BEST lattice QCD 2017

The BEST plan for lattice QCD

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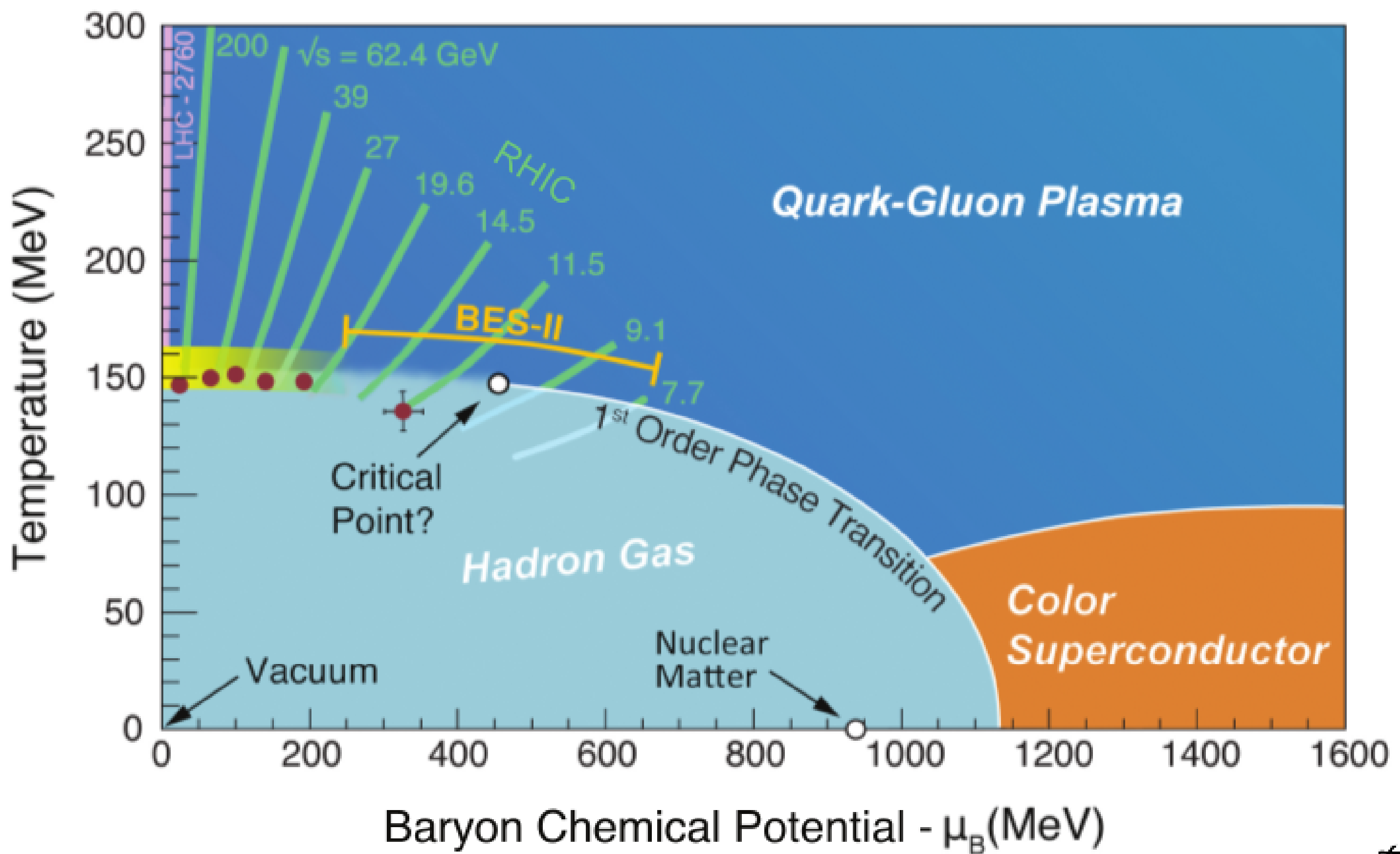
- characterize bulk thermodynamics and fluctuations of conserved charges on the freeze-out line
- provide the equation of state for BES
- (constrain) the location of the critical point

August 2017

Exploring the phase diagram of strong-interaction matter

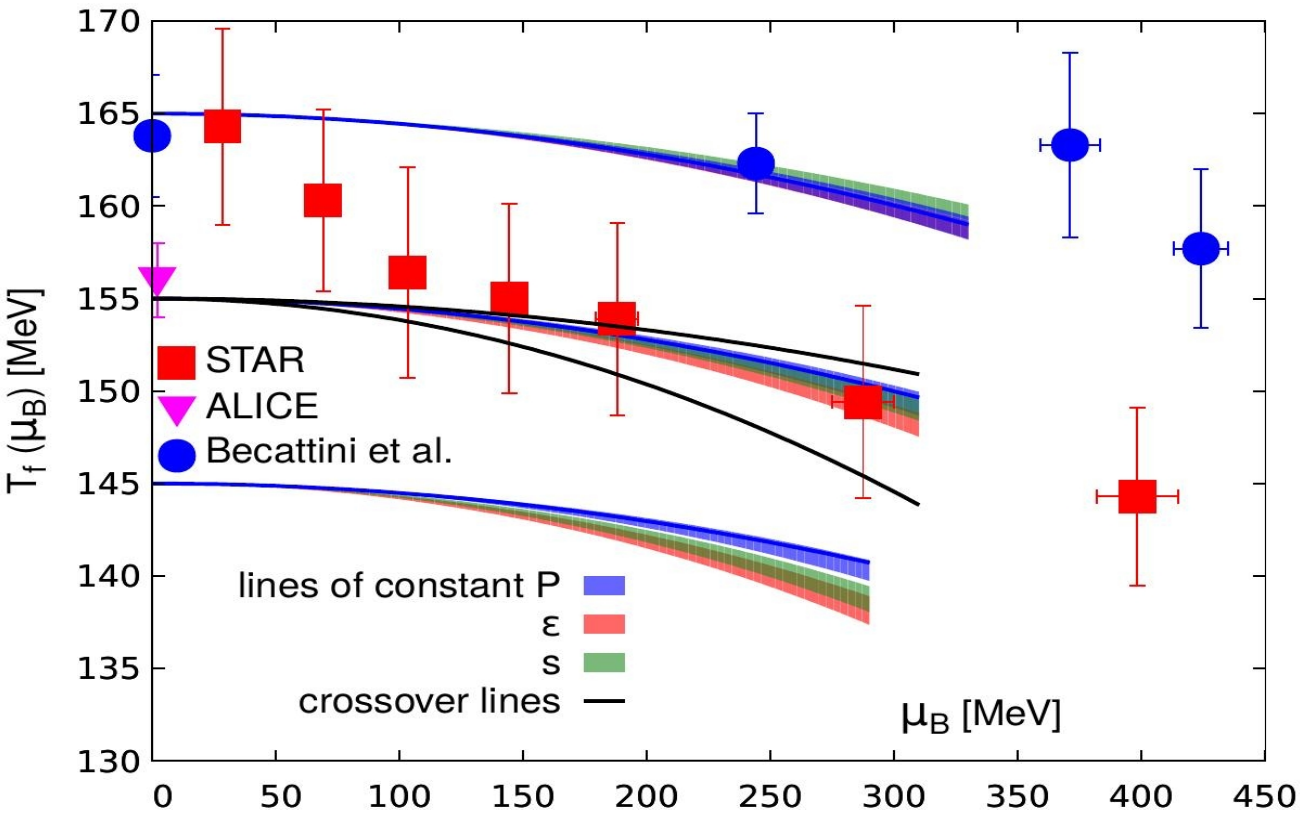


Where is the
critical point?

Chiral transition, hadronization and freeze-out

- pseudo-critical temperature $T_c = 154(9)\text{MeV}$
- hadronization temperatures $T_h = 164(2)\ \text{MeV}$
- freeze-out temperatures: $T_{fo} = 156(3)\ \text{MeV}$

$$T_{fo} = [164(5) - 168(4)]\ \text{MeV}$$



Where does hadronization set in?

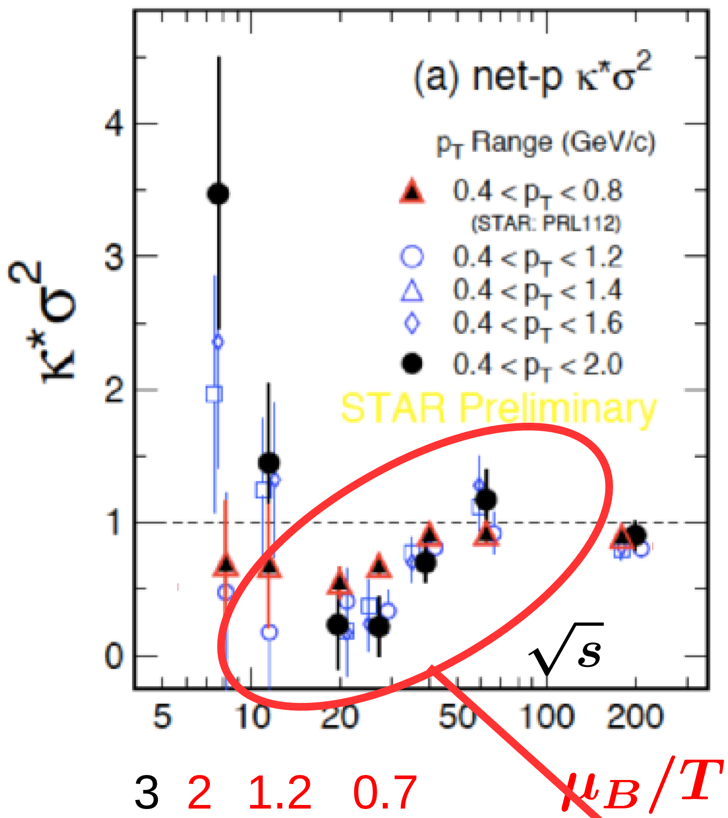
physics is quite different at lower and upper end of the current error bar on Tc

probed with net-charge correlations&fluctuations

crossover transition lines:
O. Kaczmarek et al., arXiv:1011.3130, C. Bonati et al., arXiv:1507.03571
R. Bellwied et al., arXiv:1507.07510 , P. Cea et al., arXiv:1508.07599

lines of constant physics
A. Bazavov et al.
(BI-BNL-CCNU...),
arXiv:1701.04325

Exploring the QCD phase diagram



?

More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

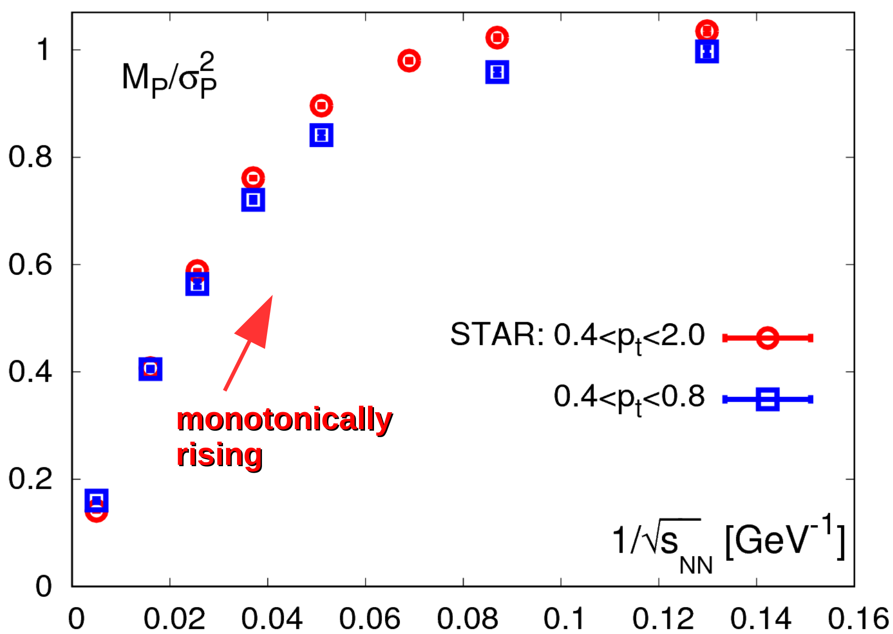
- For $\sqrt{s} \geq 19.6$ GeV : Structure of net-electric charge and net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

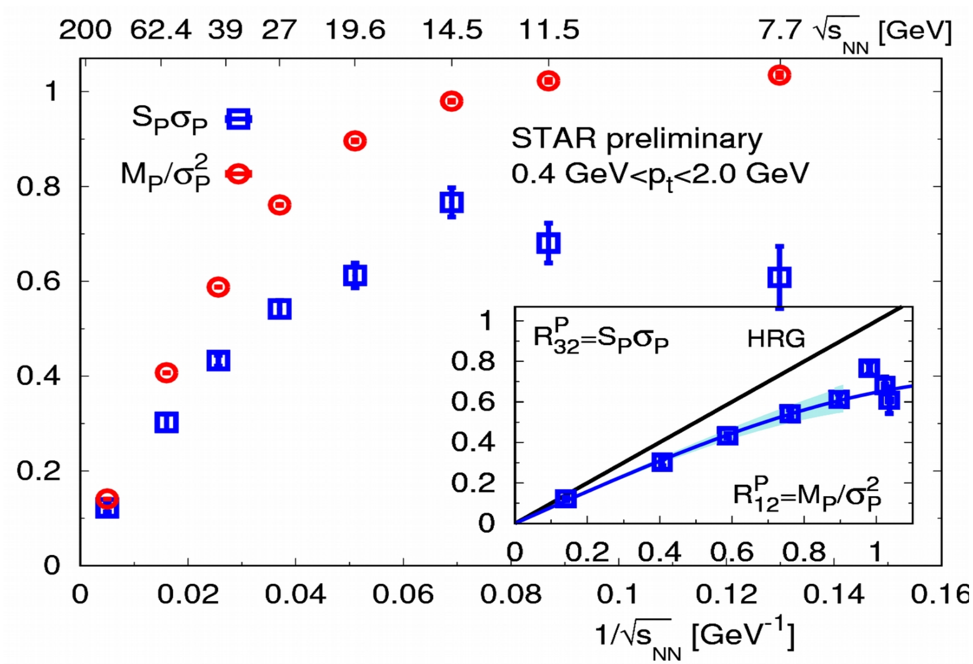
Cumulant ratios of conserved net-charge fluctuations

cumulant ratios: $\frac{\chi_n^X}{\chi_m^X}$, $X = B,\ Q,\ S$ $\quad (n,m) = (1,2):\ M_X/\sigma_X^2$

$(3,2):\ S_X\sigma_X$

$\chi_n^X = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$ $\quad (4,2):\ \kappa_X\sigma_X^2$





