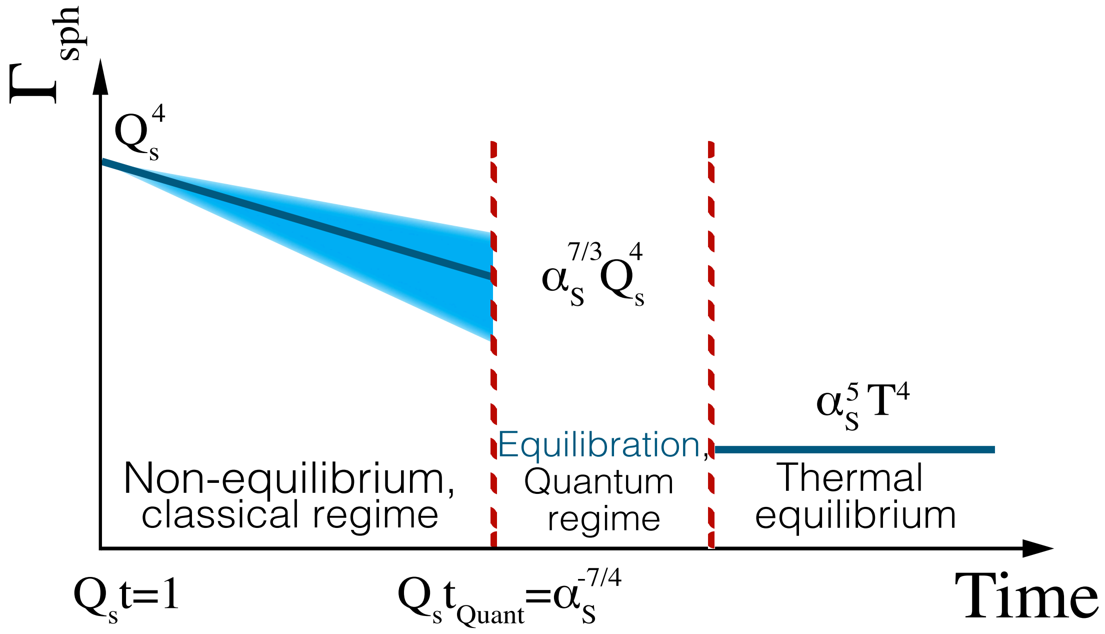
From dynamical separation of scales, non-equilibrium sphaleron rate controlled by modes of order magnetic screening

Glasma (early times): $\Gamma^{neq}_{sph} \approx Q_s^4 \rightarrow \Gamma^{neq}_{sph}(t) \sim Q_s^4 (Q_s t)^{-\frac{4}{3}} \sim \sigma(t)^2$

(MM, Schlichting, Venugopalan PRD 93 2016)

Equilibrium (late times): $\Gamma^{eq}_{sph} \approx \alpha_S^5 T^4 \sim \alpha_S^4 Q_s^4$

(Arnold, Son, Yaffee; Moore et al.)



Dominant amount of axial charge (should be) generated at early times

Real Time Lattice QCD

To study full CME dynamics, need to extend non-equilibrium gauge field studies to include fermions

Cannot resort to classical approximation for fermions because of Pauli principle

Expand initial fermion field in operator basis, evolve wave function by solving Dirac equation

$$(i\not{D} - m)\hat{\psi} = 0$$

SU(N$_c$)+U(1)

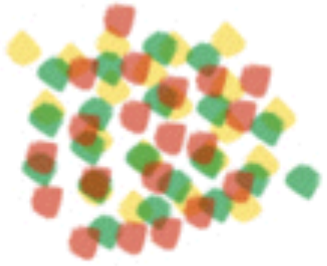
$$\hat{\psi}_x(t) = \frac{1}{\sqrt{V}} \sum_\lambda \left(\hat{b}_\lambda(t=0)\phi^u_\lambda(t,x) + \hat{d}^\dagger_\lambda(t=0)\phi^v_\lambda(t,x) \right)$$

Measure operator expectation values, i.e $j^\mu_V = q\langle \hat{\bar{\psi}}(x)\gamma^\mu \bar{\psi}(x)\rangle$

Lattice fermions have well known doubling problem

Axial Charge from Sphaleron Transition

Want to simulate in clean, controlled environment to establish theoretical techniques, using knowledge from over-occupied plasma

