Chiral magnetohydrodynamics & machine-learning CME

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Chiral magnetic effect

$$\boxed{\boldsymbol{j}_{\mathrm{anom}} = \kappa_B \boldsymbol{B} + \kappa_\omega \boldsymbol{\omega}}$$

$$\boxed{\boldsymbol{j}_{5,\mathrm{anom}} = \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega}}$$

Chiral Fluid

EM fields

Chiral MHD

MHD = Magnetohydrodynamics

Systems described by chiral MHD

- Heavy-ion collisions
  - For the CME search, reliable estimate of the life-time of **B** is important

- Early Universe

- Weyl/Dirac semimetals

CME currents from magnetic reconnections

[Hirono-Kharzeev-Yin, PRL'16]

Magnetic & fermionic helicities

$$\partial_\mu j^\mu_A = C_A \boldsymbol{E} \cdot \boldsymbol{B}$$

$$\Rightarrow \boxed{\frac{d}{dt}\left[\mathcal{H} + \mathcal{H}_F\right] = 0}$$

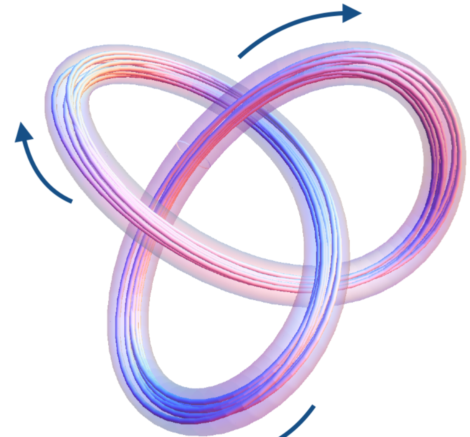
$$\mathcal{H} = \int d^3x \boldsymbol{A} \cdot \boldsymbol{B}$$