- M_P/σ_P^2 is monotonically rising
- $S_P\sigma_P < M_P/\sigma_P^2$

replace $\sqrt{s_{NN}}$ in favor of M_P/σ_P^2

Cumulant ratios of conserved net-charge fluctuations

cumulant ratios: $\frac{\chi_n^X}{\chi_m^X}$, $X = B,\ Q,\ S$

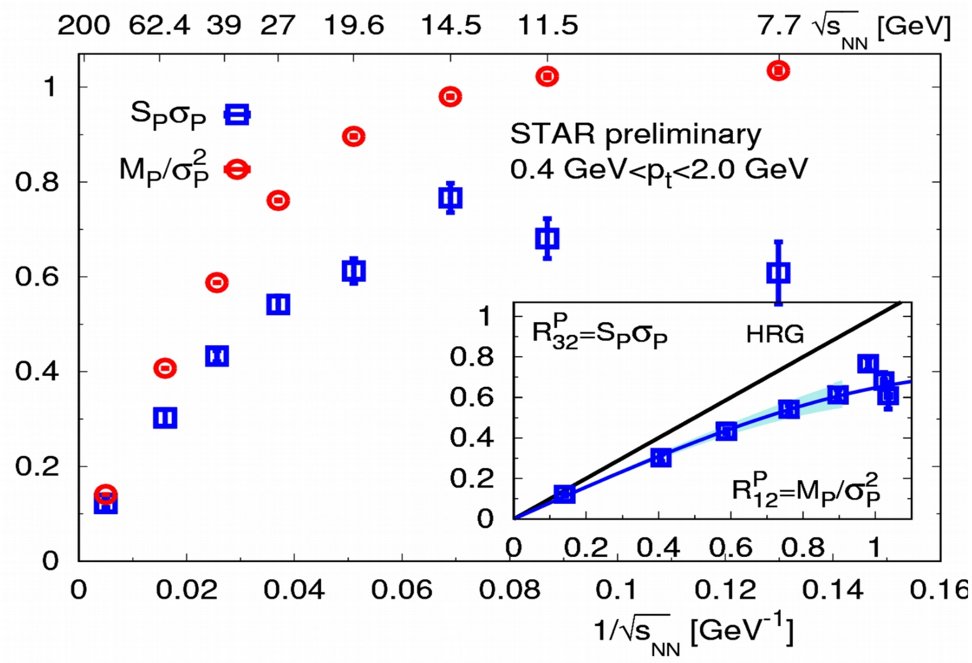
$$\chi_n^X = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$$

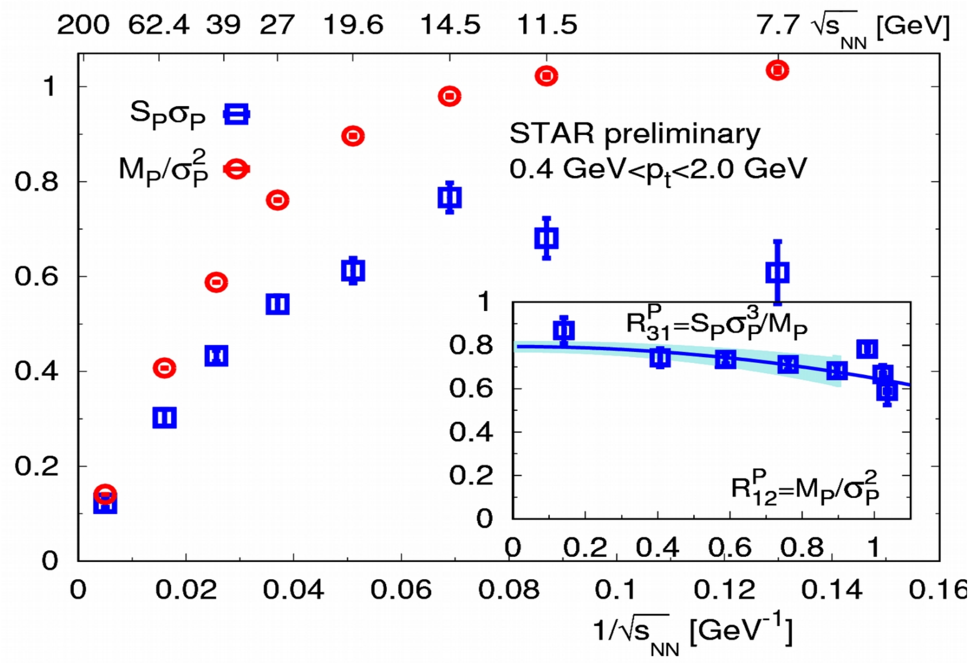
$(n,m) = (1,2):\quad M_X/\sigma_X^2$

$(3,2):\quad S_X\sigma_X$

$(4,2):\quad \kappa_X\sigma_X^2$

$(3,1):\quad S_X\sigma_X^3/M_X$





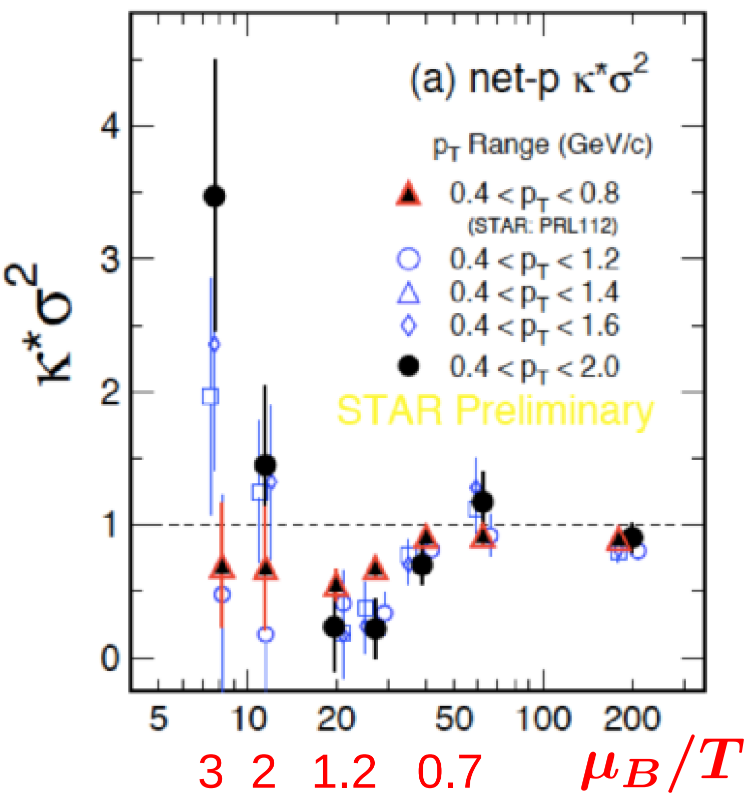
- $S_P\sigma_P < M_P/\sigma_P^2$
- $R_{31}^P = R_{32}^P R_{12}^P$

identical information

Cumulant ratios of conserved net-charge fluctuations

kurtosis*variance: $R_{42}^X = (\kappa\sigma^2)_X = \frac{\chi_4^X}{\chi_2^X}$

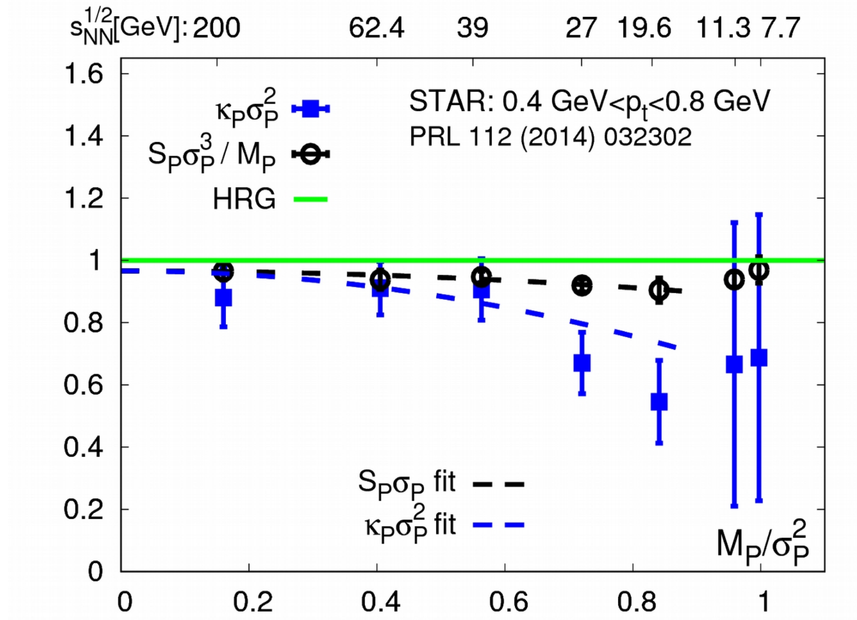
skewness*variance[1/2]: $R_{32}^X = (S\sigma)_X = \frac{\chi_3^X}{\chi_2^X}$

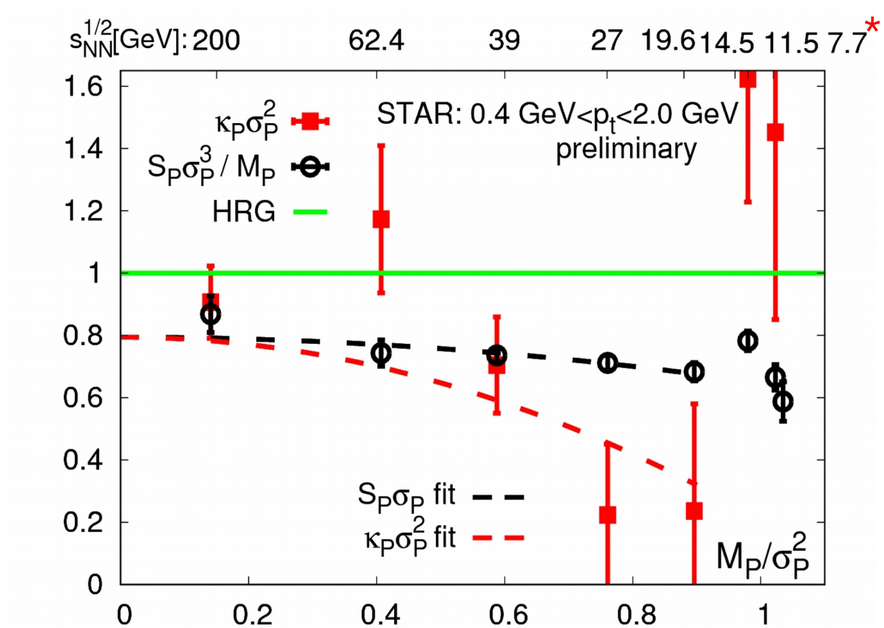


X. Luo (STAR Collaboration),
PoS CPOD2014 (2014) 019

at $\mu_B = 0$
$(\kappa\sigma^2)_P \simeq S_P\sigma_P^3/M_P$

slope of
$(\kappa\sigma^2)_P \simeq 3S_P\sigma_P^3/M_P$





* not shown

Conserved charge fluctuations and freeze-out

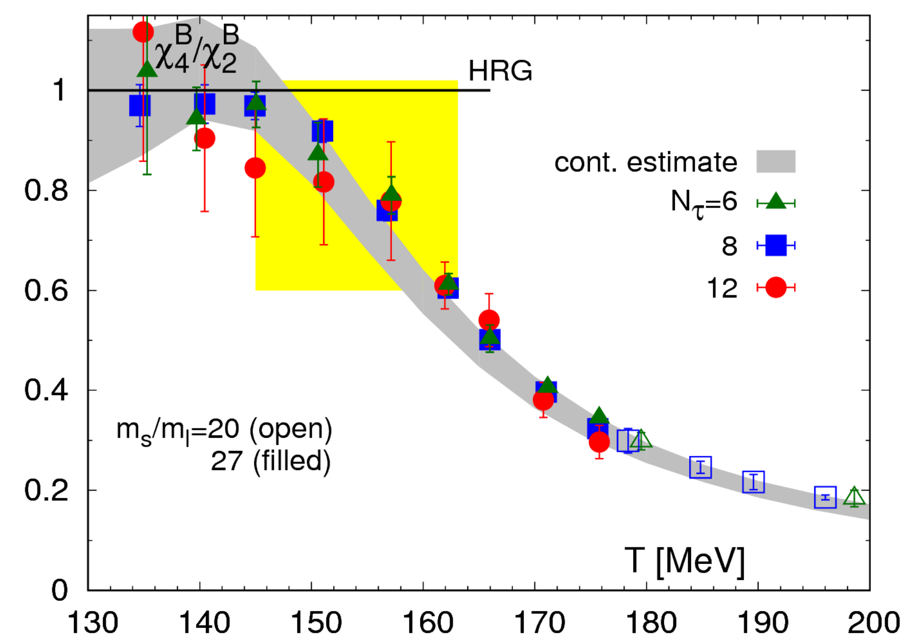
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

no need for talking about a chemical potential

$$S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



formulas are given for the case $\mu_S = \mu_Q = 0$

However, entire analysis is done for $n_S = 0$, $n_Q/n_B = 0.4$

Conserved charge fluctuations and freeze-out

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

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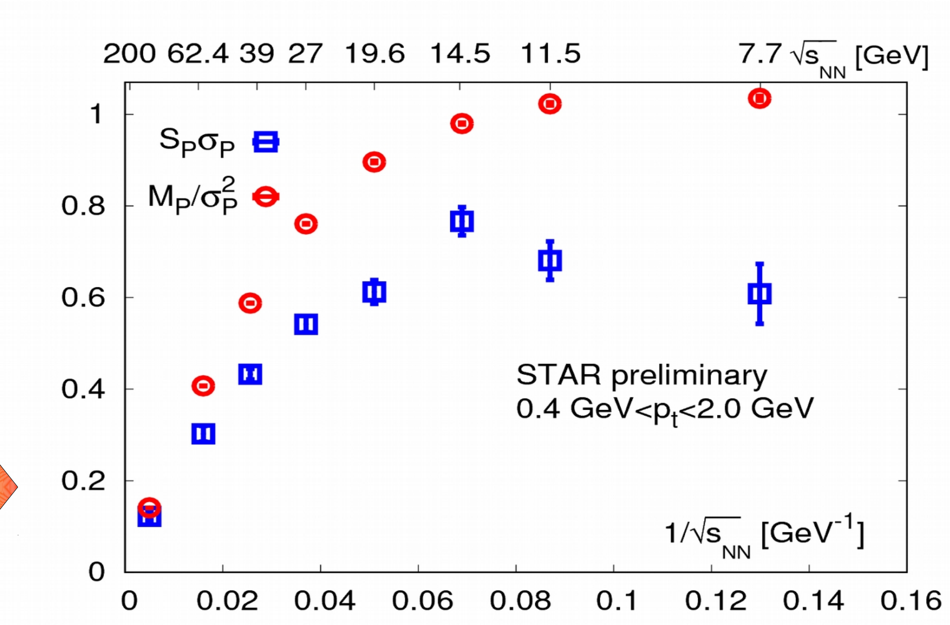
$$S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



$$\frac{\chi_4^B}{\chi_2^B} < 1 \quad \Longleftrightarrow \quad S_B\sigma_B < M_B/\sigma_B^2$$