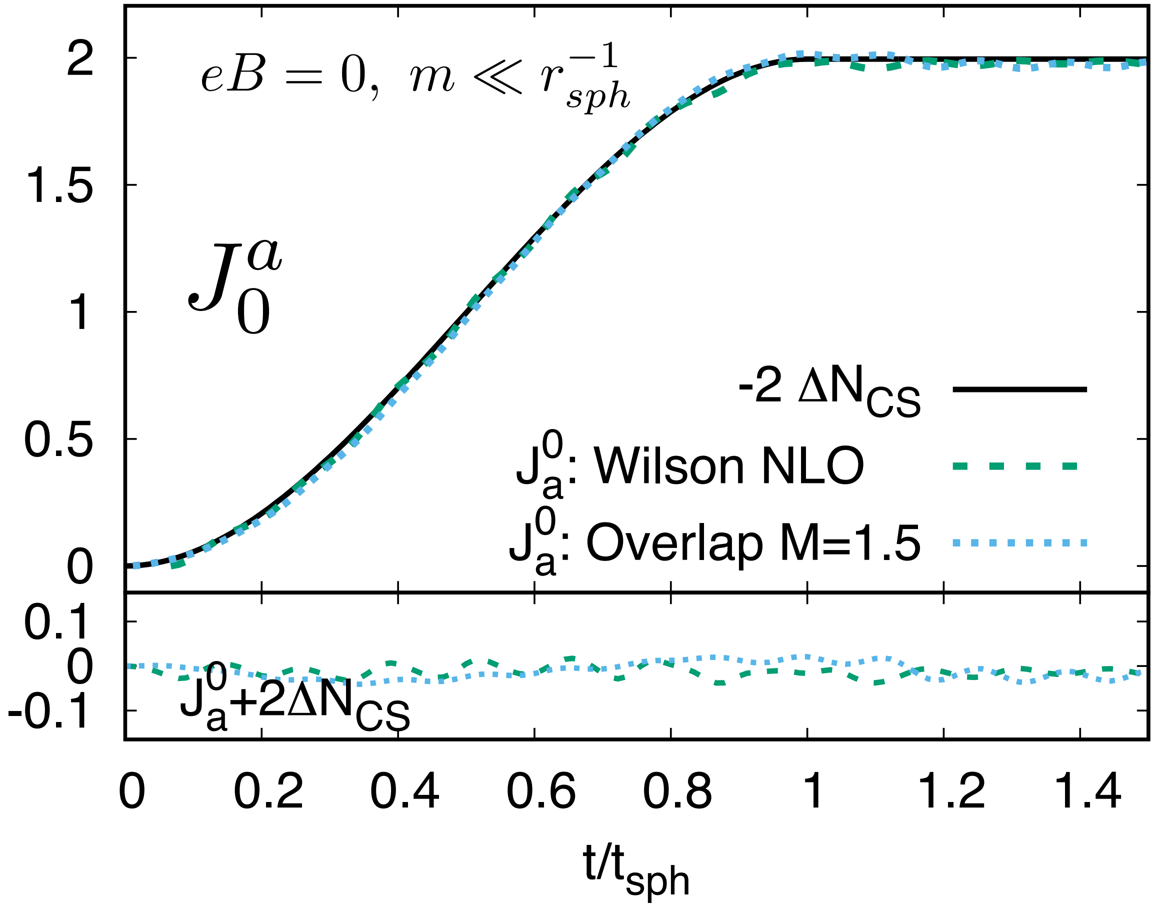

Over occupied plasma

Consider single sphaleron of size r_{sph}, timescale t_{sph}

Unit change in Chern-Simons number induces axial charge imbalance

$$\Delta N_{CS} = \frac{g^2}{8\pi}\int d^4x \mathbf{E}^a \cdot \mathbf{B}^a$$

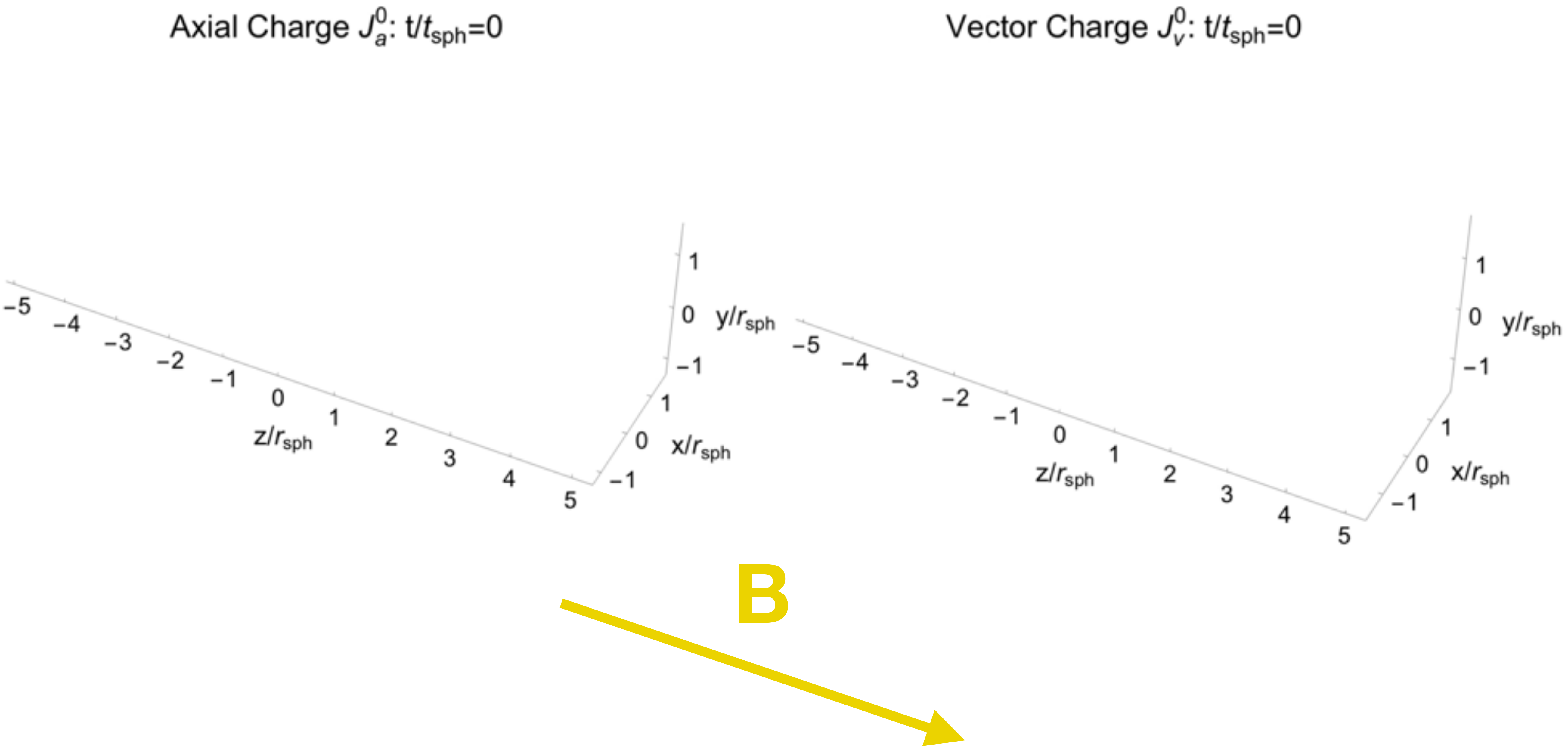
$$\Delta J_5^0 = -2\Delta N_{CS}$$



Simulate with both Wilson and Overlap fermions

MM, Mueller, Schlichting, Sharma PRD 95 2017