Magnetic helicity

$$\mathcal{H}_F = \frac{2}{C_A} \int d^3x \; n_A$$

Fermionic helicity

Magnetic helicity knows topology

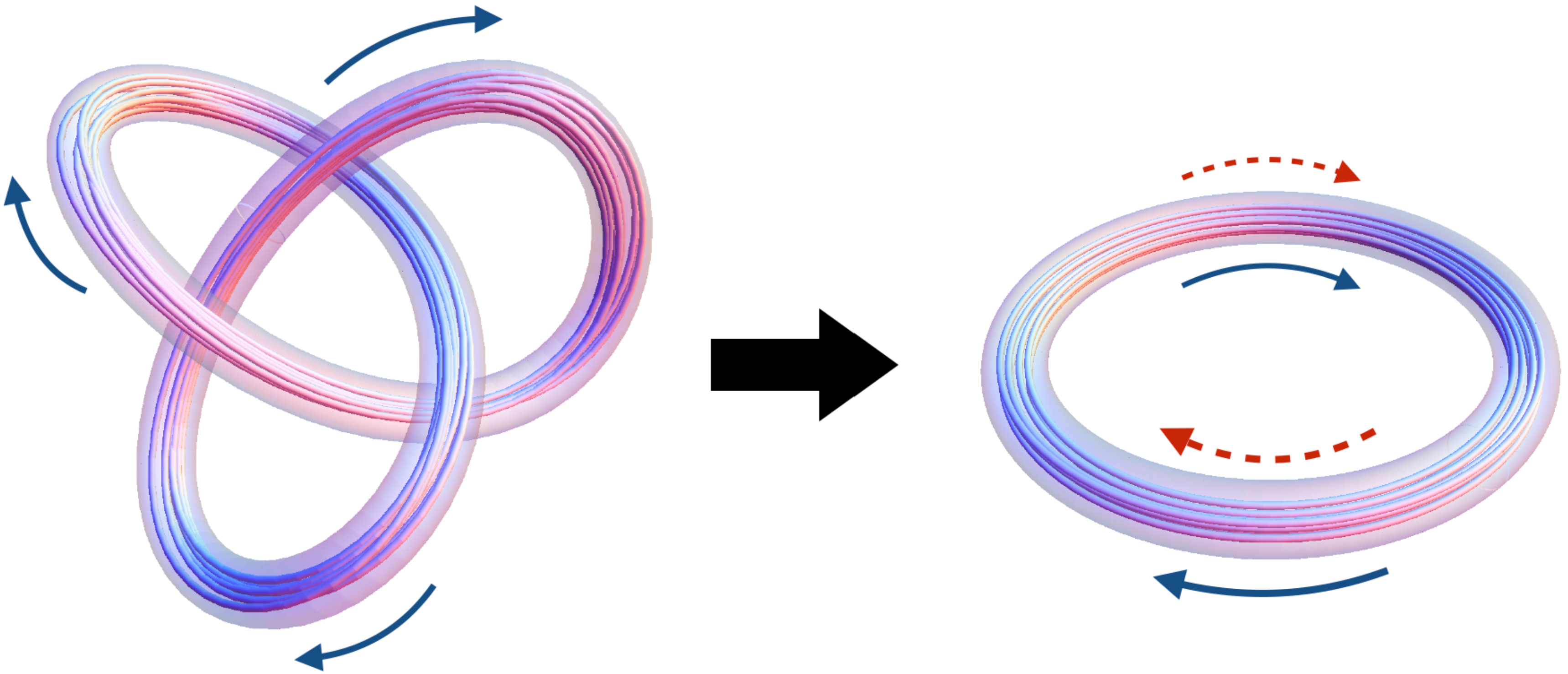


$$\mathcal{H} = \int d^3x \, \boldsymbol{A} \cdot \boldsymbol{B}$$

$$= \sum_i \mathcal{S}_i \varphi_i^2 + 2 \sum_{i,j} \mathcal{L}_{ij} \varphi_i \varphi_j$$

Self-linking number

Linking number



CME currents from reconnections of **B**

[Hirono-Kharzeev-Yin PRL'16]

$$\sum_i \oint_{C_i} \Delta \boldsymbol{J} \cdot d\boldsymbol{x} = -\frac{e^3}{2\pi^2} \Delta \mathcal{H}$$

Change of topology induces CME currents!

Formulation & waves of chiral MHD

[Hattori-Hirono-Yee-Yin, in preparation]

Chiral MHD

- MHD & chiral MHD can be understood as a low-energy effective theory basing on derivative expansion

- A new anomaly-induced instability

EOM of MHD

$$\partial_\mu T^{\mu\nu}_{\text{fluid}} = F^{\nu\rho} j_\rho$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

EOM of MHD

$$\partial_\mu T^{\mu\nu}_{\mathrm{tot}} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$T^{\mu\nu}_{\mathrm{tot}} = T^{\mu\nu}_{\mathrm{fluid}} + T^{\mu\nu}_{\mathrm{EM}}$$

$$T^{\mu\nu}_{\mathrm{EM}} = -F^{\mu}{}_{\alpha} F^{\nu\alpha} + \frac{1}{4}\eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

EOM of MHD

$$E^\mu \equiv F^{\mu\nu} u_\nu, \quad B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$$

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu - \epsilon^{\mu\nu\rho} B_\rho$$

$$\tilde{F}^{\mu\nu} = B^\mu u^\nu - B^\nu u^\mu + \epsilon^{\mu\nu\rho} E_\rho$$

$$\epsilon^{\mu\nu\alpha} \equiv \epsilon^{\mu\nu\alpha\beta} u_\beta$$

Hydrodynamic variables

- Parameters characterizing local thermal equilibrium

- Neutral fluid with a conserved charge $\{T(\boldsymbol{x}),\ u^\mu(\boldsymbol{x}),\ \mu(\boldsymbol{x})\}$

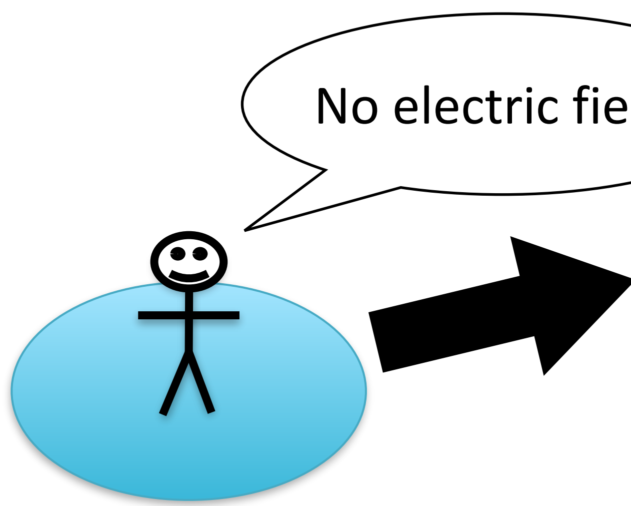
- MHD $\{T(\boldsymbol{x}),\ u^\mu(\boldsymbol{x}),\ B^\mu(\boldsymbol{x})\}$

# of hydro variables = # of equations = 7

No electric field in the fluid frame in ideal MHD

$$E^{\mu}_{(0)} = 0$$

Correspond to large conductivity limit



Constitutive relation for ideal MHD

$$T^{\mu\nu}_{\mathrm{tot}(0)} = (e+p)u^\mu u^\nu - p\eta^{\mu\nu} + \boldsymbol{B}^2\left[u^\mu u^\nu - b^\mu b^\nu - \frac{1}{2}\eta^{\mu\nu}\right]$$

$$B^\mu = |\boldsymbol{B}| b^\mu \quad b_\mu b^\mu = -1$$

$$F^{\mu\nu}_{(0)} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

CME doesn't play any role in ideal MHD!

Why?