Conserved charge fluctuations and freeze-out mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B \sigma_B^3 / M_B$ and $R_{42}^B \equiv \kappa_B \sigma_B^2$ are closely related

$$R_{31}^B = r_{31}^{B,0} + r_{31}^{B,2} \left(R_{12}^B\right)^2$$

$$R_{42}^B = r_{42}^{B,0} + r_{42}^{B,2} \left(R_{12}^B\right)^2$$

$\underline{\mu_S = \mu_Q = 0:}$

$$r_{42}^{B,0} = r_{31}^{B,0} = \frac{\chi_4^B}{\chi_2^B}$$

$$r_{42}^{B,2} = 3 r_{31}^{B,2} = \frac{1}{2} \left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B} \right)^2 \right)$$

Conserved charge fluctuations and freeze-out mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B^3/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

$$R_{31}^B = r_{31}^{B,0} + r_{31}^{B,2}\left(R_{12}^B\right)^2$$

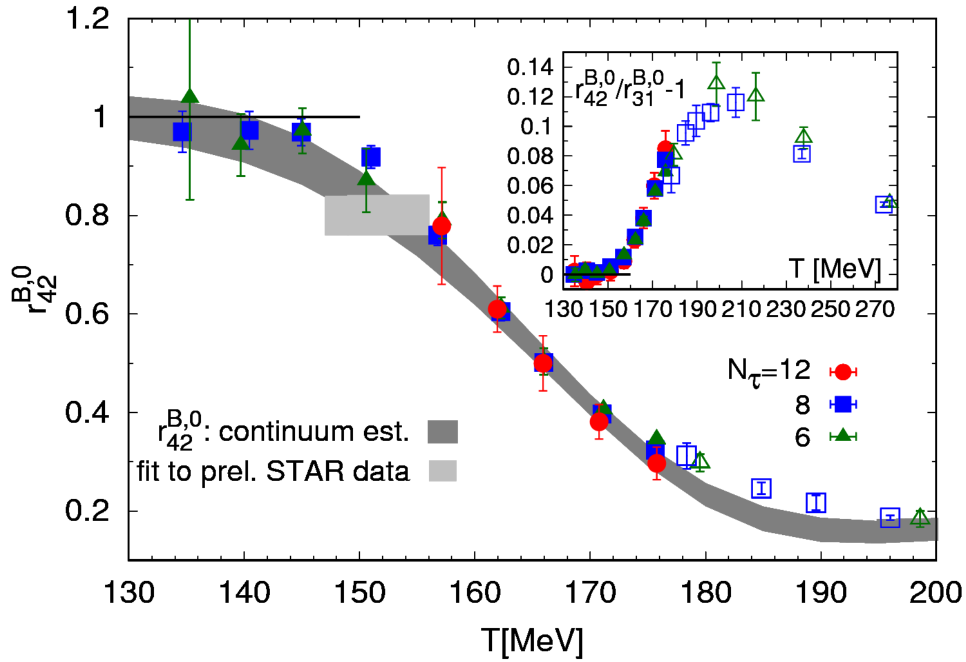
$$R_{42}^B = r_{42}^{B,0} + r_{42}^{B,2}\left(R_{12}^B\right)^2$$

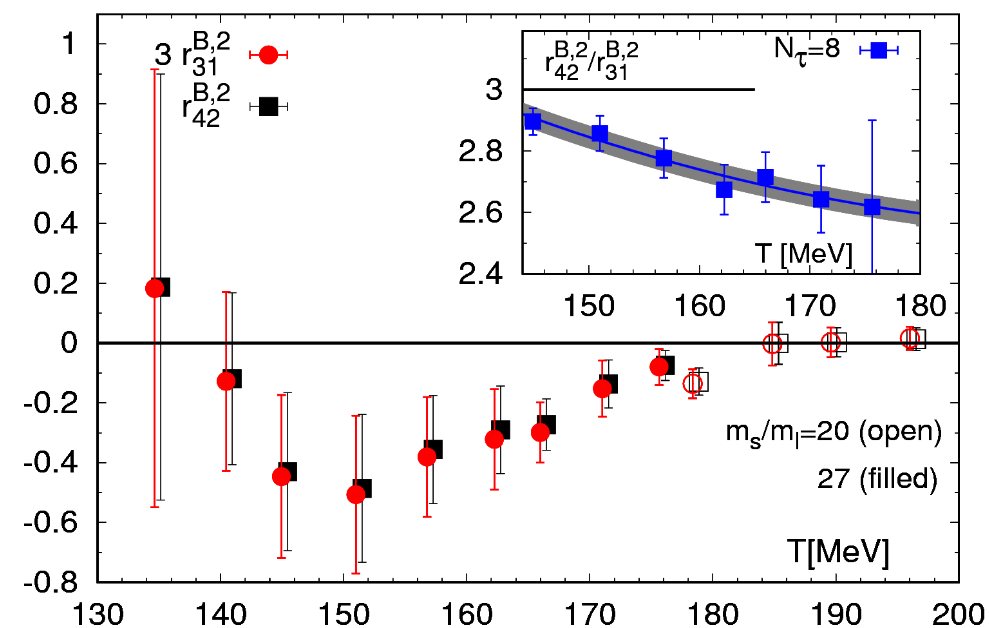
$\Rightarrow$ $n_S = 0\ ,\ n_Q/n_B = 0.4:$

$$r_{42}^{B,0} \simeq r_{31}^{B,0}$$

$$r_{42}^{B,2} \simeq 3r_{31}^{B,2}$$

A. Bazavov et al. (HotQCD Collaboration), in preparation





Skewness and kurtosis on a "line of constant physics"

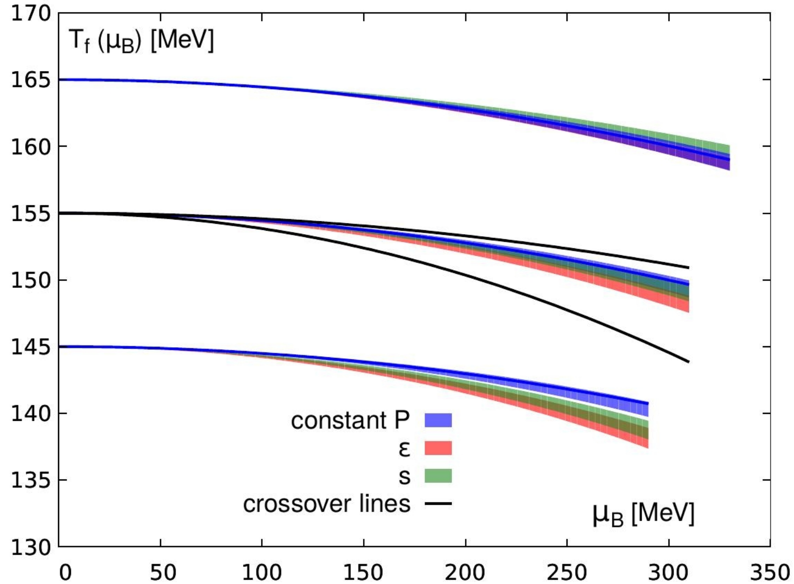
– Temperature on the "freeze-out" line changes with increasing μ_B

– consider cumulant ratios on lines $T_f(\mu_B) = T_f(0)(1 - \kappa_2^f(\mu_B/T_f(0))^2) + \mathcal{O}(\mu_B^4)$

– Taylor expansion in T and μ_B

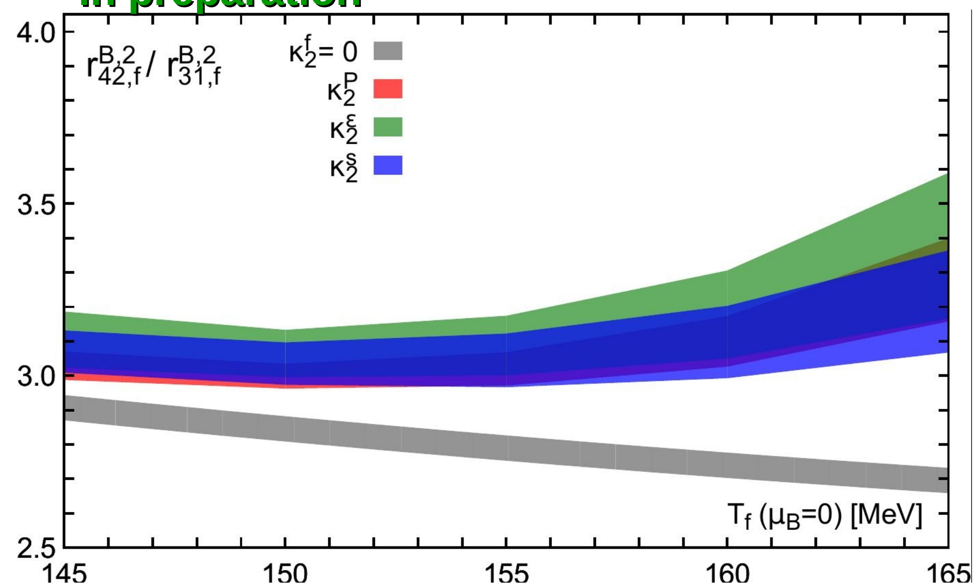
$$r_{nm}^{B,k} \Rightarrow r_{nm,f}^{B,k} \equiv r_{nm}^{B,k}(T_{f,0}) - \kappa_2^f T_{f,0} \left. \frac{\mathrm{d}r_{nm}^{B,k-2}}{\mathrm{d}T} \right|_{T=T_{f,0}}$$

A. Bazavov et al. arXiv:1701.04325



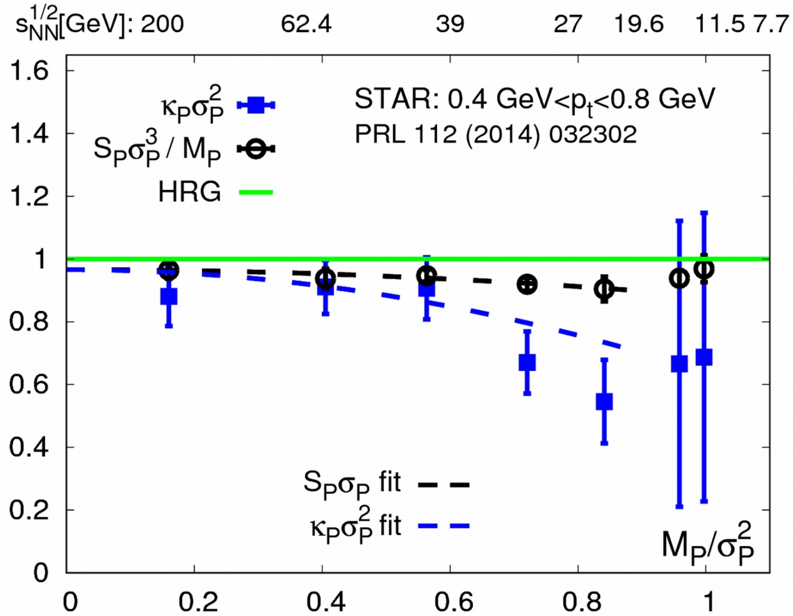
$$0.006 < \kappa_2^f < 0.015$$

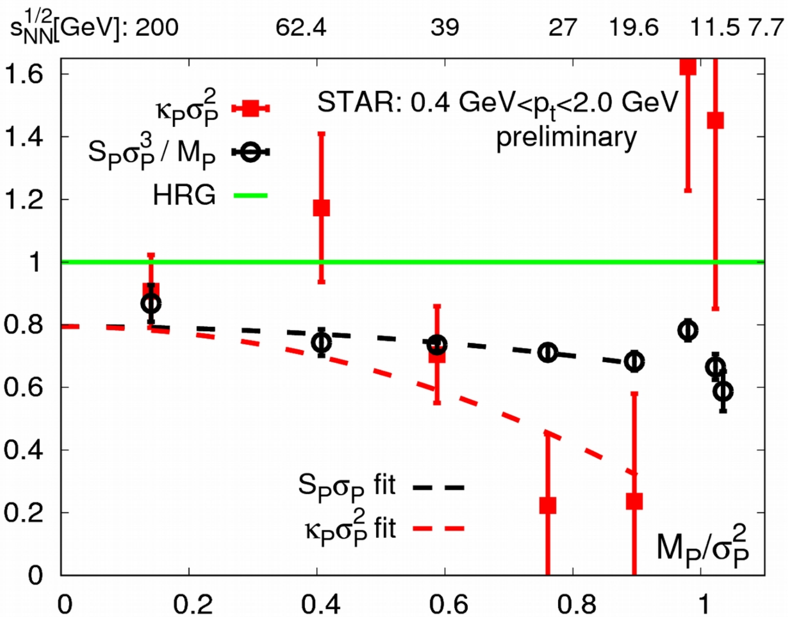
A. Bazavov et al. (HotQCD Collaboration), in preparation



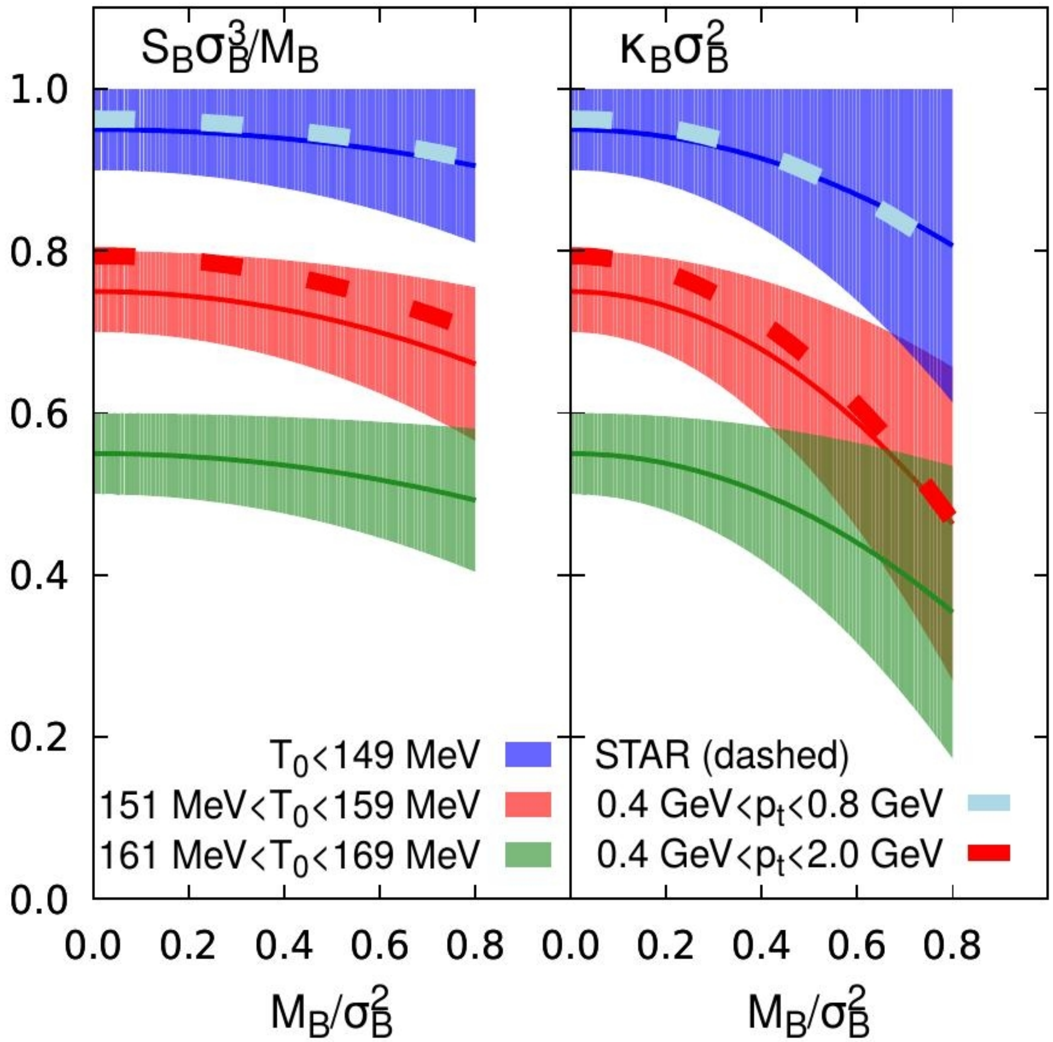
$$\frac{r_{42,f}^{B,2}}{r_{31,f}^{B,2}} = 3 - 3.6$$