Anomalous transport in real time



Axial Charge J_a^0: $t/t_{\mathrm{sph}}=0$

Vector Charge J_v^0: $t/t_{\mathrm{sph}}=0$

For animation, see: https://sites.google.com/stonybrook.edu/markmace/graphics

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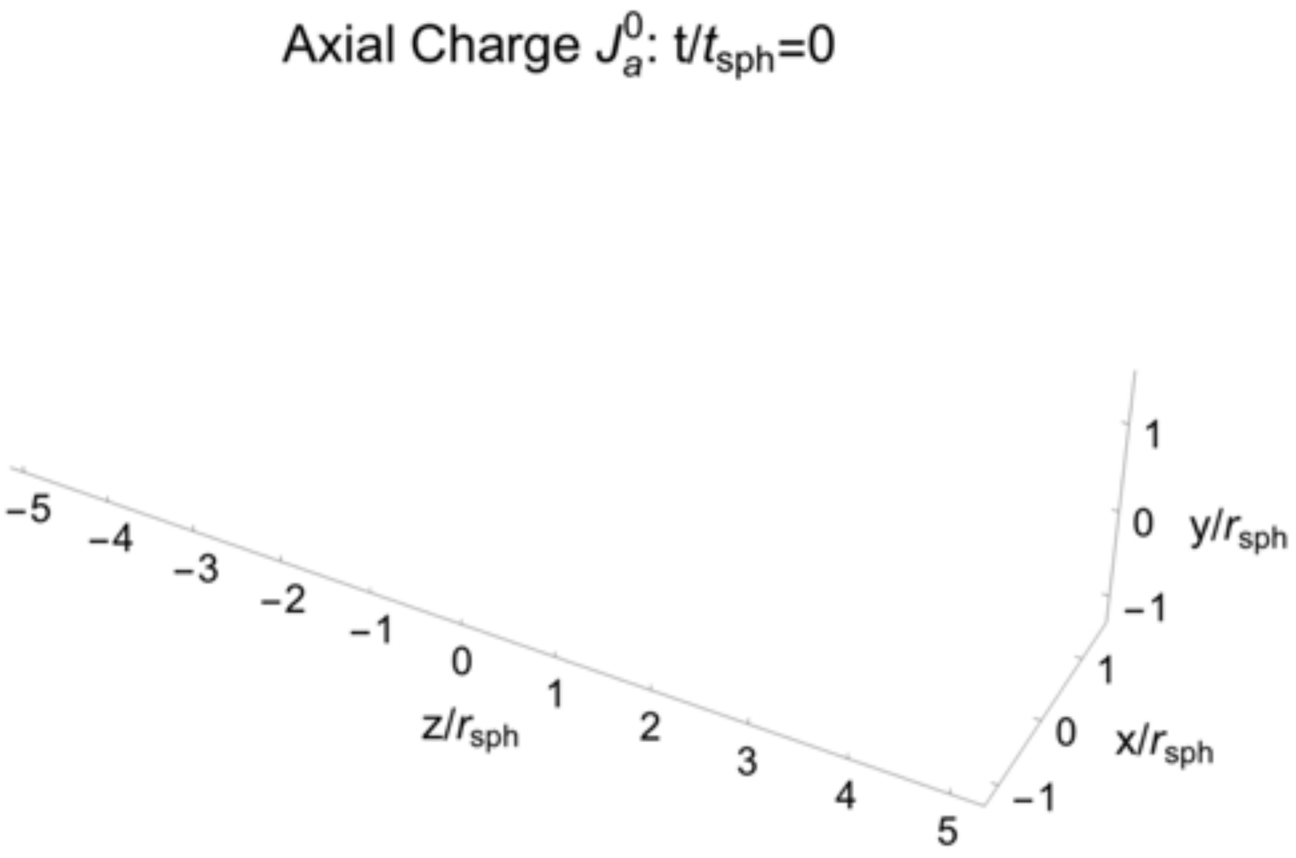

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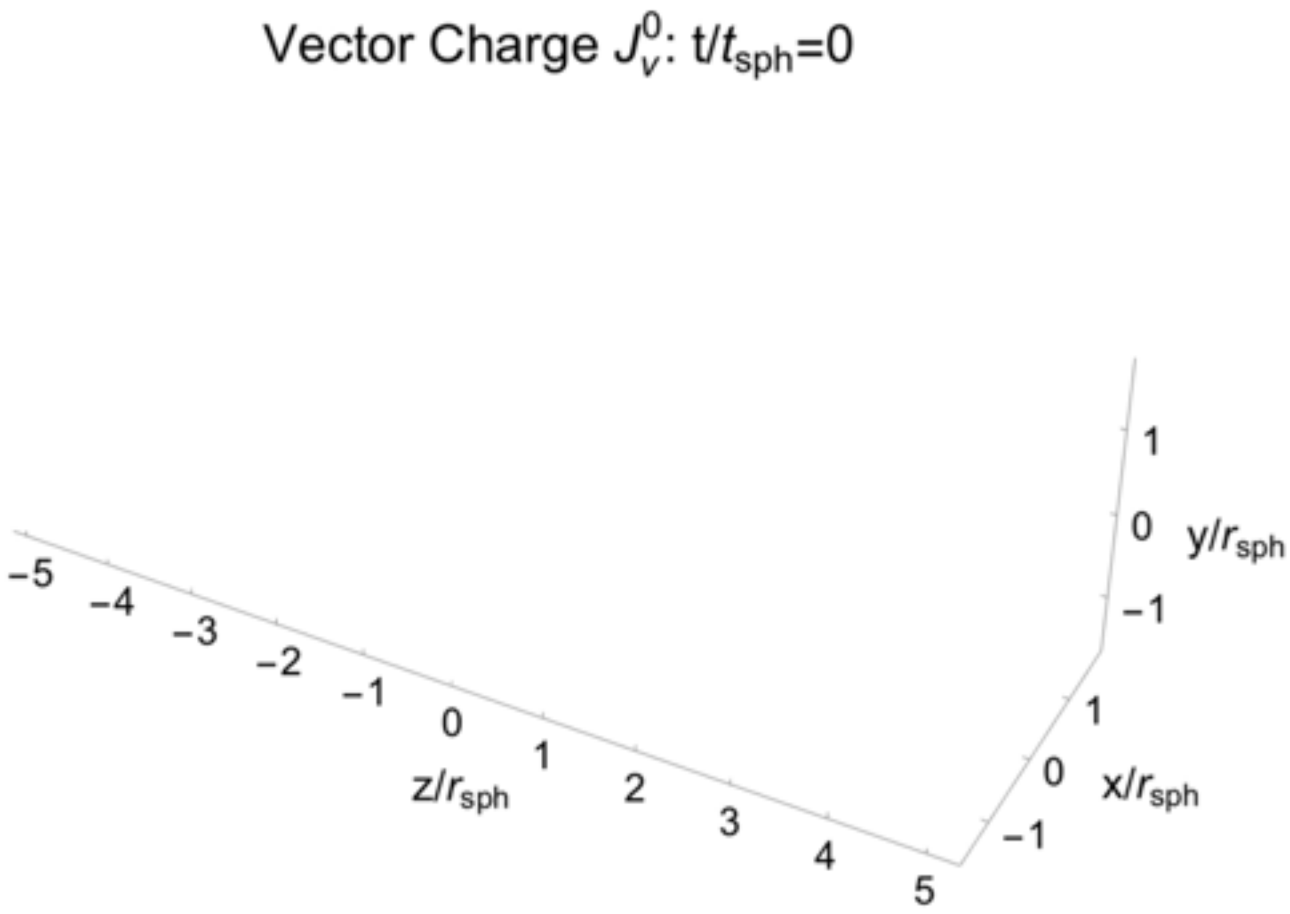
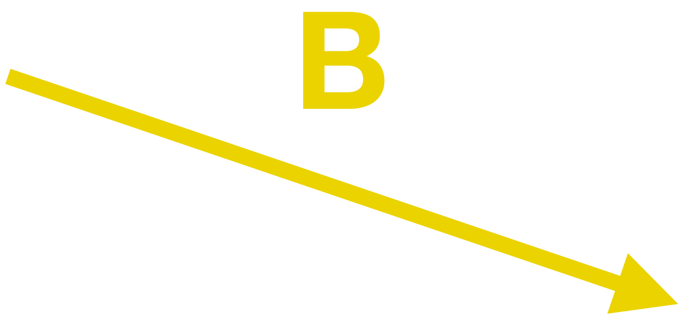