Constitutive relation doesn't determine the current

$$j^\mu = \underset{\substack{\downarrow \\ \infty}}{\sigma}\,\underset{\substack{\downarrow \\ 0}}{E^\mu} + \sigma_B B^\mu$$

Current is determined by the Maxwell equation

$$j^\mu = \partial_\nu F^{\mu\nu} \qquad F^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

Chiral magnetic conductivity never appears in EOM

$$\partial_\mu T^{\mu\nu}_{\mathrm{tot}} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

Conservation of topology of B

- Flux is “frozen in” to the fluid
- Magnetic helicity is conserved in ideal MHD
  - **No reconnection**

$$h_{\mathrm{B}}^{\mu} = \tilde{F}^{\mu\nu} A_{\nu}$$: helicity current

$$\partial_{\mu} h_{\mathrm{B}}^{\mu} = 2\tilde{F}^{\mu\nu} F_{\mu\nu} = 8E^{\mu} B_{\mu} = 0$$

$$\mathcal{H} = \int d^3x h_{\mathrm{B}}^{0}$$ is conserved

First order in derivative expansion

Using the second law,

$$T^{\mu\nu}_{\mathrm{tot}(1)} = \zeta \Delta^{\mu\nu} \partial\cdot u + 2\eta \nabla^{<\mu} u^{\nu>}$$

E^{μ}: C-odd, P-odd

$$E^{\mu}_{(1)} = \frac{1}{\sigma\beta} \epsilon^{\mu\nu\alpha\beta} u_{\nu} \partial_{\alpha}(\beta B_{\beta}) - \underbrace{\epsilon_B B^{\mu}}_{\text{CME}}$$

σ : electric conductivity

$\epsilon_B = \dfrac{\sigma_B}{\sigma}$ $\quad$ σ_B : chiral magnetic conductivity

First order in derivative expansion

$$\tilde{F}^{\mu\nu}_{(1)} = \epsilon^{\mu\nu\rho\sigma} E_{(1)\rho} u_\sigma$$

$$\Rightarrow \quad \begin{aligned} &\partial_\mu \left[T^{\mu\nu}_{\mathrm{tot}(0)} + T^{\mu\nu}_{\mathrm{tot}(1)} \right] = 0 \\ &\partial_\mu \left[\tilde{F}^{\mu\nu}_{(0)} + \tilde{F}^{\mu\nu}_{(1)} \right] = 0 \end{aligned}$$

Waves in chiral MHD

- Linear fluctuations - 6 modes in total

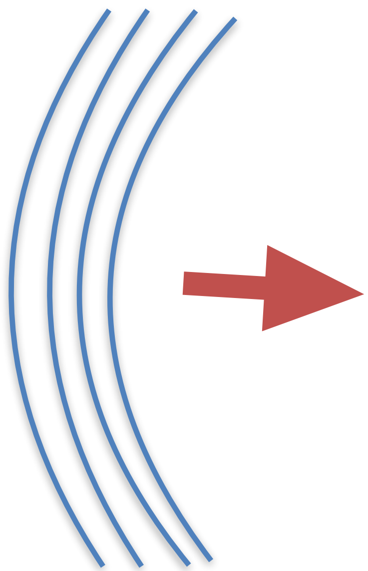
$$e \to e + \delta e,$$

$$B \to B + \delta B,$$

$$u^\mu \to u^\mu + \delta u^\mu,$$

$$b^\mu \to b^\mu + \delta b^\mu.$$

Alfven wave



restoring force

magnetic field line

momentum along the background **B**

Dispersion relation $\omega = \pm v_A k_{||}$

$$v_A^2 = \frac{\boldsymbol{B}^2}{e + p + \boldsymbol{B}^2}$$ Alfven velocity

Alfven wave in dissipative MHD

Dispersion relation

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2$$

damping

$$\lambda = \frac{1}{\sigma}$$

σ : electric conductivity

$$\bar{\eta} \equiv \frac{\eta}{e + p + \boldsymbol{B}^2}$$

η : shear viscosity

Alfven wave in chiral MHD

Including CME (when $\boldsymbol{k} \propto \boldsymbol{B}$)