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis



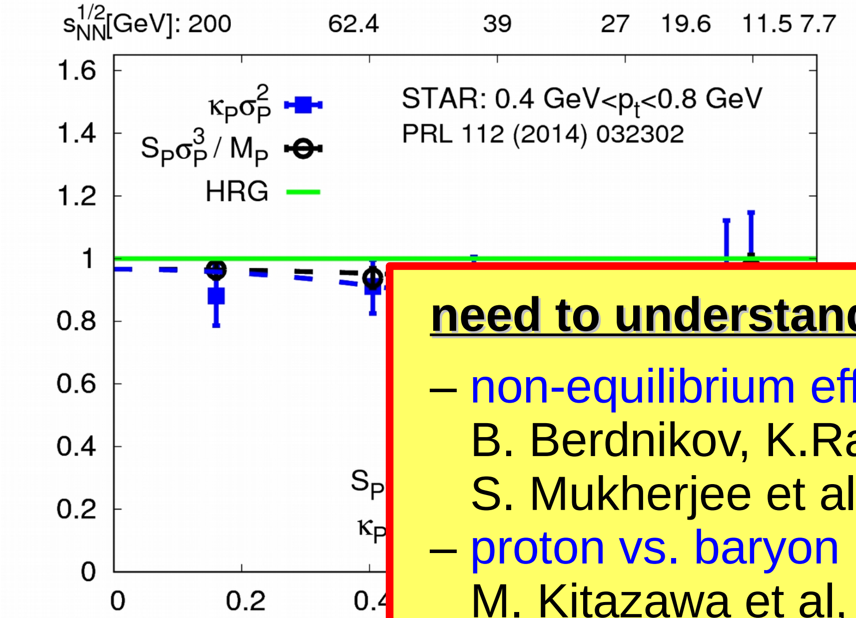


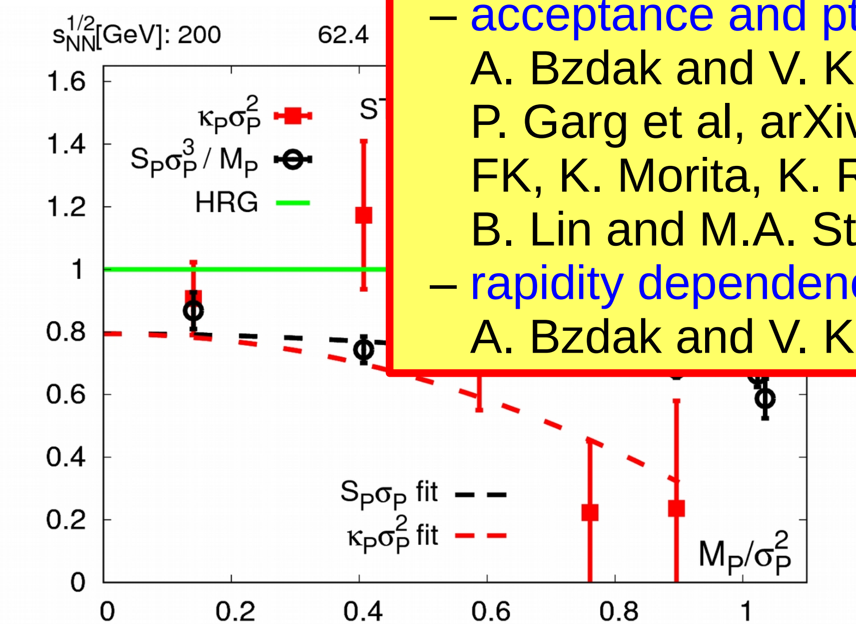
NLO lattice QCD
(not yet continuum extraplated)



hotQCD in preparation

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis





NLO lattice QCD
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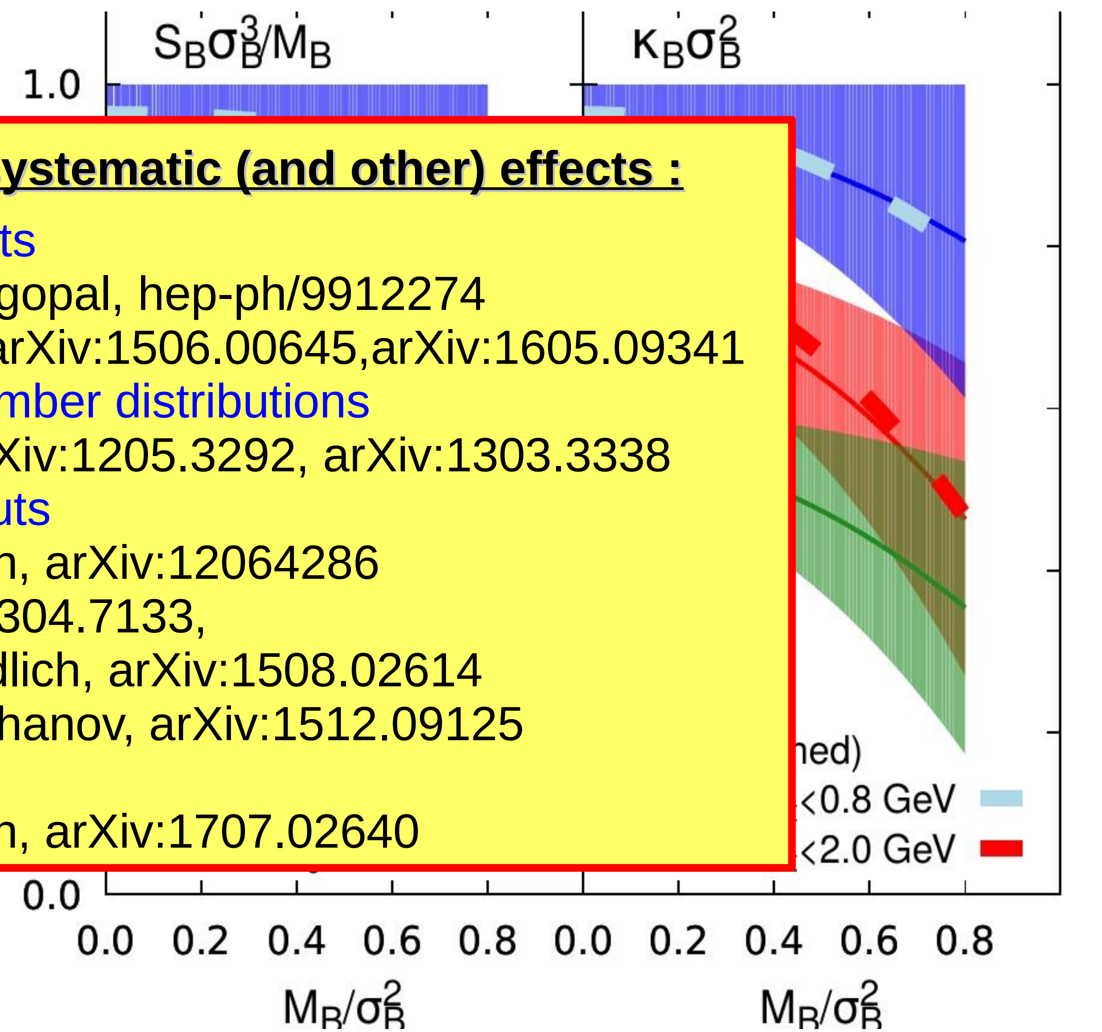

need to understand systematic (and other) effects :

– non-equilibrium effects
  B. Berdnikov, K.Rajagopal, hep-ph/9912274
  S. Mukherjee et al., arXiv:1506.00645,arXiv:1605.09341
– proton vs. baryon number distributions
  M. Kitazawa et al, arXiv:1205.3292, arXiv:1303.3338
– acceptance and pt-cuts
  A. Bzdak and V. Koch, arXiv:12064286
  P. Garg et al, arXiv:1304.7133,
  FK, K. Morita, K. Redlich, arXiv:1508.02614
  B. Lin and M.A. Stephanov, arXiv:1512.09125
– rapidity dependence
  A. Bzdak and V. Koch, arXiv:1707.02640

hotQCD in preparation

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

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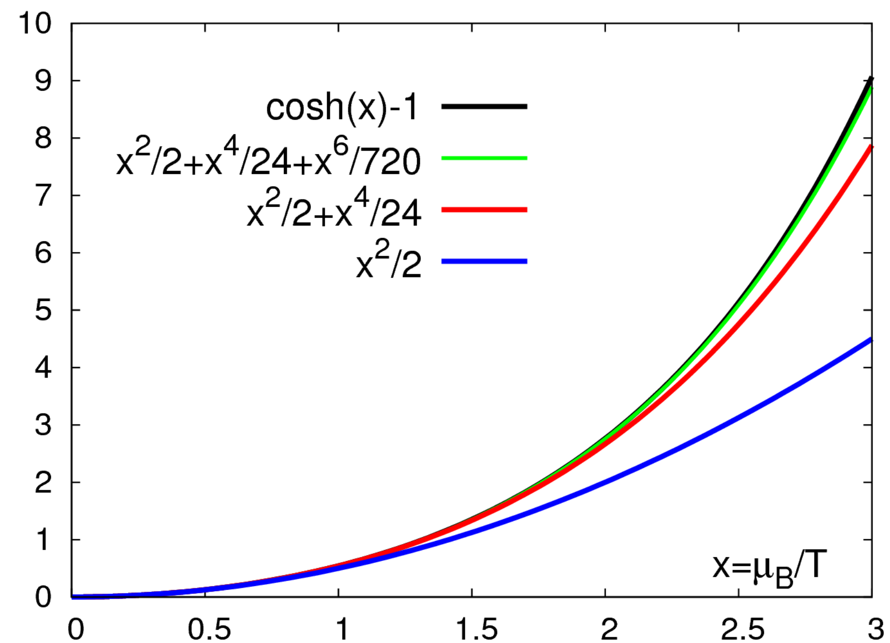
An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

HRG vs. QCD:

$\mathcal{O}((\mu_B/T)^4)$: difference is less than 3% at $\mu_B/T = 2$

$\mathcal{O}((\mu_B/T)^6)$: difference is less than 2% at $\mu_B/T = 3$



Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

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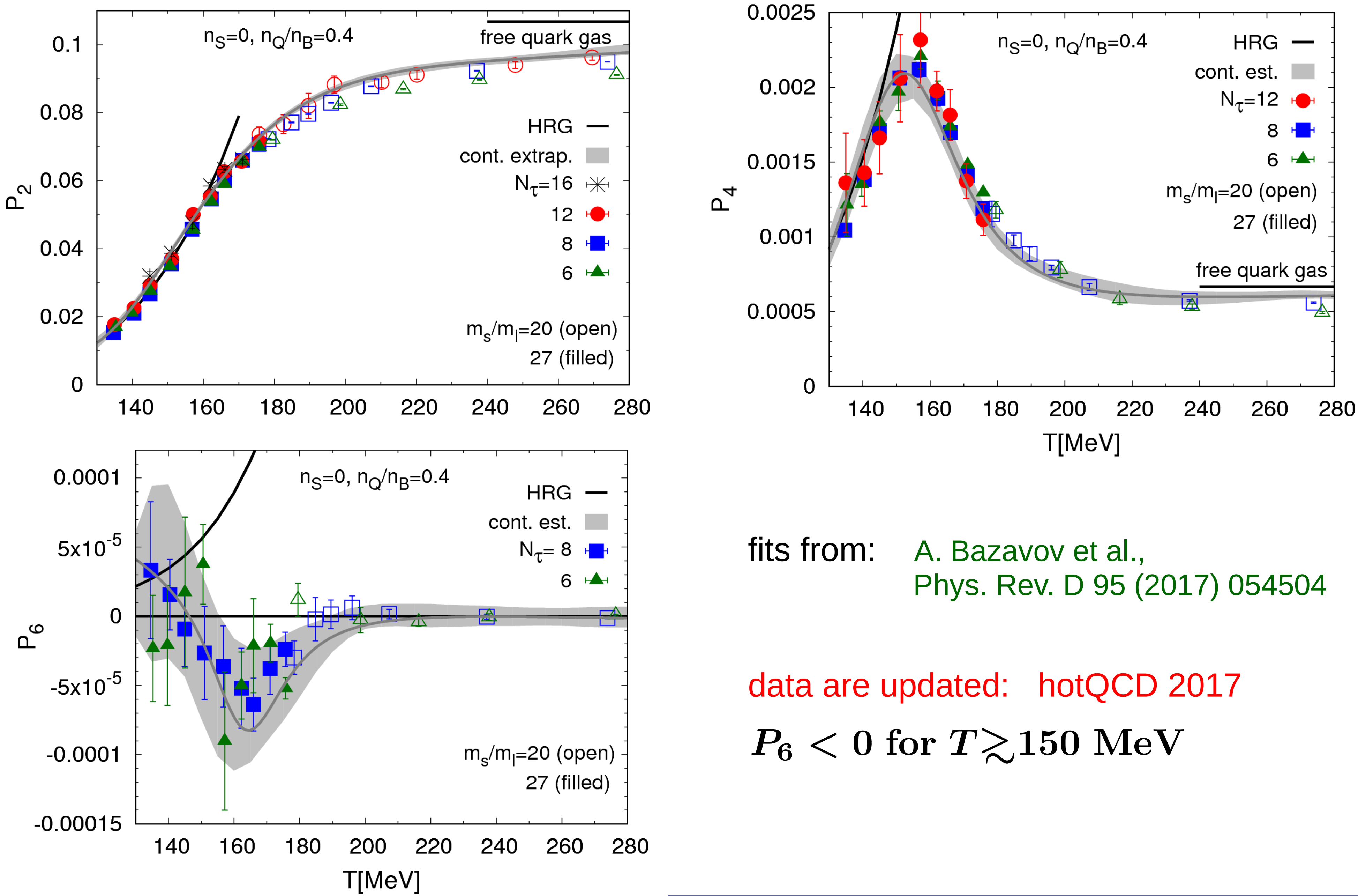
$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2} \left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24} \left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

strangeness neutral: $n_S = 0 \;,\; n_Q/n_B = 0.4$

$$\frac{P(T,\mu_B)}{T^4} = P_0(T) + P_2(T) \left(\frac{\mu_B}{T}\right)^2 + P_4(T) \left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