Vector Charge J_v^0: $t/t_{\mathrm{sph}}=0$



For animation, see: https://sites.google.com/stonybrook.edu/markmace/graphics

Quark mass dependence

Light quarks $m_f \ll r_{sph}^{-1}$

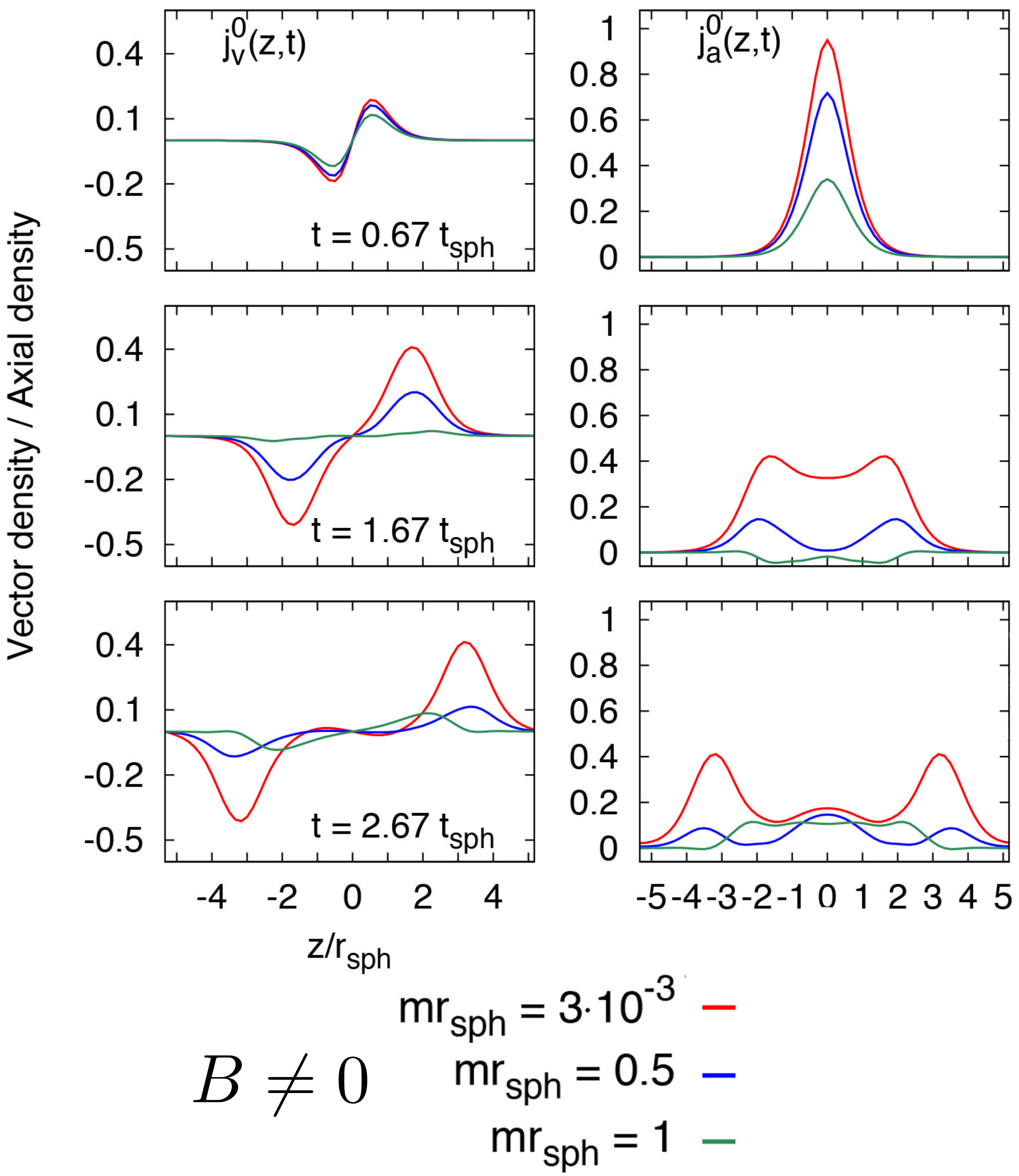
Non-dissipative transport from Chiral Magnetic Wave

Well described by anomalous hydrodynamic “shock-wave”

Heavy quarks $m_f \sim r_{sph}^{-1}$

Dissipation of axial charge leads to reduced axial charge density

How to include in macroscopic picture?