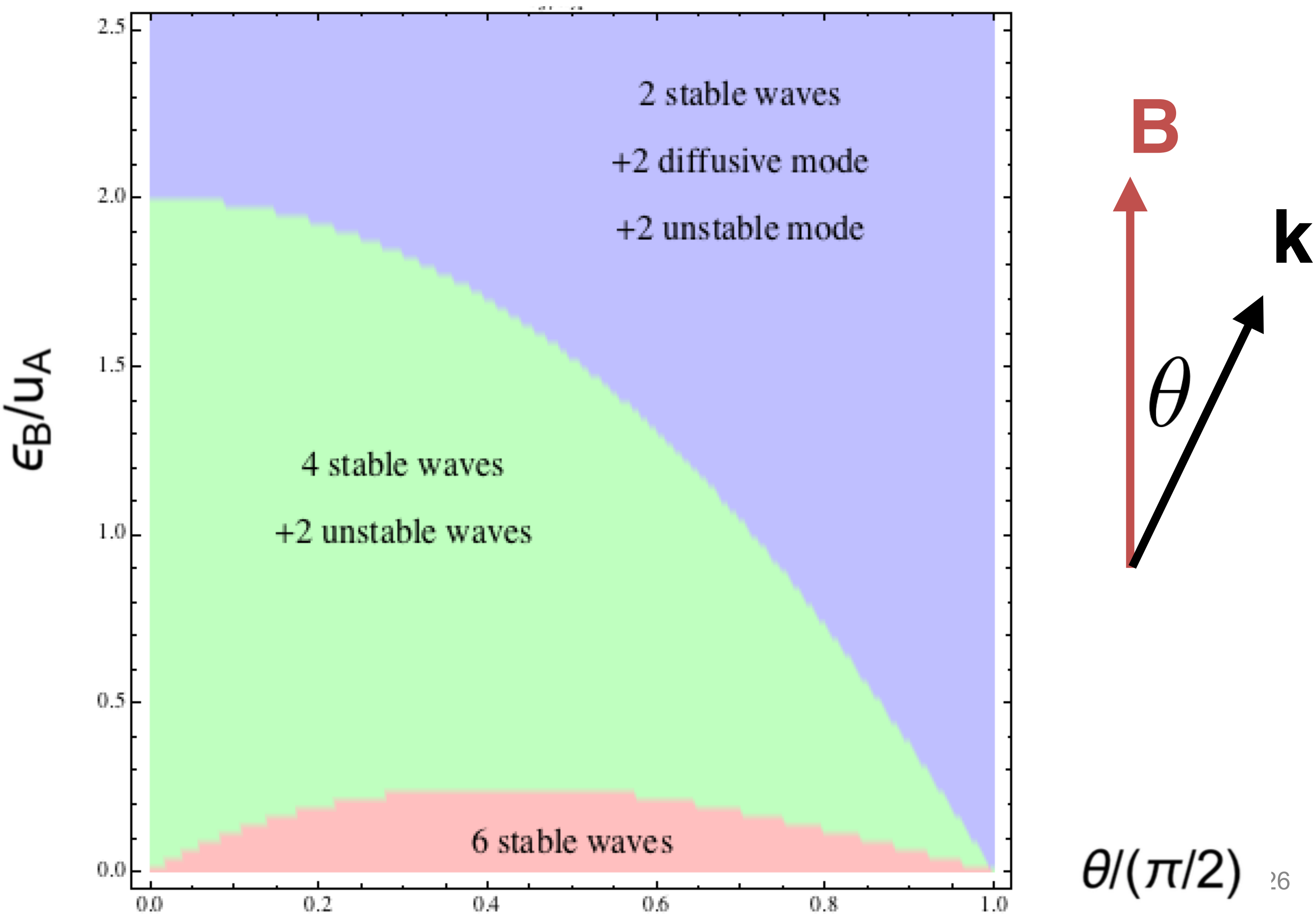
$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[(\bar{\eta} + \lambda)\, k_{||}^2 - \underbrace{s\epsilon_B k_{||}}_{\text{CME}}\right]$$

$s = \pm 1$ indicates the helicity of the mode

$$i\boldsymbol{k} \times \underbrace{\boldsymbol{e}^{(s)}}_{\text{helicity eigenstate}} = sk\boldsymbol{e}^{(s)}$$