Taylor expansion coefficients

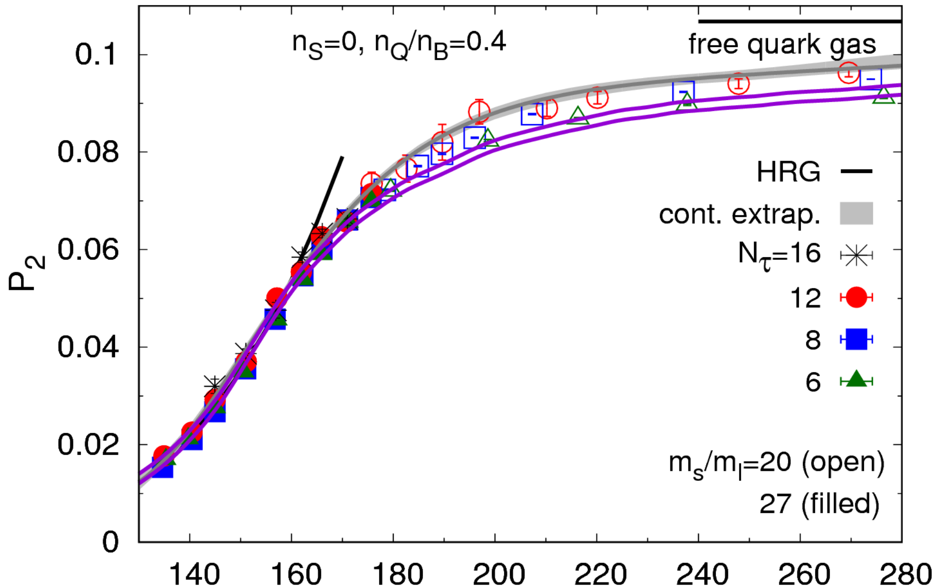


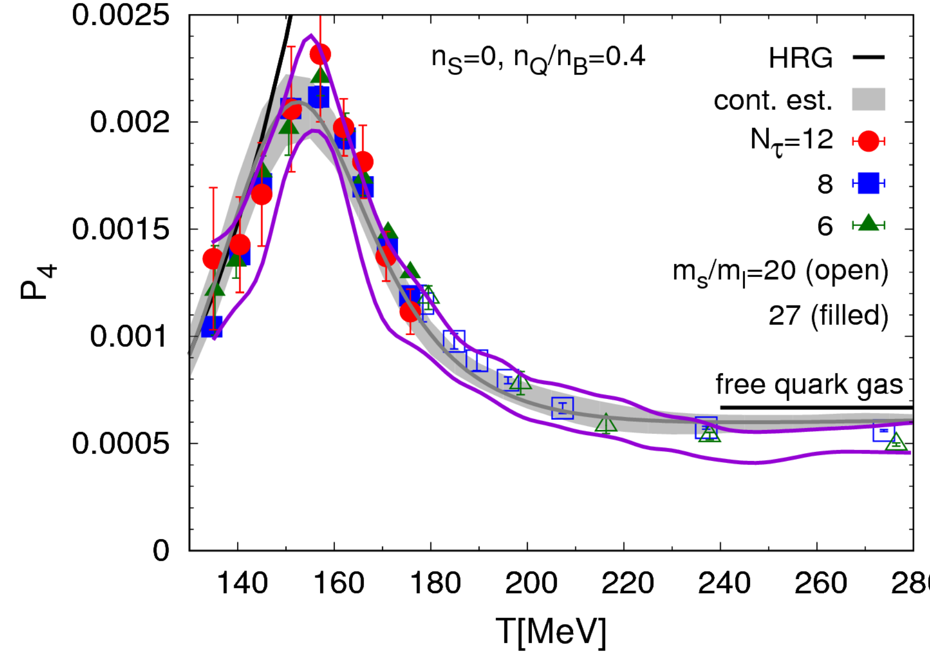
fits from: A. Bazavov et al., Phys. Rev. D 95 (2017) 054504

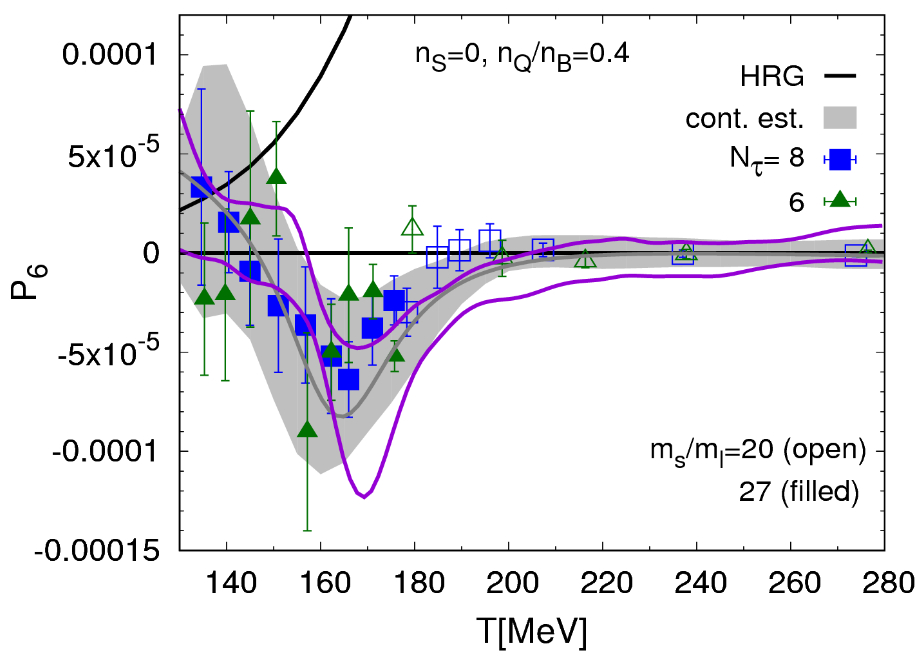
data are updated: hotQCD 2017

$P_6 < 0$ for $T \gtrsim 150$ MeV

Taylor expansion coefficients







fits from: A. Bazavov et al., Phys. Rev. D 95 (2017) 054504

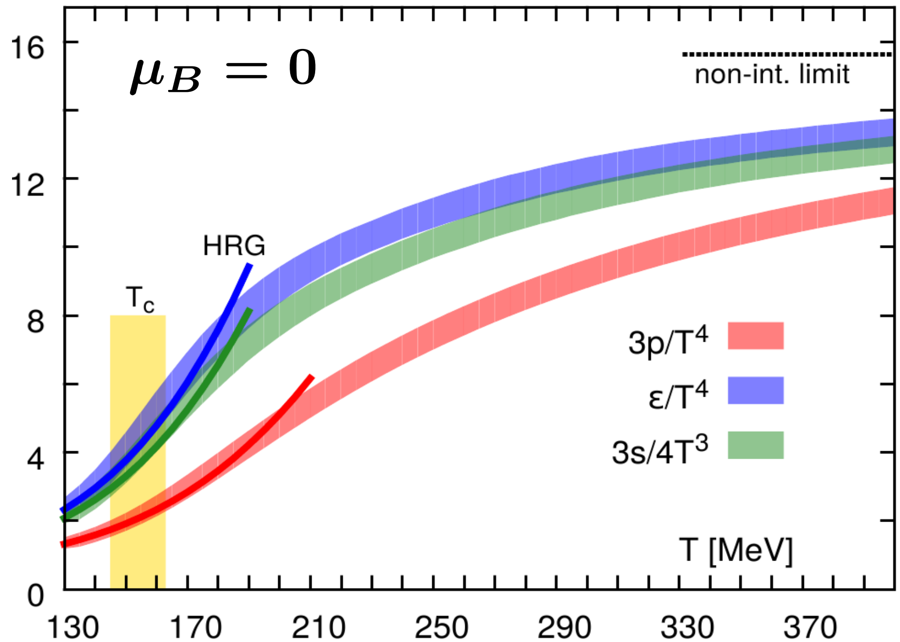
data are updated: hotQCD 2017

$$P_6 < 0 \text{ for } T \gtrsim 150 \text{ MeV}$$

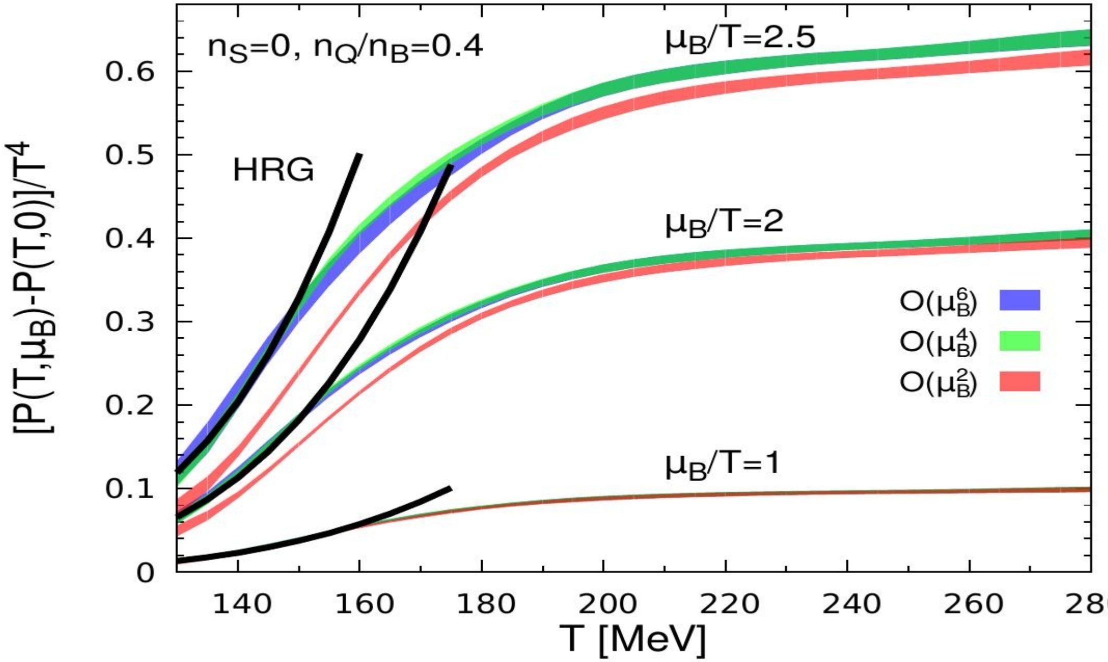
consistent with results obtained from analytic continuation:
J. Gunther et al., EPJ Web Conf. 137 (2017) 07008

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B}{24}\left(\frac{\mu_B}{T}\right)^4 + \frac{\chi_6^B}{720}\left(\frac{\mu_B}{T}\right)^6 + \ldots$$



(10-30)% contribution to total pressure at $\mu_B/T = 2$



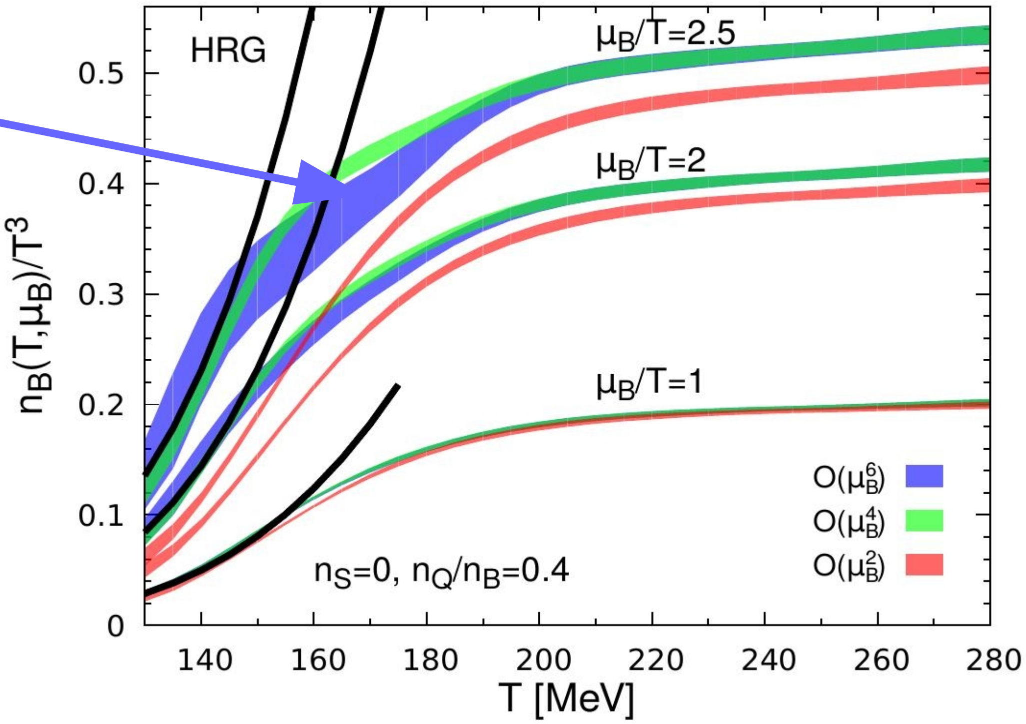
The EoS is well controlled for $\mu_B/T \leq 2$
or equivalently $\sqrt{s_{NN}} \geq 19.6\ \mathrm{GeV}$

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

net baryon-number density:

$$\frac{n_B}{T^3} = \chi_2^B \frac{\mu_B}{T} + \frac{\chi_4^B}{6}\left(\frac{\mu_B}{T}\right)^3 + \frac{\chi_6^B}{120}\left(\frac{\mu_B}{T}\right)^5 + \ldots$$

dip in $\chi_6^B(T),\ P_6(T)$
calls for inclusion of
higher order corrections
and/or better statistics!!