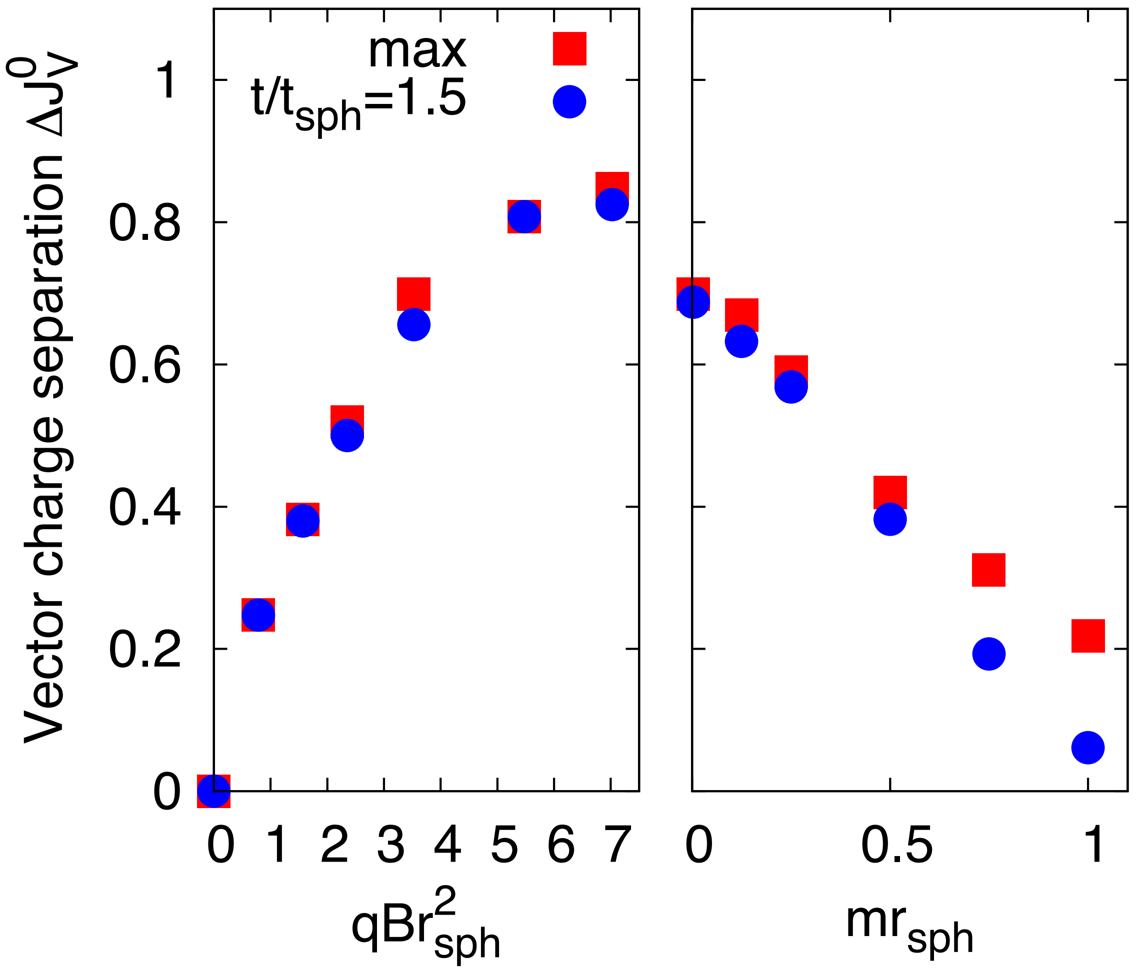


$B \neq 0$

MM, Mueller, Schlichting, Sharma PRD 95 2017

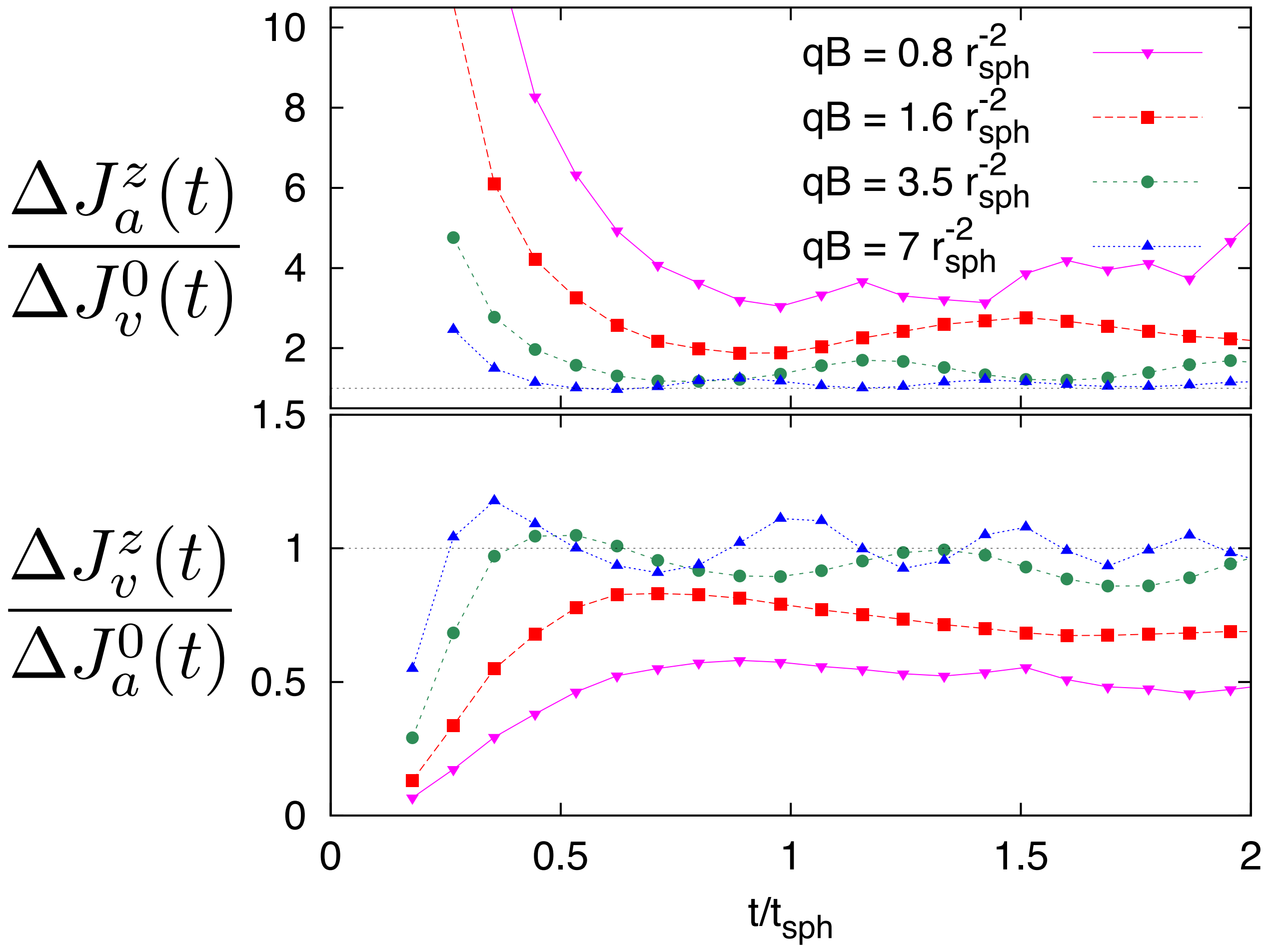
Transport

- Mass, **B** dependence on charge separation



Vector charge separation vanishes for large m

- B dep. for ratios of charge separation



Nontrivial behavior away from asymptotic limit

Finite "relaxation time" should be taken into account

MM, Mueller, Schlichting, Sharma PRD 95 2017

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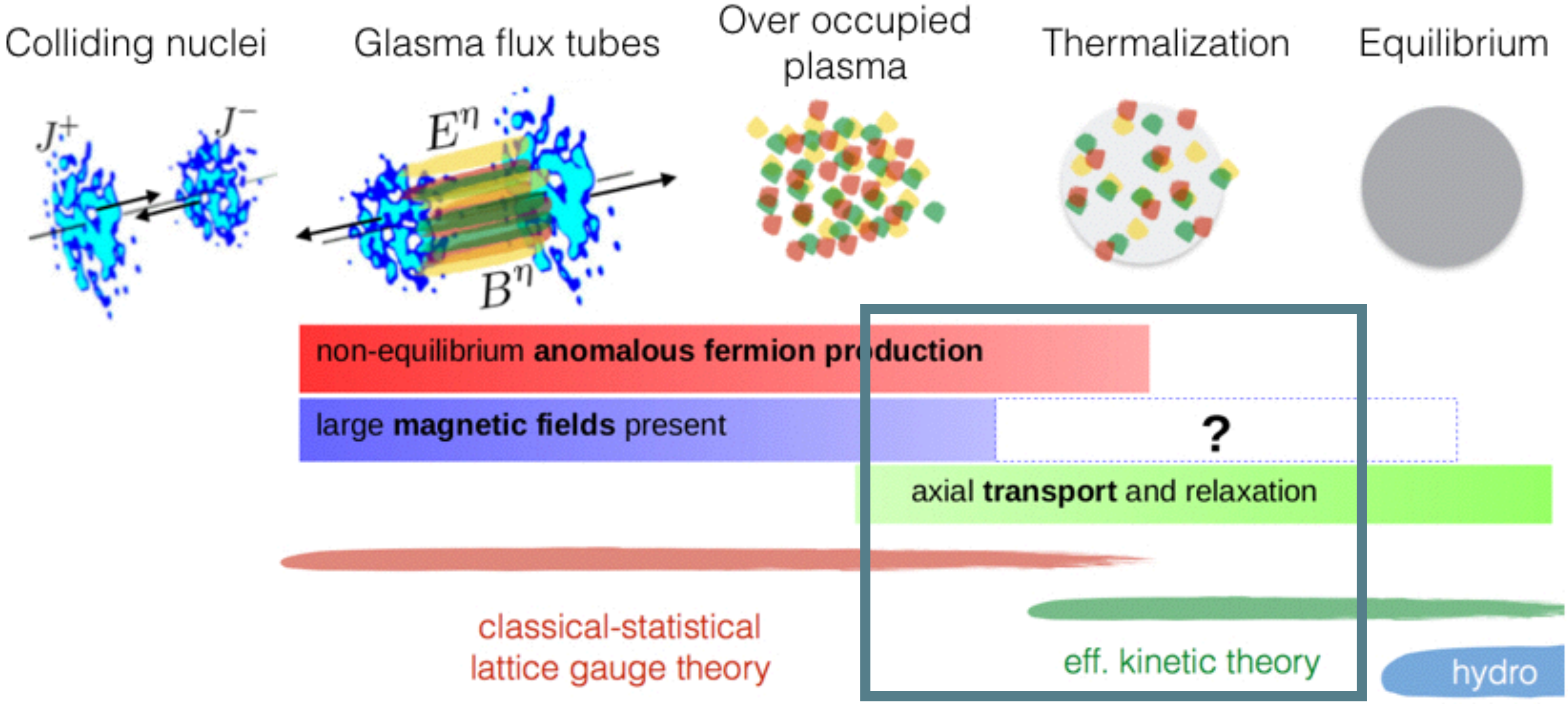