Instability in one of the helicity modes

Stability of the waves

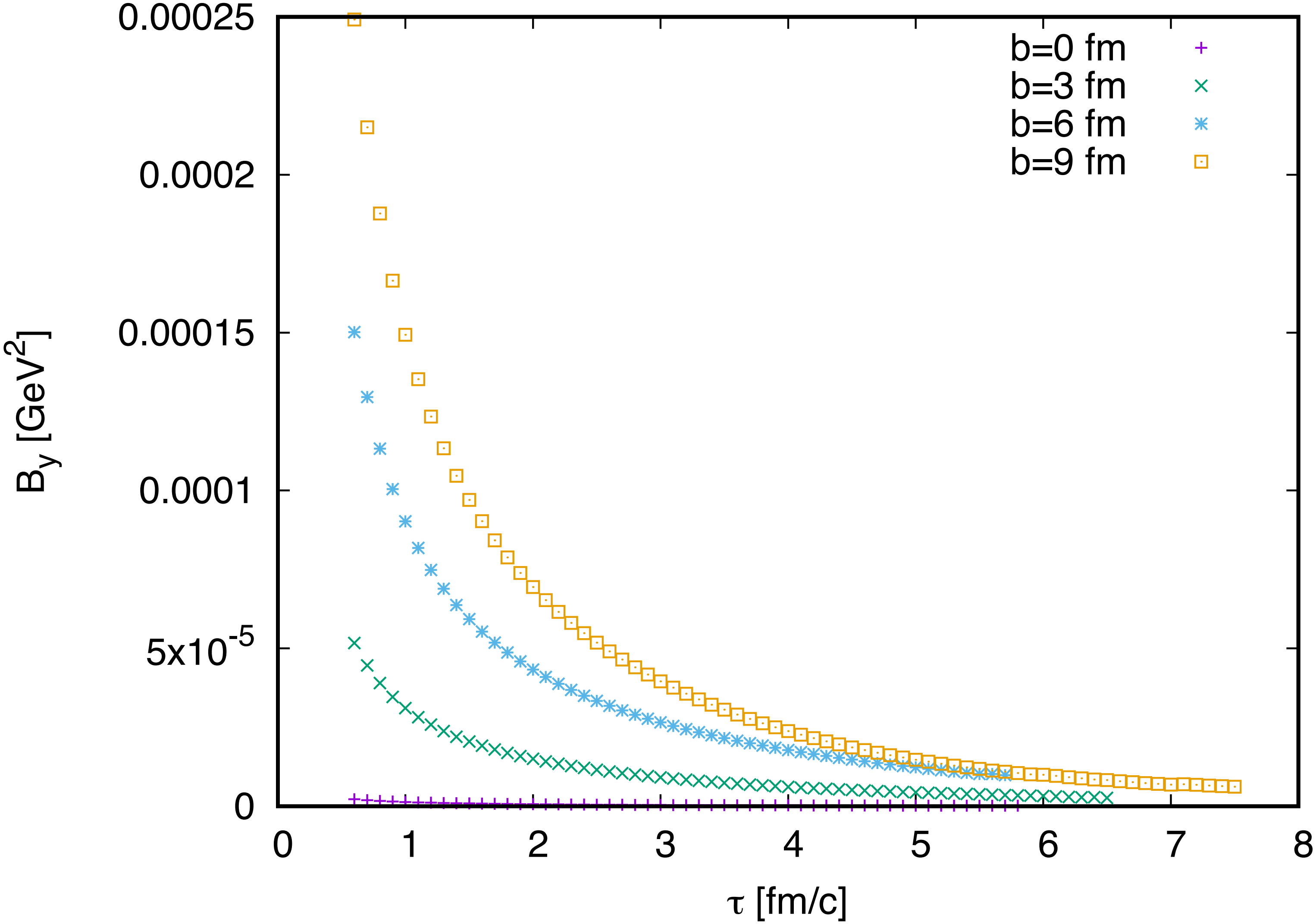


Toward numerical impl. of chiral MHD

with Mace, Kharzeev, Inghirami et.al.

- Collaborating with ECHO-QGP
- Implement:
  - Finite resistivity + anomalous effects
  - Pre-equilibrium CME currents

B-field evolution



Machine-learning CME

[Hirono-Kharzeev-Mace, in progress]

Observables for CME

- gamma correlation $\gamma_{\alpha\beta} = \left\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}}) \right\rangle$

- Can pick up two-particle correlations unrelated to charge separation ("non-flow")

$$f_{\alpha\beta}(\boldsymbol{p}_1, \boldsymbol{p}_2) = f_\alpha(\boldsymbol{p}_1) f_\beta(\boldsymbol{p}_2) + \underbrace{f^c_{\alpha\beta}(\boldsymbol{p}_1, \boldsymbol{p}_2)}_{\sim \frac{1}{N}}$$

Backgrounds

- Flowing resonances **[Voloshin PRC'04]** **[F. Wang PRC'10]**
- Local charge conservation + flow **[Schlichting-Pratt, PRC'11]** **[Pratt-Schlichting-Gavin PRC'11]**
- Transverse momentum conservation **[Bzdak-Koch-Liao PRC'10, PRC'11]** **[Pratt-Schlichting-Gavin PRC'11]**