⇒ The EoS is well controlled for $\mu_B/T \leq 2$
or equivalently $\sqrt{s_{NN}} \geq 19.6$ GeV

HRG vs. QCD
net baryon-number fluctuations

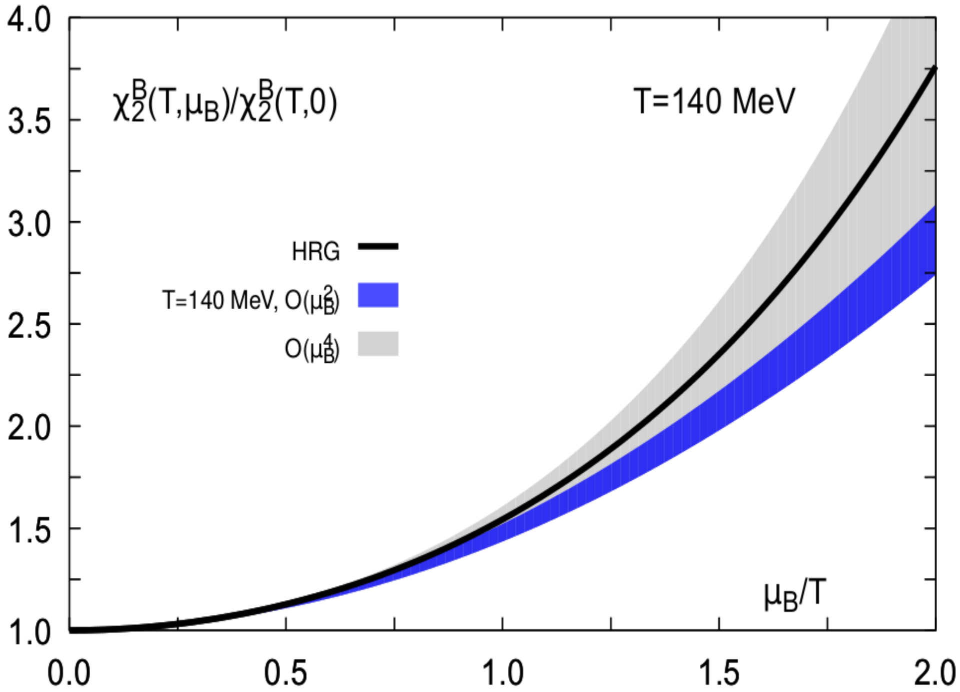
$$\mu_B/T > 0$$

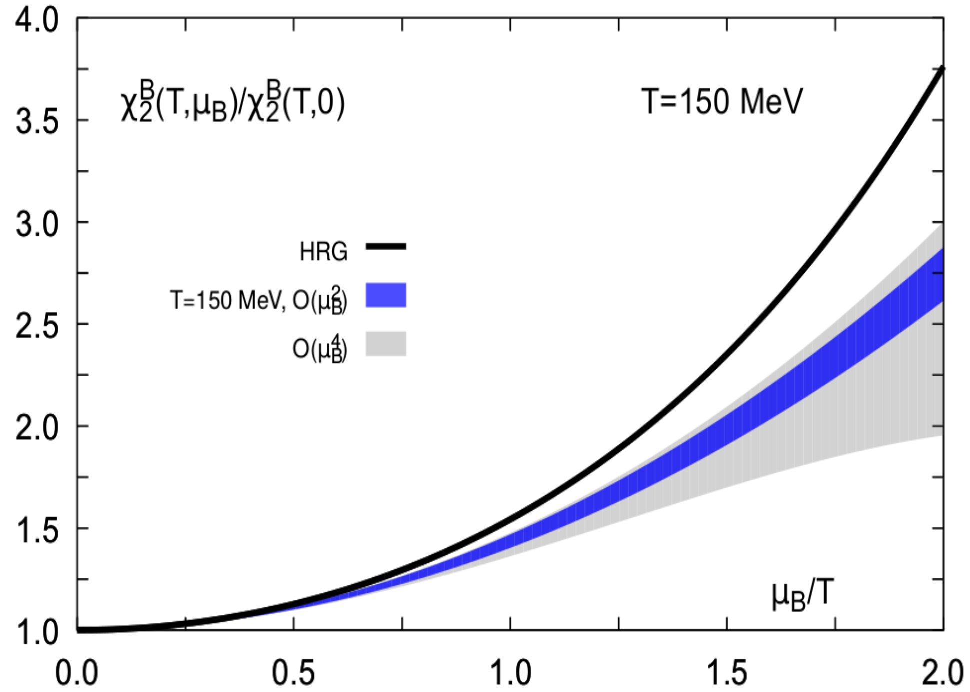
for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**





HRG vs. QCD
net baryon-number fluctuations

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for simplicity: $\mu_Q = \mu_S = 0$

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– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

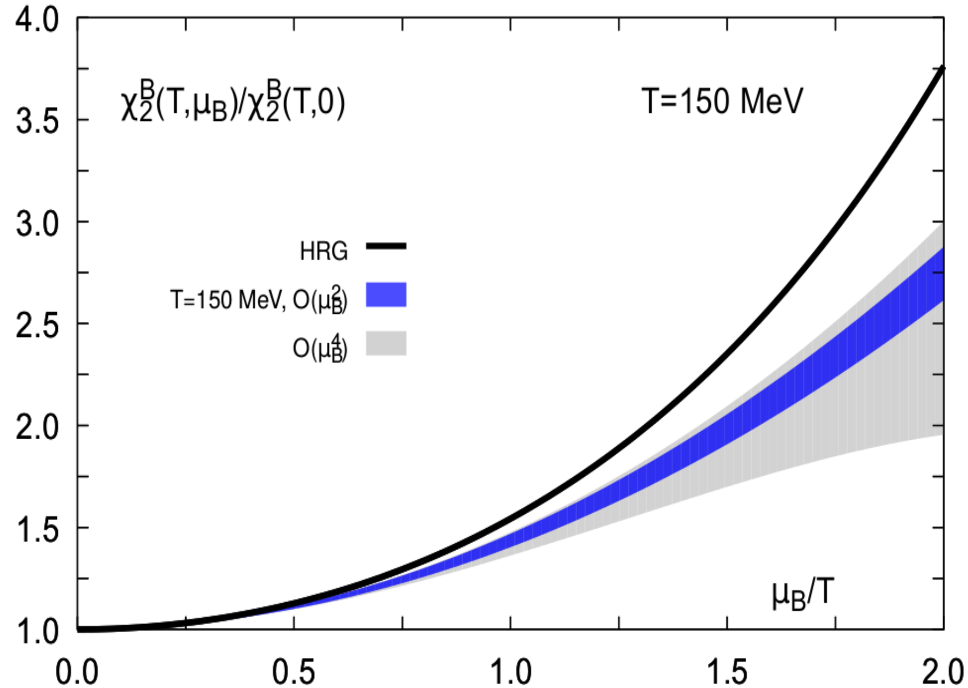
– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**

no evidence for enhanced net baryon-number fluctuations for

$$T \geq \ 135\text{MeV}\ ,\ \mu_B \leq 2T$$

↓

no evidence for getting closer to a "critical region"



Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left| \frac{n(n-1)\chi_n^B}{\chi_{n+2}^B} \right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T,\mu_B)$ to be monotonically growing with μ_B/T

⇕

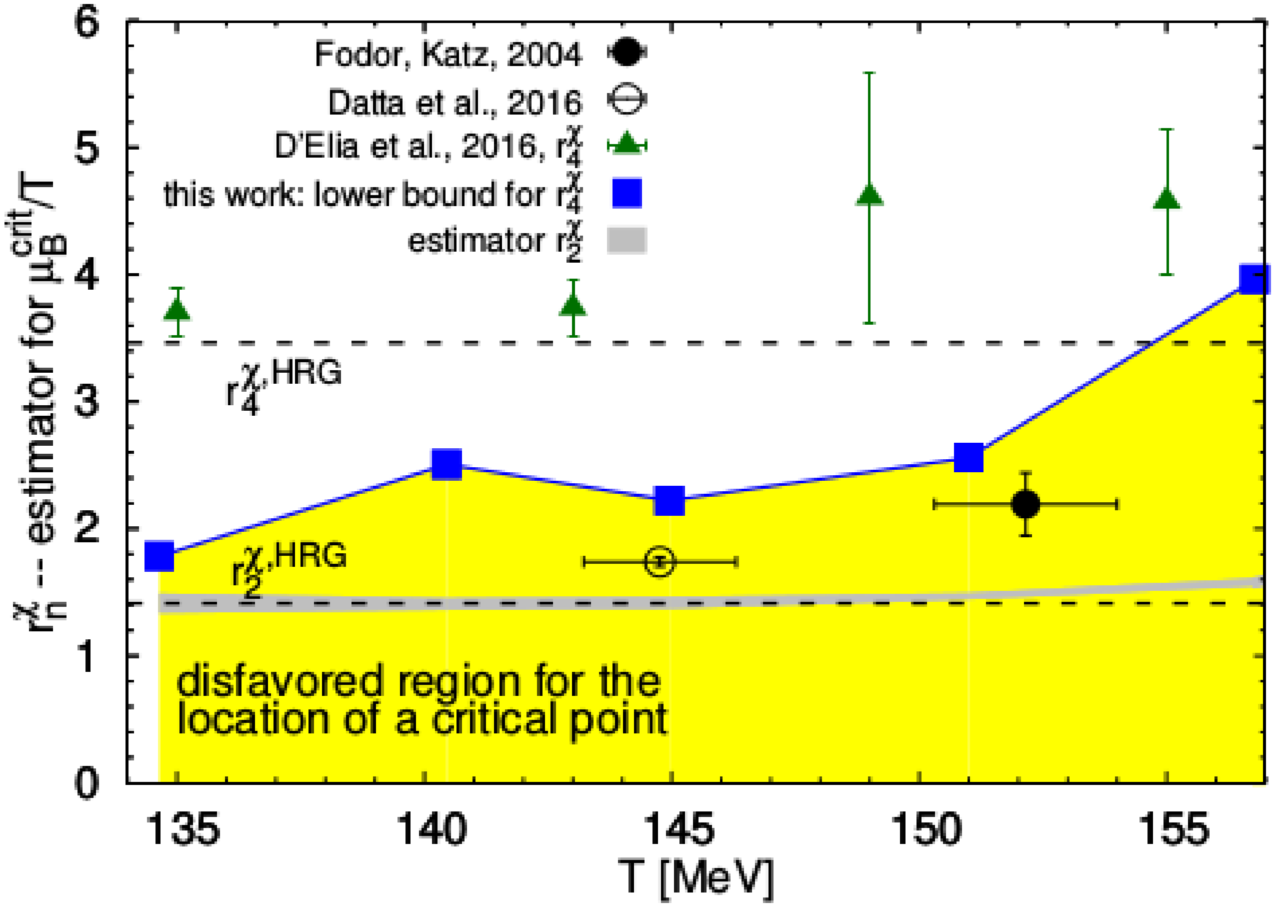
at T_{CP} : $\kappa_B \sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

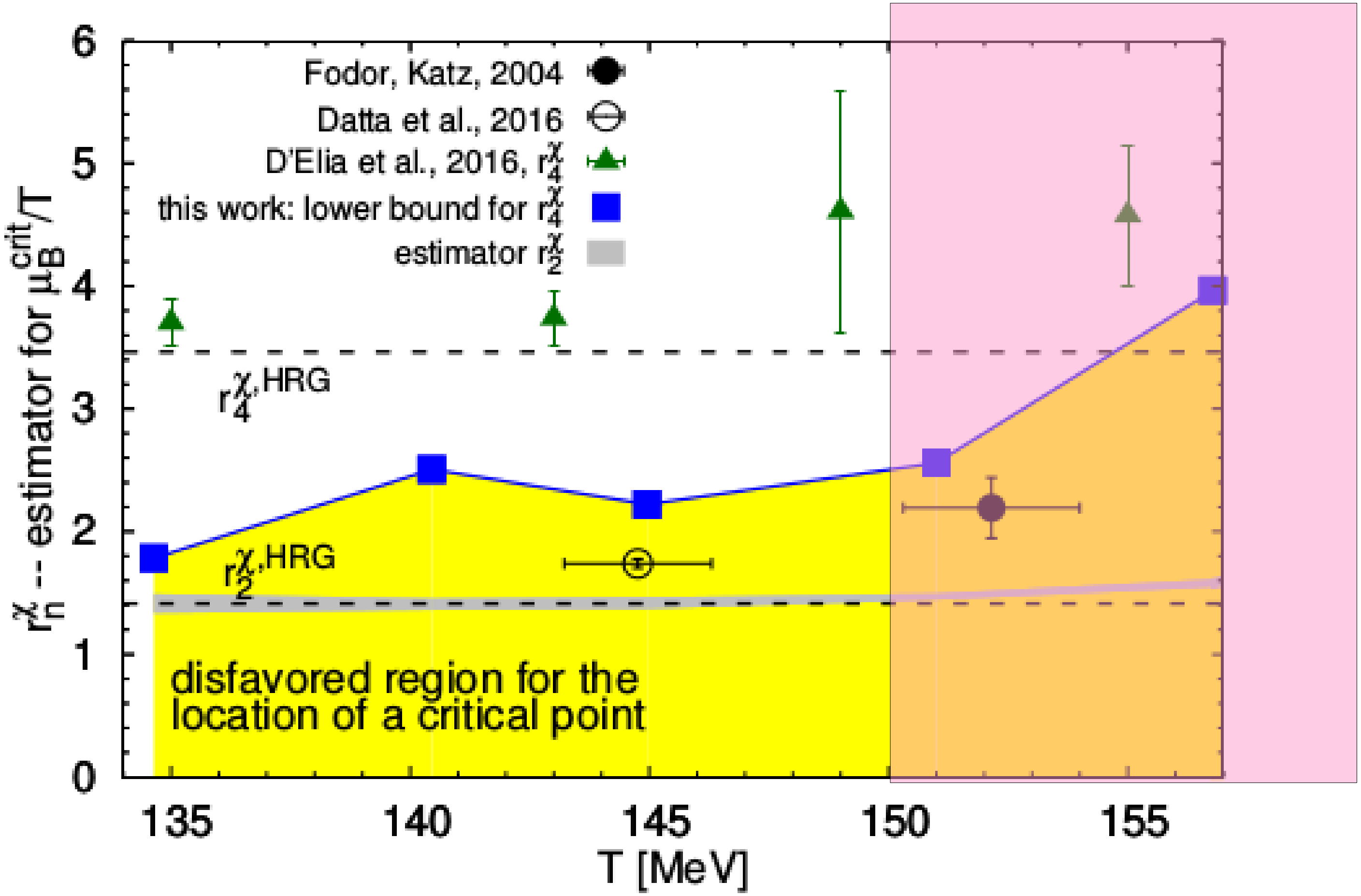
– Taylor expansion can not be used close to the critical point