Adapted from S.Schlichting/N. Mueller 2016

Chiral Kinetic Theory

Classical simulations only applicable for large occupancies

Low occupancies but still far from equilibrium - Kinetic Theory

Start with Liouville equation description of phase space

Stephanov, Yin PRL 109 162001 (2012), Son, Yamamoto PRL 109 181602 (2012)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}}\dot{\mathbf{x}} + \frac{\partial f}{\partial \mathbf{p}}\dot{\mathbf{p}} = C[f]$$

Consider Weyl equations $(\sigma \cdot \mathbf{p})u_{\mathbf{p}} = \pm|\mathbf{p}|u_{\mathbf{p}}$

Take into account spinning particle via Berry monopole

$$i\mathcal{A}_{\mathbf{p}} \equiv u_p^{\dagger}\nabla_{\mathbf{p}}u_{\mathbf{p}}$$

Action of spinning particle in background field

$$S = \int dt\left[p^i\dot{x}^i + A^i(x)\dot{x}^i - \mathcal{A}^i(p)\dot{p}^i - \epsilon_{\mathbf{p}}(x) - A^0(x)\right]$$

Chiral Kinetic Theory

From action get equations of motion in terms of Berry curvature

Stephanov, Yin PRL 109 162001 (2012), Son, Yamamoto PRL 109 181602 (2012)

$$\sqrt{G}\dot{\mathbf{x}} = \hat{\mathbf{p}} + \mathbf{E} \times \mathbf{b} + \mathbf{B}(\hat{\mathbf{p}} \cdot \mathbf{b}) \qquad \sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \hat{\mathbf{p}} \times \mathbf{b} + \mathbf{b}(\mathbf{E} \cdot \mathbf{B})$$

$$G \equiv (1 + \mathbf{B} \cdot \mathbf{b})^2$$

Berry Curvature: $\mathbf{b} = \pm \frac{\hat{p}}{2|\mathbf{p}|^2}$

From this can get anomaly equation and CME

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = \frac{1}{4\pi^2} \mathbf{E} \cdot \mathbf{B} f|_{\mathbf{p}=0}$$

$$\boldsymbol{j} = \int_{\boldsymbol{p}} \sqrt{G} f\, \dot{\boldsymbol{x}} = \underbrace{\underbrace{\int_{\boldsymbol{p}} f \frac{\partial \mathcal{E}}{\partial \boldsymbol{p}}}_{\text{normal current}} + \underbrace{\boldsymbol{E} \times \int_{\boldsymbol{p}} f \boldsymbol{b}}_{\text{anom. Hall current}}}_{=0 \text{ in equilibrium}} \underbrace{+ \boldsymbol{B} \int_{\boldsymbol{p}} f (\hat{\boldsymbol{p}} \cdot \boldsymbol{b})}_{\text{CME}}$$

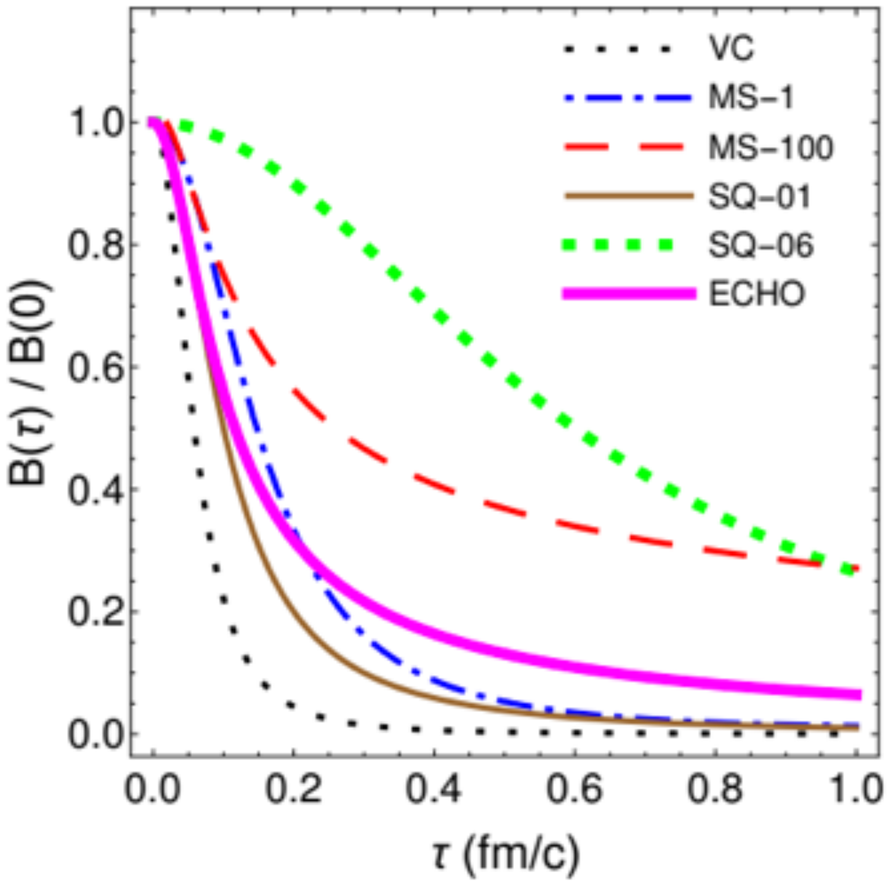
Modeling Chiral Kinetic Theory

First modeling studies of CKT: solve Liouville eqn for f(x,p,t) and EOM for x(t), p(t) using relaxation time approximation