Those all contribute as $\sim \dfrac{v_2}{N}$

Machine-learning CME

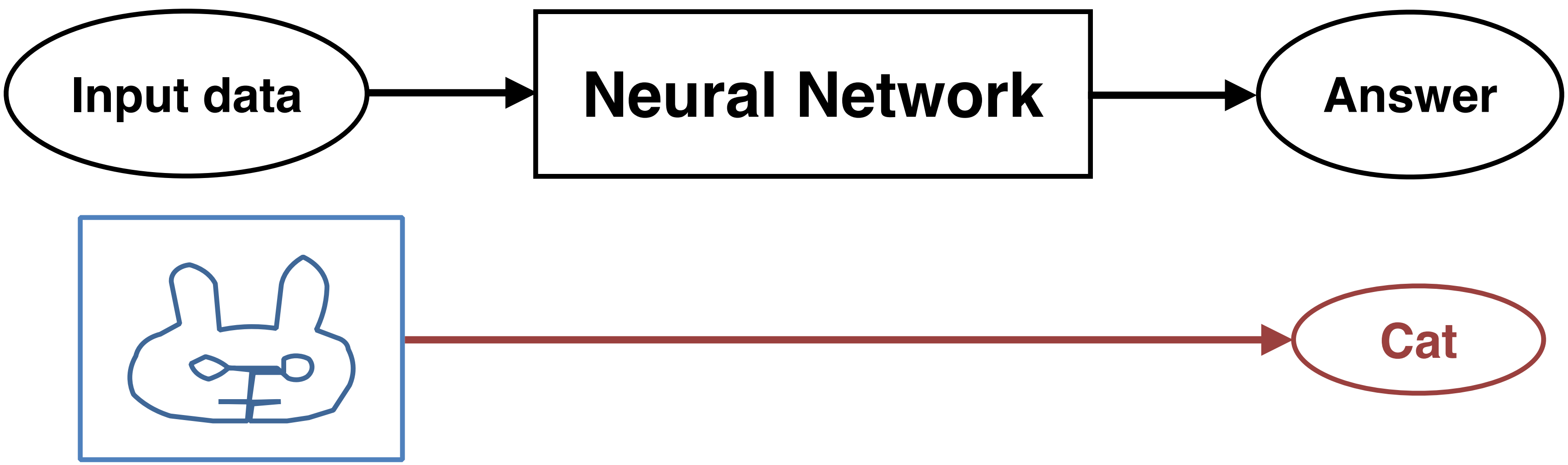
[Hirono-Kharzeev-Mace, in progress]

- How to separate of signal & background is a big problem
- Gammas - include contributions from background effects
- There can be better observables
    - Less sensitive to background effects

Machine-learning CME

[Hirono-Kharzeev-Mace, in progress]

- Use **neural network** to learn the feature of CME
  - Training (parameter tuning)