estimates/constraints on critical point location



01/01/17:
based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

estimates/constraints on critical point location



strongly disfavored as

$$\chi_6^B < 0$$

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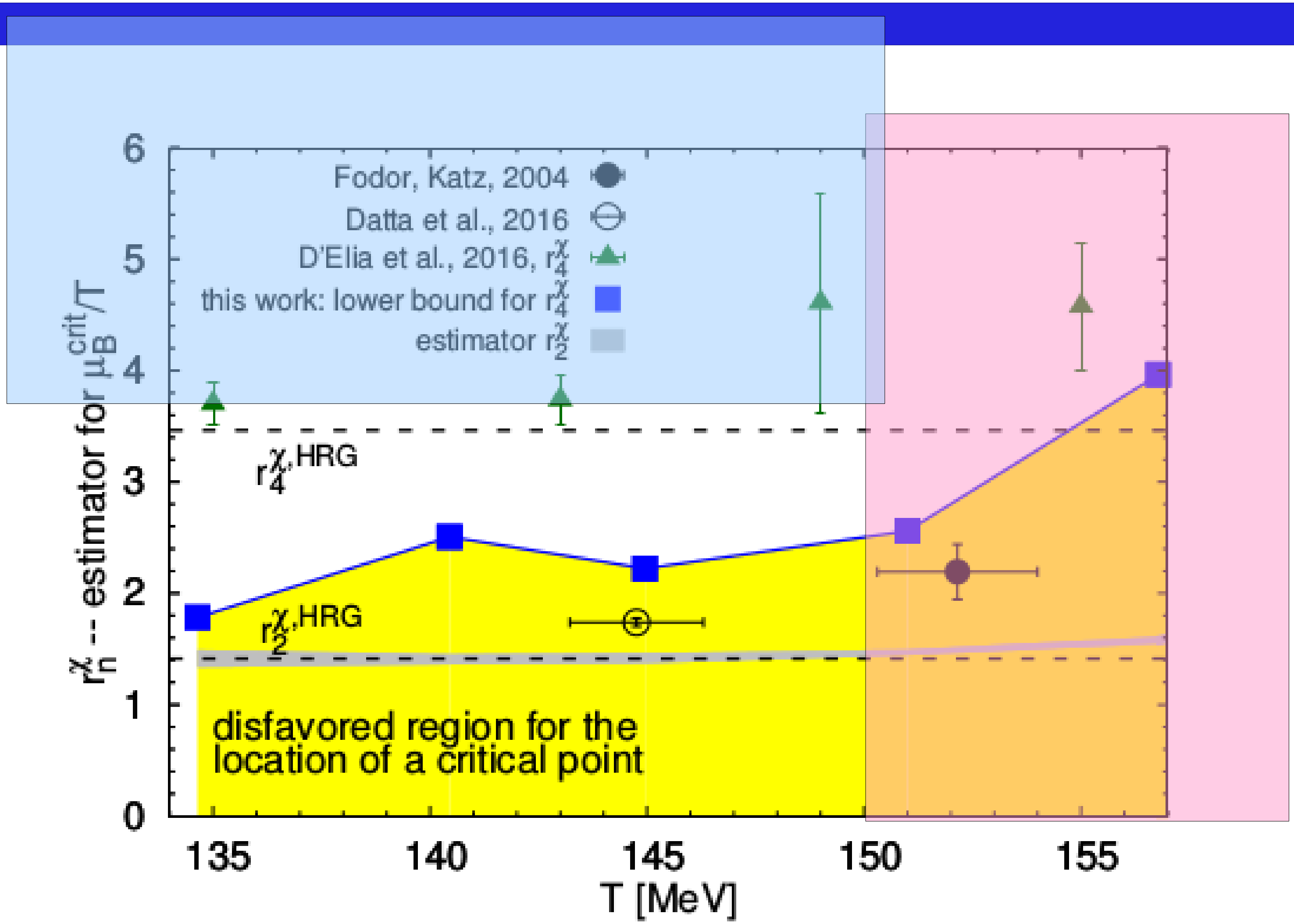
estimates/constraints on critical point location

not accessible in BES@RHIC collider mode



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To do list

What is needed to understand equilibrium properties of conserved charge fluctuations on the freeze-out line?

– improve lattice QCD results on 6th (and 8th) order cumulants of conserved charge fluctuations

– self-consistent determination of freeze-out parameters based on equilibrium QCD calc.: $T_f(\mu_B),\ \mu_B,\ [\mu_S(\mu_B),\ \mu_Q(\mu_B)]$

What can be done about "locating the critical point"?

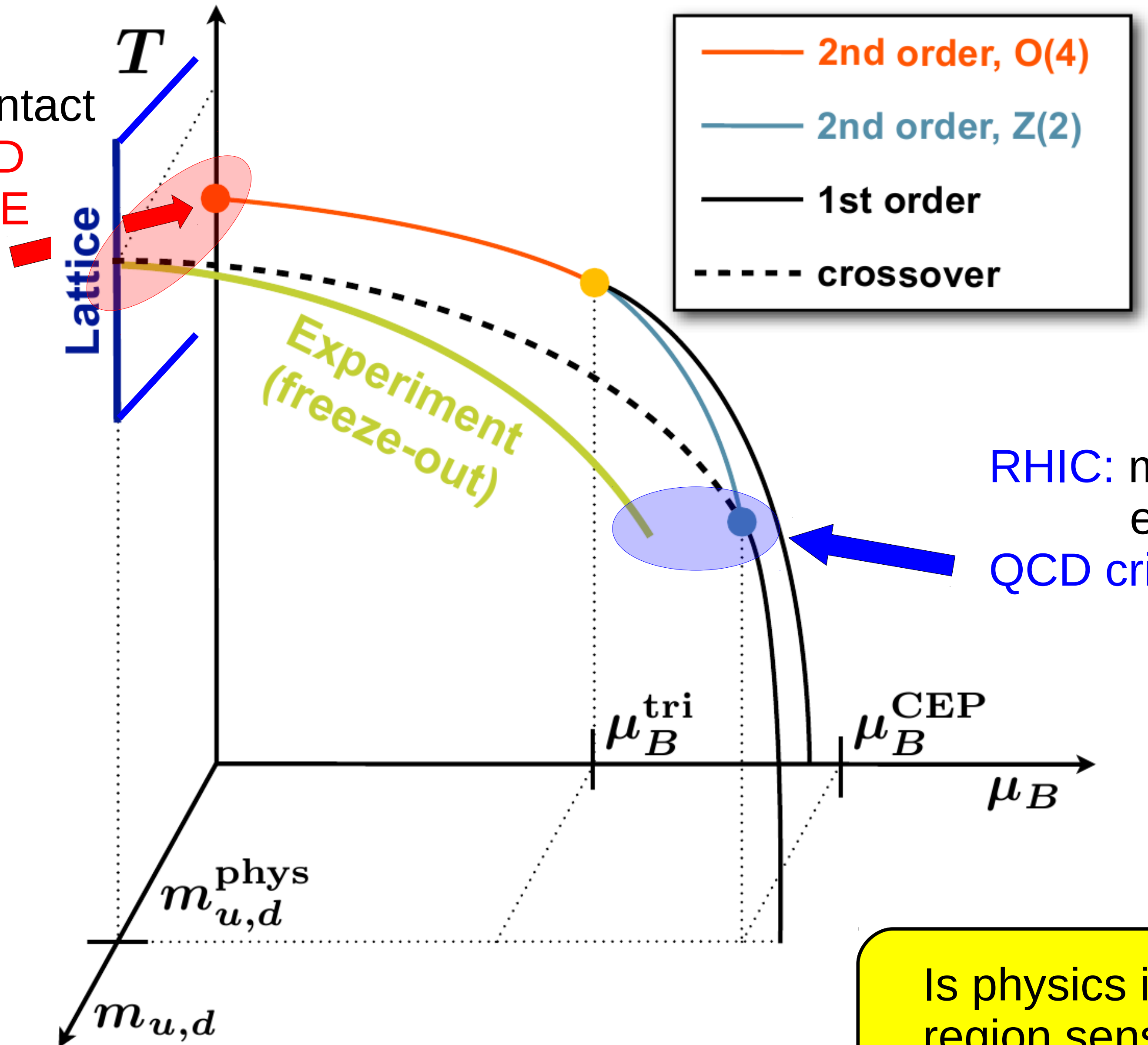
– use 6th (and 8th) order cumulants to put bounds on its location

– keep working on new simulation techniques

By-product: EoS in the entire parameter range accessible to the RHIC BES-II

Chiral critical point and QCD critical endpoint

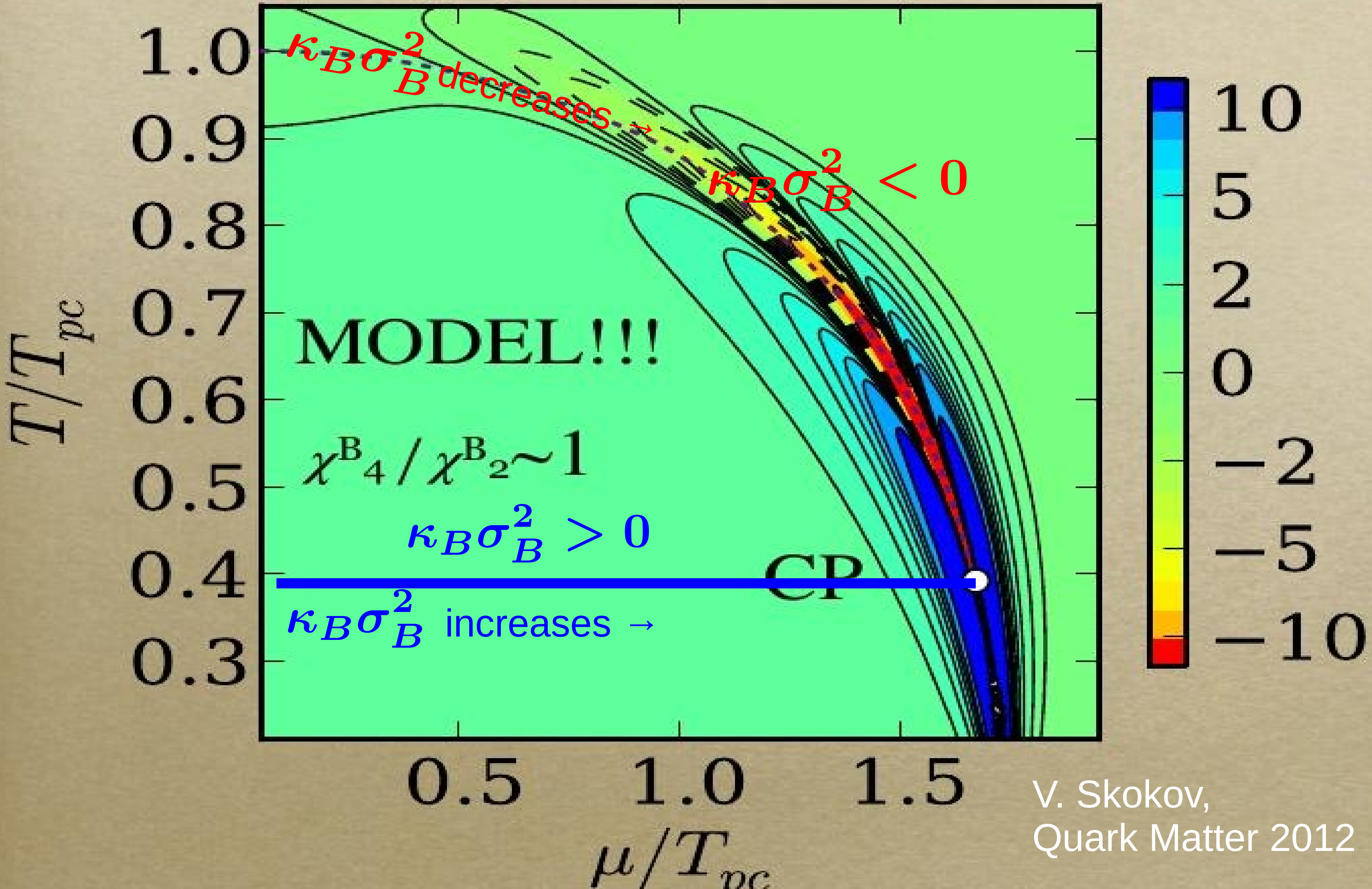
LHC: may establish contact with the QCD chiral PHASE transition



RHIC: may establish evidence for the QCD critical end point

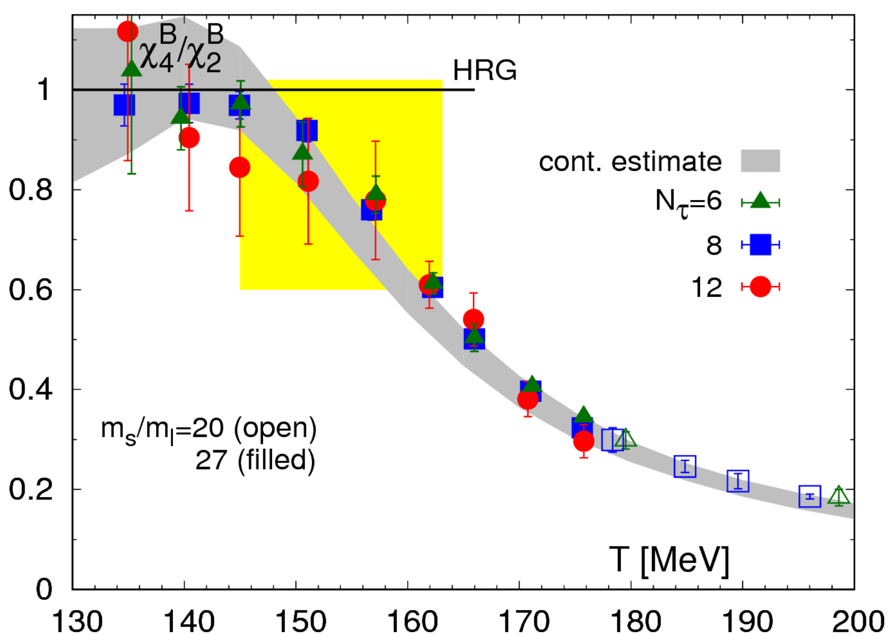
Is physics in the freeze-out region sensitive to critical behavior?