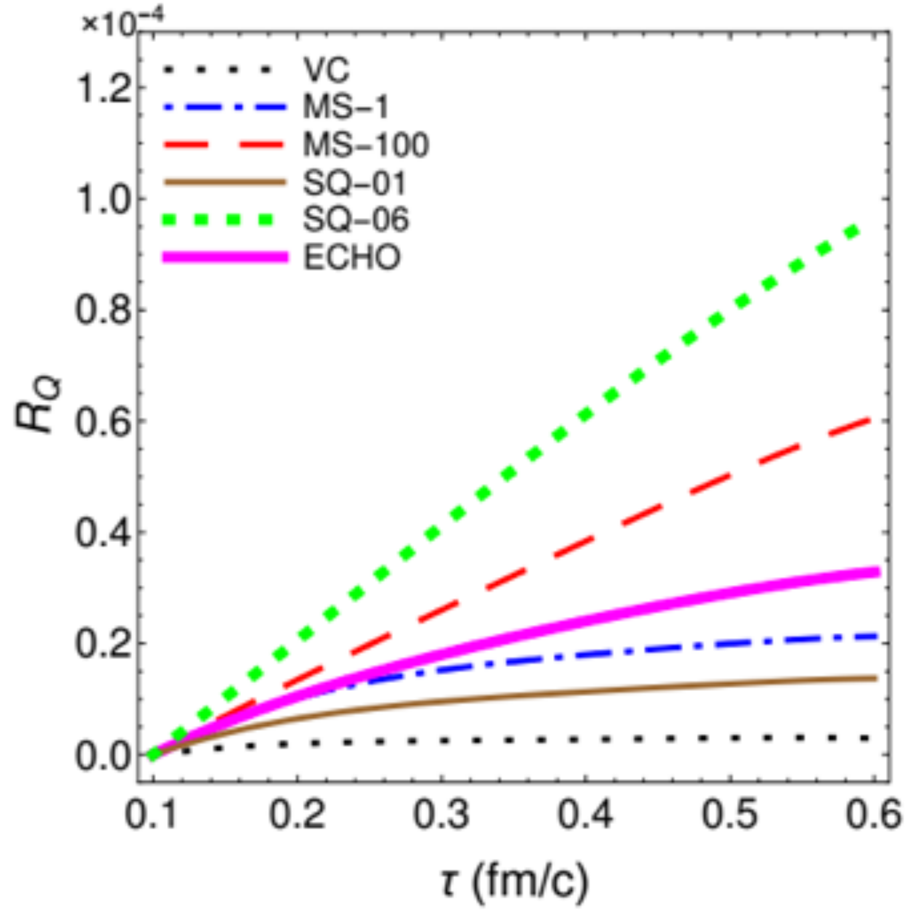
Huang, Jiang, Shi, Liao, Zhuang, arXiv:1703.08856