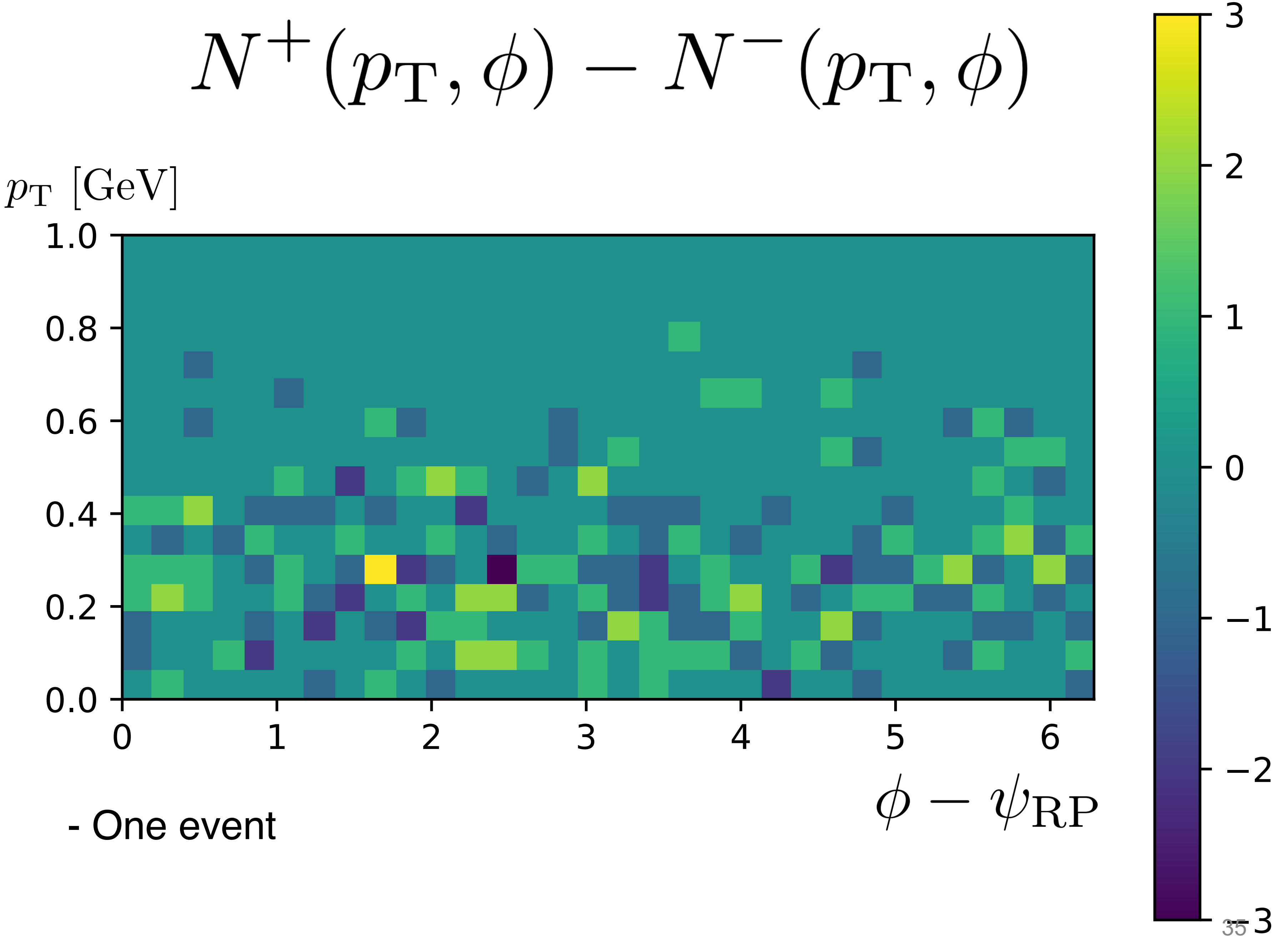


- Applied for detection of first-order PT

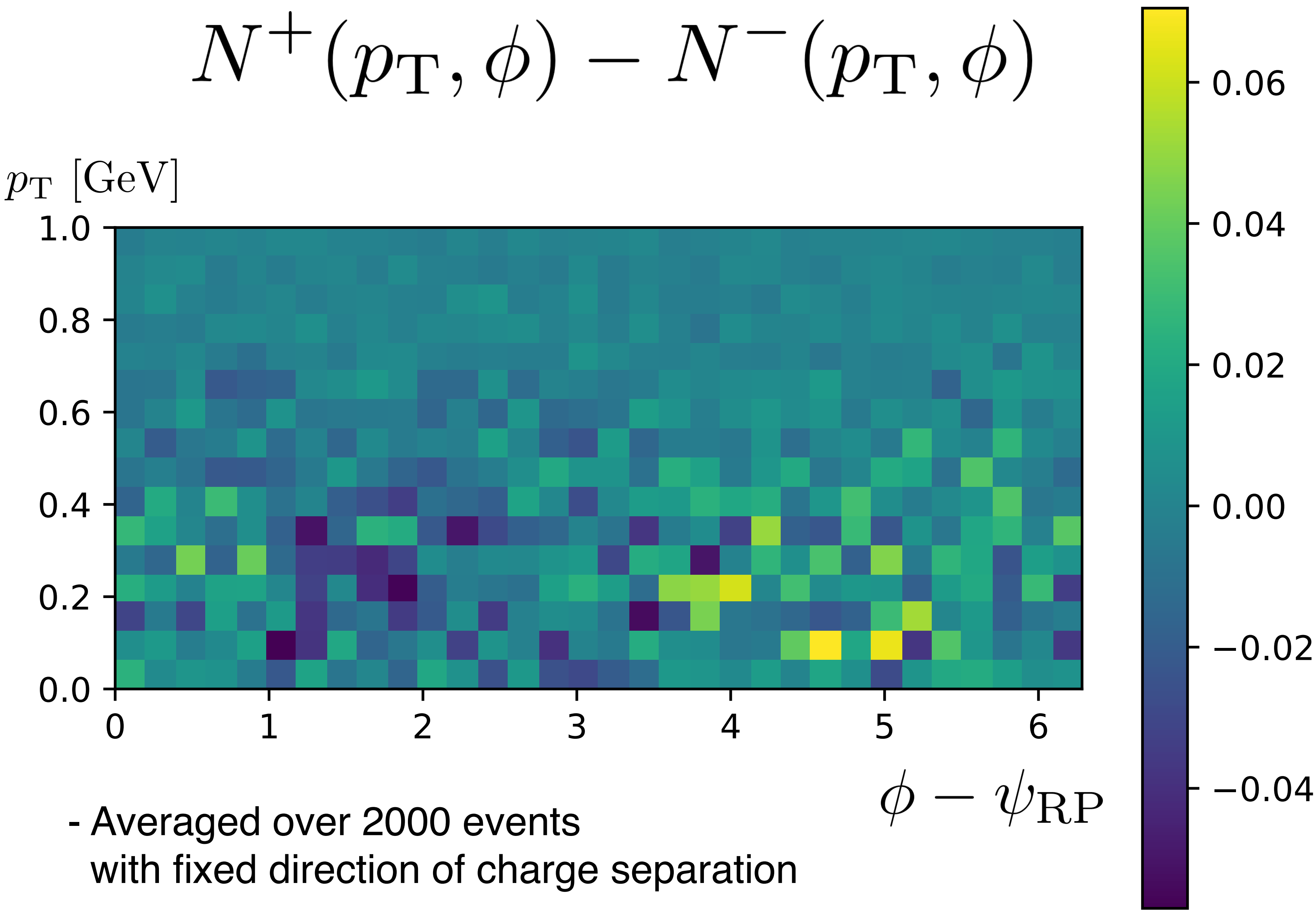
[Pang-Zhou-Su-Petersen-Stöcker-Wang 1612.04262]

Data preparation & learning

- Supervised learning
- Raw event data (momenta of each particle)
  - Fireball
    - With charge separation (about 1% in a1)
    - Without charge separation
  - Produce particle via Cooper-Frye prescription
  - Perform resonance decay
- Charge-dependent distributions
  - One-particle distributions
  - Multi-particle distributions
- Use Keras + Tensorflow