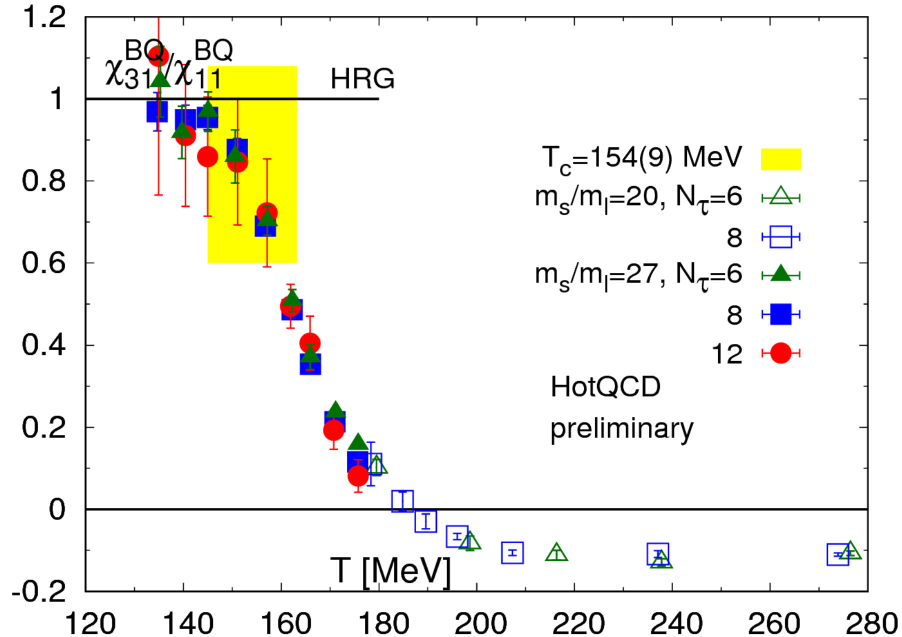
Chiral model and negative χ^B_4/χ^B_2:

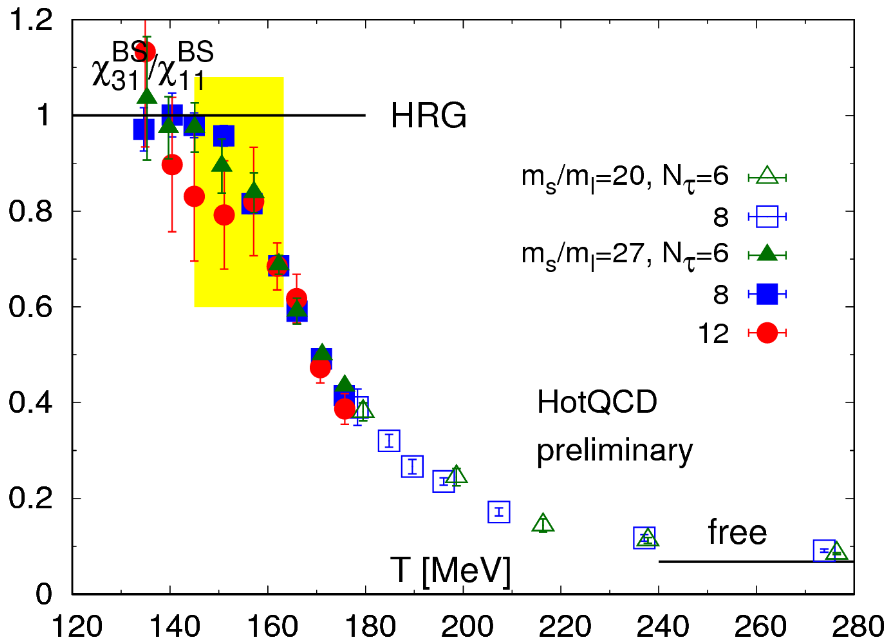


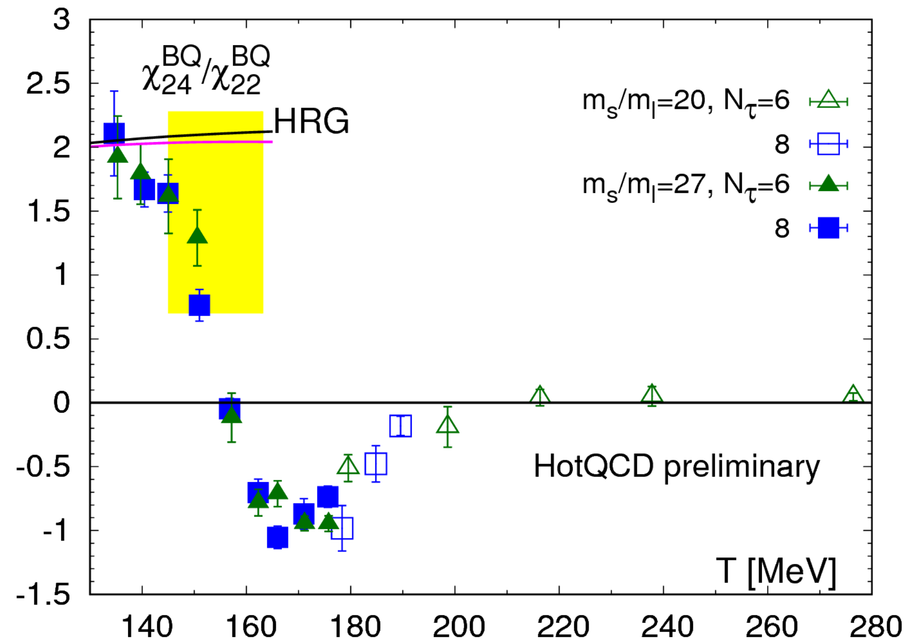
V. Skokov,
Quark Matter 2012

Some 4th and 6th order cumulants

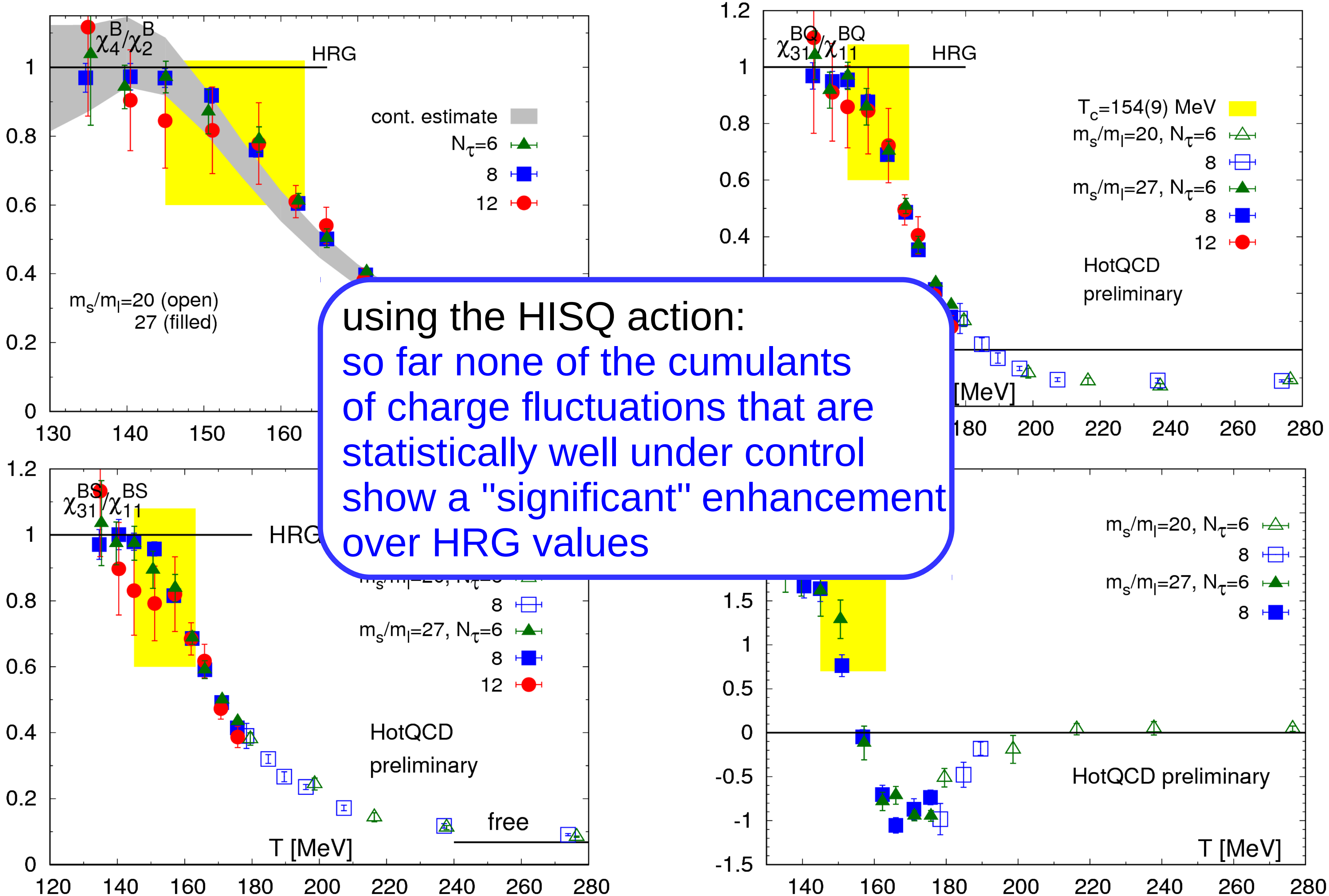








Some 4th and 6th order cumulants



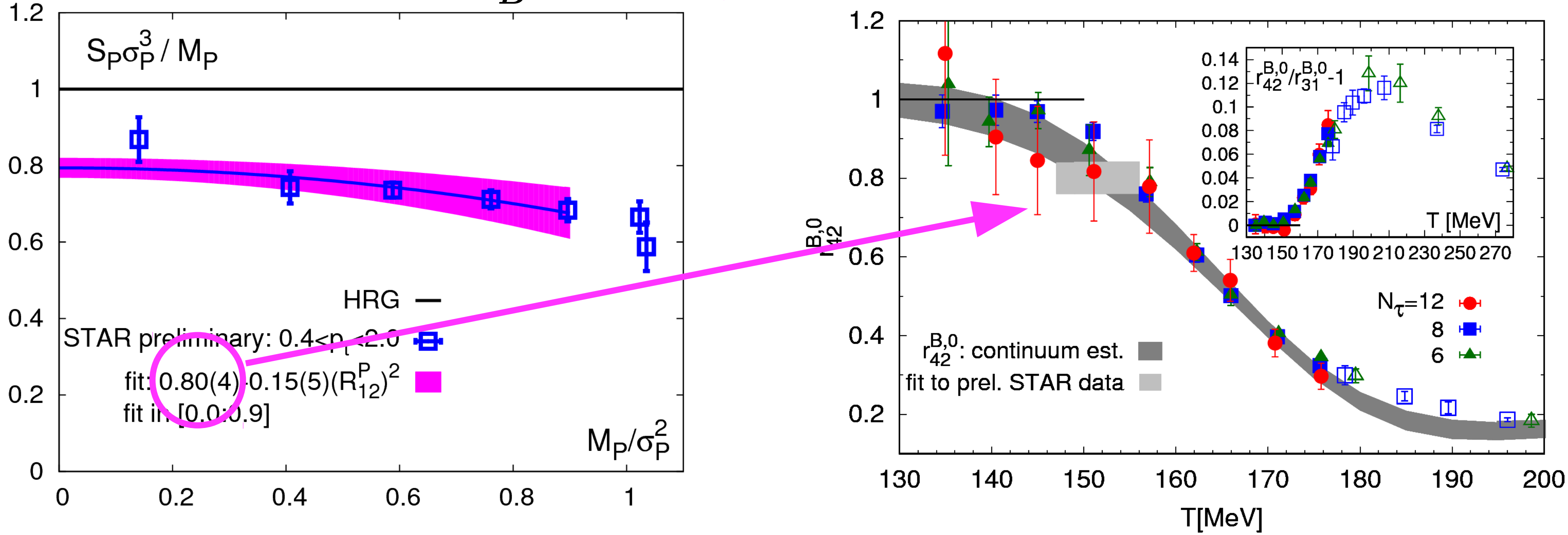
using the HISQ action:
so far none of the cumulants of charge fluctuations that are statistically well under control show a "significant" enhancement over HRG values

Conserved charge fluctuations and freeze-out
mean, variance and skewness

NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \dots$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = r_{31}^{B,0} + r_{31}^{B,2}\left(R_{12}^B\right)^2$$



intercept consistent with QCD result,

– used $T_{f,0}$ determined from fit to $(M_Q/\sigma_Q^2)/(M_P/\sigma_P^2)$

Conserved charge fluctuations and freeze-out
mean, variance and skewness

NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \dots$$

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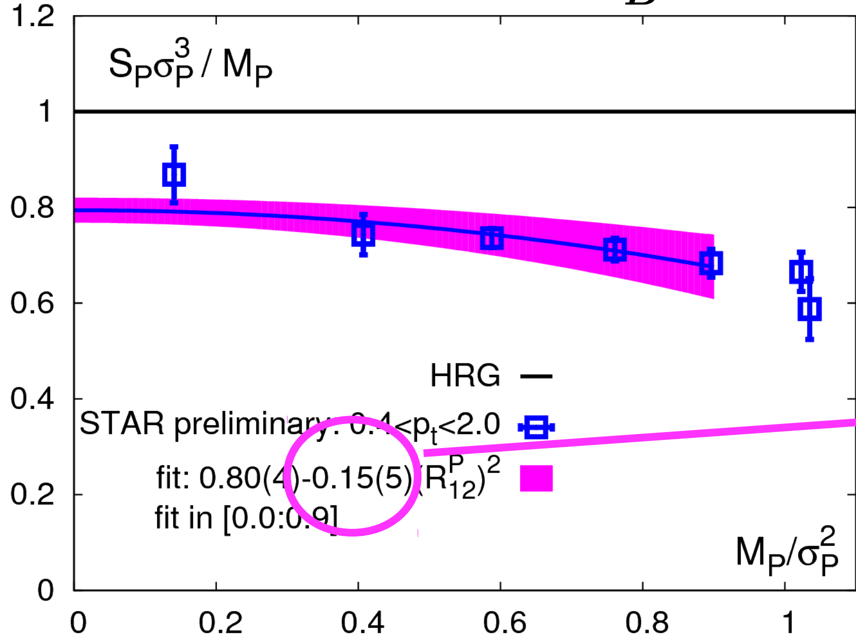
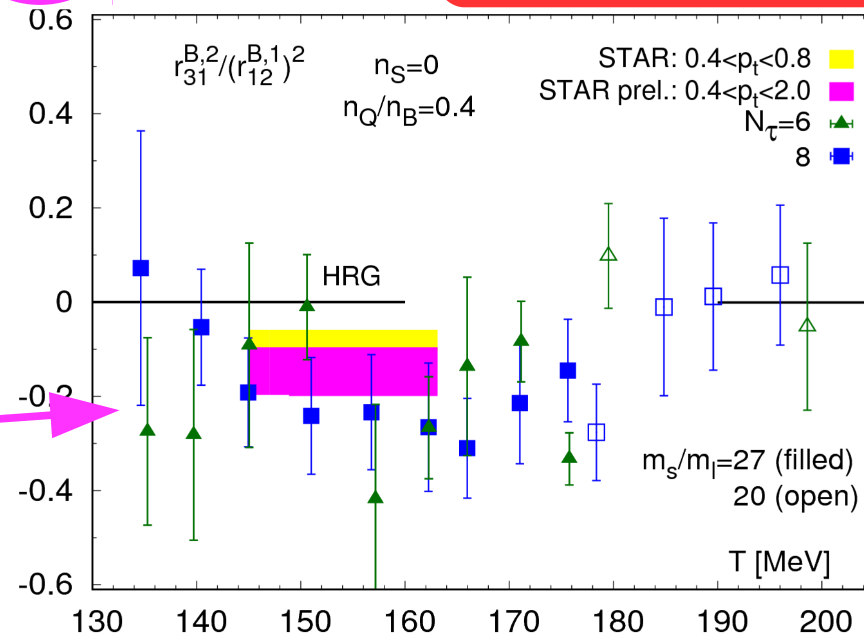
lattice QCD calculation involves 6th order cumulants





curvature consistent with (still preliminary) QCD result (noisy, coarse lattice)

used $T_{f,0}$ determined from fit to $(M_Q/\sigma_Q^2)/(M_P/\sigma_P^2)$