Study magnetic field model dependence on charge separation

Background B field



Charge Sep. wrt Reaction Plane

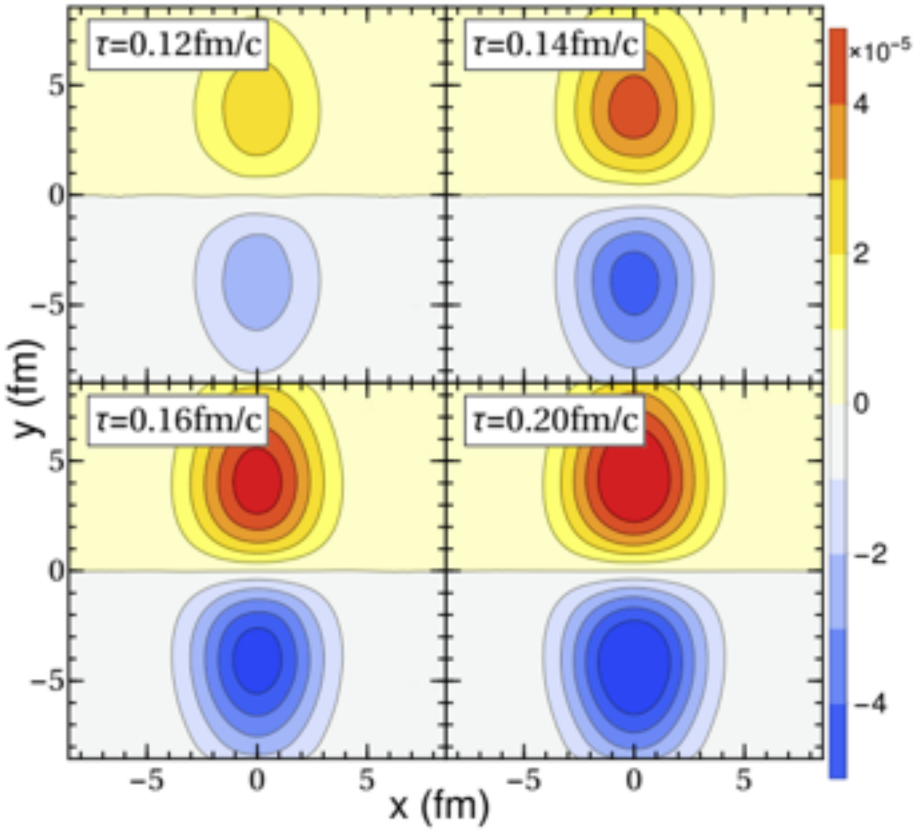


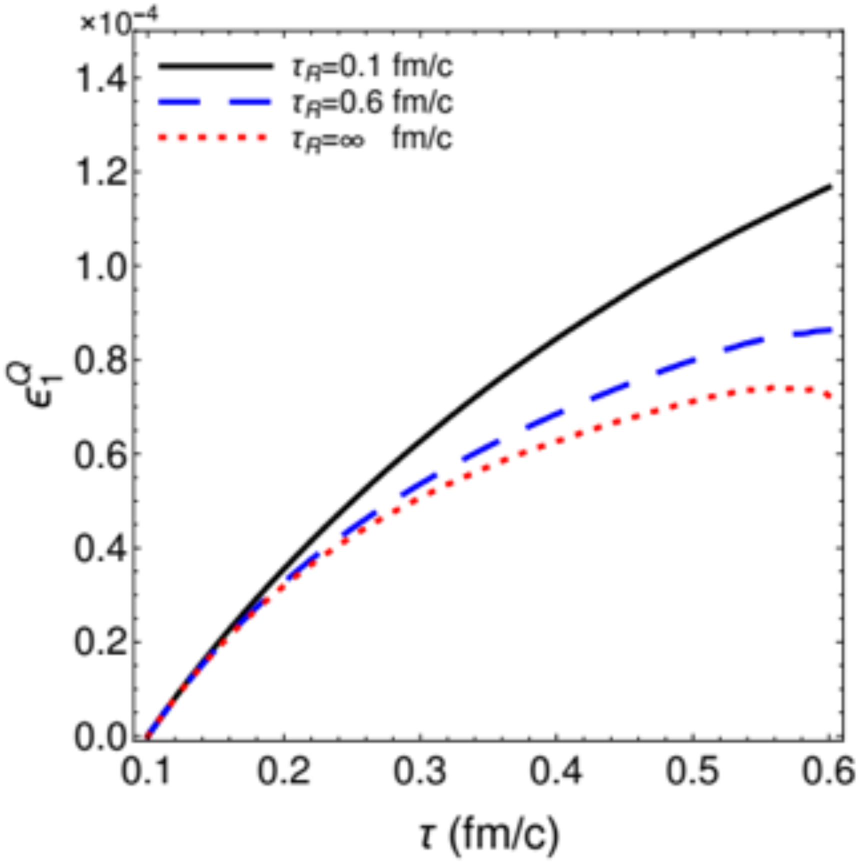
Modeling Chiral Kinetic Theory

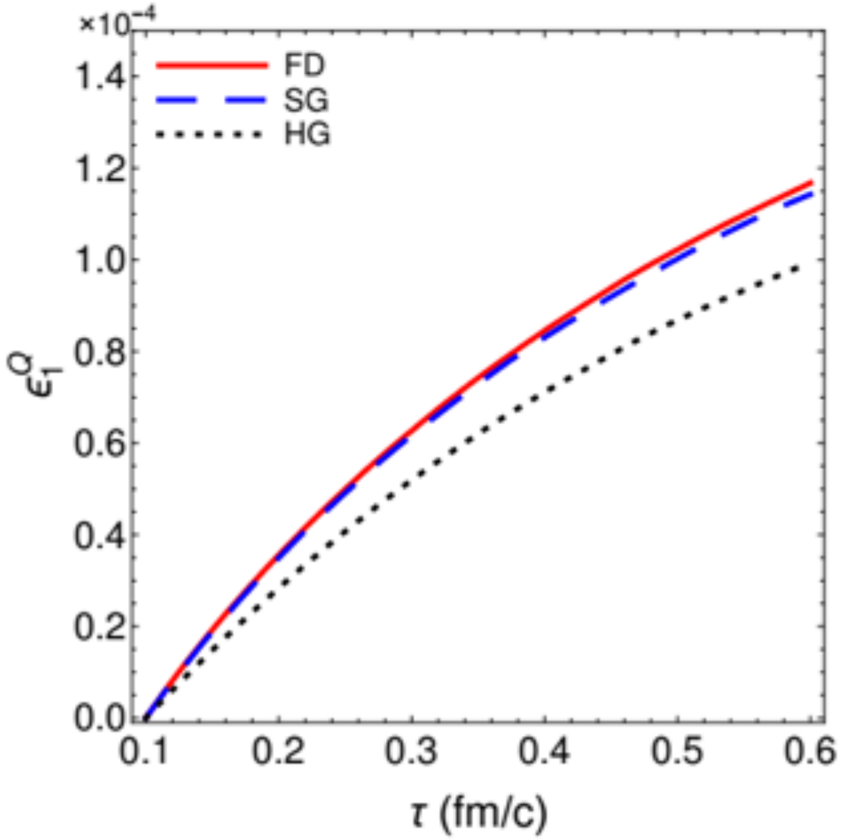
$$\epsilon_1^Q = \frac{\int dz dr_\perp^2 \, d\phi \, r_\perp^2 \; \sin\phi \, n^Q}{\int dz dr_\perp^2 \, d\phi \, r_\perp^2 \; n^{tot.}}$$

Net charge density

Dipole moment of the net charge density







Relaxation time dep.

Initial distr.

Huang, Jiang, Shi, Liao, Zhuang, arXiv:1703.08856

Other *recent* developments in CKT

Formulate in terms of world-lines, anomaly not be directly tied t
Berry monopole

Mueller, Venugopalan arXiv:1701.03331, arXiv:1702.01233

Can formulate in terms of Wigner functions

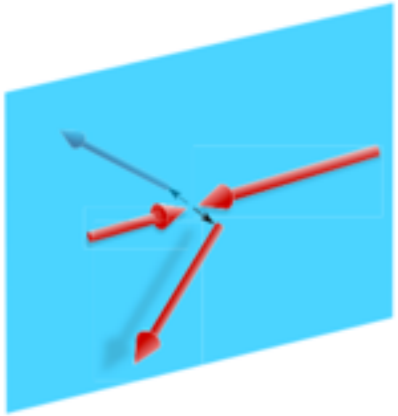
Hidaka, Pu, Yang Phys. Rev. D 95, 091901 (2017),Gao, Pu, Wang PRD 96 (2017)

Boost invariant formulation possible

Ebihara, Fukushima, Pu PRD 96 (2017)

Can include collisions, however subtleties exist with "side-jumps"

Chen, Son, Stephanov, Yee, Yin PRL 113 (2014), Chen, Son, Stephanov PRL 115 (2015), Hidaka, Pu, Yang Phys. Rev. D 95, 091901 (2017)



Stephanov, Nuclear Physics A 956 (2016)

Greater understanding of contributions to CME current (in and) out of equilibrium using CKT

Kharzeev, Stephanov, Yi PRD 95 (2017)

Conclusion

This year

First calculation of axial charge production from weakly coupled pre-equilibrium dynamics

Successful microscopic description of simple system thanks to improvements and chiral lattice fermions

Results suggest heavy quarks may not play significant role in anomalous transport

Finite relaxation time for the generation of the CME and CSE must be taken into account

Developed fully back-reacted classical QCDxQED code with chiral Fermions

Theoretical progress in Chiral Kinetic Theory and first steps towards using it for modeling

Moving forward

Lattice field theory:

Understand sphalerons in more realistic setting, compare to axial charge from flux-tubes

End goal is to have complete dynamical early time description

More realistic **B** field (dynamical QCDxQED) and gauge field configurations

Pre-equilibrium conductivity, other transport possible

Address initial quark production from the initial stage in CGC

Work in progress on initial condition model for axial/vector charge and currents for anomalous magnetohydrodynamics using results from real time lattice QCD simulations

Can also address chiral plasma instabilities

Moving forward

Chiral Kinetic Theory:

- Formulate effective chiral kinetic description which includes fluctuations
  - Boltzmann formulation with background fields needed
- Better understanding of side-jump/collisions/etc.
- Continue pushing numerics

A unified effort:

- **Like for bulk thermalization, match Classical Yang-Mills to CKT/ECKT for complete non-equilibrium description**

Thanks!

From Flickr user digitalworldofpaul