$N^+(p_T, \phi) - N^-(p_T, \phi)$
p_T [GeV]
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
$\phi - \psi_{RP}$
- One event
3
2
1
0
−1
−2
−3

$$N^+(p_\mathrm{T}, \phi) - N^-(p_\mathrm{T}, \phi)$$

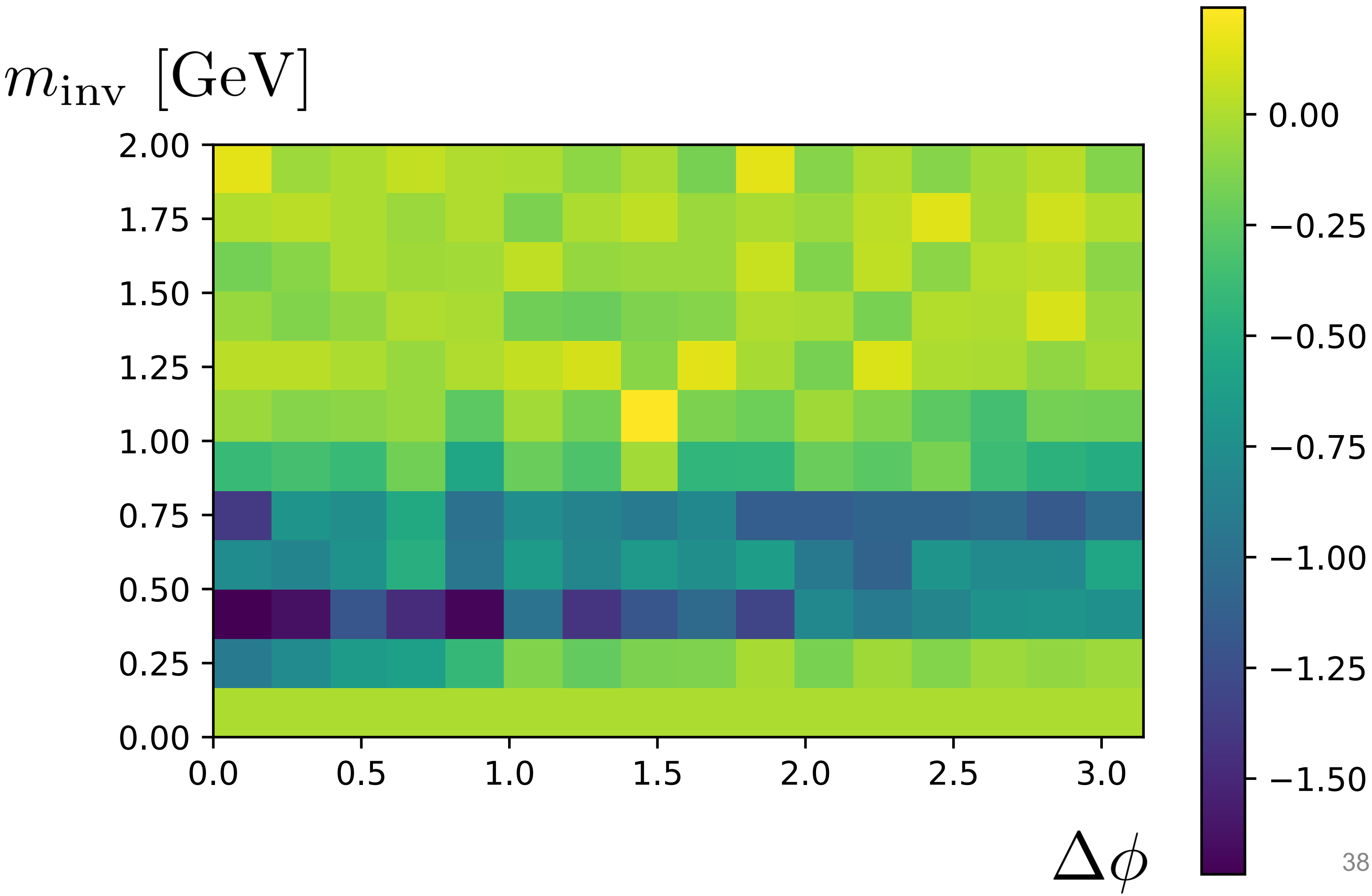


- Averaged over 2000 events with fixed direction of charge separation

Machine-learning CME

- Machines can detect CME well for the single-particle net-charge distribution (averaged over a certain number of events), when the direction of charge separation is fixed
- We are testing this method for two-particle charge-dependent corr.

$$N^{++} + N^{--} - N^{+-} - N^{-+}$$



$m_{\rm inv}$ [GeV]
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\Delta\phi$
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50

Summary

1. **B-field topology & CME currents**

    - Reconnections generate CME currents

2. **Chiral MHD**

    - CME appears in the first order P-odd correction

    - Helicity dependent instability

    - Numerical implementation is under way

3. **Machine learning CME**

    - Use of neural network for a better observable